Structured Learning for Control of Networked Systems : Stability and Steady-State Tracking Guarantees

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2023.12.05

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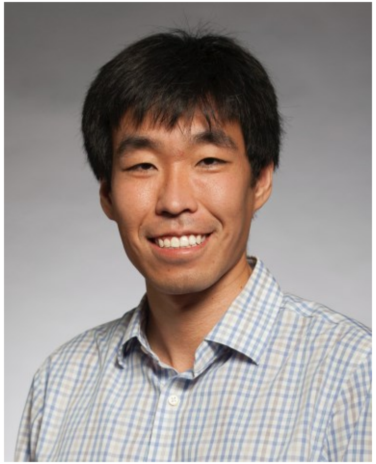
Acknowledgements



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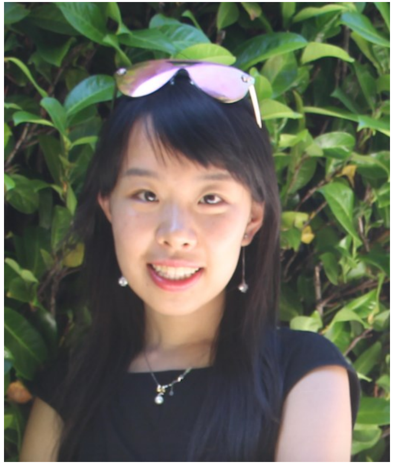

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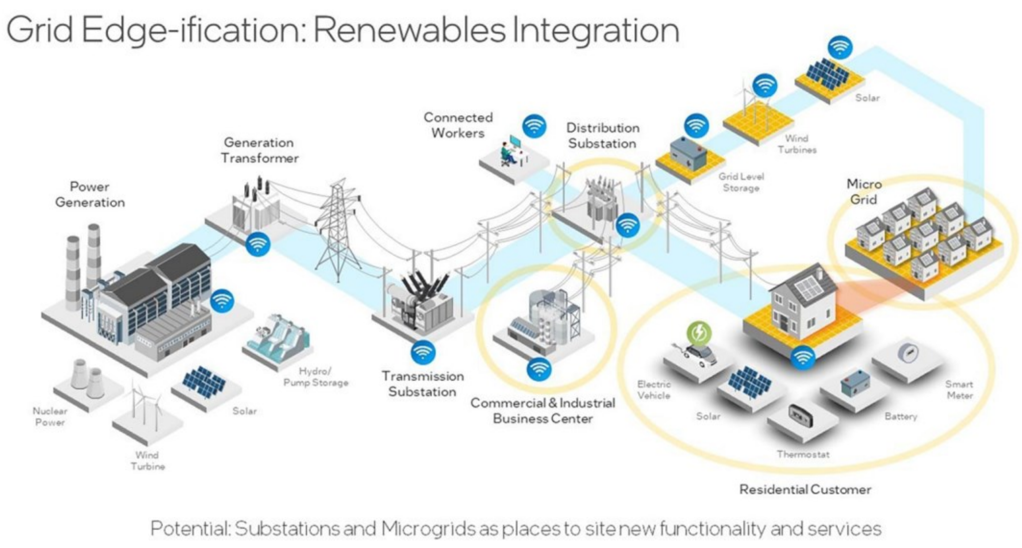
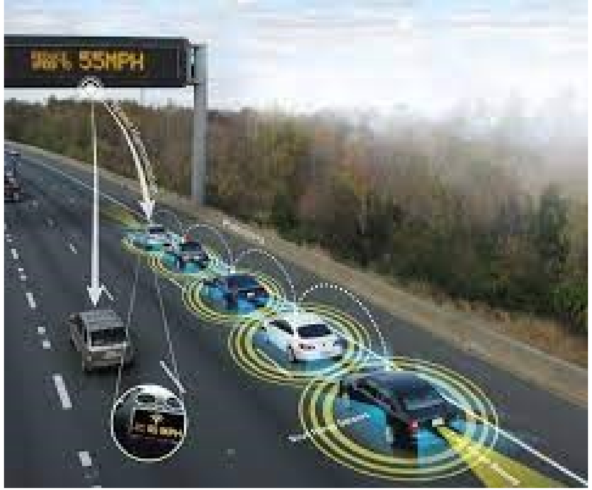
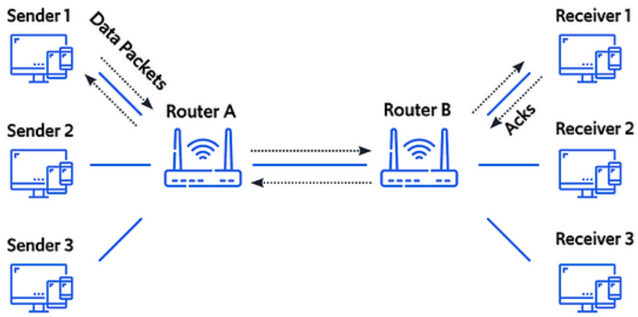
1 Networked Systems



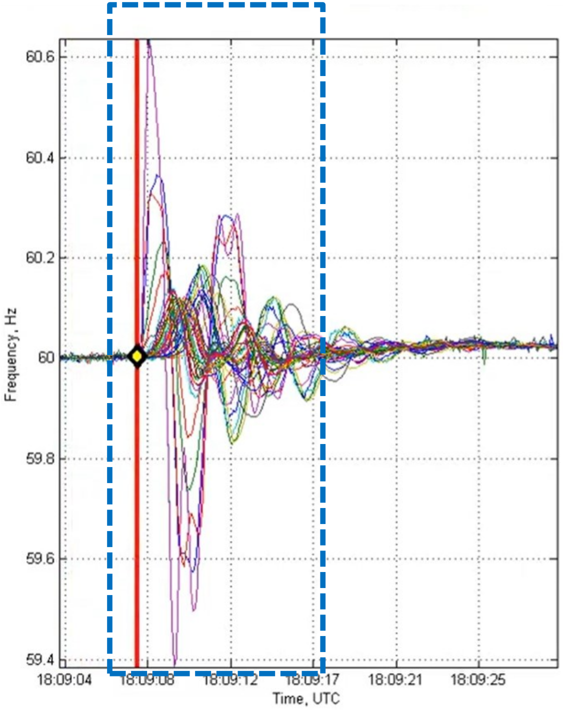


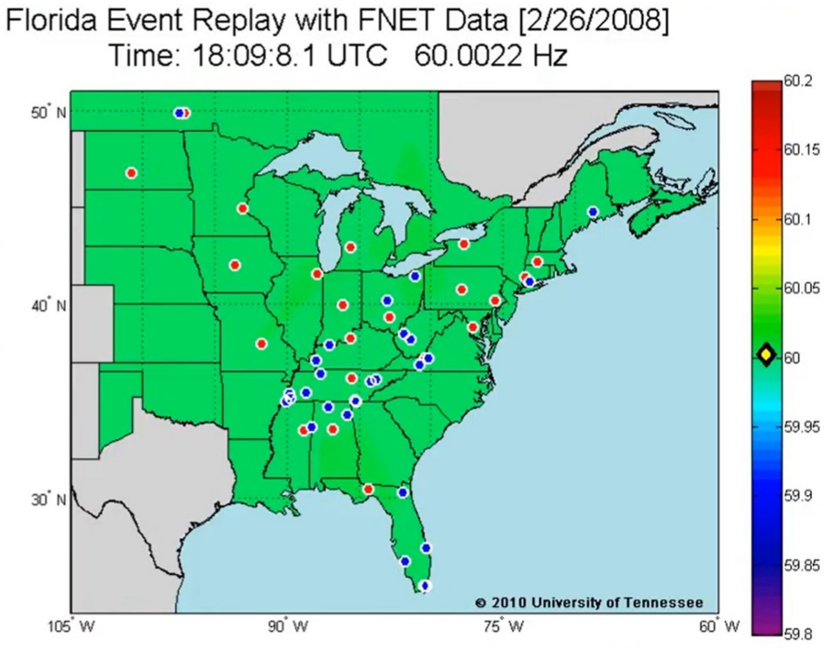


https://www.intel.com/content/www/us/en/newsroom/opinion/tech-can-solve-grids-climate-change-challenge.html

1 Performance metrics

Transient Period: Reduce the largest frequency deviations





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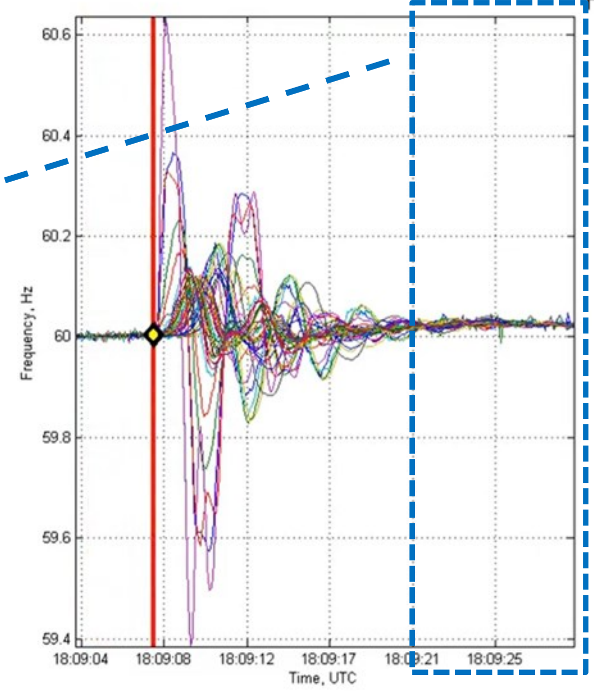
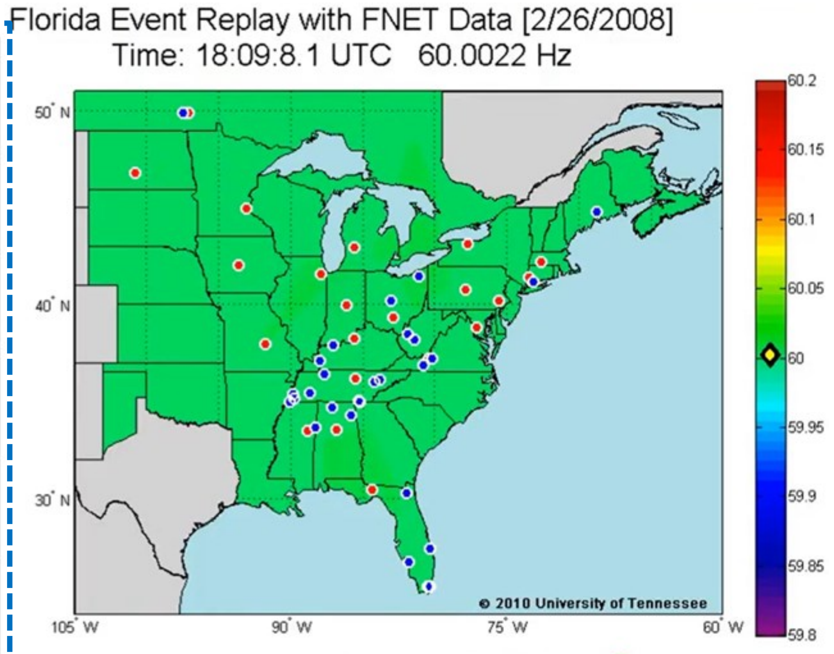
https://www.youtube.com/watch?v=H7y-oJYpDkM

1 Performance metrics

Steady state:

- ❑ Frequency reaches agreement at 60Hz





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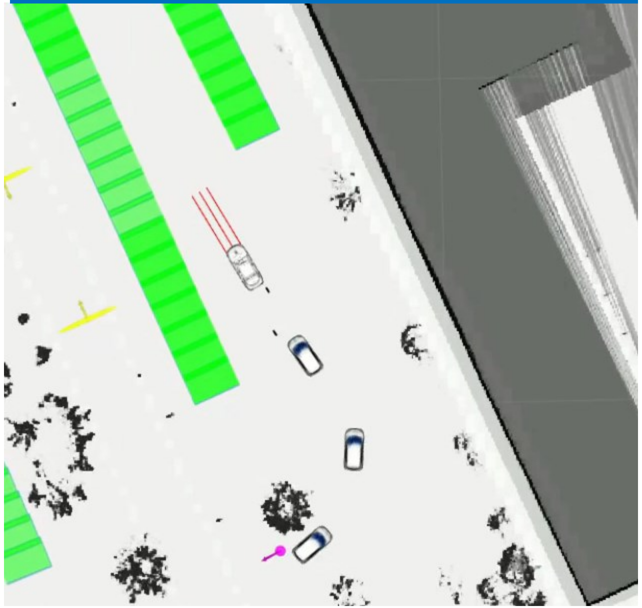
https://www.youtube.com/watch?v=H7y-oJYpDkM

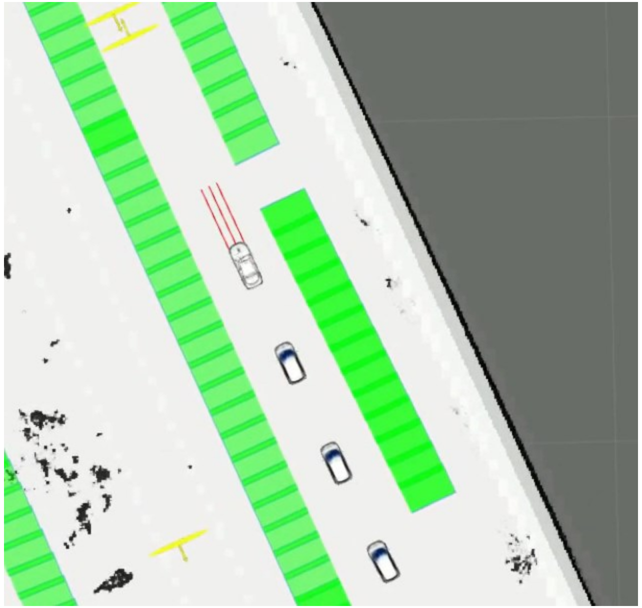
1 Performance metrics

Transient period performance



- ❑ Keep the relative distance

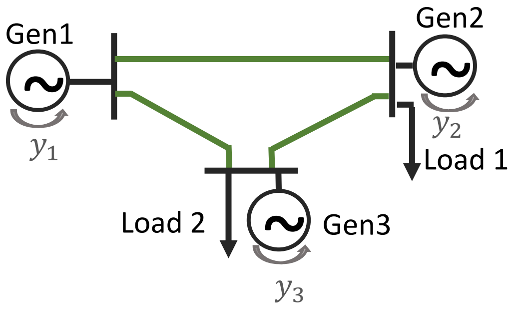
Steady-state performance



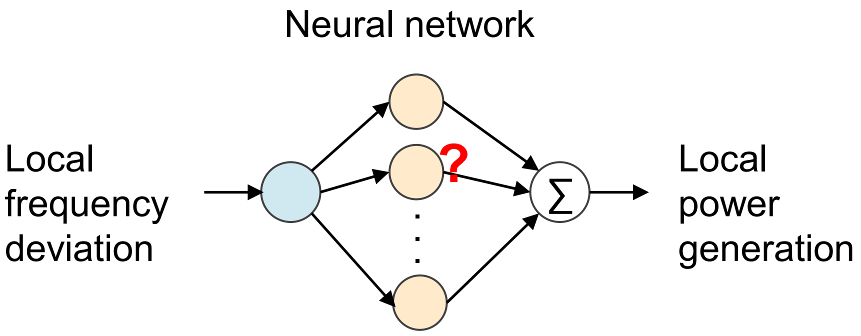
- ❑ The desired velocity

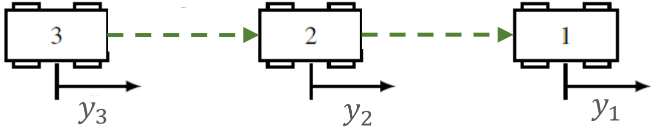
https://www.youtube.com/watch?v=F90K-rYQRAY

1 Learning for Control

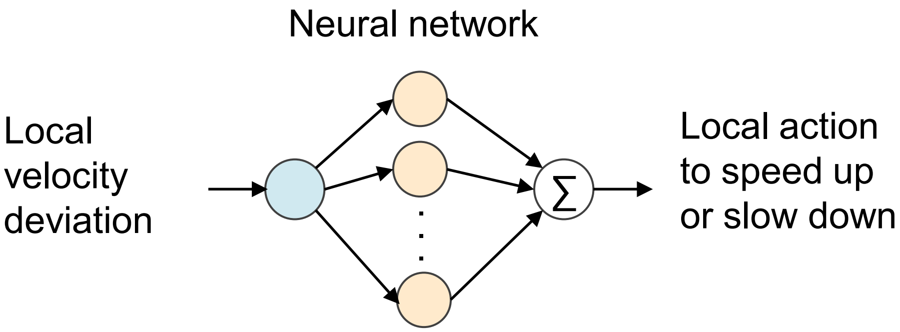


(a) Example 1: A power grid





(b) Example 2: A vehicle platoon



1 Learning for Control

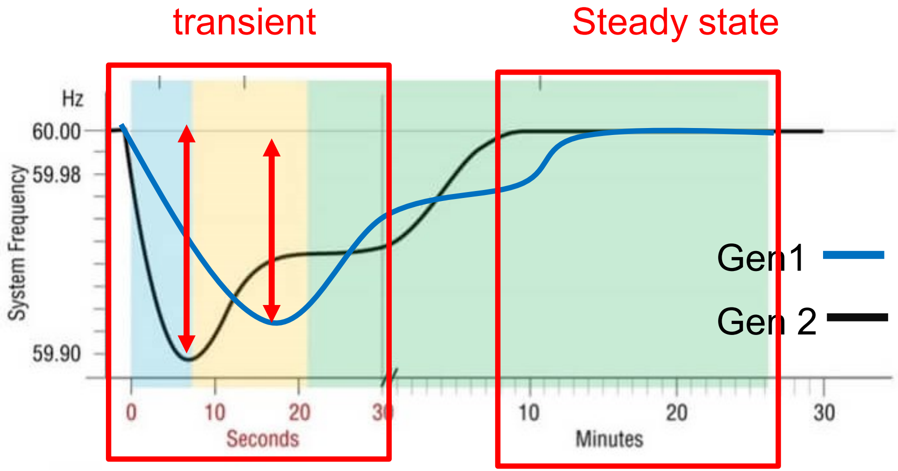
Transient Optimization

Training neural networks

$$\min_{u} \sum_{t=0}^{T} \textit{Transient Cost}$$

$$s.t. \quad \textit{dynamics of the system}$$

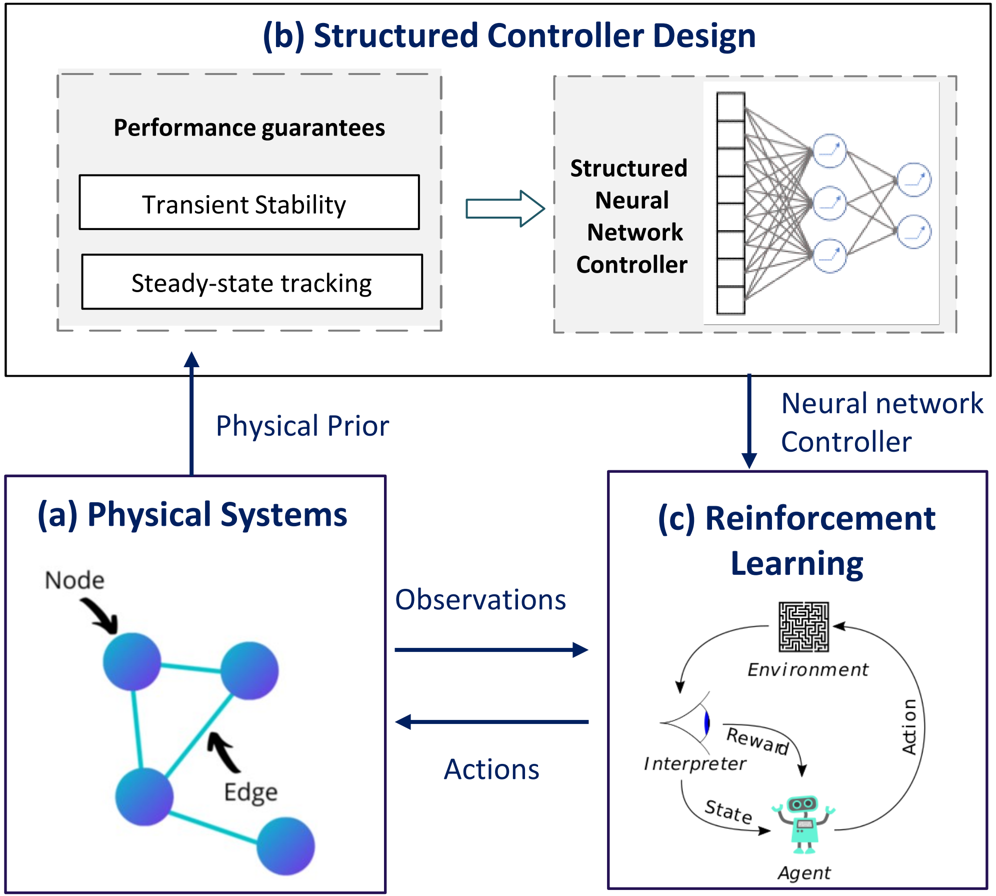
$$\textit{transient stability and output tracking}$$



Provable guarantees for a range of tracking point?

1 Key idea

- ❑ Derive structural properties of controllers satisfying performance guarantees
- ❑ Enforce the structures in the design of neural networks



2 Power System Transient Dynamics

The swing equation for power system transient dynamics

Angle

Frequency deviation

nonlinear neural networks

Local static controller that change active power for frequency control

$$\dot{\delta}_i = \omega_i,$$

$$M_i\dot{\omega}_i = -D_i\omega_i - d_i - u_i\,(\omega_i) - \sum_{(i.j)\in\mathcal{E}} \underbrace{B_{ij}\sin(\delta_i - \delta_j)}_{\text{Power Flow}}$$

Inertia constant

Damping coefficient

Net power extraction

The susceptance of the line (i, j)

2 Transient and Steady-state Optimization

Transient performance

$$\min_{u(\omega)} \sum_{i=1}^{n} \sum_{t=0}^{T} J_i\left(\omega_i(t), u_i(\omega_i(t))\right)$$

$$s.t. \qquad dynamics\ of\ the\ system$$

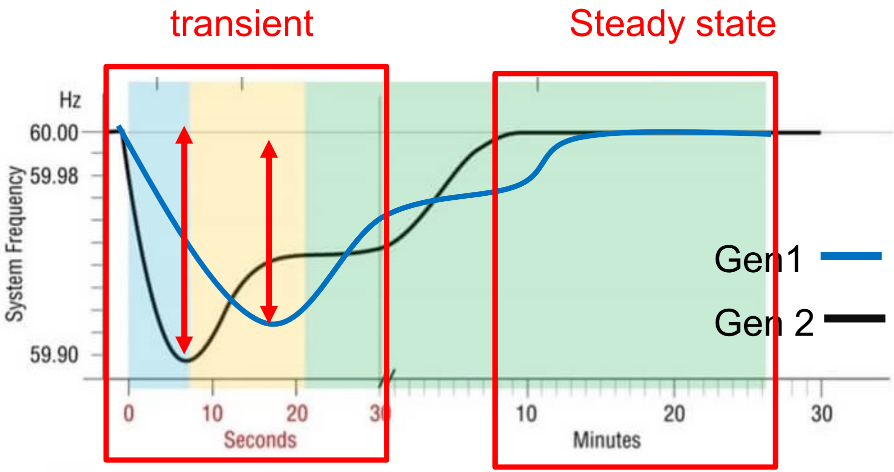
$$\underline{a}_i \le a_i(\omega_i(t)) \le \overline{a}_i$$

$$\boxed{a_i(\omega_i(t)) \text{ is stabilizing}}$$

Steady-state performance

- Frequency Restoration

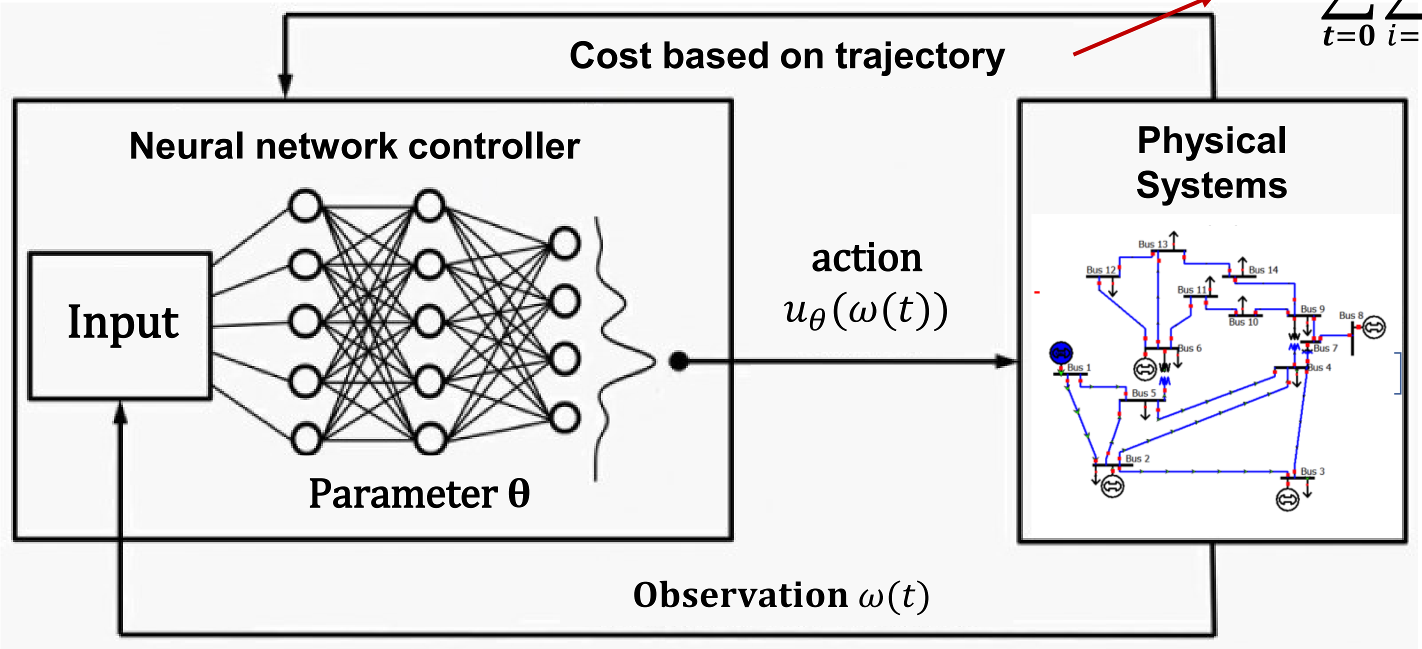
$$\omega_i^* = 0$$



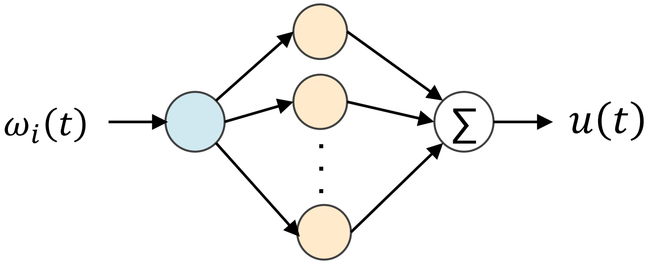
2 Transient Optimization with RL

- Parameter the controller with neural network controller
- Reinforcement learning (RL) for training the neural network

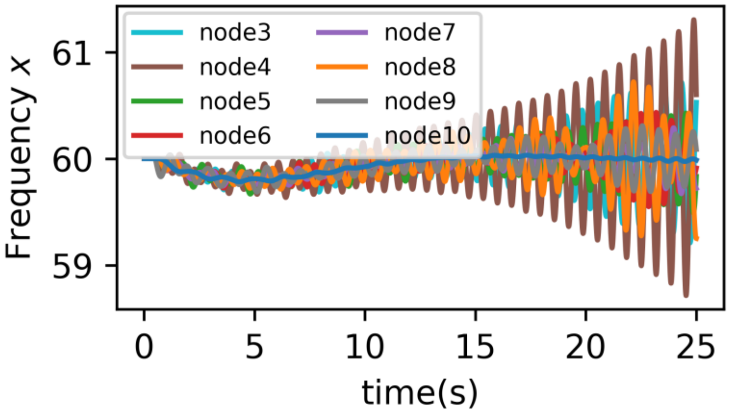
$$\boldsymbol{Loss} = \sum_{t=0}^{T} \sum_{i=1}^{n} J_i(\omega_i(t), u_i(t))$$



2 Hard Constraint on Stability



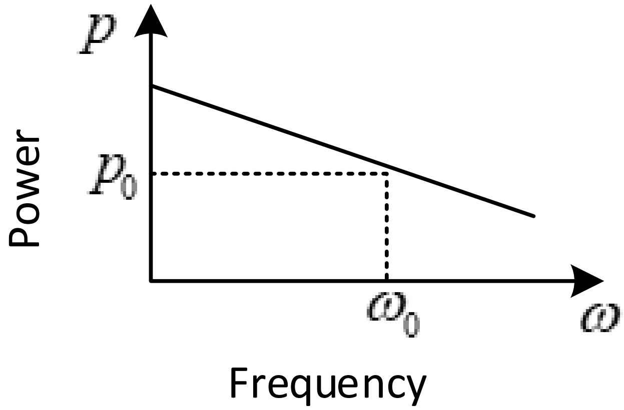
$$\boldsymbol{Loss} = \sum_{t=0}^{T} \sum_{i=1}^{n} J_i(\omega_i(t), u_i(t))$$



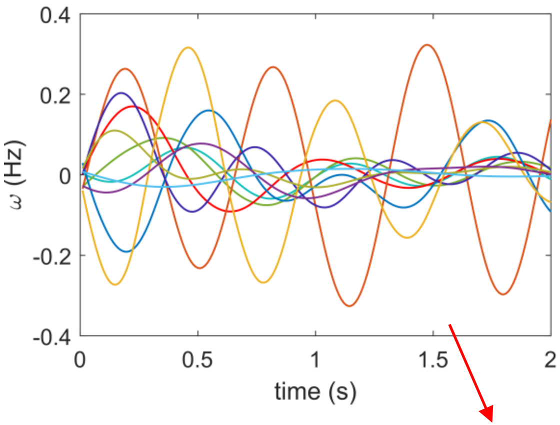
(a) Dynamics for system with generic neural network controllers

2 Hard Constraint on Stability

Current approaches in literature



Option 1 : linearized control law and system dynamics



Option 2 : RL without stability requirement, or simply add soft penalties

2 Lyapunov Approach for a Stabilizing Controller

A local Lyapunov function $V(\delta, \omega)$ for the dynamic system is

$$V(\boldsymbol{\delta}, \boldsymbol{\omega}) = \sum_{(i.j)\in\mathcal{E}} \underbrace{-B_{ij}\cos(\delta_{ij}) + B_{ij}\cos(\delta_{ij}^*) - B_{ij}\sin(\delta_{ij}^*)(\delta_{ij} - \delta_{ij}^*)}_{\text{Potential Energy (Bregman distance of } -B_{ij}\cos(\delta_{ij}))} + \frac{1}{2}\sum_{i=1}^{n} \underbrace{M_i(\omega_i - \omega_i^*)^2}_{\text{Kinetic Energy}}$$

It is a valid Lyapunov function in $\Theta = \left\{(\boldsymbol{\delta}, \boldsymbol{\omega}) | \delta_{ij} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \forall i.j\right\}$, where $-B_{ij}\cos(\delta_{ij})$ is convex and therefore

$$-B_{ij}\cos(\delta_{ij}) \geq \left(-B_{ij}\cos(\delta_{ij}^*)\right) + \left(-\nabla B_{ij}\cos(\delta_{ij}^*)\right)(\delta_{ij} - \delta_{ij}^*)$$

with equality holding if and only if $\delta_{ij} = \delta_{ij}^*$.

2 Lyapunov Approach for a Stabilizing Controller

Sufficient condition for asymptotic stability

$$\dot{V}(\boldsymbol{\delta}, \boldsymbol{\omega}) = \sum_{i=1}^{n} \frac{\partial V}{\partial \delta_i} \dot{\delta}_i + \frac{\partial V}{\partial \omega_i} \dot{\omega}_i$$

Independent with system parameter and topologies!

$$= \sum_{i=1}^{n} -D_i(\omega_i - \omega_i^*)^2 \boxed{- (\omega_i - \omega_i^*)(u_i(\omega_i) - u_i(\omega_i^*))}$$

$$\leq 0$$

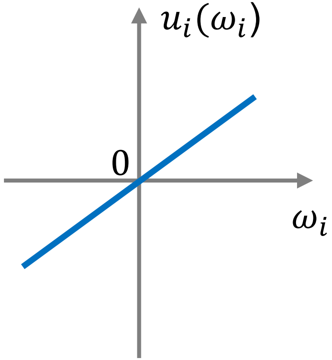
The condition $\dot{V}(\boldsymbol{\delta}, \boldsymbol{\omega}) \leq 0$ holds if

$$(\omega_i - \omega_i^*)(u_i(\omega_i) - u_i(\omega_i^*)) \geq 0 \qquad \forall i = 1, \cdots n$$

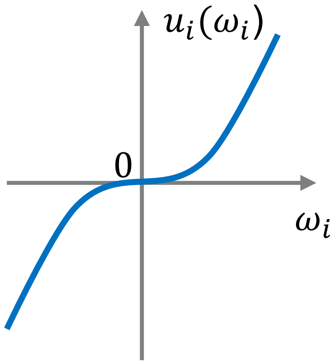
A unique equilibrium if $u_i(\omega_i)$ is monotonic increasing and cross the origin

2 Structural Property for a Stabilizing Controller

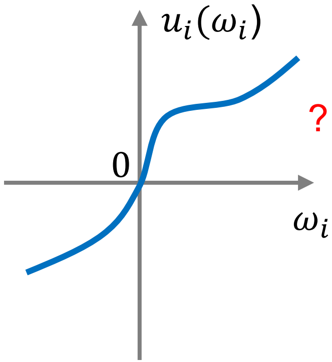
Monotonic increasing functions cross the origin



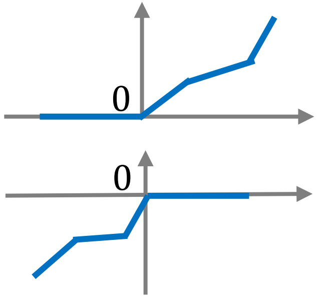
Linear



Polynomial



General form?



Neural Network?

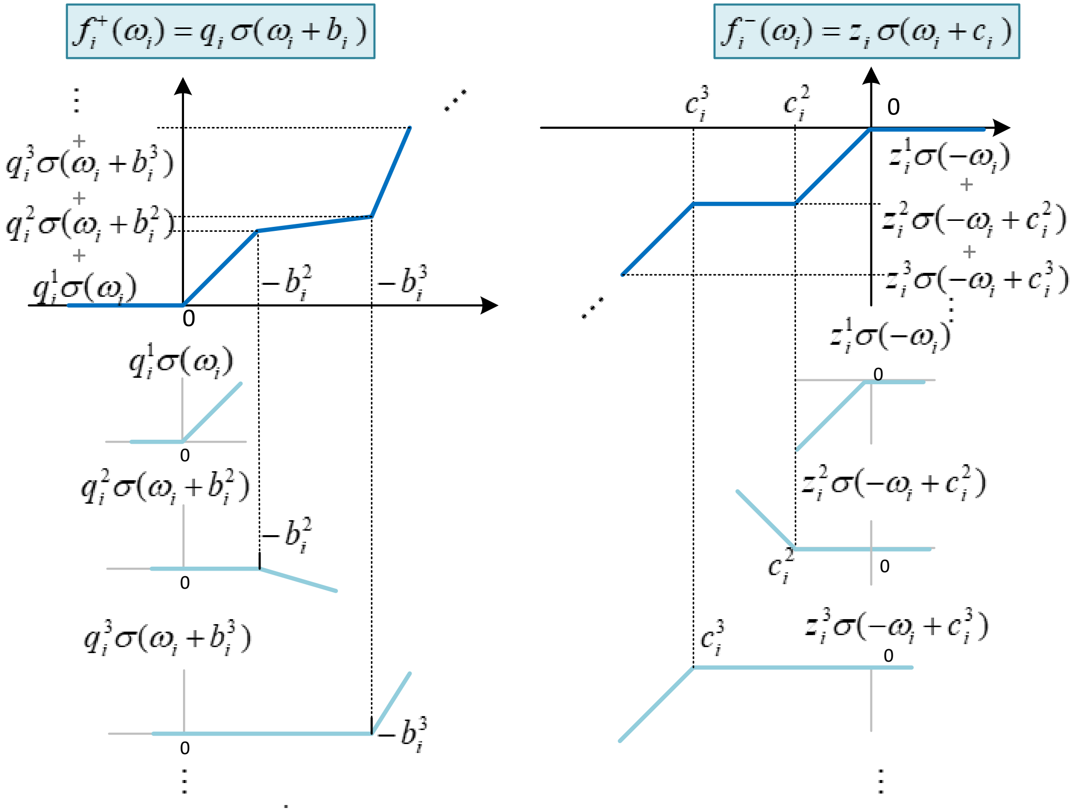
2 Structured Neural Network Design

ReLU function

$$\sigma(x) = \max(x, 0)$$

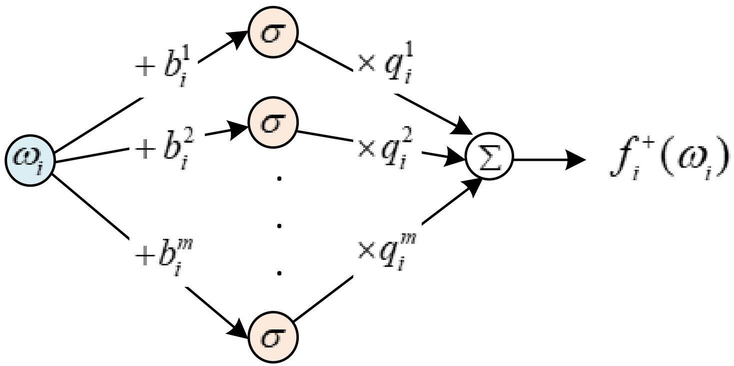
$$q\sigma(x+b) = \begin{cases} q(x+b) & if \;\; x > -b \\ 0 & otherwise \end{cases}$$

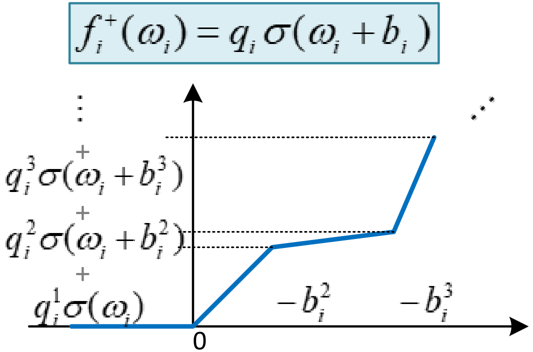
Explicitly engineer the structure of neural network controllers by stacking the ReLU function



2 Structured Neural Network Design

Structured one-layer neural network for parameterizing controller





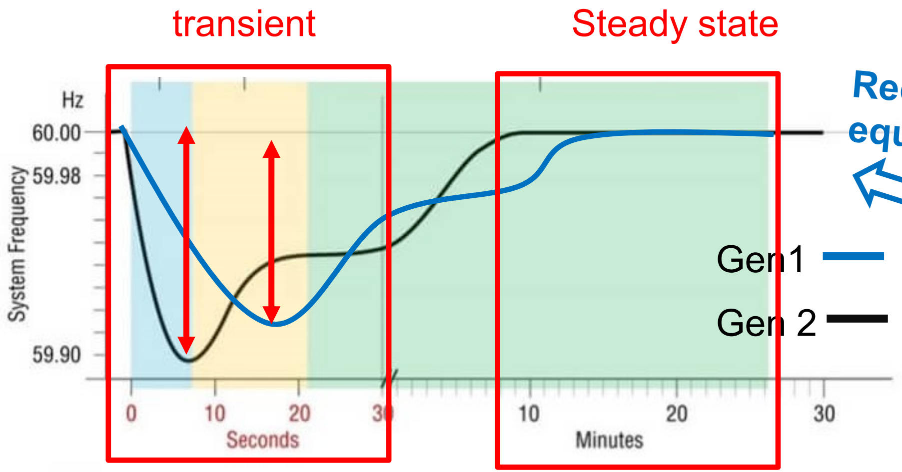
$$f_i^+(\omega_i) = \boldsymbol{q}_i\sigma(\mathbf{1}\omega_i + \boldsymbol{b}_i)$$

$$where \sum_{j=1}^{l} q_i^j \geq 0, \quad \forall l = 1, 2, \cdots, m$$

$$b_i^1 = 0, b_i^l \leq b_i^{(l-1)}, \quad \forall l = 2, 3, \cdots, m$$

We proved that $u_i(\omega_i) = f_i^+(\omega_i) + f_i^-(\omega_i)$ is a universal approximation of any bounded, Lipschitz continuous and monotonically increasing function through the origin.

3 Steady-State Tracking



Required equilibrium

Steady-state metrics

- ❑ Frequency Restoration

$$\omega_i^* = 0$$

- ❑ Structural property of controllers stabilize towards the required equilibrium
- ❑ Engineer the neural networks to satisfy these structures

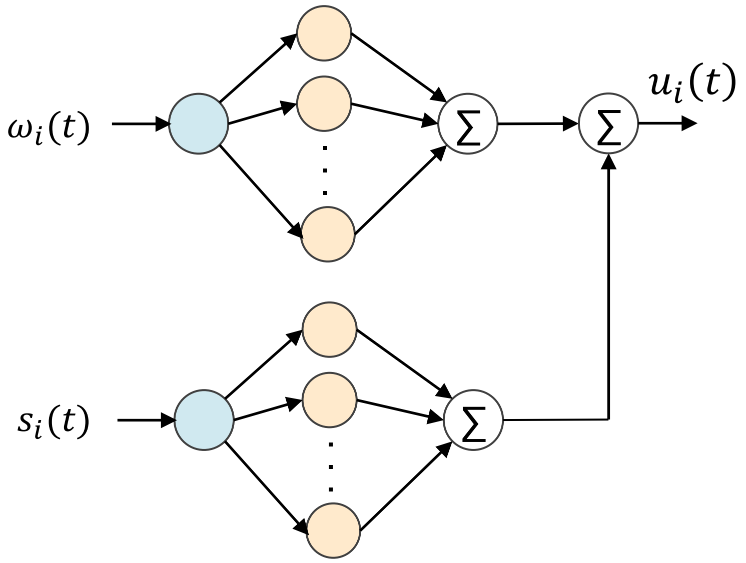
Satisfy by design

3 Control theoretic motivations

- Output agreement to the required value

Integral Control

$$\dot{s}_i = \overline{y} - y_i \quad \Longleftrightarrow \quad s_i(t) = \int_{\tau=0}^{t} \left(\overline{y} - y_i(\tau)\right) d\tau$$



3 Generalized PI Controller

$\dot{s}_i = \omega_i$

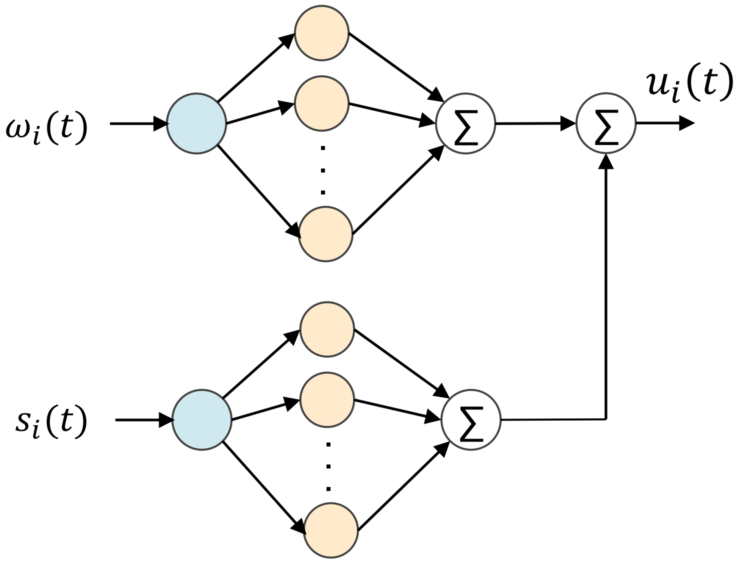
$u_i(t) = \pi_i^P(\omega_i) + \pi_i^I\big(s_i(t)\big)$

Generalized proportional control

Generalized integral control

Check equilibrium: If $\omega_i^* = \overline{y}$, then $\dot{s}_i = 0$

Whether trajectories from any possible initial states converges to this equilibrium?



3 Monotonicity →Convexity

Define the integral function $L_i(s_i) = \int_0^{s_i} \pi_i^I(z)dz$, which is strictly convex since $\nabla^2 L_i(s_i) = \frac{d\pi_i^I(s_i)}{ds_i} > 0$ by strictly monotonicity.

A local Lyapunov function $V(\eta, \omega, s)$ for the dynamic system is

$$V(\eta,\omega,s) = \sum_{(i.j)\in\mathcal{E}} -B_{ij}\cos(\delta_{ij}) + B_{ij}\cos(\delta_{ij}^*) - B_{ij}\sin(\delta_{ij}^*)(\delta_{ij}-\delta_{ij}^*) + \frac{1}{2}\sum_{i=1}^{n} M_i(\omega_i-\omega_i^*)^2$$

$$+\sum_{i=1}^{n} \underbrace{L_i(s_i) - L_i(s_i^*) - \nabla L_i(s_i^*)(s_i - s_i^*)}_{\text{Bregman distance with } L_i(s_i)}$$

3 Convergence of ω_i

The time derivative of the energy function is

$$\dot{V}(\eta, \omega, s) = \sum_{i=1}^{n} \frac{\partial V}{\partial \omega_i} \dot{\omega}_i + \frac{\partial V}{\partial s_i} \dot{s}_i + \sum_{l=1}^{m} \frac{\partial V}{\partial \eta_l} \dot{\eta}_l$$

$$\leq \textstyle\sum_{i=1}^{n} -\rho_i (\omega_i - \omega_i^*)^2 + \left(\pi_i^P(\omega_i) - \pi_i^P(\omega_i^*)\right)(\omega_i - \omega_i^*)$$

$$\leq \textstyle\sum_{i=1}^{n} -\rho_i (\omega_i - \omega_i^*)^2 \quad \text{(by monotone of } \pi_i^P\text{)}$$

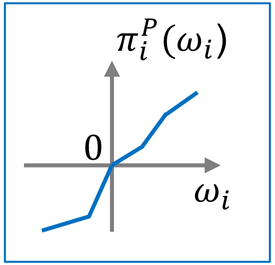
3 Generalized Neural-PI Controller with Guarantees

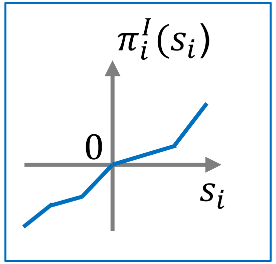
In summary, the control law given as follows stabilizes the system and also guarantees frequency restoration at the steady state

$$\dot{s}_i = \omega_i$$

$$u_i(\omega_i) = \pi_i^P(\omega_i) + \pi_i^I(s_i)$$

Monotone Neural Network Implementation



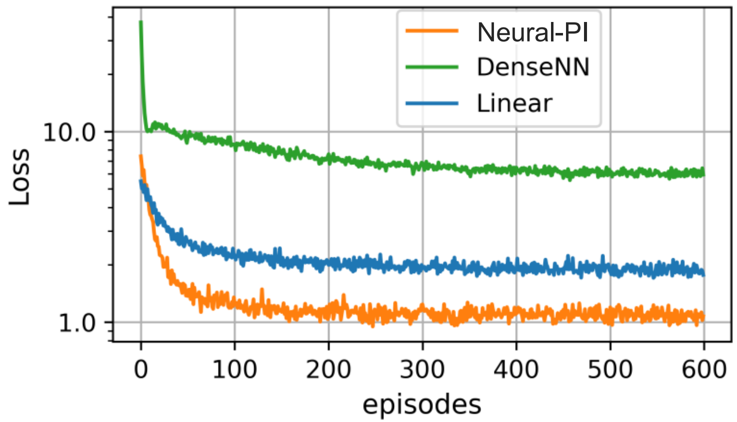


3 Case study : Power systems

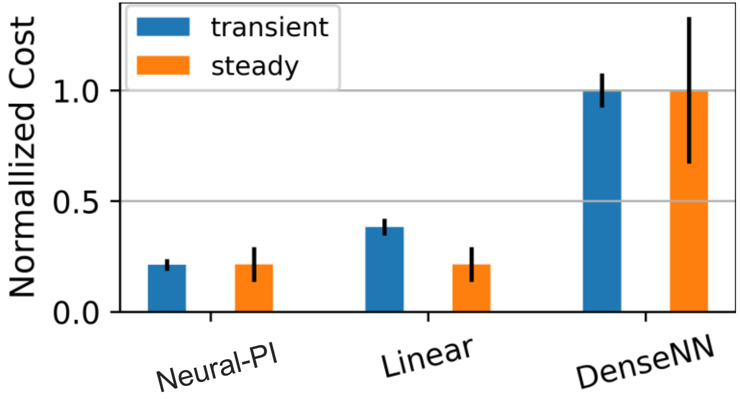
Case study is conducted on IEEE-39 bus test system.

Compare the proposed Neural-PI controller with

1) Linear PI control with the coefficient optimized by training
2) Dense neural network

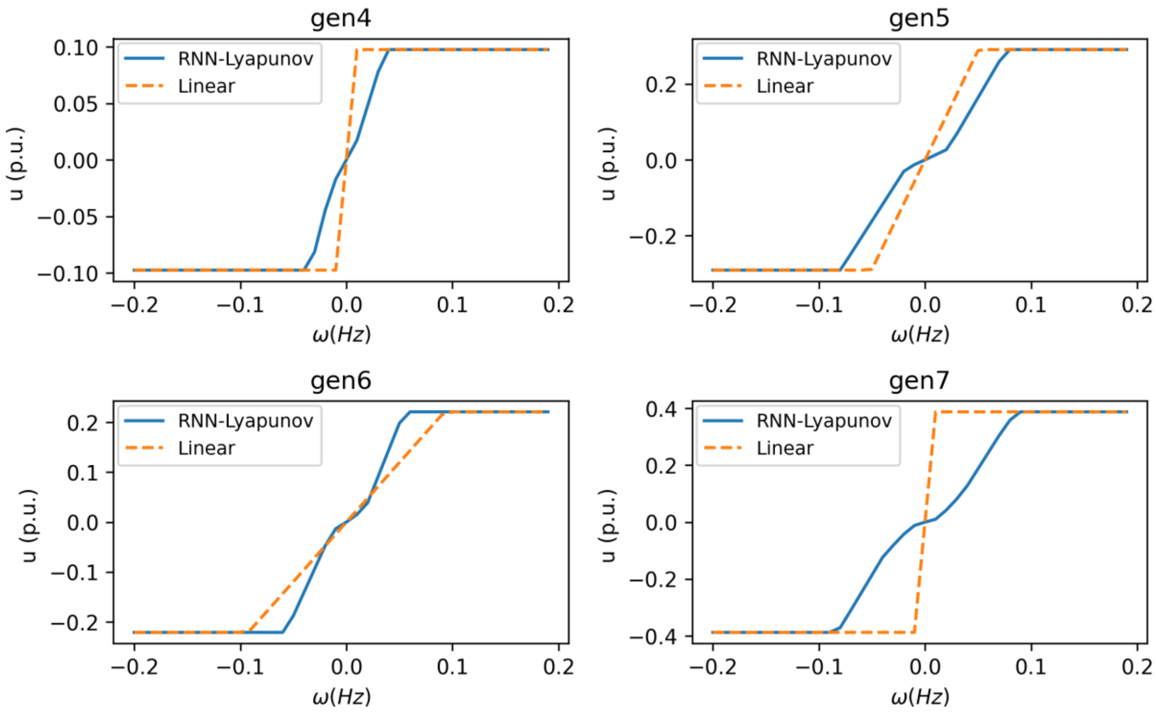


Training loss



Transient and steady-state cost

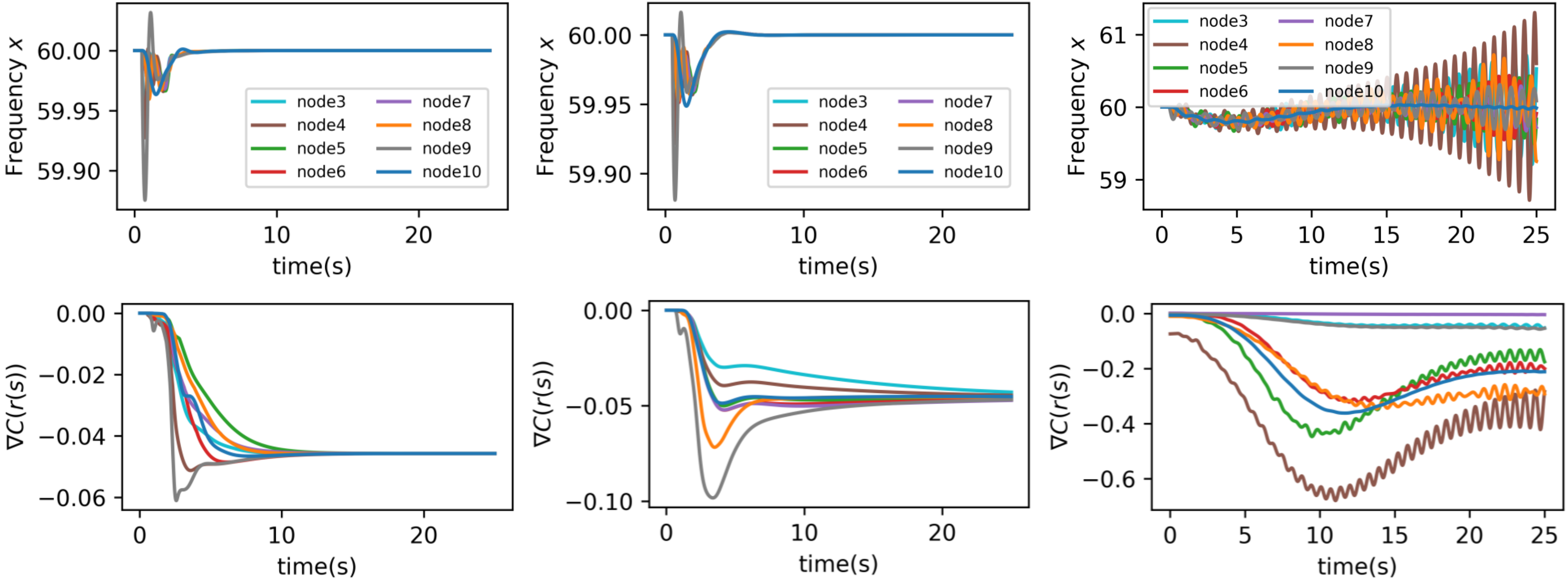
3 Case Study



Control Action *u* obtained by different approaches

3 Case study : Power systems

The dynamics after the same step load changes



(a) Neural-PI (b) Linear (c) DenseNN

4 Generalization to More Dynamic Systems

We consider a dynamic system described by

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}),\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}),$$

where state $\boldsymbol{x} \in \mathbb{R}^n$, output $\boldsymbol{y} \in \mathbb{R}^m$, control action $\boldsymbol{u} \in \mathbb{R}^m$.

Assumption: Equilibrium-Independent Passivity

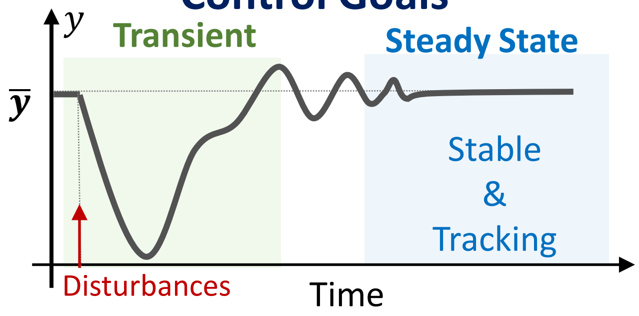
The system described by $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}),\ \boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x})$ is strictly equilibrium-independent passive (EIP) if it satisfies:

(i) for every equilibrium $\boldsymbol{u}^*$, there exists a unique $\boldsymbol{x}^*$ such that $\boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{u}^*) = \boldsymbol{0}$, and

(ii) there exists a positive definite storage function $S(\boldsymbol{x}, \boldsymbol{x}^*)$ a constant ρ such that

$$S(\boldsymbol{x}^*, \boldsymbol{x}^*) = 0 \ \text{ and } \ \dot{S}(\boldsymbol{x}, \boldsymbol{x}^*) \leq (\boldsymbol{y} - \boldsymbol{y}^*)^T(\boldsymbol{u} - \boldsymbol{u}^*) - \rho||\boldsymbol{y} - \boldsymbol{y}^*||^2$$

4 Generalized PI Controller



The control law is

$$\dot{s} = \bar{y} - y$$

$$u = \underbrace{\pi^P(\bar{y} - y)}_{\text{Proportional control}} + \underbrace{\pi^I(s)}_{\text{Integral control}}$$

Strictly Monotone Functions

Gradient of convex function $\nabla g(z)$ is strictly monotone

Definition: Strictly Monotone Functions

A continuous function $q\colon \mathbb{R}^m \mapsto \mathbb{R}^m$ is strictly monotone on $D \subset \mathbb{R}^m$ if $\left(q(\eta) - q(\xi)\right)^T(\eta - \xi) \geq 0,\ \forall \eta, \xi \in D$, with the equality holds if and only if $\eta = \xi$.

4 Stability

Lyapunov function := Storage function + Bregman Distance of Convex Functions

Comes from $\boldsymbol{\pi}^I(\boldsymbol{s}) = \nabla g(\boldsymbol{z})$

Definition: Bregman Distance

The Bregman Distance of a strictly convex function g: $\mathbb{R}^m \mapsto \mathbb{R}$ at the point s^* is defined as $B(\boldsymbol{s}, \boldsymbol{s}^*) = g(\boldsymbol{s}) - g(\boldsymbol{s}^*) - \left(\nabla g(\boldsymbol{s}^*)\right)^T(\boldsymbol{s} - \boldsymbol{s}^*)$, which is positive definite with equality holds only when $s = s^*$.

4 Stability

Lyapunov function := Storage function + Bregman Distance of Convex Functions

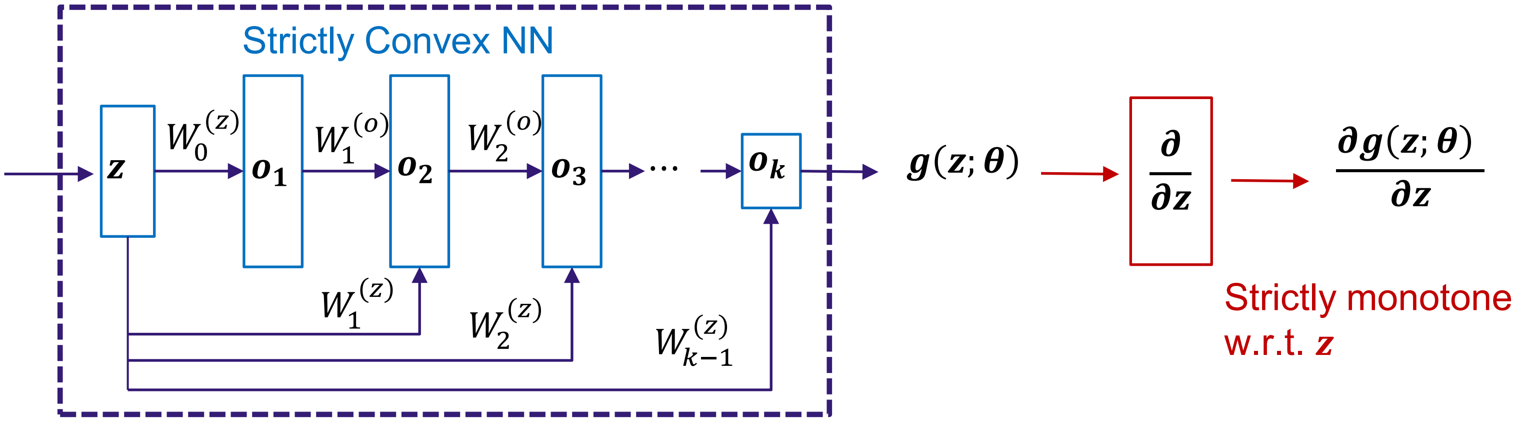
Comes from $\boldsymbol{\pi}^I(\boldsymbol{s}) = \nabla g(\boldsymbol{z})$

$$\dot{V} \leq -\rho||\boldsymbol{y} - \boldsymbol{y}^*||^2 + \underbrace{\boxed{(\boldsymbol{y} - \boldsymbol{y}^*)^T \left(\boldsymbol{\pi}^P(\overline{\boldsymbol{y}} - \boldsymbol{y}) - \boldsymbol{\pi}^P(\overline{\boldsymbol{y}} - \boldsymbol{y}^*)\right)}}_{\leq 0 \text{ by monotonicity}}$$

Definition: Bregman Distance

The Bregman Distance of a strictly convex function $g\colon \mathbb{R}^m \mapsto \mathbb{R}$ at the point s^* is defined as $B(\boldsymbol{s}, \boldsymbol{s}^*) = g(\boldsymbol{s}) - g(\boldsymbol{s}^*) - \left(\nabla g(\boldsymbol{s}^*)\right)^T (\boldsymbol{s} - \boldsymbol{s}^*)$, which is positive definite with equality holds only when $s = s^*$.

4 Strictly Convex NN $\rightarrow$ Monotone NN



A strictly convex function $g(z;\theta)$ parameterized by k -layer neural network, with o_l being the output of the l-th layer

$$o_{l+1} = \sigma_l\left(W_l^{(o)} o_l + W_l^{(z)} z + b_l\right), \qquad \mathrm{g}(\mathrm{z};\theta) = o_k$$

Strictly convex and increasing

Positive except for $o_0, W_0^{(o)} \equiv 0$

4 Strictly Convex NN - Universal approximation

Convex function ← Max of affine function ← DenseNN with ReLU ← DenseNN with softplus-beta

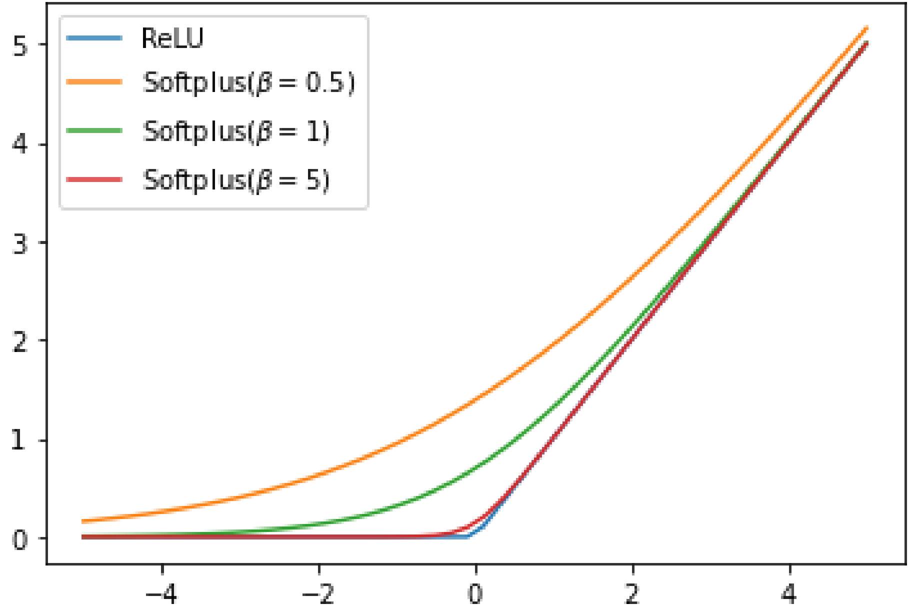
Universal approximation of any strictly convex functions

$$o_{l+1} = \sigma_l \left(W_l^{(o)} o_l + W_l^{(z)} z + b_l \right),$$

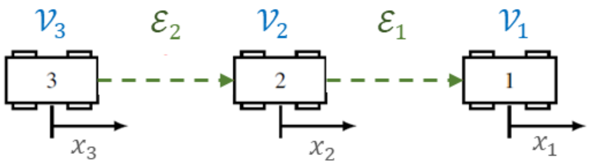
$$\mathrm{g}(\mathrm{z}; \theta) = o_k$$

when the activation is

$$\sigma_l^{softplus}(x) \coloneqq \frac{1}{\beta} \log\left(1 + e^{\beta x}\right)$$



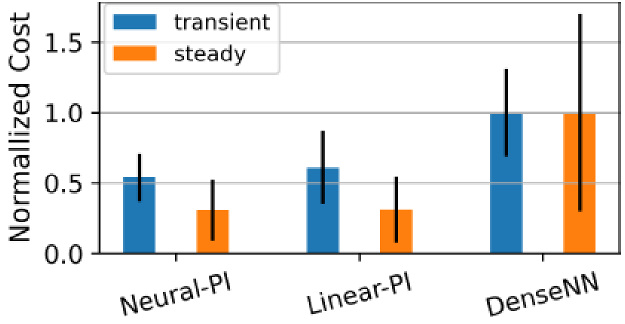
4 Case study : Vehicle platooning



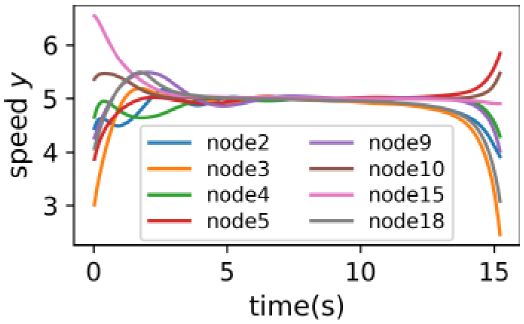
Vehicle platooning problem with 20 vehicles

Compare the proposed Neural-PI controller with

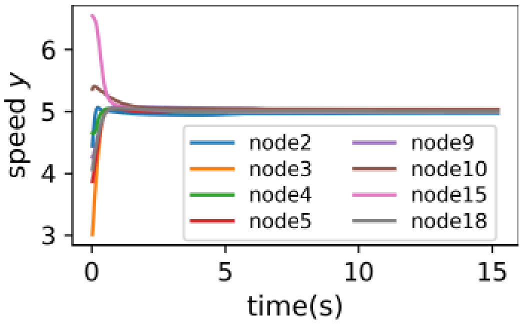
1) Linear PI control with the coefficient optimized by training
2) Unstructured dense neural network



(a) Transient and steady-state cost



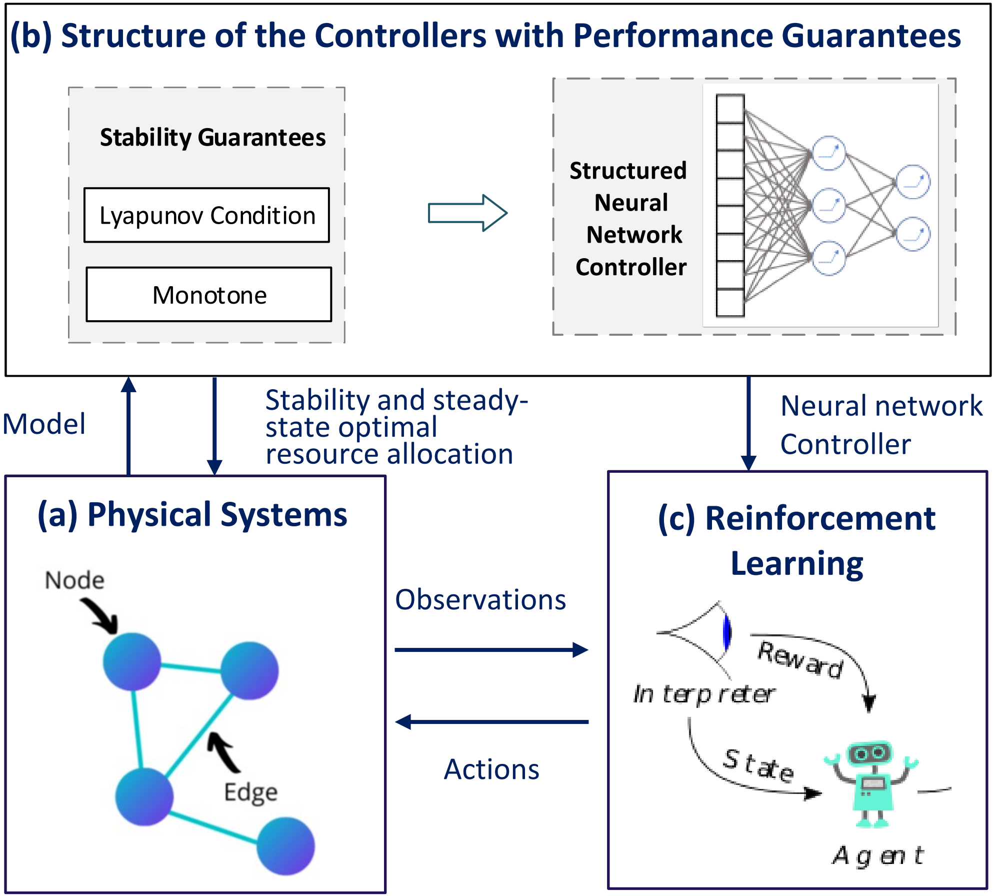
(b) Velocities under DenseNN



(c) Velocities under Neural-PI

5 Conclusions

- Derive structural properties of controllers satisfying performance guarantees
- Enforce the structures in the design of neural networks



Thank you!

- Feel free to contact me at wenqicui@uw.edu
- Online version and code of the above works can be found in

[1] Wenqi Cui, Yan Jiang, and Baosen Zhang. Reinforcement learning for optimal primary frequency control: A Lyapunov approach. IEEE Transactions on Power Systems, 2022

[2] Wenqi Cui, Yan Jiang, Baosen Zhang and Yuanyuan Shi. Structured Neural-PI Control for Networked Systems: Stability and Steady-State Optimality Guarantees. NeurIPS, 2023.