SIMULATION AND ANALYSIS OF BIT ERROR RATE VS. SIGNAL-TO -NOISE RATION IN M-ARAY PULSE AMPLITUDE MODULATION

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Table of Contents

1.Abstract

In digital communication systems, reliably detecting transmitted signals in the presence of noise is a central challenge. This project addresses this issue through the implementation of correlation receivers and matched filtering techniques, which are theoretically optimal for identifying known signals corrupted by Additive White Gaussian Noise (AWGN). By modeling the transmission and reception of Pulse Amplitude Modulation (PAM) signals, we analyze the statistical behavior of bit and symbol errors across varying noise conditions. Our simulations demonstrate how matched filtering enhances the signal-to-noise ratio and improves detection accuracy by maximizing the likelihood of correct symbol identification. The results validate the effectiveness of correlation-based detection methods in minimizing bit error rates and illustrate the probabilistic relationship between noise, modulation levels, and system performance.

2.Important Definitions

2.1 Additive White Gaussian Noise (AWGN)

It is a type of random noise that is added to a signal during transmission or processing

Additive: The noise is simply added to the signal—it does not multiply or distort the signal in other complex ways.

$$x(t) = s\_i(t) + w(t)$$

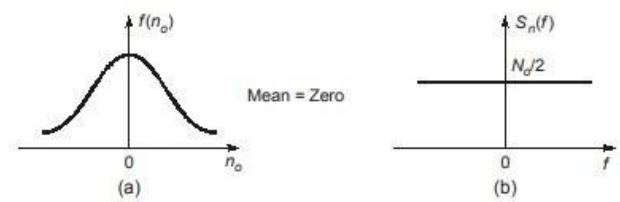


Figure: *(a) PDF of white noise (b) PSD of white noise*

PSD of white noise $S_n(f) = \frac{N_o}{2}$

White: The noise has equal power across all frequencies, like white light containing all colors—this means it affects all parts of the signal spectrum equally.

Gaussian: The noise values follow a normal probability distribution, meaning most values are close to the average with fewer extreme deviations.

Noise: Random unwanted variations that interfere with the original signal, making it harder to detect or interpret correctly.

2.2 Bit Error Rate (BER)

The ratio of the number of bits received in error to the total number of bits transmitted. It measures the accuracy of a digital communication system; a lower BER indicates better performance.

$$BER = \frac{number\ of\ bits\ received\ in\ error}{total\ number\ of\ bits}$$

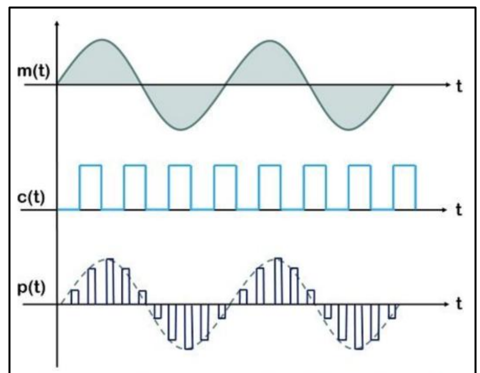
2.3 Signal-to-Noise Ratio (SNR)

A measure of signal strength relative to background noise, usually expressed in decibels (dB). A higher SNR means the signal is much stronger than the noise, resulting in better detection and lower error rates.

2.4 M-Pulse Amplitude Modulation (M-PAM)

M-PAM is a digital modulation scheme in which each symbol represents one of **M distinct amplitude levels** of a pulse. Instead of transmitting a single bit per symbol (as in binary PAM), M-PAM can represent multiple bits per symbol by using more amplitude levels. The number of bits k represented by each symbol is related to M by the formula:

$$k = M$$



Waveform representation of PAM signal

For example:

- 4-PAM transmits 2 bits per symbol $(4) = 2$)
- 8-PAM transmits 3 bits per symbol ($(8) = 3$))

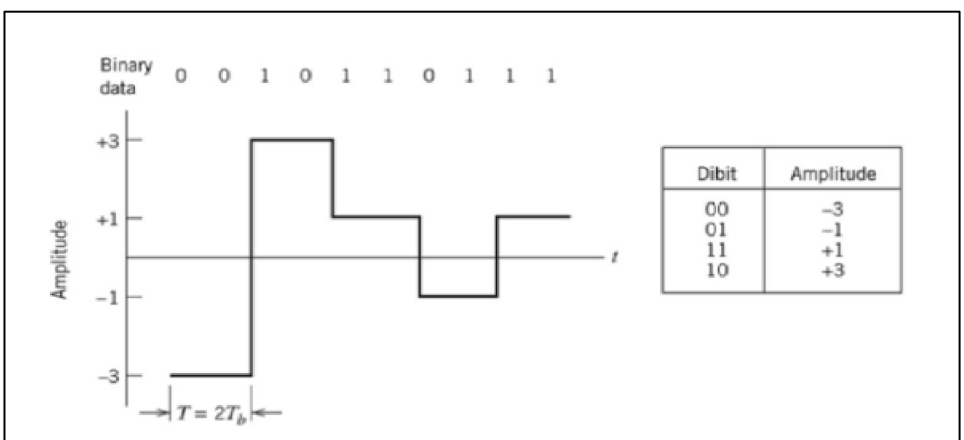


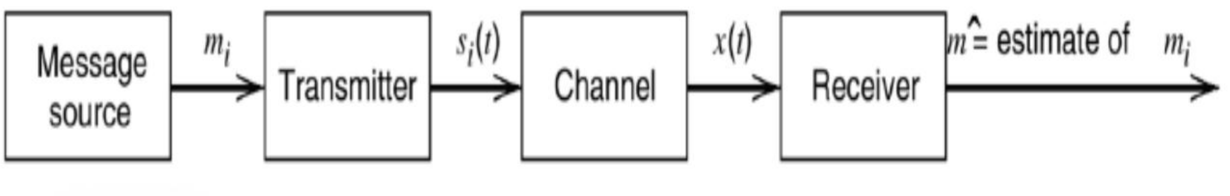
Figure 1: 4-PAM System

The general idea is to increase data rate without increasing the signal bandwidth. However, increasing **M** makes the amplitude levels closer together, which makes the system **more sensitive to noise** (especially AWGN), and increases the probability of symbol errors.

The performance of M-PAM is typically evaluated in terms of **Symbol Error Rate (SER)** and **Bit Error Rate (BER)**, which both depend on the **Signal-to-Noise Ratio (SNR)**. Higher M values allow more data per symbol but require higher SNR for reliable communication.

3.Correlation Receiver

In digital communication systems, accurately detecting transmitted signals in the presence of noise, particularly Additive White Gaussian Noise (AWGN) is a central challenge. To address this, we employ a **correlation receiver**, an optimal detection system designed to maximize the probability of correctly identifying a known signal. It works by comparing the received noisy waveform with a set of expected templates, enabling reliable estimation of the transmitted message. The following explanation, along with the system diagram, breaks down the role of each stage in this process.



Block diagram of a generic digital communication system.

Overview of the Digital Communication System

1. Message Source

- Output: m_i a digital message (e.g., a bit or bit sequence).
- This is the original information the system wants to transmit (like "0", "1", or more complex symbols).

2. Transmitter

- Function: Maps each message m_i to a unique signal waveform $s\_i(t)$.
- This is modulation. For example, using M-PAM, a message like "10" might be mapped to a specific amplitude pulse.

- Output: $s\_i(t)$, the modulated signal.

3. Channel

- Function: Introduces distortion, mainly noise, during transmission.
- In this case, the noise is modeled as Additive White Gaussian Noise (AWGN).
- Equation:

$$x(t) = s\_i(t) + w(t)$$

where:

- x(t): received signal,
- $s\_i(t)$: transmitted signal,
- w(t): random noise added by the channel.

4. Receiver

- Function: Analyzes the noisy received signal $x(t)$ to estimate the original message $\hat{m}$.
- This includes the correlation receiver, which processes $x(t)$ and determines the most likely transmitted signal $s\_i(t)$, and thus the original message m_i.

Inside the Correlation Receiver

The correlation receiver is used to determine which signal s_k(t) (from a known set of possible signals) most closely matches the received noisy signal **x(t)**. It consists of two main parts:

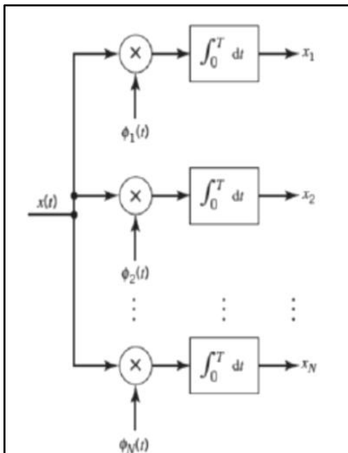
Part 1: Detector (Demodulator)

The **detector** computes how closely the received signal resembles each of the known signal templates using a set of orthonormal basis functions $\phi_1(t)$, $\phi_2(t)$, …, ϕ_N(t). The received signal x(t) is projected onto each basis function to obtain the correlation values:

$$\mathbf{x_k = \int_0^T x(t) \cdot \phi_k(t)\, dt}, \quad \text{for } k = 1, 2, \ldots, N$$

Where:

- **x(t)** is the received noisy signal,
- $\boldsymbol{\phi_k(t)}$ is the k-th basis function (template),



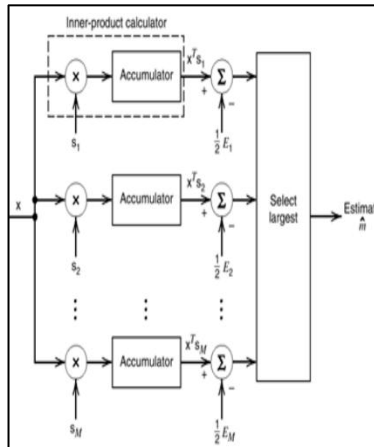
Detector or demodulator.

- $\mathbf{x_k}$ is the scalar projection (correlation value) of x(t) onto $\phi_k(t)$,
- **T** is the signal duration.

The set of values $\{x_1, x_2, \ldots, x_N\}$ forms the vector representation of the received signal in signal space. These correlation values are passed to the decision device for further processing.

Part 2: Decoder

The decoder receives the set of correlation values $\{x_1, x_2, \ldots, x_N\}$ from the correlator and determines which transmitted signal is the most likely. Each possible transmitted signal is also represented as a vector $s_i = \{s_{i1}, s_{i2}, \ldots, s_{iN}\}$, where the components correspond to the same basis functions $\phi_k(t)$.



Signal transmission decoder.

The decision device computes the distance between the received vector $x = \{x_1, x_2, \ldots, x_N\}$ and each possible signal vector s_i, and selects the one with the minimum Euclidean distance:

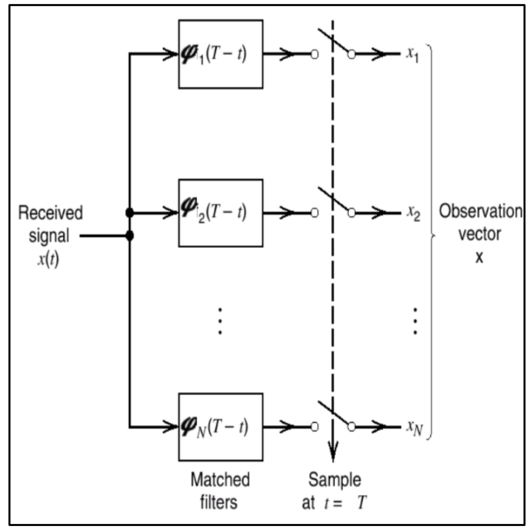
$$\hat{\mathbf{s}} = \min_i \|\mathbf{x} - \mathbf{s}_i\|$$

Where:

- $\hat{s}$ is the estimated transmitted symbol,
- x is the vector of correlation values,
- s_i is the i-th known signal vector,
- $\|x - s_i\|$ denotes the Euclidean distance between x and s_i.

This decision rule minimizes the probability of error and selects the signal that is geometrically closest to the received signal in the signal space.

4.Matched Filter



The correlation receiver can also be implemented using a bank of **matched filters**, which offer an equivalent and often more practical approach for real-time signal detection. Instead of explicitly computing the correlation integrals, each received signal is passed through a filter that is matched to the expected transmitted waveform.

A **matched filter** is a linear time-invariant (LTI) filter whose **impulse response h(t)** is designed to match the time-reversed version of the signal template it is detecting:

$$h(t) = \phi(T - t)$$

Where:

- **ϕ(t)** is the known signal waveform (or basis function),
- **T** is the symbol duration.

The matched filter maximizes the **output signal-to-noise ratio (SNR)** at the sampling instant

t = T, making it optimal for detecting known signals in AWGN.

When the received signal **x(t)** passes through the matched filter, the output at time **T** is:

$$y(T) = \int_0^T x(\tau) \cdot \phi(T - \tau)\, d\tau$$

By the properties of convolution and time reversal, this output is equivalent to computing the inner product between **x(t)** and **ϕ(t)**, as done in the detector:

$$y(T) = \int_0^T x(t) \cdot \phi(t)\, dt$$

This shows that **matched filtering and correlation** produce identical results when properly aligned. Therefore, in practical systems, the detector can be replaced by a matched filter followed by a simple sampler at the correct instant.

5.Graphical Interpretation of ML Decision Rule

Let Z donate the N-dimensional space (observation space) of all possible observation vectors x, because we have assumed that the decision rule must say $\hat{m} = m_i$, where i =1, 2 ,......,M, the observation space Z is correspondingly partitioned into M=decision regions.Accordingly , we may state the decision rule as:

$$\boxed{\begin{array}{c}\text{Observation vector } x \text{ lies in the region } Z_i \\ \text{if } l(m_k) \text{ is maximum for } k = i\end{array}}$$

for the log-likelihood function for an AWGN channel is defined as:

$$\boxed{l(m_i) = -\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{ij})^2, \; i = 1, 2, \ldots, M}$$

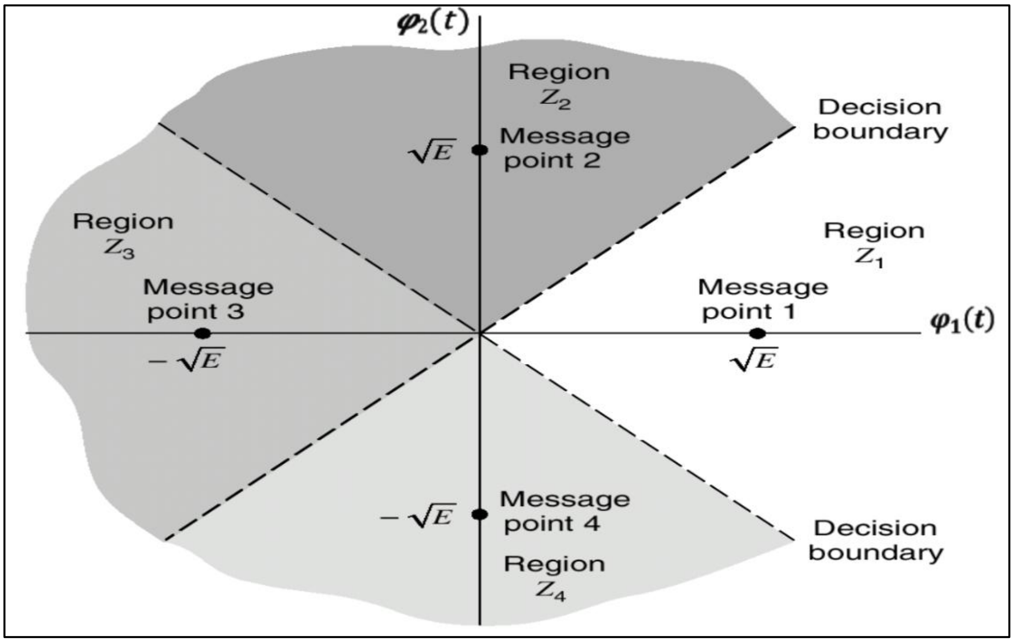
Accordingly, we may formulate the maximum likelihood decision rule for an AWGN channel as:

$$\boxed{\begin{array}{c}\text{Observation vector } x \text{ lies in the region } Z_i \\ \text{if } \sum_{j=1}^{N}(x_j - s_{kj})^2 \text{ is minimum for } k = i\end{array}}$$

Accordingly, we may formulate the ML decision rule as

$$\boxed{\begin{array}{c}\text{Observation vector } x \text{ lies in the region } Z_i \text{ if} \\ \left[\sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2}E_k\right] \text{ is maximum for } k = i, \text{where } E_k \text{ is the} \\ \text{transmitted energy}\end{array}}$$

Thus, for an AWGN channel, the M decision regions are bounded by linear hyperplane boundaries .



Illustrating the partitioning of the observation space into decision regions

for the case when N = 2 and M = 4; it is assumed that the M transmitted

symbols are equally likely.

6.Methodology and Results

6.1Simulink:

6.1.1 Block Diagram

1. **Random Integer Generator:** The source used for the whole channel, generating random integers ranging from[0,M-1].

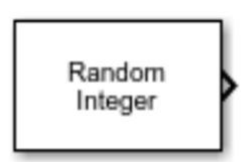


Figure 5: Random Integer Generator

1. **M-PAM Modulator:** modulates the Random generated signal following PAM technique. Technically speaking the M-PAM modulator here encodes or map the generated integers(bits) into known amplitudes where M is the number of amplitudes based on the number of bits per symbol. (bit width)

Figure 6: M-PAM Modulator

2. **AWGN Channel:** The channel used to add additive white Gaussian noise to the modulated random generated signal

3. **M-PAM Demodulator:** demodulates the received modulated signal, decoding each amplitude to a set of bits their width is dependent on the level M used in the modulator.

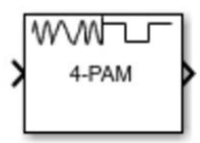


Figure 8: M-PAM Demodulator

4. **Error Rate Calculation:** calculates the bit error rate (BER) simply by comparing the original signal values(bits) and the received demodulated signal, the two signals are synchronized with each other, and the width is maintained across the process.

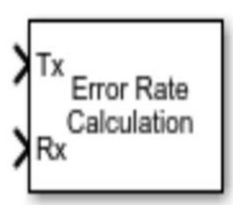


Figure 9: Error Rate Calculation

5. **To Workspace:** A block used to add the variable ber as an array to be used in the plotting process.

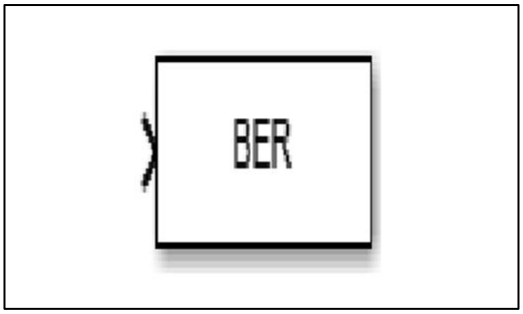


Figure 10: To Workspace

Here is the Diagram of the Used Simulink Blocks:

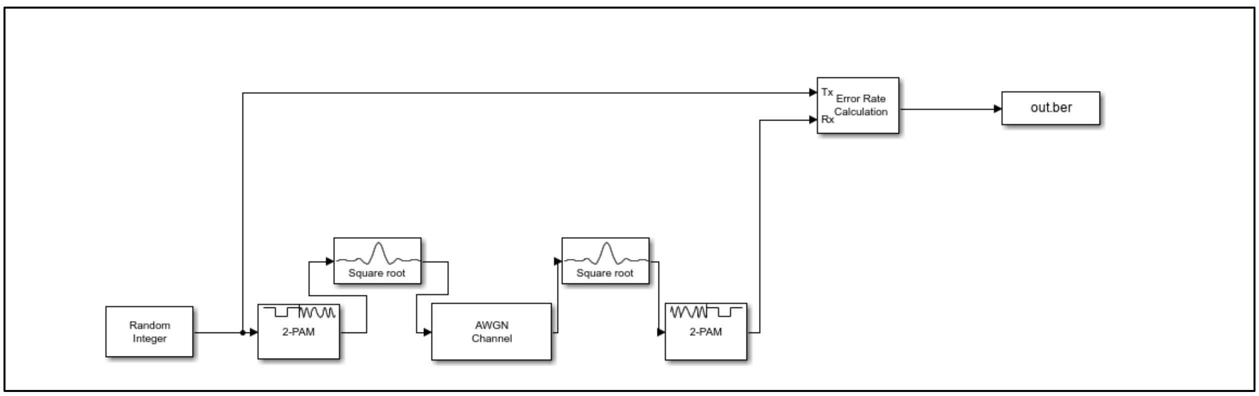


Figure 11: Block Diagram

6.1.2 Results:

The three continuous curves show the relation between the bit error rate(BER) and the Eb/N0 ratio or signal to noise ratio(SNR), The dotted curves represent the practical insights for varying M-levels demonstrating two relations:-

1.increasing the SNR ratio decreases the error in compared bits in general and results in more successful bit comparing processes

As values of M increase, more errors occur and that is due to increasing the number of bits per symbol(bit width) which makes the comparison process even harder as there are more amplitude levels used resulting in more median values for the same SNR ratio.

For instance, at the beginning of the three dotted graphs for different M-levels, although the SNR ratio is almost the same the BER is higher for higher order modulation techniques.

The graph also shows that the lower order levels tend to lower BERs much faster than the higher ones, which possess a slower decrease in BER rate while increasing the SNR.

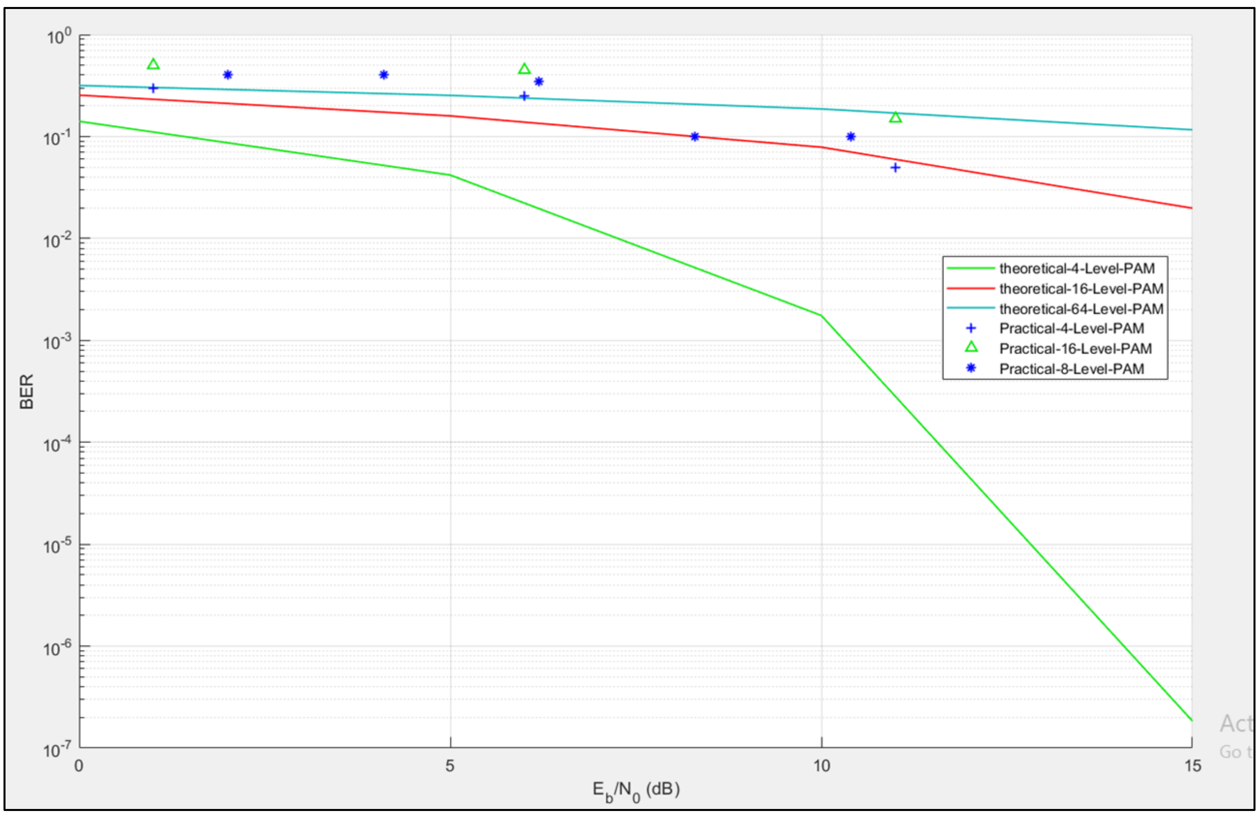


Figure 12: Simulink Results

2.MATLAB Code

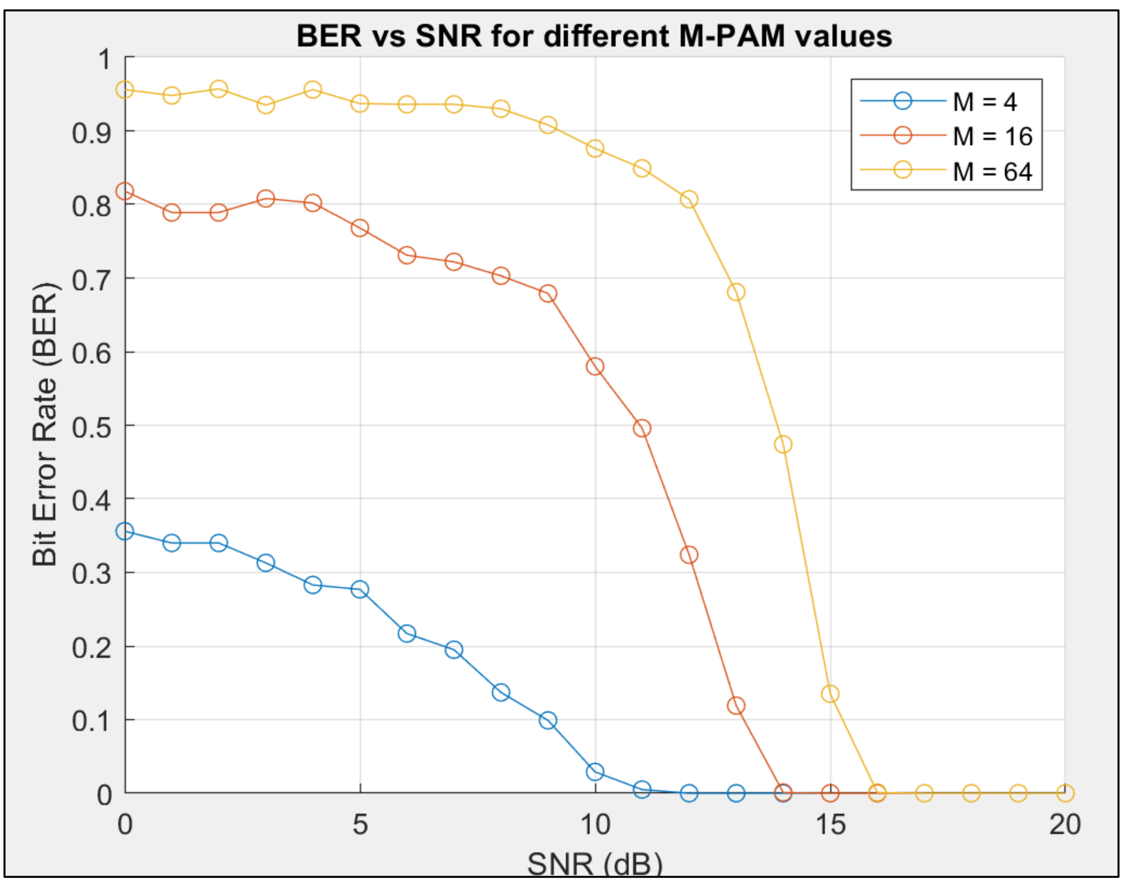
6.2.1 Code:

```
% Define the modulation levels for M-PAM
M_values = [4, 16, 64];

% Define the range of SNR values in dB to simulate over
SNR_db = 0:1:20;

% Define number of bits to transmit per simulation
num_of_Bits = 1000;

% Preallocate BER results matrix: rows for M values, columns for SNR values
BER_M = zeros(length(M_values), length(SNR_db));

% Loop over each M-PAM modulation value
for m_index = 1:length(M_values)

    % Get the current modulation level (e.g., 4, 16, or 64)
     M = M_values(m_index);

    % Generate random symbols (integers) between 0 and M-1
    transmitted_Bits = randi([0, M-1], 1, num_of_Bits); % Each symbol
represents log2(M) bits

    % Modulate the symbols using PAM modulation
    symbols = pammod(transmitted_Bits, M);

    % Loop over each SNR value
    for i = 1:length(SNR_db)

        snr = 10^(SNR_db(i)/10); % Convert SNR from dB to linear scale for
internal use

        % Add white Gaussian noise to the modulated signal at the given SNR
        receivedSignal = awgn(symbols, snr, 'measured');

        % Demodulate the received noisy signal
        receivedBits = pamdemod(receivedSignal, M);

        % Compare received symbols with transmitted ones to count bit errors
        bit_Errors = sum(receivedBits ~= transmitted_Bits);

        % Compute and store the Bit Error Rate (BER)
        BER_M(m_index, i) = bit_Errors / num_of_Bits;
    end
end

% Plotting BER vs SNR for each M value
figure;
hold on;
for m_index = 1:length(M_values)
    semilogy(SNR_db, BER_M(m_index, :), '-o', ...
```

```
        'DisplayName', ['M = ' num2str(M_values(m_index))]); % Use log scale
for BER
end

grid on;
xlabel('SNR (dB)');
ylabel('Bit Error Rate (BER)');
title('BER vs SNR for different M-PAM values');
legend show;
hold off;
```

6.2.2 Results

The simulation investigates the effect of modulation order MMM and signal-to-noise ratio (SNR) on the bit error rate (BER) for M-ary Pulse Amplitude Modulation (M-PAM). Three modulation levels were tested: 4-PAM, 16-PAM, and 64-PAM, over an SNR range from 0 to 20 dB. As expected, the BER decreases as SNR increases for all values of MMM, indicating improved reliability in higher-SNR environments. However, for a given SNR, higher-order modulations (e.g., 64-PAM) exhibit significantly higher BER compared to lower-order modulations (e.g., 4-PAM), due to the reduced distance between adjacent signal levels. This highlights the trade-off between data rate and error performance in digital modulation schemes.



The MATLAB code showed the same results of the Simulink Model, as the value of M increases, the BER decreases at a slower rate with the increasing of Eb/N0, which shows a higher probability of error per symbol.

10. Conclusion:

In this experiment, we explored signal modulation using the Pulse amplitude modulation. This process involves segmenting input signals into blocks and normalizing the resulting signal. Notably, using more levels (M levels) and achieving higher Signal-to-Noise Ratio (SNR) ratios can significantly impact communication quality, this can be implemented using a matched filter.

A matched filter is a signal processing technique that optimally detects signals in the presence of noise. It's designed to maximize the Signal-to-Noise Ratio (SNR), improving signal detection accuracy. By aligning the filter's response with the shape of the expected signal, it enhances the ability to extract weak signals from noisy backgrounds, this will greatly enhance the channel output resulting in far fewer BERs for the same M-level Modulator used.

11. References:

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3. *Sakib, Meer & Liboiron-Ladouceur, Odile. (2013). A Study of Error Correction Codes for PAM Signals in Data Center Applications. Photonics Technology Letters, IEEE. 25. 2274-2277. 10.1109/LPT.2013.2286105.*
4. Bit Error Rate and Signal to Noise Ratio Performance Evaluation of OFDM System with QPSK and QAM M-array Modulation Scheme in Rayleigh, Rician and AWGN Channel Using MATLAB/Simulink

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