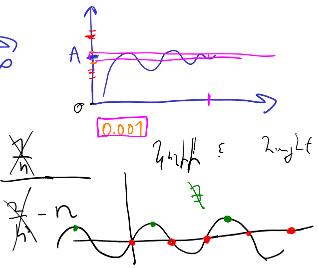
1. Find if exists the limit of the sequence as $n \to \infty$

1. $\frac{1}{n^2}$
2. $\frac{n^2}{2 - n^3}$
3. $(0.99)^n$
4. $(1.01)^n$
5. $\sin(\pi n)$

Romantic interpretation of 3 and 4

$\frac{1}{n^2}$ $n \to \infty$

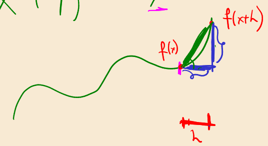
$\frac{\frac{n^2}{n^3}}{\frac{2-n^3}{n^3}} = \frac{\frac{1}{n}}{\frac{2}{n^3} - 1}$



2. Derivatives

Calculate $f'(x)$ 1. $x^2 + 4$ 2. $3x^4 - \frac{1}{x}$ 3. $5\sin^2(x)$ 4. xe^x

$(x^2+4)' = 2x$



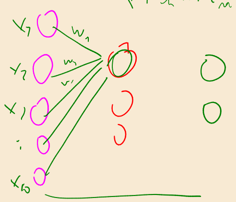
$\frac{f(x+h) - f(x)}{x+h-x}$ $h \to 0$

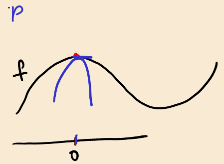
$+\frac{1}{x}$ $-x^{-1} \to +1 \cdot x^{-1-1} = +\frac{1}{x^2}$

xe^x

$x' \cdot e^x + x(e^x)'$

$\sigma(x_1 w_1 + x_2 w_2 + \dots + x_{10} w_{10})$





$f \approx P$

$f(0) = P(0)$

$f'(0) = P'(0)$

$f''(0) = P''(0)$

$f(x) = P(a) + P'(a)(x-a) + P''(a)\frac{(x-a)^2}{2!} + \dots + P^{(n)}(a) \cdot \frac{(x-a)^n}{n!}$

$P(x) = k_0 x^0 + k_1 x^1 + k_2 x^2 + k_3 x^3 + \dots$

$f(0) = P(0) = k_0 + k_1 \cdot 0 + \dots$

$k_0 = f(0)$

$f'(0) = P'(0) = 0 + 1 \cdot k_1 + 1 \cdot k_2 \cdot 2x + k_3 \cdot 3x^2 \dots$

$1 \cdot k_1 = f'(0) \Rightarrow k_1 = \frac{f'(0)}{1}$

$f''(0) = P''(0) = 0 + 0 + 2k_2 + \cancel{x \dots}$

$1 \cdot 2 k_2 = f''(0) \Rightarrow k_2 = \frac{f''(0)}{2!} \to \dots$

$\cos x$

$-\sin x$

$-\cos x$

$\sin x$

$\cos x$

$-\sin x$

e^x $x^T x$ ∇

Let $f : [-1, 2] \to \mathbb{R}, x \mapsto \exp(x^3 - 2x^2)$

(a) Compute f'

(b) Plot f and f' with R

(c) Find all possible candidates x^* for maxima and minima.
Hint: exp is a strictly monotone function.

(d) Compute f''

(e) Determine if the candidates are local maxima, minima or neither.

(f) Find the global maximum and global minimum of f

$\left(e^{x^3 - 2x^2}\right)' = \boxed{e^{x^3-2x^2}(3x^2 - 4x)}$

$f(x) = e^x$

$g(x) = x^3 - 2x^2$

$f(g(x))$

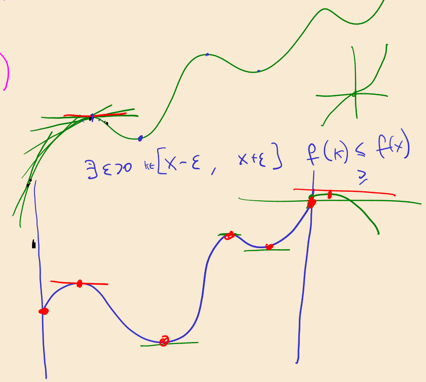
$f'(x) \stackrel{?}{=} 0$

$e^x \neq 0 \Rightarrow 3x^2 - 4x = 0$

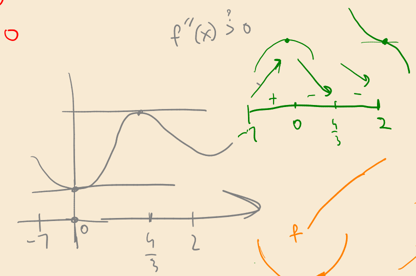
$x(3x-4) = 0$

$x = 0$, $x = \frac{4}{3}$

$\exists \varepsilon > 0 \; \forall x \in [x - \varepsilon, x + \varepsilon] \; f(x) \leq f(x)$

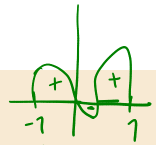


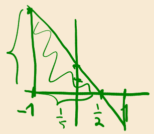
$f''(x) \stackrel{?}{>} 0$



What is the signed area between the curve $f(x)$ and the x-axis on the interval $[-1, 1]$?

1. $2-4x$
2. x^2+2
3. e^{-x}
4. $\sin(x)$





$(2x)'$ $\quad 2$

$-2x^2 \to -2x$ $\quad -4x$

$\int 2-4x\,dx = 2x - \frac{4x^2}{2} + C$

$2x - 2x^2 \Big|_{-1}^{1} = 2\cdot 1 - 2\cdot 1^2 - 2\cdot(-1) - 2(-1)^2$

$\int_a^c = \int_a^b + \int_b^c$

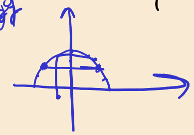
$\int x^2+2$ $\quad \frac{x^3}{3} + 2x + C$

e^{-x} $\quad \boxed{-e^{-x}}$ $\quad +e^{-x}$

$\sin x$ $\quad -\cos x$

$2x$ $\quad x^2+c$

$-\pi, \pi$



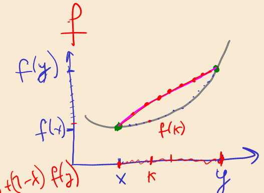
$f(-x) = f(x)$

$f(x) = -f(-x)$

$\boxed{8} \cdot \boxed{\sin x}$

$\int a \cdot f(x)$

$a \in \mathbb{R}$



$\lambda f(x)+(1-\lambda) f(y)$

$f(x) = \underline{k}x+b$

$\dfrac{f(y)-f(x)}{y-x}$

$x+t(y-x) = X \qquad x+\ (y-x)$

$t \in [0,1] \qquad x+\frac{1}{2}(y-x)$

$\underline{\lambda x+(1-\lambda) y} \qquad x+ty-tx = x(1-t)+yt$

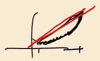
$\lambda \hat{=} 1-t$

$\forall \lambda \in [0,1] \quad \forall x,y \in D(f)$

$$\boxed{f(\lambda x+(1-\lambda) y) \leq \lambda f(x)+(1-\lambda) f(y)}$$

$f \quad f+g \quad f,g \quad y$

$g \qquad f(g(x))$



$(f+g)(x) \qquad h = f+g$

$h(x)$

$\boxed{h(\lambda x+(1-\lambda) y)} = f(\lambda x+(1-\lambda) y)+g(\lambda x+(1-\lambda) y)$

$\lambda f(x)+\lambda g(x)+(1-\lambda) f(y)+(1-\lambda) g(y)$

$\lambda f(x)+(1-\lambda) f(y)+\lambda g(x)+(1-\lambda) g(y) \ (\leq) \geq$

$f(\lambda x+(1-\lambda) y) \quad + \quad g(\cdots\cdots)$