Practice 3: Calculus

1. Find if exists the limit of the sequence as $n \to \infty$

1. $\frac{1}{n^2}$
2. $\frac{n^2}{2-n^3}$
3. $(0.99)^n$
4. $(1.01)^n$
5. $\sin(\pi n)$

Romantic interpretation of 3 and 4

2. Derivatives

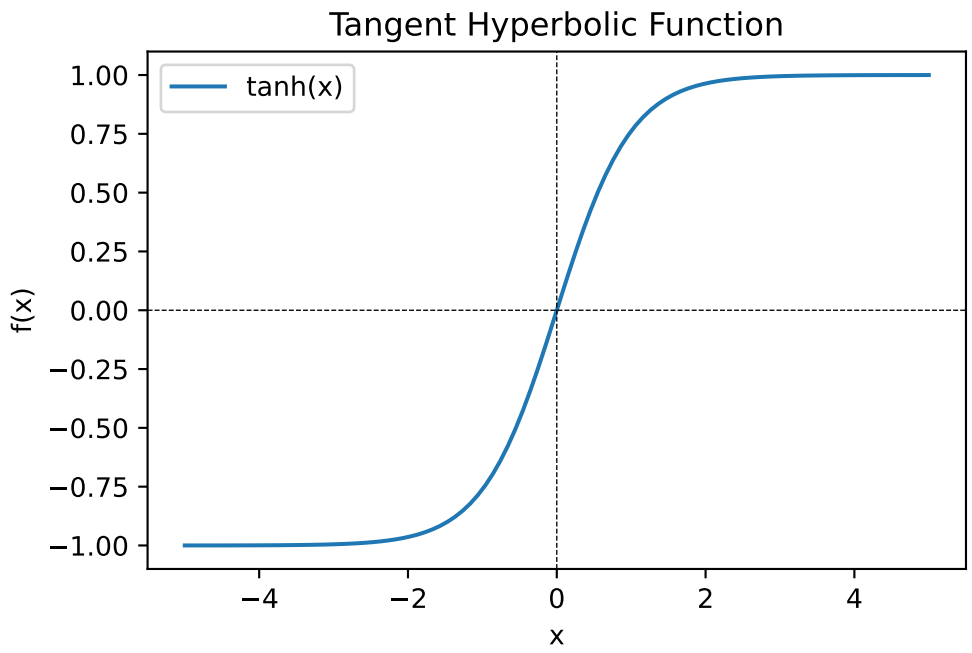
Calculate $f'(x)$

1. $x^2 + 4$
2. $3x^4 - \frac{1}{x}$
3. $5\sin^2(x)$
4. xe^x

3. TanH derivative

Calculate the derivative of tangent hyperbolic function $f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

Here is the plot:



4. Minima, Maxima

Let $f : [-1, 2] \to \mathbb{R},\ x \mapsto \exp(x^3 - 2x^2)$

(a) Compute f'

(b) Plot f and f' with R

(c) Find all possible candidates x^* for maxima and minima. *Hint:* exp is a strictly monotone function.

(d) Compute f''

(e) Determine if the candidates are local maxima, minima, or neither.

(f) Find the global maximum and global minimum of f.

5. Taylor Series

Find the Taylor polynomial of the function $f(x) = \sin(x)$ around $x = 0$.

6. Signed area

What is the signed area between the curve $f(x) = x^2$ and the x-axis on the interval $[-1, 1]$?

1. $2 - 4x$
2. $x^2 + 2$
3. e^{-x}
4. $\sin(x)$

7. Convexity

Consider two convex functions $f, g : \mathbb{R} \to \mathbb{R}$.

(a) Show that $f + g : \mathbb{R} \to \mathbb{R}$, $x \mapsto f(x) + g(x)$ is convex.

(b) Now, assume that g is additionally non-decreasing, i.e. $g(y) \geq g(x)$ for all $x \in \mathbb{R}$ and all $y \in \mathbb{R}$ with $y > x$. Show that $g \circ f$ is convex.