Determinants

- Prove that $\det(B^{-1}AB) = \det(A)$ if B is invertible.
- Suppose Q is a 3×3 real matrix such that $Q^T Q = I$. What values can $\det(Q)$ take?.

$\det(B^{-1}AB) = \det(A)$

$A, B \in M^n$

$\exists B^{-1}$

$\det(B^{-1}) = \dfrac{1}{\det(B)}$

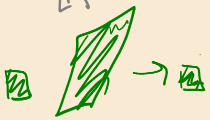
$n \times k$ $\quad A \in M_{n\times k}$ $\quad \mathbb{R}^{n \times k}$

$|B^{-1}|\,|A|\,|B| = |A|$

1

$|A| = 1.5$

$|A^{-1}| = \dfrac{1}{1.5}$



$|A \cdot B| = |A| \cdot |B|$

$3 \quad 2 \quad 6$

$Q \in \mathbb{R}^{n \times n}$

$\boxed{Q^T Q = I} \Rightarrow |Q^T Q| = |I| = 1$

$\boxed{|Q^T| = |Q|} = x$

$x^2 = 1 \Rightarrow \begin{cases} x = 1 \\ x = -1 \end{cases}$

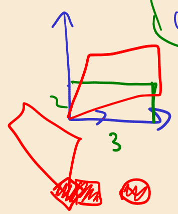
$|Q|$

$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$Q^T = Q^{-1}$

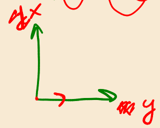
orthogonal

$Q^T Q$



$\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

$\begin{vmatrix} 3 & 1 \\ 8 & 2 \end{vmatrix} \neq \begin{vmatrix} 3 & 8 \\ 1 & 2 \end{vmatrix}$



Linear dependence, basis, rank

We have the following dataset of cheeses

Cheese	Price in USD	Price in AMD	Year Made	Years Waited	Year Eaten
v_1	300	150,000	2013	5	2018
v_2	450	225,000	2010	9	2019
v_3	350	175,000	2014	3	2017
v_4	400	200,000	2013	5	2018
v_5	280	140,000	2011	6	2017

Year Made + Years Waited = Year Eaten

$X_2 = cX_1$

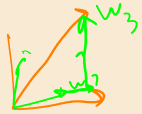
Tasks

Determine the maximum possible dimension of the subspace spanned by $v_1, \dots, v_5$.

Identify a maximal linearly independent subset (a basis) of these vectors.

Which features would you use in a Machine Learning and why? If not all, why?

(w_1, w_2, w_3)



$W_3 = c_1 W_1 + c_2 W_2$

$c_1, c_2 \in \mathbb{R}$

$\sum c_i^2 \neq 0$

$\langle v_1, v_2 \dots v_5 \rangle = \{c_1 v_1 + c_2 v_2 + \dots + c_5 v_5 \mid \forall c_1, c_2 \dots c_5 \in \mathbb{R}\}$

5x3

$y = Gx$

3x4

3x3

$X \to m \times n$

$X^T \to n \times m$

$X^T X \to n \times n$

X y Normal Eqn

MSE

$(X^T X)^{-1} X^T y$

non singular

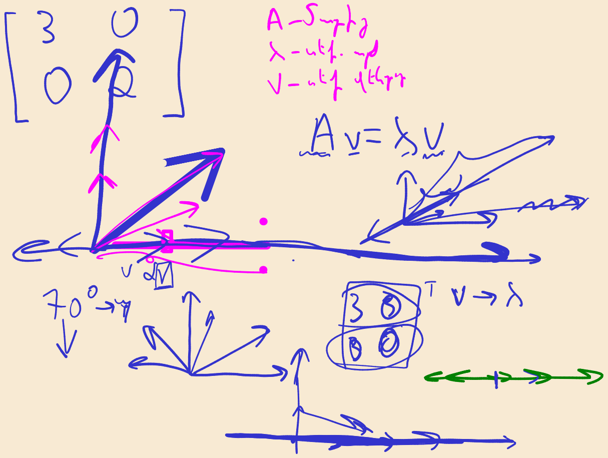
Full rank

$|X| = 0$

$|X^T| = 0$

$|X^T X| = 0 \Rightarrow \exists$

$$A = \begin{pmatrix} 5 & 8 \\ 2 & 5 \end{pmatrix}$$



$$Av = \lambda_1 v$$

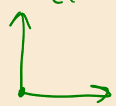
$$Av - \lambda I v = \Theta$$

$$|(A - \lambda I)v| = |\Theta|$$

$$|A - \lambda I| = 0 \; ?$$

f ... charact. polyn.

$$Av = 3v$$



A^n

λ_1, λ_2

v_1, v_2

v_{11} v_{21}
v_{12} v_{22}

$$A\begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = \lambda_1 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

$$A\begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = \lambda_2 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

$$A\begin{bmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{bmatrix} = \begin{bmatrix} \lambda_1 v_{11} & \lambda_2 v_{21} \\ \lambda_1 v_{12} & \lambda_2 v_{22} \end{bmatrix} = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

P $\quad \lambda \quad \Lambda$

3×4

$$\begin{bmatrix} \lambda_1^{40} & 0 \\ 0 & \lambda_2^{40} \end{bmatrix}$$

$$AP = P\Lambda$$

$$APP^{-1} = \boxed{P\Lambda P^{-1}}$$

$$A^n = P\Lambda \cancel{P^{-1}} \cancel{P}\Lambda \cancel{P^{-1}} \cancel{P}\Lambda P^{-1}$$

$$A^n = P\Lambda^n P^{-1}$$

1. Vector
2. Norm
3. Dot product
4. Cosine similarity
5. Matrix geometrical interpretation
6. Linear dependence/independence
7. Basis
8. Rank
9. Inverse
10. Determinant
11. Eigenvector, eigenvalue
12. Subspace, Linear space