# Vectors

Hayk Aprikyan, Hayk Tarkhanyan

- Communication:
  - For questions, memes, announcements, homeworks: Slack
  - For lecture slides and books: Google Drive, GitHub
- Main books (in English):
  - Poole, "Linear Algebra: A Modern Introduction"
  - Johnston, "Introduction to Linear and Matrix Algebra"
  - Stewart, "Calculus"
  - Blitzstein, Hwang, "Introduction to Probability"
  - Grimmett, Welsh, "Probability: An Introduction"
- Supplementary books (in Armenian):
  - Ohanyan, "Probability Theory", Lectures
  - Gevorgyan, Sahakyan, "Algebra and Elements of Mathematical Analysis 11, 12"
  - Musoyan, "Mathematical Analysis", parts 1-2
  - Movsisyan, "Higher Algebra and Number Theory"

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- Or by taking the two columns of the table: $\begin{bmatrix} 10 \\ 20 \\ 50 \\ 100 \\ 200 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

# Vectors

## Definition

An ordered set of $n$ real numbers is called a **vector** (or **column vector**) in $\mathbb{R}^n$:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

where $v_1, v_2, \ldots, v_n$ are the **components** of the vector.
A vector written horizontally is called a **row vector**:

$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}$$

We will denote $\mathbf{v} \in \mathbb{R}^n$ to indicate that $\mathbf{v}$ is a vector in $\mathbb{R}^n$.

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Vectors in $\mathbb{R}^1$

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Vectors in $\mathbb{R}^1$ are real numbers: $[v] \in \mathbb{R}$.

# Examples of Vectors in $\mathbb{R}^n$

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad \text{(3-dimensional column vector)}$$

$$\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{(4-dimensional column vector)}$$

$$\mathbf{v}_3 = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \quad \text{(2-dimensional column vector)}$$

$$\mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(Zero vector in 3-dimensional space)}$$

$$\mathbf{v}_5 = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \quad \text{(3-dimensional row vector)}$$

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We would have the following coins:

$$\mathbf{b} + \mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 1+0 \\ 2+0 \\ 0+3 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

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To add two vectors $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ in $\mathbb{R}^n$, add their corresponding components:

$$\mathbf{v} + \mathbf{u} = \begin{bmatrix} v_1 + u_1 \\ v_2 + u_2 \\ \vdots \\ v_n + u_n \end{bmatrix}$$

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Note that we can only add two vectors if they are of the same length!

# Multiplication of vector by scalar

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What if the money in our pockets doubled? We would have:

$$2 \cdot \mathbf{b} = 2 \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 0 \\ 2 \end{bmatrix}$$

from each coin.

# Multiplication of vector by scalar

## Definition

To multiply a vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ by a scalar $c$ in $\mathbb{R}^n$, multiply each component of the vector by the scalar:

$$c \cdot \mathbf{v} = \begin{bmatrix} c \cdot v_1 \\ c \cdot v_2 \\ \vdots \\ c \cdot v_n \end{bmatrix}$$

# Properties of Vectors

## Associativity and Commutativity of Vector Addition

For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, the vector addition is commutative and associative:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$$

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## Associativity and Commutativity of Scalar Multiplication

For any scalar $c$ and vectors $\mathbf{v}$ and $\mathbf{u}$ in $\mathbb{R}^n$, scalar multiplication is associative and commutative:

$$c \cdot (\mathbf{v} + \mathbf{u}) = c \cdot \mathbf{v} + c \cdot \mathbf{u}$$

$$(c \cdot d) \cdot \mathbf{v} = c \cdot (d \cdot \mathbf{v})$$

# Vector Subtraction

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## Definition

For a vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$, the **negative** of $\mathbf{v}$, denoted as $-\mathbf{v}$, is obtained by negating each component:

$$-\mathbf{v} = \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_n \end{bmatrix}$$

# Vector Subtraction

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The subtraction of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$ is defined as the sum of $\mathbf{u}$ and the negative of $\mathbf{v}$:

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \\ \vdots \\ u_n - v_n \end{bmatrix}$$

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## Example

$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 2-1 \\ -1-4 \\ 3-0 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$$

# Vector Transposition

## Definition

For a column vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$, the **transpose**, denoted as $\mathbf{v}^T$, is a row vector:

$$\mathbf{v}^T = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$$

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For a row vector $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}$ in $\mathbb{R}^n$, the **transpose**, denoted as $\mathbf{u}^T$, is a column vector:

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## Transpose Properties

- For any vector $\mathbf{v}$ in $\mathbb{R}^n$, $(\mathbf{v}^T)^T = \mathbf{v}$
- For any scalar $c$, $(c \cdot \mathbf{v})^T = c \cdot \mathbf{v}^T$

# Dot Product of Vectors

In our example we had $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ coins of $\mathbf{a} = \begin{bmatrix} 10 \\ 20 \\ 50 \\ 100 \\ 200 \end{bmatrix}$ nominations (values) respectively.

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How much money do we have in total?

**Definition**

The **dot product** of two vectors $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$ is:

$$\mathbf{u} \cdot \mathbf{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + \cdots + u_n \cdot v_n$$

# Dot Product of Vectors

## Example

If $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$, then:

$$\mathbf{u} \cdot \mathbf{v} = (2 \cdot 1) + (-1 \cdot 4) + (3 \cdot 0) = 2 - 4 + 0 = -2$$

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Going back to our example, we can calculate our money with the dot product of **a** and **b**:

$$\mathbf{a} \cdot \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 20 \\ 50 \\ 100 \\ 200 \end{bmatrix} = 2 \cdot 10 + 1 \cdot 20 + 2 \cdot 50 + 0 \cdot 100 + 1 \cdot 200 = 340$$

# Dot Product of Vectors

**Remark 1**

The dot product of two vectors is defined if and only if the vectors have the same number of components (i.e. are of the same length).

**Remark 2**

The dot product of two vectors is a *number* (scalar), not a vector.

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The dot product of two vectors is defined if and only if the vectors have the same number of components (i.e. are of the same length).

## Remark 2

The dot product of two vectors is a *number* (scalar), not a vector.

This is why the dot product is often called **scalar product**.

# Properties of Dot Product

## Properties

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^n$, and let $c$ be a scalar. The dot product has the following properties:

1. Commutativity:
$$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$
2. Distributivity over Vector Addition:
$$(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$$
3. Scalar Multiplication:
$$(c \cdot \mathbf{u}) \cdot \mathbf{v} = c \cdot (\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (c \cdot \mathbf{v})$$
4. Non-negativity:
$$\mathbf{u} \cdot \mathbf{u} \geq 0 \text{ and } \mathbf{u} \cdot \mathbf{u} = 0 \text{ if and only if } \mathbf{u} = \mathbf{0}$$

# Examples

Consider vectors $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$.

Let's calculate $(5\mathbf{u} - \mathbf{v}) \cdot \mathbf{w}$:

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Let's calculate $(5\mathbf{u} - \mathbf{v}) \cdot \mathbf{w}$:

$$
\begin{aligned}
(5\mathbf{u} - \mathbf{v}) \cdot \mathbf{w} &= \left( 5 \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} \right) \cdot \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \\
&= \left( \begin{bmatrix} 5 \\ -10 \\ 15 \end{bmatrix} - \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} \right) \cdot \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 5 \\ -14 \\ 16 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = 5 \cdot (-2) + (-14) \cdot 1 + 16 \cdot 2 = 8
\end{aligned}
$$

# Geometric interpretation of vectors

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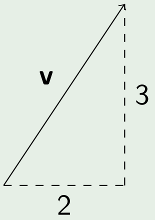
## Geometric Interpretation

- In addition to their algebraic representation, vectors have a geometric interpretation.
- We can think of a vector $\mathbf{v}$ as a point in the 2d space,
- Or we can imagine it as an arrow in space, starting from the origin $(O(0,0))$ and pointing to the mentioned point.
- The components of $\mathbf{v}$ are the **coordinates** of the point in the plane.

## Example

- Consider the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.
- This vector points to the point (2, 3) in the plane.

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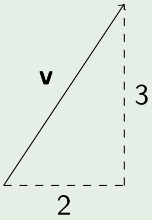
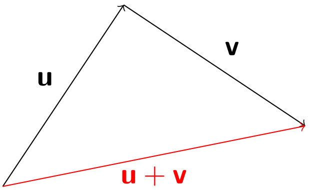



In general, the vector with coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$ is represented by the point with coordinates $(x, y)$.
What do you think happens in the 3d space?

# Addition of vectors

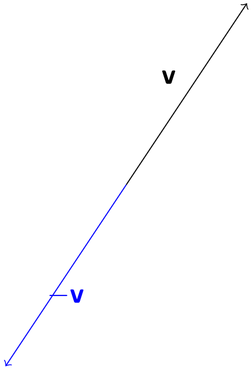
Let's interpret some of our vector operations geometrically.

- **Addition:** To add vectors $\mathbf{u}$ and $\mathbf{v}$, place the tail of $\mathbf{v}$ at the head of $\mathbf{u}$. The sum $\mathbf{u}+\mathbf{v}$ is the vector pointing from the tail of $\mathbf{u}$ to the head of $\mathbf{v}$.



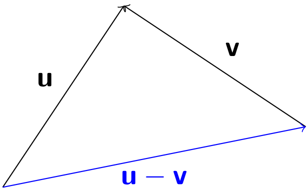
# Negative of vectors

- **Negation:** The negative of a vector $\mathbf{v}$, denoted $-\mathbf{v}$, is a vector with the same magnitude but opposite direction.



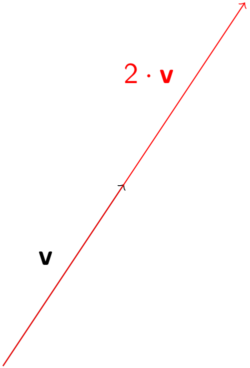
## Subtraction of vectors

- **Subtraction:** To subtract $\mathbf{v}$ from $\mathbf{u}$, place the tail of $\mathbf{v}$ at the head of $\mathbf{u}$. The result $\mathbf{u} - \mathbf{v}$ is the vector pointing from the head of $\mathbf{v}$ to the head of $\mathbf{u}$.



# Multiplication by scalar

- **Scalar Multiplication:** Scaling a vector $\mathbf{v}$ by a scalar $c$ stretches or compresses the vector. The result $c \cdot \mathbf{v}$ has the same direction as $\mathbf{v}$ but a different magnitude.



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## Vector Operations

- $3\mathbf{a} + \mathbf{b} = 3 \cdot [3, 2] + [2, 0]$
- $3\mathbf{a} + \mathbf{b} = [9, 6] + [2, 0]$
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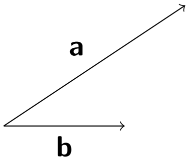
How can we interpret it geometrically?

# Example



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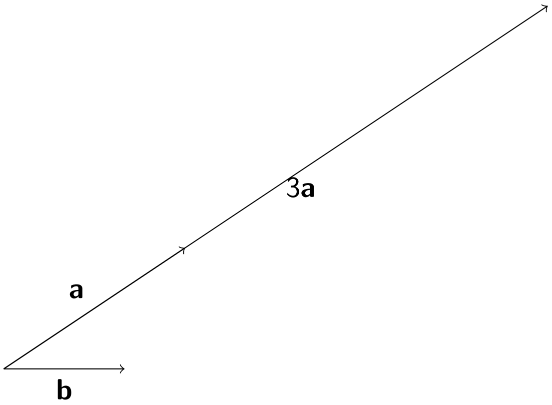

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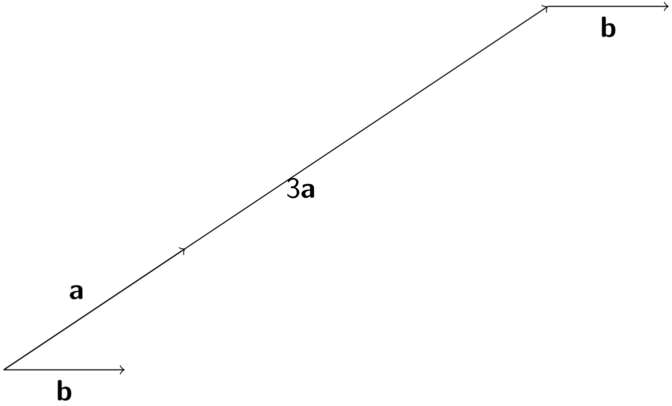

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