Conv. Men.

**Problem 3.** Find the gradient and the Hessian of the following functions:

a) $f(x, y) = 6x - y^2$,

b) $f(x, y) = x^2y^2 - 4xy + 1$,

c) $f(x, y) = e^{\pi x} - \sin(\pi y) - \pi xy$

$$\nabla f(x,y) = \begin{bmatrix} f_x' \\ f_y' \end{bmatrix} = \begin{bmatrix} 6 \\ -2y \end{bmatrix}$$

$$y = kx + b$$

$$-\nabla f(x,y)$$

Gradient

$$\theta^{new} = \theta^{old} - \alpha \nabla f$$

$$\begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

**Problem 5.** Does the following function have local extrema? If so, find them:

a) $f(x, y) = 3xy$

b) $f(x, y) = x^2 - xy$

c) $f(x, y) = 2x^2 - x^3 - y^2$

You can plot the graph or use the $D$ on the last slide.

$$\nabla f(x,y) = \begin{bmatrix} 2x - y \\ -x \end{bmatrix}$$

$$\nabla^2 f(x,y) = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix}$$

For the functions of one variable we looked at the sign of $f''$.
In case of two variables, we look at:

- $f_{xx}$
- and $D = f_{xx}f_{yy} - f_{xy}^2$

$$\begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \geq 0$$

$f'' > 0$

**Theorem**

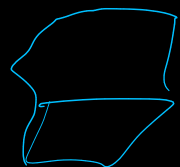
If at some point $(a, b)$

- $D > 0$ and $f_{xx} > 0$ $\Rightarrow$ local minimum
- $D > 0$ and $f_{xx} < 0$ $\Rightarrow$ local maximum
- $D < 0$ $\Rightarrow$ saddle point

$vAv^T \geq 0$

$ax^2 > 0$

$a > 0$



$$\nabla f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\|v\| = 1$

**Problem 4.** Consider the bivariate function $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x^2 + 0.5y^2 + xy$.

a) Find the direction of greatest increase of $f$ at $(x, y) = (1, 1)$.

b) Find the direction of greatest decrease of $f$ at $(x, y) = (1, 1)$.

c) Calculate the directional derivative at the point $(x, y) = (1, 1)$ along the vector $\mathbf{v} = [0.6, 0.8]^T$

d) Find a direction in which $f$ does not instantly change at $(x, y) = (1, 1)$.

$$f_x \cdot v_1 + f_y \cdot v_2 = 0$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x^2 + \frac{1}{2}y^2 + xy$$

$$\nabla f(x,y) = \begin{bmatrix} 2x + y \\ y + x \end{bmatrix} \quad -\nabla f(1,1) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

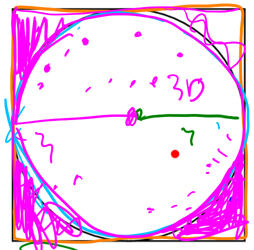
$3 \cdot 0.6$

$2 \cdot 0.8$

$\|v\| = 1$

**Problem 5.** What is the probability that a randomly generated point withing the square will lay in the circle:

$$\frac{S_\circ}{S_\square} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$



$S_\square = 4r^2$

$S_\circ = \pi r^2$

$\pi$ 80%

$2r$

$$\frac{S_\circ}{S_\square} = \pi$$

**Problem 6.** Bertrand's paradox

**Problem 7.** A fair coin is tossed 3 times. What is the probability of getting at least one tail?

**Problem 8.** An urn contains 3 red balls and 5 blue balls. Two balls are drawn at random without replacement. What is the probability that both balls are red?

**Problem 9.** A standard deck of 52 playing cards is shuffled. What is the probability that the top card is a heart or a queen?

$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}$

$1-\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{7}{8}$

$\frac{3}{3+5}\cdot\frac{2}{7}=\frac{3}{8}\cdot\frac{2}{7}$

$C_8^2$

$A_8^2$

$\frac{8\cdot 7}{2}$

$3\cdot 2$

$C_3^2$

$\frac{\#succ}{\#total}=\frac{C_3^2}{C_8^2}=\frac{\frac{3\cdot 2}{2}}{\frac{8\cdot 7}{2}}$

52

$\heartsuit$ 4 or K

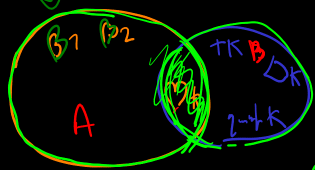
$\frac{1}{4}$ $\quad \frac{4}{52} = \frac{1}{13}$

$\frac{1}{4} + \frac{1}{13} - \frac{1}{52}$

$\heartsuit$ K

Inclusion

$P(A) + P(B) - P(A \cap B)$




inclusion ex

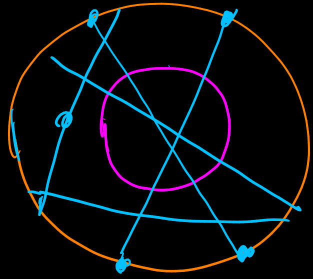
$A \cup B = A + B - A \cap B$

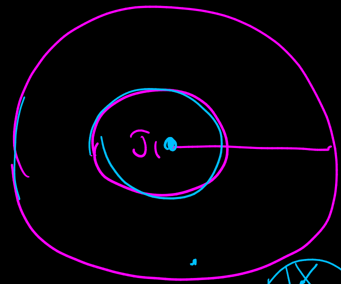
$A + B + C - A \cap B - A \cap C - B \cap C + A \cap B \cap C$

$\heartsuit$ K

B1 B2 B3 ... BK | +B

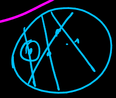
~~BK~~



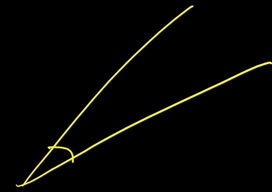



$2r$

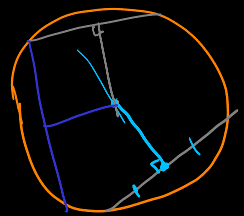
$\pi (2r)^2$ $\quad 4$

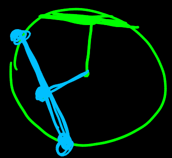


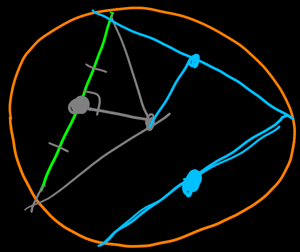
$$\frac{\pi r^2}{\pi (2r)^2} = \frac{1}{4}$$



$\frac{1}{3}$





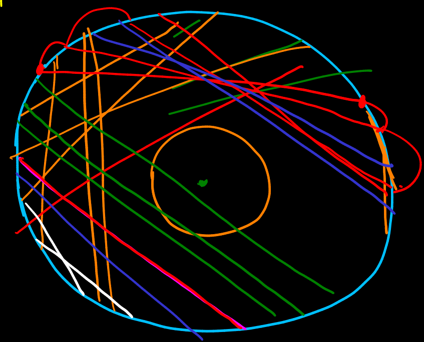


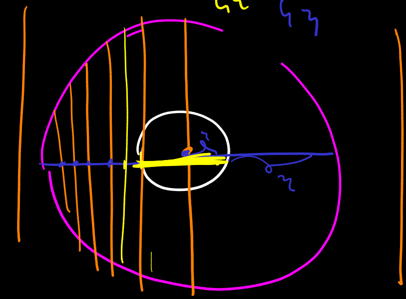
$\Omega \quad P(A) \to [0,1]$

$y =$

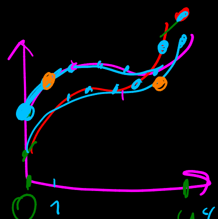
$\frac{2r}{4r} = \frac{1}{2}$

$4r$





$$\int f(x)\cdot g(y-x)\,dx$$




$f(0)\cdot g(6) +$

$+ f(1)\cdot g(5) +$

$+ f( g( \ldots$

$x, y \quad 7 \quad x \to f \quad y \to g$

| x | y |
|---|---|
| 1 | 6 |
| 2 | 5 |
| 3 | 4 |
| 4 | 3 |

$f(1)\cdot g(6) + f(2)\cdot g(5) +$

$+ f(3)\cdot g(4) \ldots$