Vectors

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- Communication:
  - For questions, memes, announcements, homeworks: Slack
  - For lecture slides and books: Google Drive, GitHub

- Main books (in English):
  - Deisenroth, "Mathematics for Machine Learning"
  - Poole, "Linear Algebra: A Modern Introduction"
  - Strang, "Introduction to Linear Algebra"
  - Mikaelian, "Linear Algebra: Theorems and Algorithms"
  - Grimmett, Welsh, "Probability: An Introduction"
  - Stewart, "Calculus"
- Supplementary books (in Armenian):
  - Ohanyan, "Probability Theory", Lectures
  - Gevorgyan, Sahakyan, "Algebra and Elements of Mathematical Analysis 11, 12"
  - Musoyan, "Mathematical Analysis", parts 1-2
  - Movsisyan, "Higher Algebra and Number Theory"

Vectors

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20	1
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100	0
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Coins	**Quantity**
10	2
20	1
50	2
100	0
200	1

- Or by taking the two columns of the table: $\begin{bmatrix} 10 \\ 20 \\ 50 \\ 100 \\ 200 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

Vectors

Definition

An ordered set of n real numbers is called a **vector** (or **column vector**) in $\mathbb{R}^n$:

$$\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

where $v_1, v_2, \ldots, v_n$ are the **components** of the vector.
A vector written horizontally is called a **row vector**:

$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \ldots & v_n \end{bmatrix}$$

We will denote $\mathbf{v} \in \mathbb{R}^n$ to indicate that v is a vector in $\mathbb{R}^n$.

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$$\mathbf{v} = \begin{bmatrix} v_1 & v_2 & \dots & v_n \end{bmatrix}$$

We will denote $\mathbf{v} \in \mathbb{R}^n$ to indicate that v is a vector in $\mathbb{R}^n$.
Vectors in $\mathbb{R}^1$ are real numbers: $[v] \in \mathbb{R}$.

Examples of Vectors in $\mathbb{R}^n$

$$\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad \text{(3-dimensional column vector)}$$

$$\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{(4-dimensional column vector)}$$

$$\mathbf{v}_3 = \begin{bmatrix} -3 \\ 2 \end{bmatrix} \quad \text{(2-dimensional column vector)}$$

$$\mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(Zero vector in 3-dimensional space)}$$

$$\mathbf{v}_5 = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \quad \text{(3-dimensional row vector)}$$

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We would have the following coins:

$$\mathbf{b} + \mathbf{c} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 1+0 \\ 2+0 \\ 0+3 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

Addition of vectors

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To add two vectors $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ in $\mathbb{R}^n$, add their corresponding components:

$$\mathbf{v} + \mathbf{u} = \begin{bmatrix} v_1 + u_1 \\ v_2 + u_2 \\ \vdots \\ v_n + u_n \end{bmatrix}$$

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$$\mathbf{v} + \mathbf{u} = \begin{bmatrix} v_1 + u_1 \\ v_2 + u_2 \\ \vdots \\ v_n + u_n \end{bmatrix}$$

Note that we can only add two vectors if they are of the same length!

Multiplication of vector by scalar

What if the money in our pockets doubled?

Multiplication of vector by scalar

What if the money in our pockets doubled? We would have:

$$2 \cdot \mathbf{b} = 2 \cdot \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 0 \\ 2 \end{bmatrix}$$

from each coin.

Multiplication of vector by scalar

Definition

To multiply a vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ by a scalar c in $\mathbb{R}^n$, multiply each component of the vector by the scalar:

$$c \cdot \mathbf{v} = \begin{bmatrix} c \cdot v_1 \\ c \cdot v_2 \\ \vdots \\ c \cdot v_n \end{bmatrix}$$

Properties of Vectors

Associativity and Commutativity of Vector Addition

For any vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, the vector addition is commutative and associative:

$$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$$

$$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$$

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Associativity and Commutativity of Scalar Multiplication

For any scalar c and vectors $\mathbf{v}$ and $\mathbf{u}$ in $\mathbb{R}^n$, scalar multiplication is associative and commutative:

$$c \cdot (\mathbf{v} + \mathbf{u}) = c \cdot \mathbf{v} + c \cdot \mathbf{u}$$

$$(c \cdot d) \cdot \mathbf{v} = c \cdot (d \cdot \mathbf{v})$$

Vector Subtraction

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Definition

For a vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$, the **negative** of $\mathbf{v}$, denoted as $-\mathbf{v}$, is obtained by negating each component:

$$-\mathbf{v} = \begin{bmatrix} -v_1 \\ -v_2 \\ \vdots \\ -v_n \end{bmatrix}$$

Vector Subtraction

Vector Subtraction

The subtraction of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$ is defined as the sum of $\mathbf{u}$ and the negative of $\mathbf{v}$:

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = \begin{bmatrix} u_1 - v_1 \\ u_2 - v_2 \\ \vdots \\ u_n - v_n \end{bmatrix}$$

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Example

$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 2-1 \\ -1-4 \\ 3-0 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}$$

Vector Transposition

Definition

For a column vector $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$, the **transpose**, denoted as $\mathbf{v}^T$, is a row vector:

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For a row vector $\mathbf{u} = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}$ in $\mathbb{R}^n$, the **transpose**, denoted as $\mathbf{u}^T$, is a column vector:

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Transpose Properties

- For any vector $\mathbf{v}$ in $\mathbb{R}^n$, $(\mathbf{v}^T)^T = \mathbf{v}$
- For any scalar c, $(c \cdot \mathbf{v})^T = c \cdot \mathbf{v}^T$

Dot Product of Vectors

In our example we had $\mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ coins of $\mathbf{a} = \begin{bmatrix} 10 \\ 20 \\ 50 \\ 100 \\ 200 \end{bmatrix}$ nominations (values) respectively.

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How much money do we have in total?

Definition

The **dot product** of two vectors $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ in $\mathbb{R}^n$ is:

$$\mathbf{u} \cdot \mathbf{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + \cdots + u_n \cdot v_n$$

Dot Product of Vectors

Example

If $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$, then:

$$\mathbf{u} \cdot \mathbf{v} = (2 \cdot 1) + (-1 \cdot 4) + (3 \cdot 0) = 2 - 4 + 0 = -2$$

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Going back to our example, we can calculate our money with the dot product of **a** and **b**:

$$\mathbf{a} \cdot \mathbf{b} = \begin{bmatrix} 2 \\ 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 20 \\ 50 \\ 100 \\ 200 \end{bmatrix} = 2 \cdot 10 + 1 \cdot 20 + 2 \cdot 50 + 0 \cdot 100 + 1 \cdot 200 = 340$$

Dot Product of Vectors

Remark 1

The dot product of two vectors is defined if and only if the vectors have the same number of components (i.e. are of the same length).

Remark 2

The dot product of two vectors is a *number* (scalar), not a vector.

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The dot product of two vectors is defined if and only if the vectors have the same number of components (i.e. are of the same length).

Remark 2

The dot product of two vectors is a *number* (scalar), not a vector.

This is why the dot product is often called **scalar product**.

Properties of Dot Product

Properties

Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be vectors in $\mathbb{R}^n$, and let c be a scalar. The dot product has the following properties:

1. Commutativity:
$$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$
2. Distributivity over Vector Addition:
$$(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$$
3. Scalar Multiplication:
$$(c \cdot \mathbf{u}) \cdot \mathbf{v} = c \cdot (\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot (c \cdot \mathbf{v})$$
4. Non-negativity:
$$\mathbf{u} \cdot \mathbf{u} \geq 0 \text{ and } \mathbf{u} \cdot \mathbf{u} = 0 \text{ if and only if } \mathbf{u} = \mathbf{0}$$

Examples

Consider vectors $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$.

Let's calculate $(5\mathbf{u} - \mathbf{v}) \cdot \mathbf{w}$:

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Let's calculate $(5\mathbf{u} - \mathbf{v}) \cdot \mathbf{w}$:

$$\begin{aligned}
(5\mathbf{u} - \mathbf{v}) \cdot \mathbf{w} &= \left(5 \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}\right) \cdot \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \\
&= \left(\begin{bmatrix} 5 \\ -10 \\ 15 \end{bmatrix} - \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}\right) \cdot \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} \\
&= \begin{bmatrix} 5 \\ -14 \\ 16 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix} = 5 \cdot (-2) + (-14) \cdot 1 + 16 \cdot 2 = 8
\end{aligned}$$

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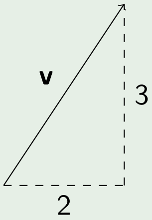
Geometric Interpretation

- In addition to their algebraic representation, vectors have a geometric interpretation.
- We can think of a vector $\mathbf{v}$ as a point in the 2d space,
- Or we can imagine it as an arrow in space, starting from the origin $(O(0,0))$ and pointing to the mentioned point.
- The components of $\mathbf{v}$ are the **coordinates** of the point in the plane.

Geometric interpretation of vectors

Example

- Consider the vector $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.
- This vector points to the point (2, 3) in the plane.



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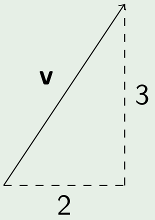
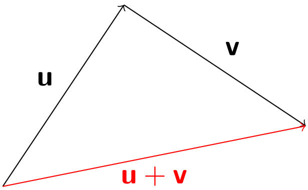


In general, the vector with coordinates $\begin{bmatrix} x \\ y \end{bmatrix}$ is represented by the point with coordinates (x, y).
What do you think happens in the 3d space?

Addition of vectors

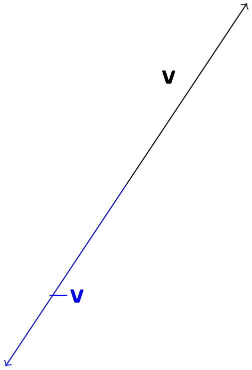
Let's interpret some of our vector operations geometrically.

- **Addition:** To add vectors $\mathbf{u}$ and $\mathbf{v}$, place the tail of $\mathbf{v}$ at the head of $\mathbf{u}$. The sum $\mathbf{u}+\mathbf{v}$ is the vector pointing from the tail of $\mathbf{u}$ to the head of $\mathbf{v}$.



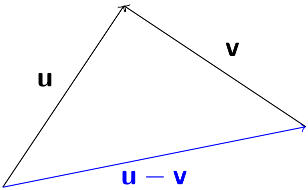
Negative of vectors

- **Negation:** The negative of a vector $\mathbf{v}$, denoted $-\mathbf{v}$, is a vector with the same magnitude but opposite direction.



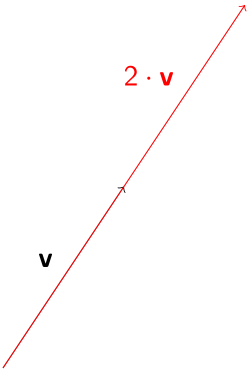
Subtraction of vectors

- **Subtraction:** To subtract $\mathbf{v}$ from $\mathbf{u}$, place the tail of $\mathbf{v}$ at the head of $\mathbf{u}$. The result $\mathbf{u} - \mathbf{v}$ is the vector pointing from the head of $\mathbf{v}$ to the head of $\mathbf{u}$.



Multiplication by scalar

- **Scalar Multiplication:** Scaling a vector $\mathbf{v}$ by a scalar c stretches or compresses the vector. The result $c \cdot \mathbf{v}$ has the same direction as $\mathbf{v}$ but a different magnitude.



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Vector Operations

- $3\mathbf{a} + \mathbf{b} = 3 \cdot [3, 2] + [2, 0]$
- $3\mathbf{a} + \mathbf{b} = [9, 6] + [2, 0]$
- $3\mathbf{a} + \mathbf{b} = [11, 6]$

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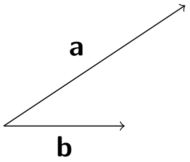
How can we interpret it geometrically?

Example



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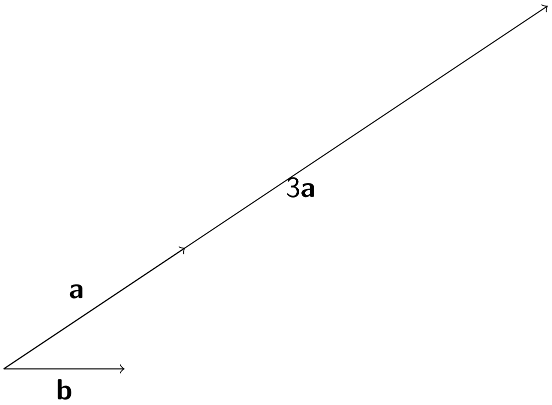

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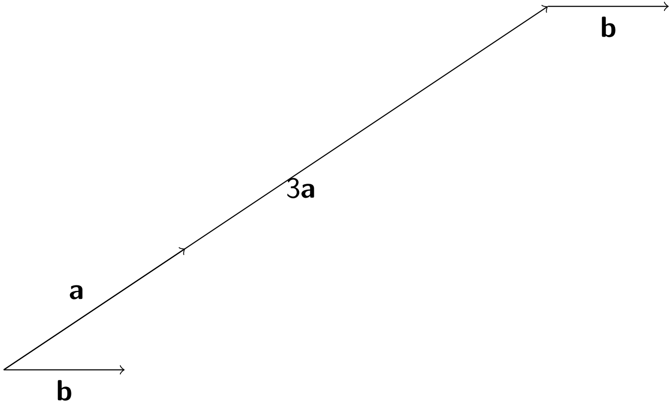

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