

Homework Problems 5 (Random Variables)

Problem 1. A spam filter tags emails as spam or not spam. Based on historical data:

- 80% of spam emails contain the word "lottery."
- 30% of non-spam emails contain the word "lottery."
- 40% of emails are spam.

What is the probability that an email containing the word "lottery" is spam?

Problem 2. There are two boxes containing $\{5, 11, 8\}$ and $\{10, 8, 6\}$ white, black, red pencils respectively. One pencil is drawn from each box. What is the probability that the pencils have the same color?

Problem 3. Three babies are given a weekly health check at a clinic, and then returned randomly to their mothers. What is the probability that at least one baby goes to the right mother?

Problem 4. Two factories produce similar weapons and deliver it to the army warehouse. The first factory's productivity is two times more than that of the second one. Moreover, 40% of the weapons produced by the first factory have some defects, while the same indicator is only 16% for the second factory. We randomly take a weapon from the warehouse, test it and it appears to have no defects. What is the probability that the weapon was produced by the first factory?

Problem 5. You ask your neighbor to water your flowers while you are on vacation. If the flowers are watered, they have about 0.85 chance of survival; otherwise, they will only survive with probability 0.2. You are 90 percent sure your neighbor will water the flowers, but when you are back, you see the flowers didn't survive. What is the probability your neighbor didn't water the flowers? Should you trust her anymore?

Problem 6. Nune uses her car 30% of the time, walks 30% of the time and rides the bus 40% of the time as she goes to work. She is late 10% of the time when she walks, 3% of the time when she drives, and 7% of the time she takes the bus.

- a) Yesterday she was late. What is the probability she took the bus?
- b) Today she was on time. Do you think she walked?

Problem 7. Rosie has ten coins. Nine of them are ordinary coins with equal chances of coming up Head and Tail when tossed, and the tenth has two Heads.

- a) If she takes one of the coins at random from her pocket, what is the probability that it is the coin with two Heads?
- b) If she tosses the coin and it comes up Heads, what is the probability that it is the coin with two Heads?
- c) If she tosses the coin one further time and it comes up Tails, what is the probability that it is one of the nine ordinary coins?

Problem 8 (additional). After a trip to Garni-Geghard, you bring your camera film to a photography shop. Unfortunately, the shop ruins 4 consecutive photos in a row from your roll of 24 photos of Garni. What is the probability that the ruined photos included the

- a) eighth or ninth or tenth photos,
- b) eighth and ninth and tenth photos

on the roll?

Problem 9. Anush and Nairi are shopping at the mall. They agree to split up for a time and then meet for lunch. They plan to meet in front of Kinopark between 12:00 and 13:00. The one who arrives first agrees to wait 15 minutes for the other to arrive. After 15 minutes, that person will leave and continue shopping. What is the probability that they will meet if each one of them arrives at any time between 12:00 and 13:00?

Hint: Try to represent the problem on the coordinate system, by letting x denote the time Anush arrives, and y , the time Nairi arrives.

Problem 10. Vahe added a dot on the $\begin{smallmatrix} \square & \square \end{smallmatrix}$ side of the die, making it $\begin{smallmatrix} \square & \square & \square \end{smallmatrix}$, and then added two dots on the $\begin{smallmatrix} \square & \square \end{smallmatrix}$ side, making it $\begin{smallmatrix} \square & \square & \square & \square \end{smallmatrix}$. What is the probability that the outcome of the die is greater than 4? Find the expectation and variance of the die.

Problem 11. The world famous gambler Vardanik from Parakar proposes the following game of chance. You roll a fair die. If you roll 1, Vardanik pays you \$25. If you roll 2, Vardanik pays you \$5. If you roll 3, you win nothing. If you roll 4 or 5, you must pay Vardanik \$10, and if you roll 6, you must pay Vardanik \$15. Do you want to play?

Problem 12. Let X be a random variable with the PDF:

$$f(x) = \begin{cases} 2x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the expectation and variance of

- a) X ,
- b) $2X$,
- c) $2X + 7$.

Problem 13 (additional). Let X and Y be two continuous random variables with uniform distribution on $(0, 2)$. Find the expectation of $X + Y$.

Problem 14 (additional). Let X be a random variable with PDF

$$f_X(x) = \begin{cases} ax^5, & \text{if } 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

where a is an unknown constant. Can you find the value of a ?