Practice 2: Matrices

Determinants

- Prove that $\det(B^{-1}AB) = \det(A)$ if B is invertible.
- Suppose Q is a 3×3 real matrix such that $Q^TQ = I$. What values can $\det(Q)$ take?.

Linear dependence, basis, rank

We have the following dataset of cheeses

Cheese	Price in USD	Price in AMD	Year Made	Years Waited	Year Eaten
v_1	300	150,000	2013	5	2018
v_2	450	225,000	2010	9	2019
v_3	350	175,000	2014	3	2017
v_4	400	200,000	2013	5	2018
v_5	280	140,000	2011	6	2017

Tasks

Determine the maximum possible dimension of the subspace spanned by $v_1, \dots, v_5$.

Identify a maximal linearly independent subset (a basis) of these vectors.

Which features would you use in a Machine Learning and why? If not all, why?

n-th power of a matrix (eigendecomposition)

Compute n-th power of the matrix

$$A = \begin{pmatrix} 5 & 8 \\ 2 & 5 \end{pmatrix}$$

Recap

1. Vector
2. Norm
3. Dot product
4. Cosine similarity
5. Matrix geometrical interpretation
6. Linear dependence/independence
7. Basis
8. Rank
9. Inverse
10. Determinant
11. Eigenvector, eigenvalue

Bbzb if time allows

- symmetric matrix eigenvectors orthogonal
- condition number

Notes: https://miro.com/app/board/uXjVIJAiBEM=/?share_link_id=18444345934