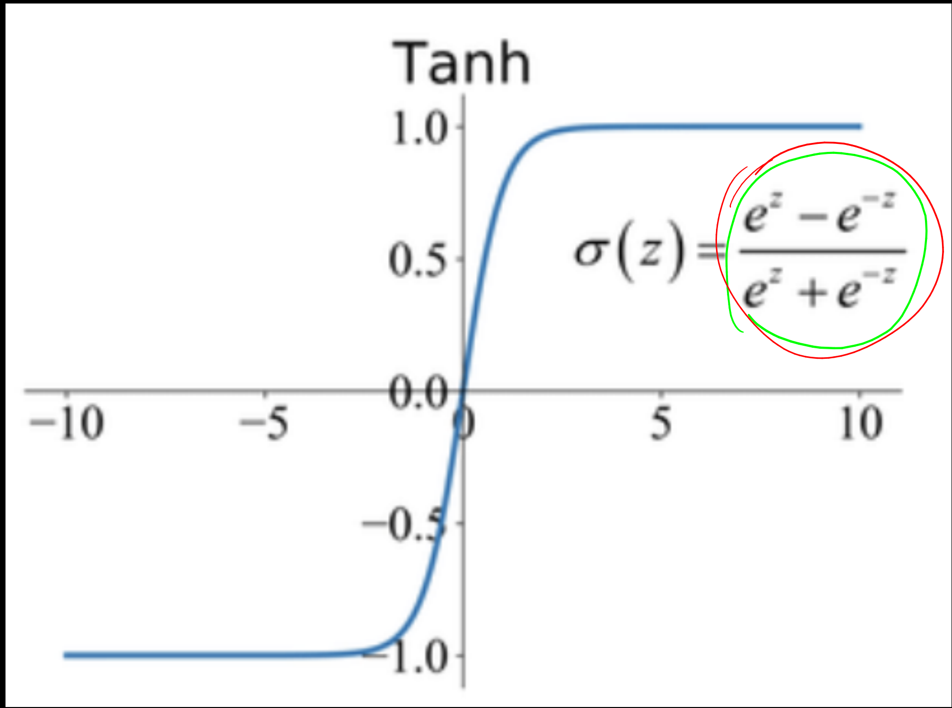


3. optim, rdus 0

1. Sigmoid, ReLU

2. Bo

$\frac{1}{1+e^{-z}}$

$x \to y$

ReLU

Sigm

tanh

$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$x^2 = 2x$

$\frac{2xh+h^2}{h} = 2x + h$

$\lim_{n\to\infty}\frac{1}{n^2}=0$

$\sin \pi l = 0$

$\sin \pi n$

$0, 0, 0, 0 \ldots \to 0$

$1\ 1\ 1\ 1 \to 1$

Problem 2. Find, if exists, the limit of the sequence, as $n \to \infty$:

a) $\frac{1}{n^2}$,

b) $\frac{n^2}{2-n^3}$,

c) $(0.99)^n$,

d) $(1.01)^n$,

e) $\sin \pi n$,

Which of the sequences above are convergent?

$1.01^{385} \approx 47$

$\forall \varepsilon > 0$

0.01

0.0001

$5+\frac{1}{n} \to 5$

$0.99^n \to 0$

$1.01^n \to$

$n^2 \to +\infty$

$1.01 \cdot 1.01 = 1.01(1+0.01) =$

$n \to \infty$

$\frac{n^2}{n} = \frac{\infty}{\infty}$

$\frac{0}{-1} = 0$

Let $f : [-1, 2] \to \mathbb{R}, x \mapsto \exp(x^3 - 2x^2)$

(a) Compute f'

(b) Plot f and f' with R

(c) Find all possible candidates x^* for maxima and minima.
Hint: exp is a strictly monotone function.

(d) Compute f''

(e) Determine if the candidates are local maxima, minima or neither.

(f) Find the global maximum and global minimum of f

$e^{x^3-2x^2}$

$e^{x^3-2x^2} \cdot (x^3 - 2x^2)' =$

$= e^{x^3-2x^2} \cdot (3x^2 - 4x)$

$3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0, \quad x = \frac{4}{3}$

$(f')' < 0$ $\quad >0 \quad f''(x) < 0$

$f''(x) > 0$

Consider two convex functions $f, g : \mathbb{R} \to \mathbb{R}$.

(a) Show that $f + g : \mathbb{R} \to \mathbb{R}, x \mapsto f(x) + g(x)$ is convex.

(b) Now, assume that g is additionally non-decreasing, i.e., $g(y) \geq g(x)\ \forall x \in \mathbb{R}, \forall y \in \mathbb{R}$ with $y > x$. Show that $g \circ f$ is convex.

$f + g \qquad g(f(x))$

a) Find the Taylor polynomial for the function $\cos x$, around point $x = 0$

$\alpha = 1 \to x$

$\alpha = 0 \to y$

$x + t(y - x)$

$f(y)$

$f(x)$

$\frac{1}{2} f(x) + \frac{1}{2} f(y)$

$\alpha f(x) + (1-\alpha) f(y) \leq$

$\leq f(\alpha x + (1-\alpha) y)$ $\quad [0,1]$

$f\left(\frac{x+y}{2}\right) \geq \frac{f(x) + f(y)}{2}$

$f\left(\frac{1}{2}x + \frac{1}{2}y\right)$

$\alpha x + (1-\alpha) y = [x, y]$

$w_0 \quad 1 - w_0 \qquad w_0 \quad w_1 \quad 1 - w_0 - w_1$

$\alpha = \frac{1}{2} \quad \frac{3}{4} \qquad \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4}$

$-e^{-1}(-e^{-(-1)}) \quad (e^x)' = e^x$

$e^{3x^2 - 2 \ldots} = e^{3x \ldots} \ldots$

$e^{-x} \cdot \frac{1}{e^{-x}}$ $\quad -e^{-x} \quad e^{-x}(-1) \quad +e^{-x}+1$

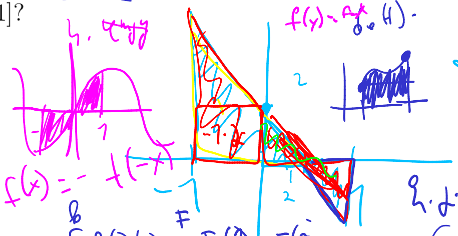
Problem 2. What is the (signed) area between the graph of $f(x)$ and the coordinate axes on the interval $[-1, 1]$?

a) $f(x) = 2 - 4x$,

b) $f(x) = x^2 + 2$,

c) $f(x) = e^{-x}$,

d) $f(x) = \sin x$.



$\int 2-4x\,dx =$

$= 2x - 2x^2 + C$

$2 - 4x = 0 \quad x = 0.5$

$\int_a^b f(x)\,dx = F(b) - F(a)$

$2 - 2\cdot 1 - (2\cdot(-1) - 2\cdot 1) =$

$= -(-2-2) = 4$

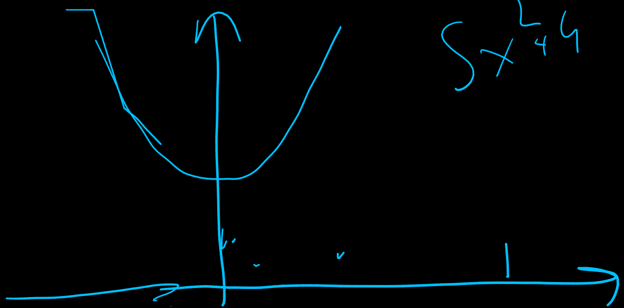
Problem 3. Calculate $f'(x)$:

a) $x^2 + 4$,

b) $3x^4 - \dfrac{1}{x}$,

c) $5\sin^2 x$,

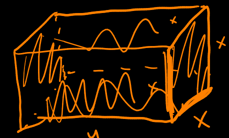
d) xe^x,



$5x^4$

10s

$2x^2 + 4xy = 10$



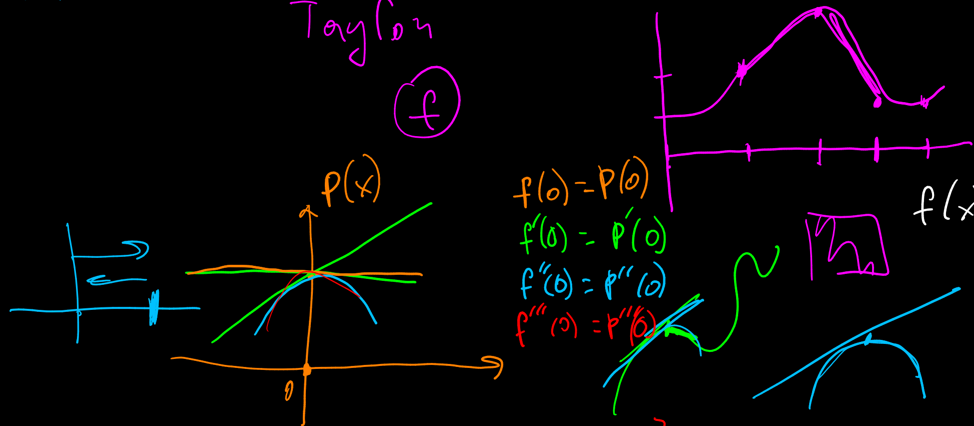
$\boxed{x^2 + 2xy = 5}$

$x^2 y$ maximize

$y = (x-4)^3$

$x^2(x-4)^3$

Taylor



$f(0) = p(0)$

$f'(0) = p'(0)$

$f''(0) = p''(0)$

$f'''(0) = p'''(0)$

$f(x) \approx f(0) + f'(0)\cdot x + \frac{f''(0)}{2!}\cdot x^2 + \dots \quad \frac{f^{(i)}(0)}{i!}x^i$

$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$(x-a)$

$f(0) = a_0$

$p'(x) = a_1 + 2\cdot a_2\cdot x + 3\cdot a_3 \cdot x^2 + \dots$

$f'(0) = a_1$

$f''(0) = 2a_2 + 3\cdot 2\, a_3 x + \dots$

$a_2 = \frac{f''(0)}{2!}$

$f'''(0) = 3\cdot 2\cdot 1\cdot a_3 + \dots$

$a_3 = \frac{f'''(0)}{3!}$

x^n

$n\cdot x^{n-1}$

$n(n-1)x^{n-2}$

$n!\, a_n$

$a_n = \frac{f^{(n)}}{n!}$

$\cos(x) = 1 + 0x - x^2 + \dots$

cos

$-\sin(x)$

$-\cos$