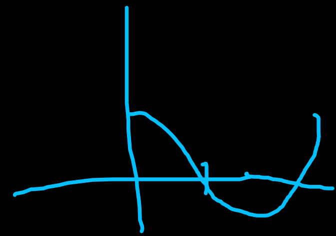
$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

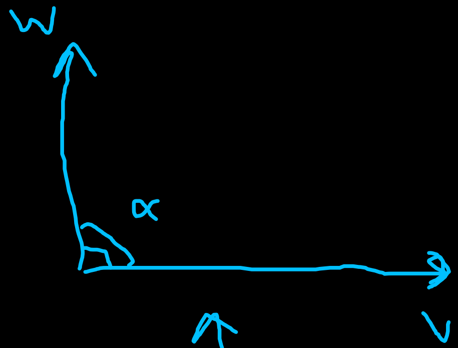
$w = \begin{bmatrix} x \\ y \end{bmatrix}$

$\boxed{2x + 3y} = 0$

$\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

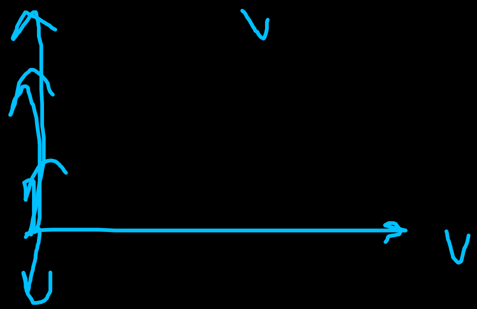
$2 \cdot 6 + 3 \cdot -2 = 0$



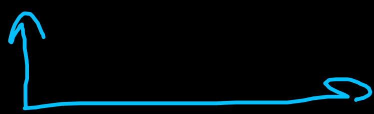


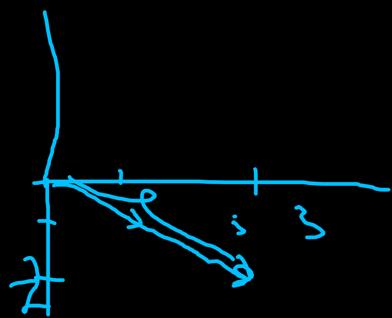
$\cos\alpha = \frac{v \cdot w}{\|v\| \cdot \|w\|} = \cos 90 = 0$

0



$w = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$





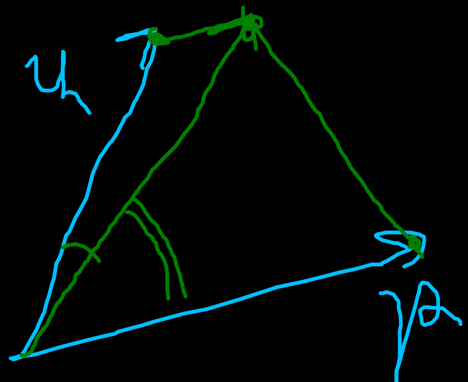
$\begin{bmatrix} 3 \\ -2 \end{bmatrix} \cdot \frac{1}{\sqrt{3^2 + (-2)^2}}$

3x+5

3+5

token

$x =$



$\sqrt{(x_2-x_1)^2 + \dots}$

Given Word Embeddings:

- **cheese:** `[1, 2]`
- **mushroom:** `[3, 1]`
- **tasty:** `[2, 2]`

Tasks:

1. **Euclidean Distance:**
   - **a.** Compute the Euclidean distance between **tasty** and **cheese**.
   - **b.** Compute the Euclidean distance between **tasty** and **mushroom**.
   - **c.** Which word is closer to **tasty** based on the Euclidean distance?
2. **Cosine Similarity:**

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

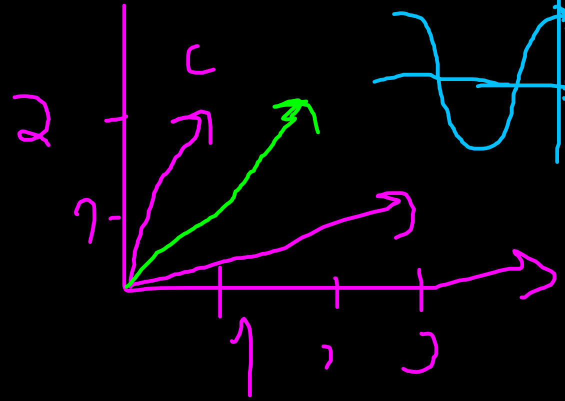
   - **a.** Compute the cosine similarity between **tasty** and **cheese** using the formula above.
   - **b.** Compute the cosine similarity between **tasty** and **mushroom**.
   - **c.** Based on cosine similarity, which word is closer to **tasty**?
3. **Discussion:**
   - Compare the outcomes from the Euclidean distance and cosine similarity calculations.
   - Discuss why one metric might be preferred over the other in different NLP applications.

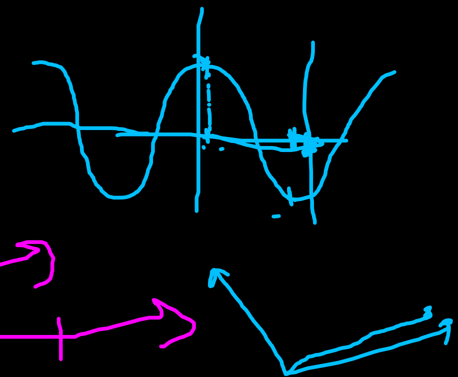
$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

harm

$\cos\alpha = 1$

$\cos\alpha = 0.5$





cheese $\sqrt{(2-1)^2+(2-2)^2} = 1$

tasty → qu

msh $\sqrt{(3-2)^2+(1-2)^2} = \sqrt{2}$

ch. $\frac{2\cdot1+2\cdot2}{\sqrt{1^2+2^2}\cdot\sqrt{2^2+2^2}} = \frac{6}{\sqrt{40}}$

$\frac{3\cdot2+1\cdot2}{\sqrt{3^2+1^2}\cdot\sqrt{2^2+2^2}} = \frac{8}{\sqrt{80}}$

$\frac{4}{3} \approx 1.33$

$\sqrt{2} \approx 1.4$

Exercise: Linear transformation matrix power

Tasks: 1. **Matrix Power:**

- Compute the matrix power of the following matrix A to the power of n:

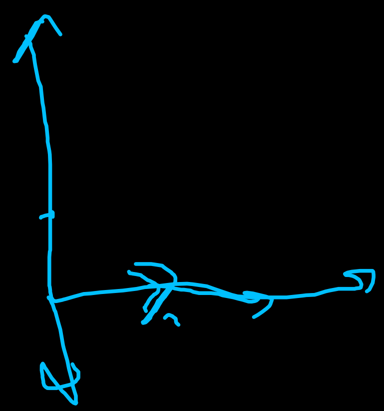
$$A = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

- What does the result represent in terms of linear transformations?

$n \in \mathbb{N}$

$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2\cdot 2+0\cdot 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4\cdot 2 & 0 \\ 0 & 1\cdot -1 \end{bmatrix}$$



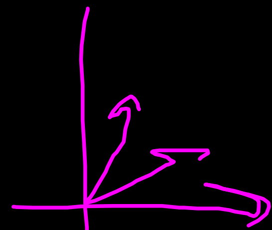
$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Հարթության կոոդինատական համակարգի սկզբնակետից ելնող վեկտորների հետևյալ բազմություններից յուրաքանչյուրի համար պարզել արդյոք այն գծային ենթատարածություն է.

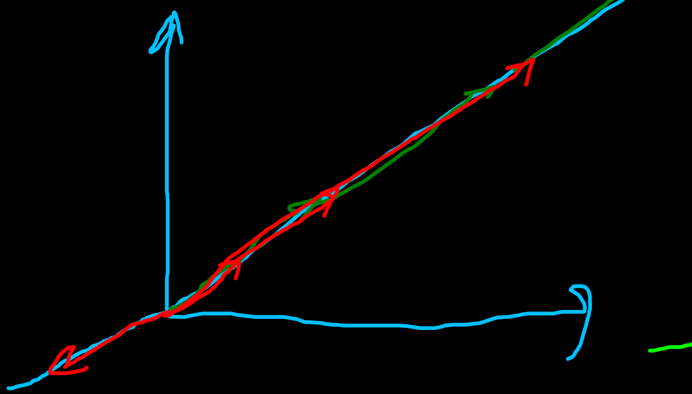
- բոլոր վեկտորները որոնց վերջնակետերը ընկած են տրված ուղղի վրա
- բոլոր վեկտորները որոնց վերջնակետերը ընկած չեն տրված ուղղի վրա
- բոլոր վեկտորները որոնց վերջնակետերը ընկած են կոորդինատական համակարգի առաջին քառորդում
- բոլոր վեկտորները որոնց վերջնակետերը ընկած են կոորդինատական համակարգի առաջին կամ երրորդ քառորդում

$(\mathbb{R}^2, +, \cdot)$

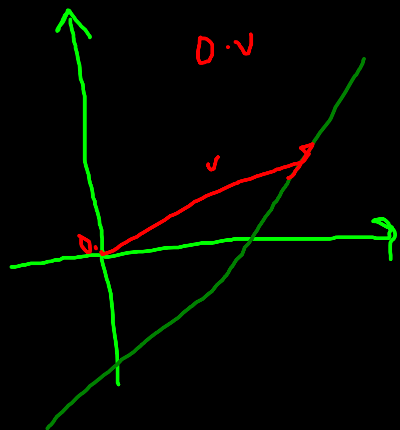


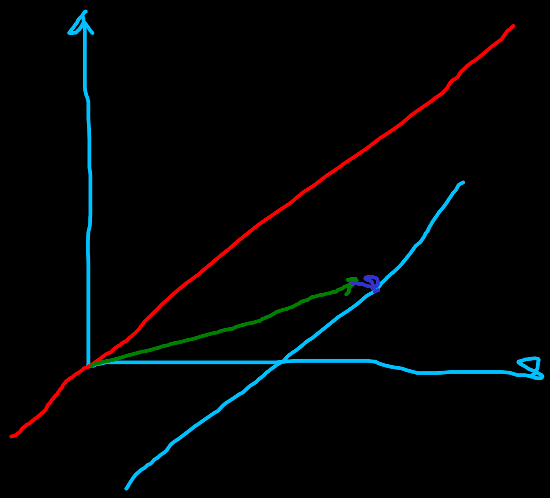
$\alpha\{[0,1],[1,0]\}$ $\in \mathbb{R}$

$[1,1]$



$(\mathbb{R}^n, +, \cdot)$

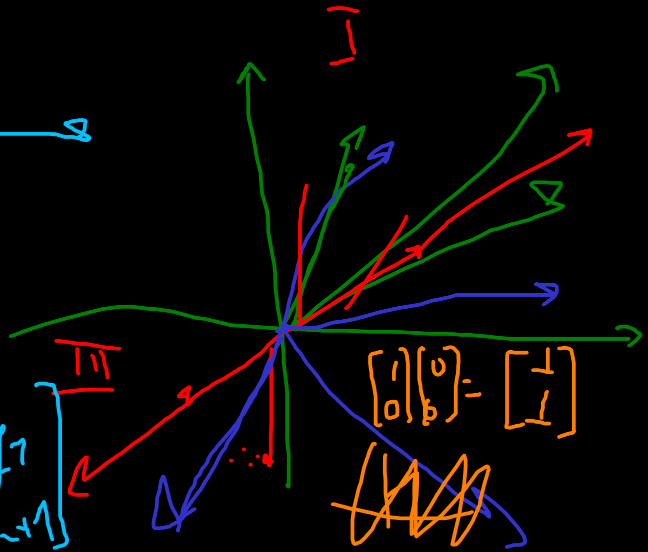




$x_1 + y_1$

$y_1 + y_2$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}x_1+y_1\\x_2+y_2+1\end{bmatrix}$$



$$\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\b\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

$$\begin{bmatrix}a\\b\\c\end{bmatrix}+\begin{bmatrix}a'\\b'\\c'\end{bmatrix}=\begin{bmatrix}a+a'\\b+b'\end{bmatrix}$$

Problem 3. Check if the following set is a vector space:

$\mathbb{R}^3$

a) $A = \mathbb{Z}$, with the usual operations $+$ and $\cdot$,

b) $B = \left\{ \begin{bmatrix}0\\0\\a\end{bmatrix} \mid \text{ for all real numbers } a \in \mathbb{R} \right\}$ with the usual operations $+$ and $\cdot$;

$\mathbb{R}^2$

$c \cdot \begin{bmatrix}a\\b\end{bmatrix} = \begin{bmatrix}ca\\cb\end{bmatrix}$

c) $C = \mathbb{R}^2 = \left\{ \begin{bmatrix}a\\b\end{bmatrix} \mid \text{ for all numbers } a, b \in \mathbb{R} \right\}$, with the usual operation $\cdot$ and the addition defined as:

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}x_1+y_1\\x_2+y_2+1\end{bmatrix},$$

$x_1 + c = 1$ $\quad c = 0$

$x_2 + d + 1 = x_2$

$d = -1$

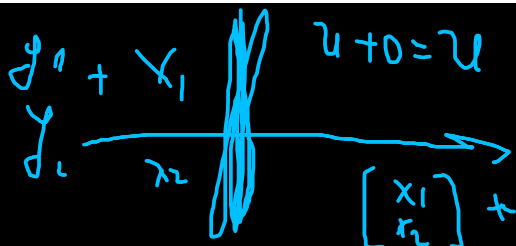
d) The set of all polynomials of degree ≤ 2, with the usual operations $+$ and $\cdot$.

$(\mathbb{Z}, +, \cdot)$

$z_1 + z_2 = \in \mathbb{Z}$

$c \in \mathbb{R}$ $\quad c \cdot z \in \mathbb{Z}$

1.5 $\quad z1$ $\quad 1 \cdot 1.5$



$u + 0 = u$

$(\mathbb{R}^3, +, \cdot)$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}c\\d\end{bmatrix}=\begin{bmatrix}x_1\\x_2\end{bmatrix} \qquad \begin{bmatrix}0\\-1\end{bmatrix}$$

$ax^2 + bx + c + 2x$

$= b(+b)$

$$\theta = \underbrace{0 \cdot x^2 + 0 \cdot x + 0 \cdot c}_{} = 0$$

$$0' \in P^2 \subset P^3$$

$$c\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}\right) =$$

$$= c\begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 + 1 \end{bmatrix} = \begin{bmatrix} c(x_1 + y_1) \\ c(x_2 + y_2 + 1) \end{bmatrix} \checkmark$$

$$cu + cv = \begin{bmatrix} cx_1 \\ cx_2 \end{bmatrix} + \begin{bmatrix} cy_1 \\ cy_2 \end{bmatrix} =$$

$$= \begin{bmatrix} cx_1 + cy_1 \\ cx_2 + cy + 1 \end{bmatrix} \checkmark$$

$$c(x_2 + y_2 + 1) \qquad cx_2 + \cancel{cy_2} + 1$$

$$\cancel{cx_2 + cy_2} + c$$

$$c = 1$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 + c &= x_1 \\ x_2 + d + 1 &= x_2 \end{aligned}\right| \Rightarrow$$

$$c = 0 \in \mathbb{R}$$

$$d = -1$$