Exercise: Finding a Perpendicular Vector

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- https://www.youtube.com/watch?v=vNePhmCMnbU
- https://www.youtube.com/watch?v=P3Y8OWkiUts

Part I

Math | Practical

2 Practice 1: Vectors

2.1 Exercise: Finding a Perpendicular Vector

Context:
In linear algebra, two vectors are perpendicular (or orthogonal) if their dot product is zero. In this exercise, you will find a vector in $\mathbb{R}^2$ that is perpendicular to a given vector.

Given:
Let $\mathbf{v} = [2, 3]$.

Tasks:

1. **Find a Perpendicular Vector:**
   - Find a non-zero vector $\mathbf{w} = [x, y]$ such that $\mathbf{v}$ and $\mathbf{w}$ are perpendicular.
2. **Verification:**
   - Show that your chosen vector $\mathbf{w}$ indeed satisfies the condition $\mathbf{v} \cdot \mathbf{w} = 0$.
3. **Unit Perpendicular Vector:**
   - Find a unit vector in the direction of $\mathbf{w}$ by computing $\frac{\mathbf{w}}{\|\mathbf{w}\|}$, where $\|\mathbf{w}\|$ is the Euclidean norm of $\mathbf{w}$.
4. **Bonus Discussion:**
   - Explain why there are infinitely many vectors perpendicular to $\mathbf{v}$ and describe the general form of all such vectors.

2.2 Exercise: Finding the Closest Word with 2D Embeddings

Context:
In NLP, words can be represented as vectors. Here, each word is represented by a 2-dimensional vector. By comparing these vectors using Euclidean distance and cosine similarity, you can determine which word is "closer" in meaning.

Given Word Embeddings:

- **cheese:** `[1, 2]`
- **mushroom:** `[3, 1]`
- **tasty:** `[2, 2]`

Tasks:

1. **Euclidean Distance:**
   - **a.** Compute the Euclidean distance between **tasty** and **cheese**.
   - **b.** Compute the Euclidean distance between **tasty** and **mushroom**.
   - **c.** Which word is closer to **tasty** based on the Euclidean distance?
2. **Cosine Similarity:**

$$\cos(\theta) = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|}$$

   - **a.** Compute the cosine similarity between **tasty** and **cheese** using the formula above.
   - **b.** Compute the cosine similarity between **tasty** and **mushroom**.
   - **c.** Based on cosine similarity, which word is closer to **tasty**?
3. **Discussion:**
   - Compare the outcomes from the Euclidean distance and cosine similarity calculations.
   - Discuss why one metric might be preferred over the other in different NLP applications.

> **Note**
>
> Cool video by 3blue1brown discussing word vectors (embeddings)

2.3 Exercise: Linear transformation matrix power

Tasks: 1. **Matrix Power:**
- Compute the matrix power of the following matrix A to the power of n:

$$A = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

- What does the result represent in terms of linear transformations?