ECM1416 | ECM1416-Computational Mathematics | Term 2
Year: 2022-2023

Coursework

Your report should consist of your IPython notebook showing what you did, the results, and what you can conclude from the exercise. Each report will be assessed on the following criteria:

- The readability and implementation of the source code for generating your results. (40%)
- The correctness, reproducibility and presentation of your results. (40%)
- The explanation of your results (what you can conclude and how does the work relate to the theoretical foundations?) (20%)

Exercise 1. Consider a diagram made up of four parts, where in each part we have points in $\mathbb{R}^2$, $P = (x, y)$, connected with a line in the order of the arrows, so for example

$$P_1 \to P_2 \to P_3, \quad \text{then} \quad P_4 \to P_1$$

means connecting P_1 to P_2 then P_2 to P_3; and then connecting P_4 to P_1. The coordinates of the points in each part are given below, where $a \in \mathbb{R}$.

- Part 1: $(a, a) \to (a, -a) \to (-a, -a) \to (-a, a) \to (a, a)$.
- Part 2: $(1 - a, a - 1) \to (-a/4, a - 1) \to (-a/4, a/2) \to (1 - a, a/2) \to (1 - a, a - 1)$.
- Part 3: $(a/4, a - 1) \to (a - 1, a/2)$, then $(a - 1, a - 1) \to (a/4, a/2)$.
- Part 4: $(a - 1, -a/4) \to (a - 1, -a/2) \to (1 - a, -a/2) \to (1 - a, -a/4)$.

Let A be a matrix defined by

$$A = \frac{1}{b}\begin{pmatrix} b & 1 \\ -1 & 1 \end{pmatrix}$$

Tasks:

(a) Write a program that performs the transformation of points in Parts 1-4 induced by matrix A.

(b) Let $a = 4$ and $b = 3$, run your program and output two figures, a figure showing the diagram produced before the transformation, and a figure showing the diagram after the transformation.

(10 marks)

Exercise 2. Consider the following initial value problem,

$$\frac{dy}{dx} = x + \frac{y}{5}, \qquad y(0) = -3$$

Tasks:

(a) Write a program to apply the Euler's method to obtain the numerical (approximated) solution, with step sizes of 1, 0.2, and 0.05 on the interval [0, 5], respectively.

(b) Work out the analytical (exact) results. Plot figures to compare the numerical results to the analytical results on the interval [0, 5]. Discuss the comparison results.

(10 marks)