Lecture 10 - GLM VI Nested models

Caspar J. van Lissa

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Nested models

Definition

> **Nested models** occur whenever you can obtain a simpler model by constraining some parameters of a more complex model to 0.

Models are *nested* if they are identical, except that some parameters are constrained to 0

- All predictors in Model 1 are also in Model 2.
- Model 1 is the 'smaller' or 'constrained' model, Model 2 is the 'larger' or unconstrained model
- Model 2 always has better fit Model 1

Example nested model

You already saw this in our first class on bivariate regression:

- 1. Null model: $\hat{Y}_i = a$, which is the same as $\hat{Y}_i = a + 0 * X_1$ (so $b_1 = 0$)
- 2. Regression model: $\hat{Y}_i = a + b_1 * X_1$

Model 1 (null model) is "nested" in model 2 (regression model)

- Model 1 is "constrained": relative to model 2, parameter b_1 is fixed to 0
- Model 2 is "unconstrained": relative to model 1, all parameters are free

Example nested model

We can apply the same principle to multiple regression:

- 1. $\hat{Y}_i = a + b_1 * X_1$
- 2. $\hat{Y}_i = a + b_1 * X_1 + b_2 * X_2$

Model 1 (one predictor) is “nested” in model 2 (multiple regression)

- Model 1 is “constrained”: relative to model 2, slope b_2 is fixed to 0
- Model 2 is “unconstrained”: relative to model 1, all parameters are free

Example

Only work hours, $R^2 = 0.07$:

Variabele	B	t	p
(Intercept)	16.90	6.26	0.00
Work_hours	0.29	2.05	0.04

Only gender roles, $R^2 = 0.13$:

Variabele	B	t	p
(Intercept)	7.57	1.50	0.14
Gender_roles	3.14	2.90	0.01

Multiple regression, $R^2 = 0.15$:

Variabele	B	t	p
(Intercept)	6.81	1.34	0.18
Work_hours	2.66	2.32	0.02
Gender_roles	0.17	1.19	0.24

Nested models

We might ask the question: Does adding gender roles to a model with only work hours significantly improve our ability to predict involvement?

- We can determine this with a nested model test

Difference in R^2

R^2 always increases when we add predictors:

Model	R^2	df1	df2
C: Hours	0.07	1	58
U: Hours and Gender roles	0.15	2	57

$\Delta R^2 = R_u^2 - R_c^2 = 0.15 - 0.07 = 0.08$

Incremental F-test

Incremental F-test

$\Delta R^2 = R_c^2 - R_u^2 = 0.15 - 0.07 = 0.08$
$F = \frac{(SSR_u - SSR_c)/(df1_u - df1_c)}{SSE_u/df2_u}$
We use an F-test to determine whether the increase in R^2 is significant:

- Increase in R-square: ΔR^2
- $SSR_u - SSR_c$: increase in variance explained by the model
- $df1_u - df1_c$: increase in the number of parameters
- $df2_u$ degrees of freedom for the residuals for the unconstrained model
- Sig. F Change = p value for the F-test for R-square change!

Compare to basic F-test

Remember the F-test for bivariate regression:

$$F = \frac{SSR/(p-1)}{SSE/(n-p)}$$

Note that this is the same as an incremental F-test comparing the null model with the regression model:

$$F = \frac{SSR/df1_u}{SSE/df2_u} = \frac{(SSR_u - SSR_c)/(df1_u - df1_c)}{SSE_u/df2_u} = \frac{(SSR_u - 0)/(df1_u - 0)}{SSE_u/df2_u}$$

Putting it together

So, if you have two nested models, you could think of three comparisons:

1. $\hat{Y}_i = a$ (the null model)
2. $\hat{Y}_i = a + b_1 X_1$ (bivariate regression)
3. $\hat{Y}_i = a + b_1 X_1 + b_2 X_2$ (multiple regression)

The F-test for the significance of the R^2 of model 2 is essentially an incremental F-test for ΔR^2 between model 1 vs model 2
Then there's also an incremental F-test for ΔR^2 between model 2 vs model 3

Reporting

The model with work hours and gender role attitudes as predictors explained significantly more variance in the data than the model with only work hours, $\Delta R^2 = 0.08, F(1, 57 = 5.38, p < .001)$.

Hierarchical regression

- Add predictors in blocks
  - If you represent a categorical predictor as dummies, all dummies must be included in the same block! Together, they represent one variable.
- Each block is added to preceding ones, so these models are nested
- Conduct incremental F-tests to determine whether each additional block significantly increases R^2

Why hierarchical regression?

To determine whether theoretically relevant factors explain significant variance *above and beyond* demographic characteristics

- E.g.: you want to predict college achievement based on high school GPA while controlling for demographic factors
  - Block 1: Demographic factors (age, sex, neighborhood quality, SES)
  - Block 2: GPA

To determine whether a previously neglected factor explains additional variance

- E.g.: you want to show that your new scale of morality explains more variance in behavioral outcomes than an existing scale of morality
  - Block 1: Add all subscales of the old morality scale
  - Block 2: Add all subscales of the new morality scale

To test the overall effect of adding dummies for a **categorical predictor** with >2 categories + You need one F-test for all dummies, together; not individual t-tests

Error

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