Lecture 9 - GLM V Multiple Regression

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2023-08-13

Recap

Two+ categories

Remember: we can use bivariate linear regression to model two categories

$\hat{Y}_i = a + b * X_i$

- a: Mean of group 1
- b: Mean difference between groups 1 and 2

For **three+** categories, we can **expand** the model:

$\hat{Y}_i = a + b_1 * X_{1i} + b_2 * X_{2i}$

- a: Mean of group 1
- b_1: Mean difference between groups 1 and 2
- b_2: Mean difference between groups 1 and 3

Multiple regression

The model with two dummies is also an example of multiple regression.
Multiple regression: Regression with more than one predictor.

- Answers the question: What is the unique effect of one predictor, controlling for the effect of all other predictors?

Multiple regression

$\hat{Y}_i = a + b_1 * X_{1i} + b_2 * X_{2i}$

Last week, X_1 and X_2 were dummies (only 0 and 1 values)

You can simply replace them with continuous predictors!

You can expand the model with as many predictors as you like:

$\hat{Y}_i = a + b_1 * X_{1i} + b_2 * X_{2i} + b_3 * X_{3i} + \cdots + b_K * X_{Ki}$

Parameter interpretation

$\hat{Y}_i = a + b_1 * X_{1i} + b_2 * X_{2i}$
a is the intercept

- Expected value when **all** predictors are equal to 0
- When using dummies, this is the mean value of the reference category
- When using continuous predictors, this is the expected value for someone who scores 0 on all predictors

b_1 and b_2 are slopes

- There's one b for each X
- b tells us how many points Y increases if X goes up by 1, while keeping **all other** X-values equal

Unique effects

Aim: predict dependent variable Y from multiple predictors $X_1, X_2, \ldots, X_k$ with a linear model:

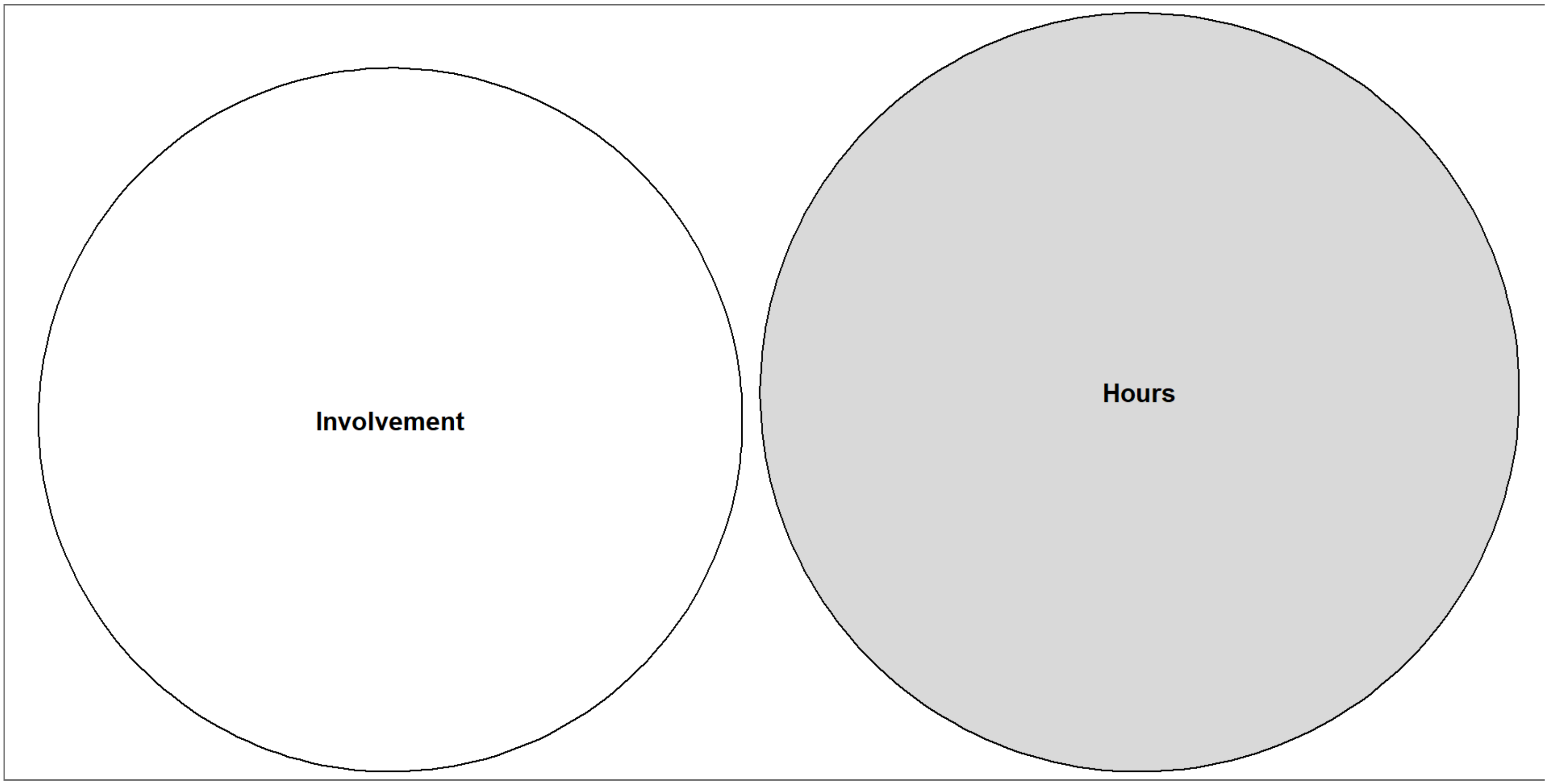
$y_i = b_0 + b_1 * x_1 + b_2 * x_2 + \ldots + b_k * x_k + \epsilon_i$

This will give you the **unique/partial effect** of each predictor, while keeping all other variables constant

A story of bubbles

A story of bubbles

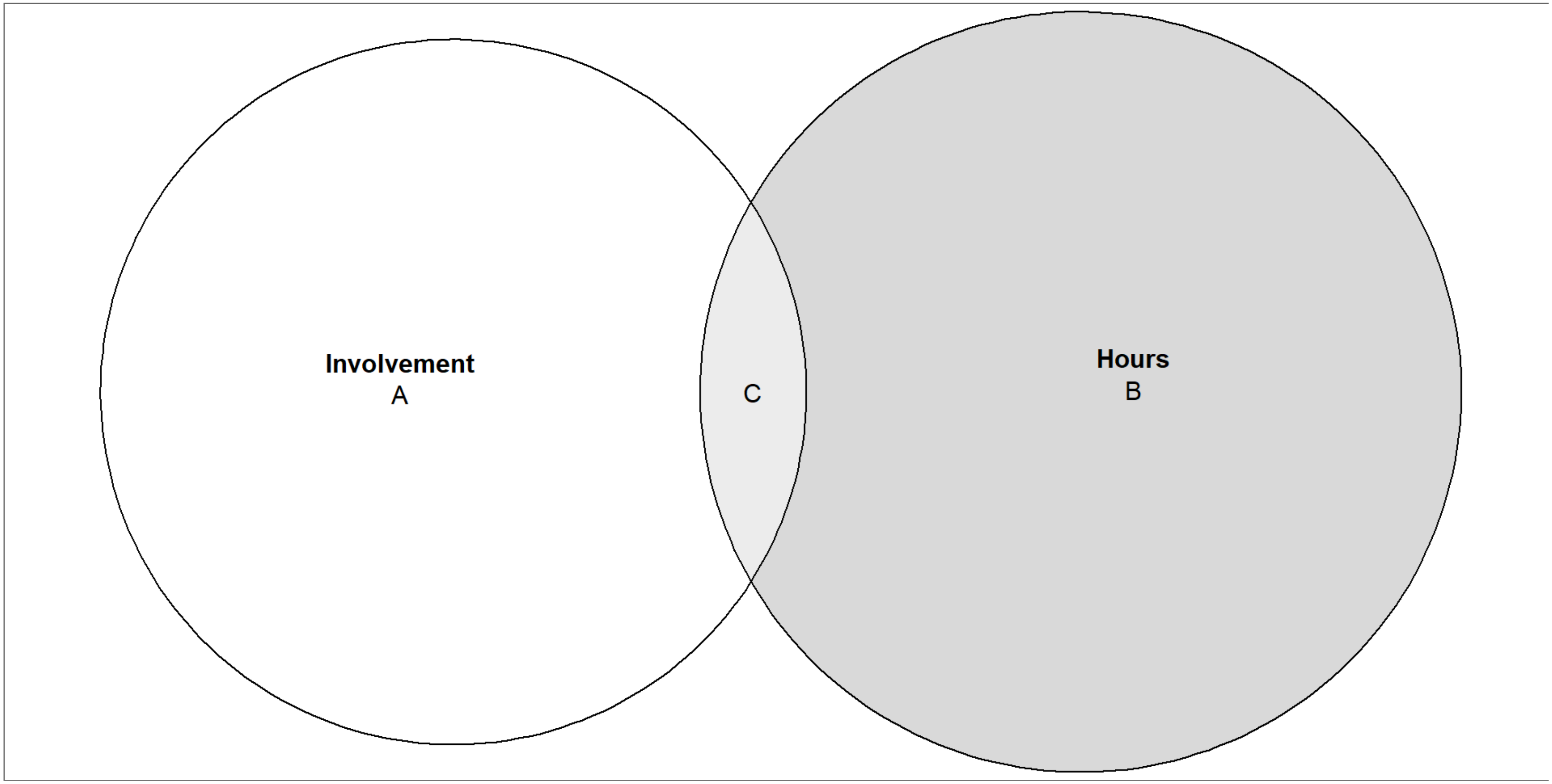
Imagine that these two circles represent the variance in two variables, for example, “Father involvement” and “Hours worked”



Visualizing covariance

Imagine that “Father involvement” and “Hours worked” covary

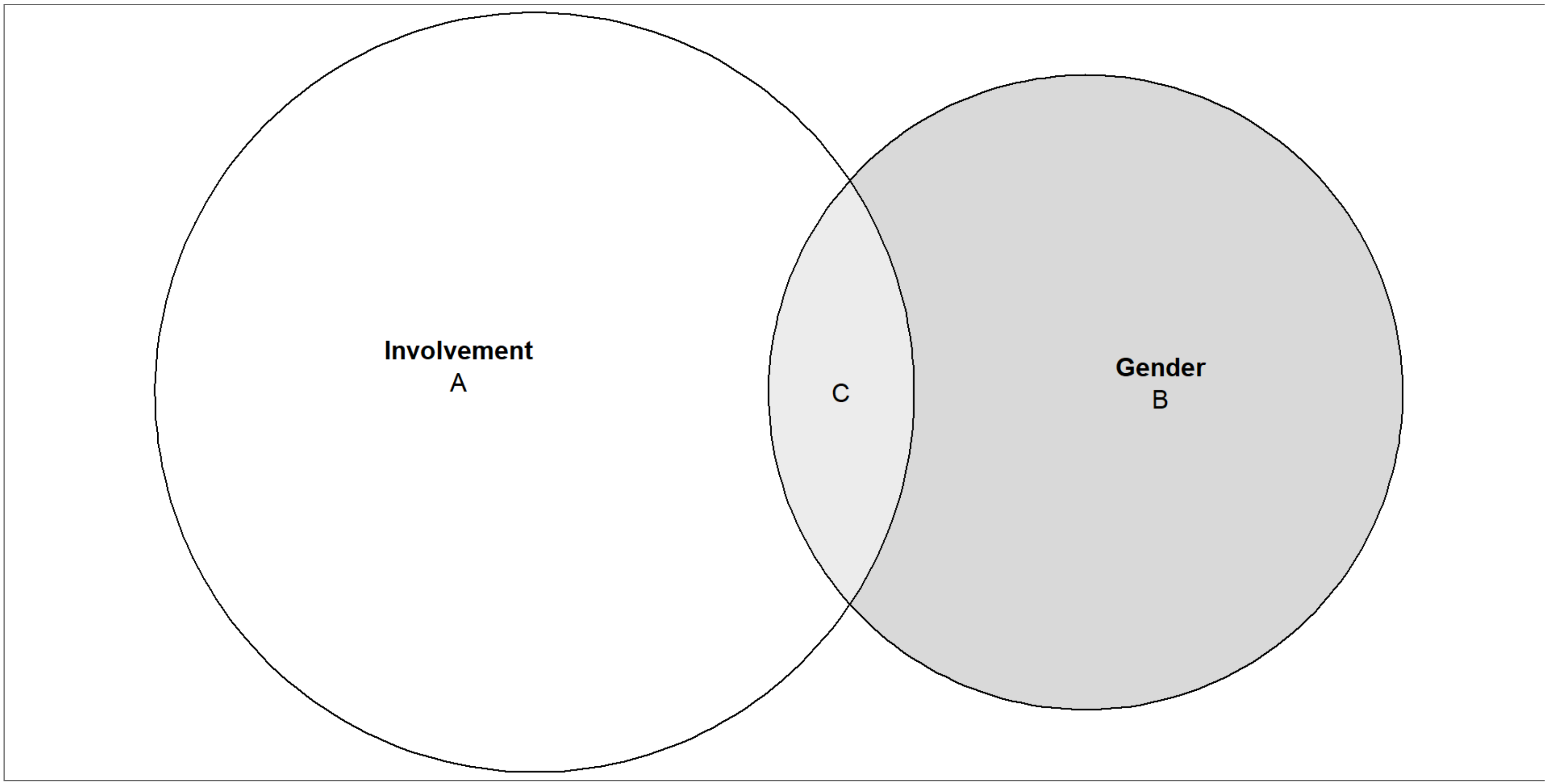
- There would be overlap in the circles (area C)
- We can describe this overlap as correlation, or as a regression coefficient


Involvement
A
C
Hours
B

Visualizing covariance

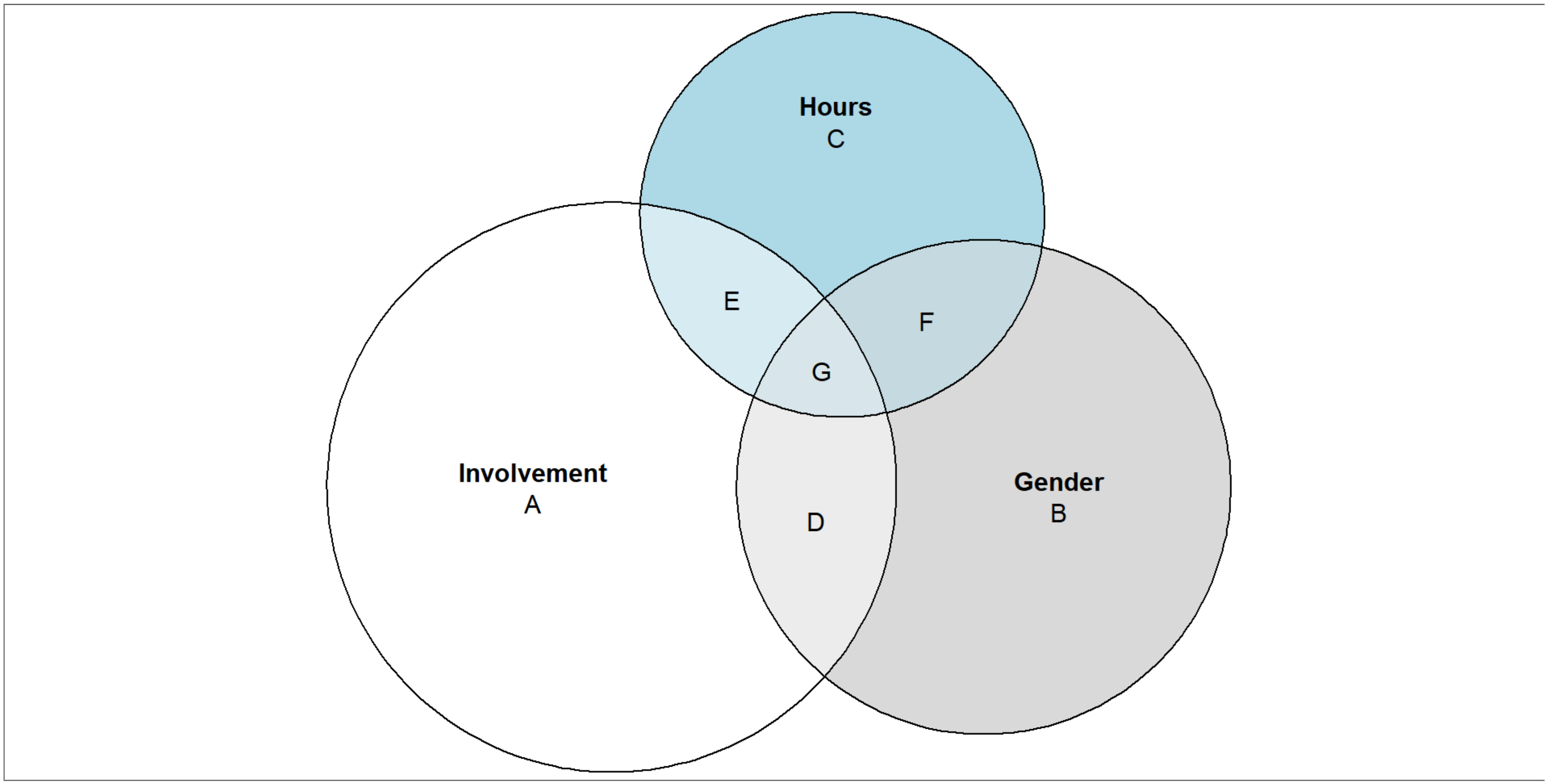
Now, let's say that "Father involvement" and "Gender role attitudes" **also** covary

- There would also be overlap in these circles



Visualizing multiple regression

Finally, imagine that Hours worked and Gender role attitudes both covary with Involvement, and also with one another (e.g., maybe more progressive fathers work fewer hours)



Coefficients

Coefficients

Only work hours:

Variabele	B	t	p
(Intercept)	16.90	6.26	0.00
Work_hours	0.29	2.05	0.04

Only gender roles:

Variabele	B	t	p
(Intercept)	7.57	1.50	0.14
Gender_roles	3.14	2.90	0.01

Multiple regression:

Variabele	B	t	p
(Intercept)	6.81	1.34	0.18
Work_hours	0.17	1.19	0.24
Gender_roles	2.66	2.32	0.02

Coefficients

Why is work hours significant on its own, but not significant when we add gender roles?

Only work hours:

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Multiple regression:

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Separate bivariate regression

Two separate bivariate regression models, $\hat{Y}_i = a + b_1 * X_1$

Multiple regression example

$$\hat{Y}_i = a + b_1 * X_1 + b_2 * X_2$$

Multiple regression example

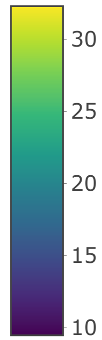
The lines are no longer “in the middle” of the data cloud?

- The effect of Work_hours is *controlled for* the effect of Gender_role, and vice versa

This is clearer when you vidualize this as a 3D plot

Multiple regression 3D plot

Multiple regression demo



Centering

Centering

> a: The intercept, expected value when **all** predictors are equal to 0

But: almost nobody works 0 hours, and nobody scores 0 on the 1-7 point Likert-scale for Gender_roles

Centering

That's why we move the zero-point:

$\text{Center}(Y_i) = Y_i - \bar{Y} = \text{observations - mean}$

Multiple regression after centering

After centering, the separate plots look like this:

Centering

By centering, you can choose a meaningful zero-point for your predictors

- For example, the mean value

When to use?

When to use multiple regression?

- To make better predictions using all available predictors
- To compare relative importance of different predictors
- When theory implies multiple causes
- To improve causal inference by controlling for confounders

Standardized regression coefficients

Standardizing regression coefficients

Problem: We want to know how important different predictors are

Problem: We want to compare the effect of the same variable across two studies

Solution: Standardize the regression coefficient to make them ~comparable

What is standardized regression coefficient

It's just the regression coefficient you would get IF you carried out the analysis after standardizing the X and Y variables

Instead of X and Y, we use Z-scores:

$Z_x = (X - \bar{X})/SD_x$

$Z_y = (Y - \bar{Y})/SD_y$

Z-scores: mean = 0, SD = 1

Z-scores lose the original units of a variable. The new unit is the SD: a Z-score of 1.3 means "1.3 standard deviations above the mean"

Interpretation

Unstandardized

A one unit increase in X is associated with a b unit increase in Y

Standardized

A one SD increase in X is associated with a β SD increase in Y

When to use (un)standardized coefficients?

Unstandardized

- If the units are meaningful/important (e.g., years, euros, centimeters, number of questions correct)
- If there are (clinical) cut-off scores

Standardized

- When units are not meaningful (e.g., depression, need to belong, job satisfaction, Likert scales).
- If you want to compare effect sizes / variable importance

Multicolinearity

MAKE ME

Causality

Causality

- Often, we want to find causal relationships: X -> Y
  - Treatment, Policy decisions, Investments
- Causality can only be established using experiments, or assumed based on theory
- If we rely on theory, it is crucial that we correctly represent our theory in our analyses
  - If the theory is wrong, or incorrectly represented by the analysis, our results will be misleading

Bivariate relationships

Possible relationships:

- There is no **statistical** way to distinguish which one is “true”
- That’s the realm of theory

Trivariate relationships

1. Common cause
2. Indirect effect
3. Collider

Common cause

- A and C share a common cause B
- Changes in B induce changes in both A and C, resulting in covariance between A and C
- If you only examine the relationship between A and C, it will be distorted by the effects of B
- In this context, B is a **confounder**
- Controlling for B is good / necessary if you want to study the relationship between A and B

Examples confounding

Examples confounding

Examples confounding

Indirect effect

- Also called mediation or chain relationship
- B is a process variable or intermediate step in the relationship between A and C
- Controlling for B hides the (indirect causal) association between A and C

Partial mediation example

Colliding

> **Colliding**: If two unrelated variables separately cause a third variable, controlling for that third variable will create a spurious statistical relationship between the unrelated variables.

- Also called “common effect” relationships: B is a common effect of A and C
- In this case, **do not** control for B when studying the relationship between A and C!

Example collider

Example collider

Example collider

Take-home message

Assuming that your model is correctly specified,

- Controlling for confounders improves causal inference
- Controlling for a collider biases (causal) inference

So don’t put EVERYTHING in the model without good reason!

Further reading

Image credit: XKCD

- Judea Pearl’s “The book of why?” - excellent holiday reading material!
- **This blog post**
- Cinelli, C., Forney, A., & Pearl, J. (2022). A crash course in good and bad controls. Sociological Methods & Research, 00491241221099552. **https://ftp.cs.ucla.edu/pub/stat_ser/r493.pdf**