GLM IV+ Advanced ANOVA

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Major CN: GLM IV+: Effect size, post-hoc tests and planned comparison in ANOVA

ANOVA

What is ANOVA

ANalysis **O**f **VA**riance

- One-way ANOVA has one categorical predictor and a continuous outcome
- Provides an omnibus (overall) test of differences between group means
  - $H_0 : \mu_1 = \mu_2 \ldots = \mu_k$
  - All k groups have the same mean
  - H_1: not H0, any means differ

Example Data

Most climbers use “chalk” (magnesium carbonate) to dry their hands

- Climbing chalk is hygroscopic (absorbs moisture)
- But does it actually improve climbing performance?

Li, Margetts, & Fowler (2010):

> [Chalk] dries the skin, decreasing its compliance and hence reducing the coefficient of friction. Secondly, [chalk] creates a slippery granular layer. […] alternative methods for drying the fingers are preferable.

Our fake data:

- Dependent variable: hangtime until faillure in seconds
- Independent variable: Method of drying hands
  - Nothing, t-shirt, chalk powder, chalk ball, liquid chalk (alcohol suspension)

Dummy Coding Scheme

- Pick one reference category
- Create $k-1$ dummy variables that indicate membership of the other groups

	D1	D2	D3	D4
nothing	0	0	0	0
t-shirt	1	0	0	0
chalk ball	0	1	0	0
chalk powder	0	0	1	0
liquid chalk	0	0	0	1

Regression with Dummies

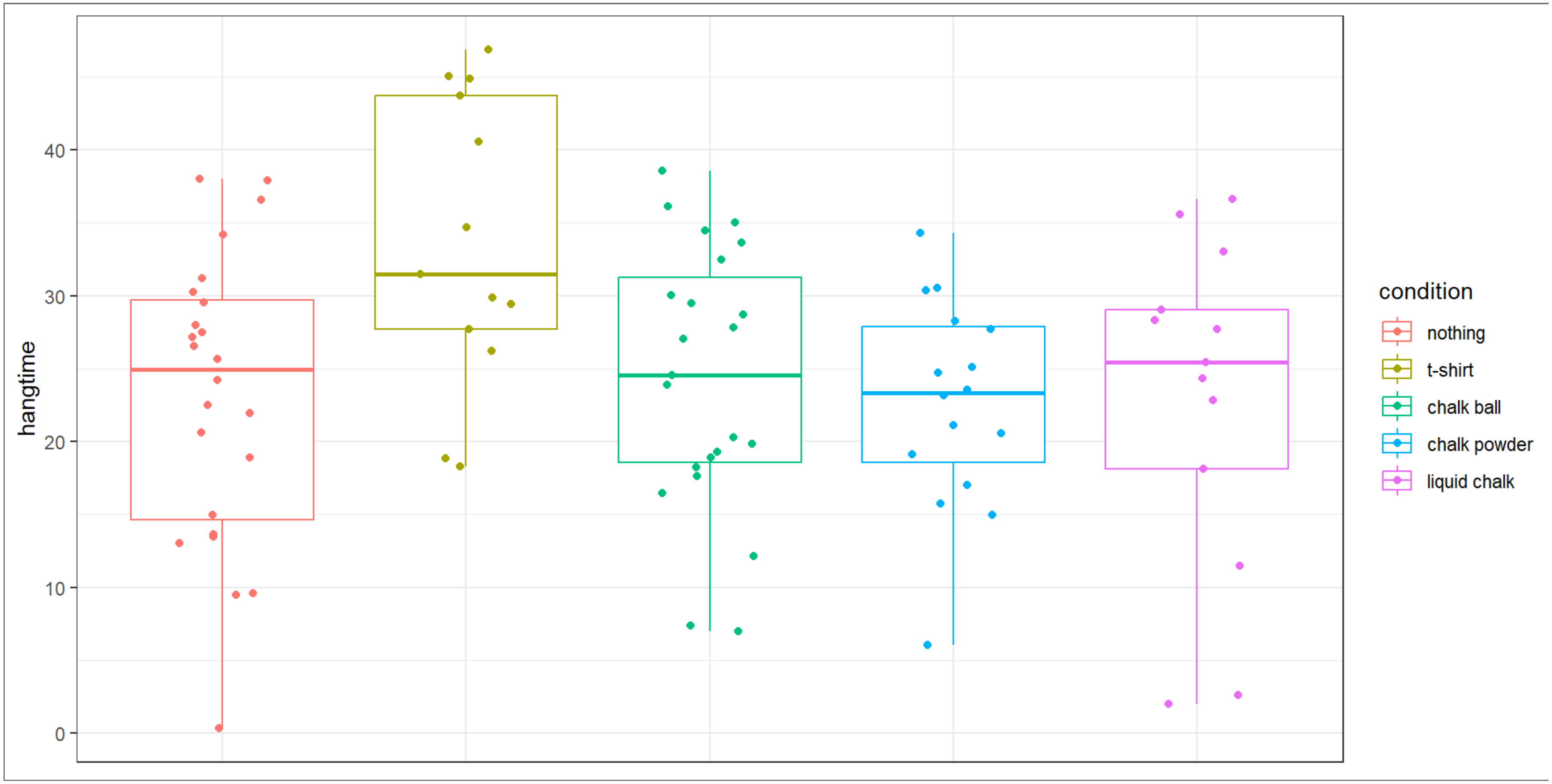
You can represent a categorical variable with k categories in a regression model using $k-1$ dummy variables:

$\hat{Y}_i = a + b_1 D_{1i} + b_2 D_{2i} + b_3 D_{3i} + b_4 D_{4i}$

- a: Intercept, mean value of the reference category “nothing”
- b_1: Difference between Nothing and t-shirt
- b_2: Difference between Nothing and chalk ball
- Et cetera

Example

	B	SE	t	p
(Intercept)	23.13	1.93	11.99	0.00
Dt-shirt	10.52	3.26	3.23	0.00
Dchalk ball	1.16	2.76	0.42	0.67
Dchalk powder	-0.49	3.05	-0.16	0.87
Dliquid chalk	-0.28	3.26	-0.08	0.93



hangtime
40
30
20
10
0
condition
nothing
t-shirt
chalk ball
chalk powder
liquid chalk

Alterntive Coding Schemes

ANOVA specification

Alternatively, you can estimate k group means using k dummy variables

	D1	D2	D3	D4	D5
nothing	1	0	0	0	0
t-shirt	0	1	0	0	0
chalk ball	0	0	1	0	0
chalk powder	0	0	0	1	0
liquid chalk	0	0	0	0	1

Regression formula:

$\hat{Y} = b_1 * D_1 + b_2 * D_2 + b_3 * D_3 + b_4 * D_4 + b_5 * D_5$

- b_1: Mean of the nothing group
- b_2: Mean of the t-shirt group
- b_3: Mean of the chalk ball group
- Et cetera

ANOVA vs Regression specification

- Both preceding models are mathematically identical
- We just replace the intercept with one additional dummy
  - Same number of parameters
- Advantage: We get a standard error for each group mean
- We can test each group mean against hypothesized values
- The default t-test tests $H_0 : \beta = 0$
  - Use the standard errors to test other hypotheses

Example Results ANOVA Specification

	Mean	SE	t	p
conditionnothing	23.13	1.93	11.99	0
conditiont-shirt	33.66	2.62	12.83	0
conditionchalk ball	24.30	1.97	12.32	0
conditionchalk powder	22.64	2.36	9.58	0
conditionliquid chalk	22.86	2.62	8.72	0

Example Custom Test

Does hang time in the T-shirt condition exceed 30 seconds? $H_0 : \beta < 30$

$t = \frac{33.66 - 30}{2.62} = 1.40$

$t_{crit}(df = 84) = 1.66, t_{test} < t_{crit}$ so cannot reject H_0

	Mean	SE	t	p
conditionnothing	23.13	1.93	11.99	0
conditiont-shirt	33.66	2.62	12.83	0
conditionchalk ball	24.30	1.97	12.32	0
conditionchalk powder	22.64	2.36	9.58	0
conditionliquid chalk	22.86	2.62	8.72	0

More Coding Schemes

Dummy coding, $k - 1$ dummies + intercept

- Gives us one group mean + difference tests with all other group means

Dummy coding, k dummies

- Gives us all group means

Other coding schemes that represent exactly the same information, but give us different information:
Deviation coding: Compare each condition to the grand mean
Contrast coding: Compare multiple group means against each other

- One control condition vs two different experimental conditions
- Effect of two positive emotions vs three negative emotions
- Instead of talking about “dummies”, we’ll talk about “indicator variables”
    - A dummy is an indicator variable that can take only 0 or 1 values

General Rules Coding Schemes

For all coding schemes:

- The possible values of each indicator must sum to 0
- Each group should be uniquely identified by a particular combination of the indicator variables
  - E.g., this is why we cannot have both an intercept and a dummy for one group; the dummy and intercept are redundant
- *Sometimes you have to account for relative group size*

Effects Coding

> **Effects Coding:** Comparing all groups to the grand mean.

- The reference category does not score 0 on all indicator variables, but receives a negative value
- In a balanced design (equal group sizes), this value is -1
- Codes for each indicator must sum to 0
- In a balanced design, the coding scheme for effects coding is:

	E1	E2	E3	E4
nothing	-1	-1	-1	-1
t-shirt	1	0	0	0
chalk ball	0	1	0	0
chalk powder	0	0	1	0
liquid chalk	0	0	0	1

Effects Coding Output

Effects coding gives us the following information:

- The grand mean for the dependent vaeriable
- The difference between each group, except the reference category, compared to the grand mean

Effects Coding Unequal Groups

We rarely have balanced designs.

- In the general way to construct effect codes, weights assigned for the reference category differ for each indicator
- They are computed as:

$-1 * n_{\text{this category}} / n_{\text{reference category}}$
Given group sizes a = 44, b = 87, c = 7:

	E1	E2
a	1	0
b	0	1
c	$\frac{-7}{44}$	$\frac{-87}{44}$

Understanding Equal Groups

Note that when group sizes are equal, we get -1 for the reference category:
Given group sizes a = 40, b = 40, c = 40:

	E1	E2
a	1	0
b	0	1
c	$\frac{-40}{40} = -1$	$\frac{-40}{40} = -1$

Example Effect Coding

We have the following sample sizes:

	nothing	t-shirt	chalk ball	chalk powder	liquid chalk
Freq	24	13	23	16	13

Which leads to this coding scheme:

	E1	E2	E3	E4
liquid chalk	1	0	0	0
chalk powder	0	1	0	0
chalk ball	0	0	1	0
t-shirt	0	0	0	1
nothing	-13/24	-16/24	-23/24	-13/24

Example Results Effects Coding

	Mean	SE	t	p
(Intercept)	24.84	1.00	24.79	0.00
E_t-shirt	8.81	2.42	3.64	0.00
E_chalk ball	-0.55	1.70	-0.32	0.75
E_chalk powder	-2.20	2.14	-1.03	0.31
E_liquid chalk	-1.98	2.42	-0.82	0.42

Contrast Coding

Another coding scheme is to compare groups of means

- E.g., is there a difference between doing nothing or using a t-shirt versus the three chalk types?
  - $H_0 : \mu_{\text{nothing, shirt}} = \mu_{\text{powder, ball, liquid chalk}}$
- And is there a difference between liquid and dry forms of chalk?
  - $H_0 : \mu_{\text{liquid chalk}} = \mu_{\text{powder, ball chalk}}$
- Complete the matrix to meet the rules of coding schemes

This is a very advanced technique!

Step 1: Plan Contrasts

Plan your contrasts. This scheme meets all requirements, **except** accounting for group size:

	nothingshirtVchalk	nothingVshirt	dryVliquid	ballVpowder
nothing	-1.5	-1	0.0	0
t-shirt	-1.5	1	0.0	0
chalk ball	1.0	0	-0.5	-1
chalk powder	1.0	0	-0.5	1
liquid chalk	1.0	0	1.0	0

Step 2: Account for Group Size

If you do not account for group size, you will be comparing means of means:

$$H_0 : \frac{\mu_{nothing} + \mu_{shirt}}{2} = \frac{\mu_{powder} + \mu_{ball} + \mu_{liquid}}{3}$$

Instead of the mean of multiple conditions:

$$H_0 : \mu_{\text{nothing, shirt}} = \mu_{\text{powder, ball, liquid chalk}}$$

If group sizes are equal, these approaches are identical and you can skip

Step 2: How To

Sample sizes:

	nothing	t-shirt	chalk ball	chalk powder	liquid chalk
Freq	24	13	23	16	13

We then get the contrast values:

	nothingshirtVchalk	nothingVshirt	dryVliquid	ballVpowder
nothing	-24/(24+13)	-1	0	0
t-shirt	-13/(24+13)	1	0	0
chalk ball	23/(23+16+13)	0	-23/(23+16)	-1
chalk powder	16/(23+16+13)	0	-16/(23+16)	1
liquid chalk	13/(23+16+13)	0	1	0

Step 3a: Do Matrix Algebra

Planned contrasts require you to invert the matrix of contrasts
Write your planned contrasts in Excel/Google Sheets:

Step 3b: Add intercept

Add an intercept, consisting of $1/k$ for each group:

- We have 5 groups, so our intercept is $1/5 = 0.2$

Step 3c: Copy-paste Formula

- Click an Empty cell
- Paste `=MINVERSE(TRANSPOSE(`
- Select your contrast matrix
- Finish the formula by typing closing brackets `))`

Step 3d: Copy-Paste Result

Copy-paste the inverted matrix

These are the values you will use for your indicators!

Example Results Contrast Coding

	Mean	SE	t	p
(Intercept)	25.32	1.04	24.38	0.00
nothingshirtVchalk	-3.40	2.03	-1.67	0.10
nothingVshirt	10.52	3.26	3.23	0.00
dryVliquid	-0.76	3.03	-0.25	0.80
ballVpowder	-1.66	3.08	-0.54	0.59

‘Post-Hoc’ Tests

You can also compare all group means to each other
With k groups, we can make $\frac{k(k-1)}{2}$ comparisons
If $k = 5$, we have $(5 * 4)/2 = 10$ comparisons.

- Historically, this is called a “post-hoc” test (=“after the fact”)
- “post-hoc” implies that this is not a hypothesized test, like a planned contrast might be
- You have to be very mindful of data dredging (false positive findings)
- This can only be done via the ANOVA interface in SPSS

Experiment-wise Type I error

Adjusting for Multiple Comparisons

- The significance level α is the probability of committing a Type I error
- We typically use $\alpha = .05$: 5% probabability of drawing a false positive conclusion
- When we conduct many tests, we run this risk each time

Experiment-wise Type I error

> **Experiment-wise Type I error α_{ew}:** The total risk of committing a Type I error (false positive conclusion) accross multiple (m) tests.

$\alpha_{ew} = 1 - (1 - \alpha)^m$
So for 3 tests: $_{ew} = 1 - (1-.05)^10 = .40
40% chance of at least one false positive conclusion may be more than we want

Bonferroni Correction

Bonferroni proposed a simple correction:
$\alpha = \alpha_{EW}/m$

- α_{EW} is the desired experiment-wise Type I error rate (e.g., .05)
- m is the number of tests
- This correction is quite conservative
- There's always a trade off: fewer false positive conclusions means it is harder to detect true effects

Error

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