GLM Repeated Measures ANOVA

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2023-08-12

Within-participants Designs

- Participants are exposed to multiple experimental conditions
  - Type of stimulus, drug dosage
- The outcome is measured at two or more measurement occasions (*longitudinal design*).
  - Test-retest designs, panel studies, diary studies, repeated physiological assessments, etc

Advantages Within-participants Designs

Many individual differences are constant

- Variability due to individual differences is thus removed from the error term
- Each subject serves as its own control
- Greater statistical power
- More information per participant = more efficient; we can use a smaller sample than if we used a between-participants design

Limitations of Within-participants Designs

For experimental designs:

- Order effects: The order of conditions may have an effect
- **Differential** order effects: Order effects may differ across different orders
    - If participants take the drug before the placebo, they may still be under the influence during the placebo condition

Solution for Order Effects

Latin Square Design

- Experimentally controls for order effects
- In a Latin square, each condition appears in one position in the ordering:

1	2	3	4
A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

This is just one out of many (in fact 256; 44) possible Latin Squares. There are tools that randomly generate a Latin square.

Limitations of Within-participants Designs

For all designs, including non-experimental:

- Learning effects: People become familiar with your questionnaire
- Historical effects: Some event may happen during your study (fire alarm goes off, global pandemic breaks out, documentary on TV about your topic of study)
- Effect of time is confounded with effect of condition
- Effect of time may have a clearly defined functional form which RM-ANOVA is ignoring
    - E.g.: PTSD probably changes *after* deployment, and then increases or decreases over time
    - Techniques like “Structural Equation Modeling” allow you to describe this

Two Repeated Measurements

An intervention is imposed on ten people
Each person is measured twice:

- Before the intervention (pretest)
- After the intervention (posttest)

Resp.	Pretest	Posttest
1	2	5
2	3	4
3	4	6
4	5	5
5	6	8
6	7	10
7	8	9
8	9	11
9	10	9
10	11	15
Mean	6.5	8.2

How to Analyze These Data?

Problem: These data violate one assumption of the general linear model:

- Independence of errors

So we can’t use linear regression, or any of its “interfaces” like the independent samples t-test

Solution: Paired Samples t-test

Solution: With just two repeated measures, the *paired samples t-test*!
This is equivalent to calculating the *difference between the two measurements* and…

- Performing a one-sample t-test on that difference
- Performing linear regression with that difference score as outcome and only an intercept as predictor

Paired Samples t-test

Resp.	Pretest	Posttest	Difference (post-pre)
1	2	5	3
2	3	4	1
3	4	6	2
4	5	5	0
5	6	8	2
6	7	10	3
7	8	9	1
8	9	11	2
9	10	9	-1
10	11	15	4
Mean	6.5	8.2	1.7

Results

t-test:

Paired Samples Test

		Paired Differences					t	df	Significance	
					95% Confidence Interval of the Difference					
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper			One-Sided p	Two-Sided p
Pair 1	t1 - t2	-1.70000	1.49443	.47258	-2.76905	-.63095	-3.597	9	.003	.006

Regression:

Coefficients[a,b]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	int	1.700	.473	.768	3.597	.006

a. Dependent Variable: dt

b. Linear Regression through the Origin

More than Two Measurements

Example: PTSD in Military Personnel

- Data on 978 Dutch military personnel who have been deployed
- 4 repeated measures of PTSD symptoms on the SCL scale
  - 1 pre-deployment, 3 every 6 months post-deployment

3+ Repeated Measurements

Research question : is there a difference between those repeated measures?
$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4; H_1$: not H_0
Two approaches:

1. Univariate Approach (aka: Linear Mixed model)
    - Just linear regression
    - The repeated measures are treated as one outcome variable
    - Each participant has multiple rows
2. Multivariate Approach
    - Treating the repeated measures as different correlated outcomes

Liner Mixed Model

Uses Linear Regression:

- Treat all repeated measurements as a single variable with repeated observations
  - Convert data to “long format”
  - 4 repeated measures -> 4 data rows per participant
- Measurement occasion (or: condition) is a “fixed effect” (= limited number of discrete values)
- Participant ID is a “random effect”; each participant may vary around a person-specific mean

This only works if the so called **sphericity** assumption holds

Sphericity Assumption

> **Sphericity:** The variances of the differences between all combinations of repeated measures are equal.

This is analogous to the homoscedasticity assumption
Closely related to the notion of compound symmetry:

- All repeated measures have equal variance
- Each pair of repeated measures have the same correlation
- You see how this justifies treating repeated measures as a single long-format dependent variable

Multivariate Approach

If we can not / do not assume sphericity, we can use the *multivariate approach*

- Repeated measurements are treated as covariates of each other
  - I.e., effect on T1 controlling for T2, T3, …
  - Effect on T2 controlling for T1, T3, …
- Because of this, this approach has more predictors and thus much smaller df
  - Needs larger sample to get the same power

Mauchly’s test of Sphericity

Mauchly's Test of Sphericity[a]

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[b] Greenhouse-Geisser	Huynh-Feldt	Lower-bound
time	.951	48.560	5	<.001	.967	.970	.333

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

- H_0: sphericity holds, H_1: sphericity does not hold
- Significant test: evidence that the assumption is violated
- This is the case here, possibly because of the qualitative difference between pre- and post-deployment measurements

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Epsilon is an estimate of departure of sphericity

- If sphericity holds, epsilon = 1
- Lower value of epsilon -> larger departure from sphericity
- Lower bound of epsilon: $1/(k-1)$
    - k is the number of repeated measures

Violation of Sphericity

- Remember: You don’t have to blindly adjust your test based on assumption checks
- You should definitely disclose it
- You can choose a test that is more robust to violations of sphericity

Mixed Model Results

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
time	Sphericity Assumed	87.432	3	29.144	7.285	<.001
	Greenhouse-Geisser	87.432	2.900	30.150	7.285	<.001
	Huynh-Feldt	87.432	2.909	30.051	7.285	<.001
	Lower-bound	87.432	1.000	87.432	7.285	.007
Error(time)	Sphericity Assumed	11726.318	2931	4.001		
	Greenhouse-Geisser	11726.318	2833.201	4.139		
	Huynh-Feldt	11726.318	2842.538	4.125		
	Lower-bound	11726.318	977.000	12.002		

- Notice error df: 2931
- You can use a corrected test
- Trade-off between Type I error (false-positives) and Type II error (false negatives) by adjusting df:
  - Sphericity assumed: Highest Type I error, lowest Type II error
  - Huynh-Feldt: Slightly lower Type I error slightly higher Type II error

- Greenhouse Geisser: Slightly lower Type I error, slightly higher Type II error
- Lower bound: Lowest Type I error, highest Type II error

Multivariate Approach

Multivariate Tests[a]

Effect		Value	F	Hypothesis df	Error df	Sig.
time	Pillai's Trace	.021	7.079[b]	3.000	975.000	<.001
	Wilks' Lambda	.979	7.079[b]	3.000	975.000	<.001
	Hotelling's Trace	.022	7.079[b]	3.000	975.000	<.001
	Roy's Largest Root	.022	7.079[b]	3.000	975.000	<.001

- Notice error df: 975 (much lower)

Mixed Design

Mixed Design

Within-participants Factor:

- Time, experimental condition, etc
- In our example: Time

Between-participants Factor:

- Sex, age, major, etc
- In our example: Whether the participant was exposed to high-intensity combat action (1) or not (0)

Mixed Design ctd

This is essentially a factorial design: 4(Time: T1, T2, T3, T4) x 2(Combat exposure: Low vs. High)

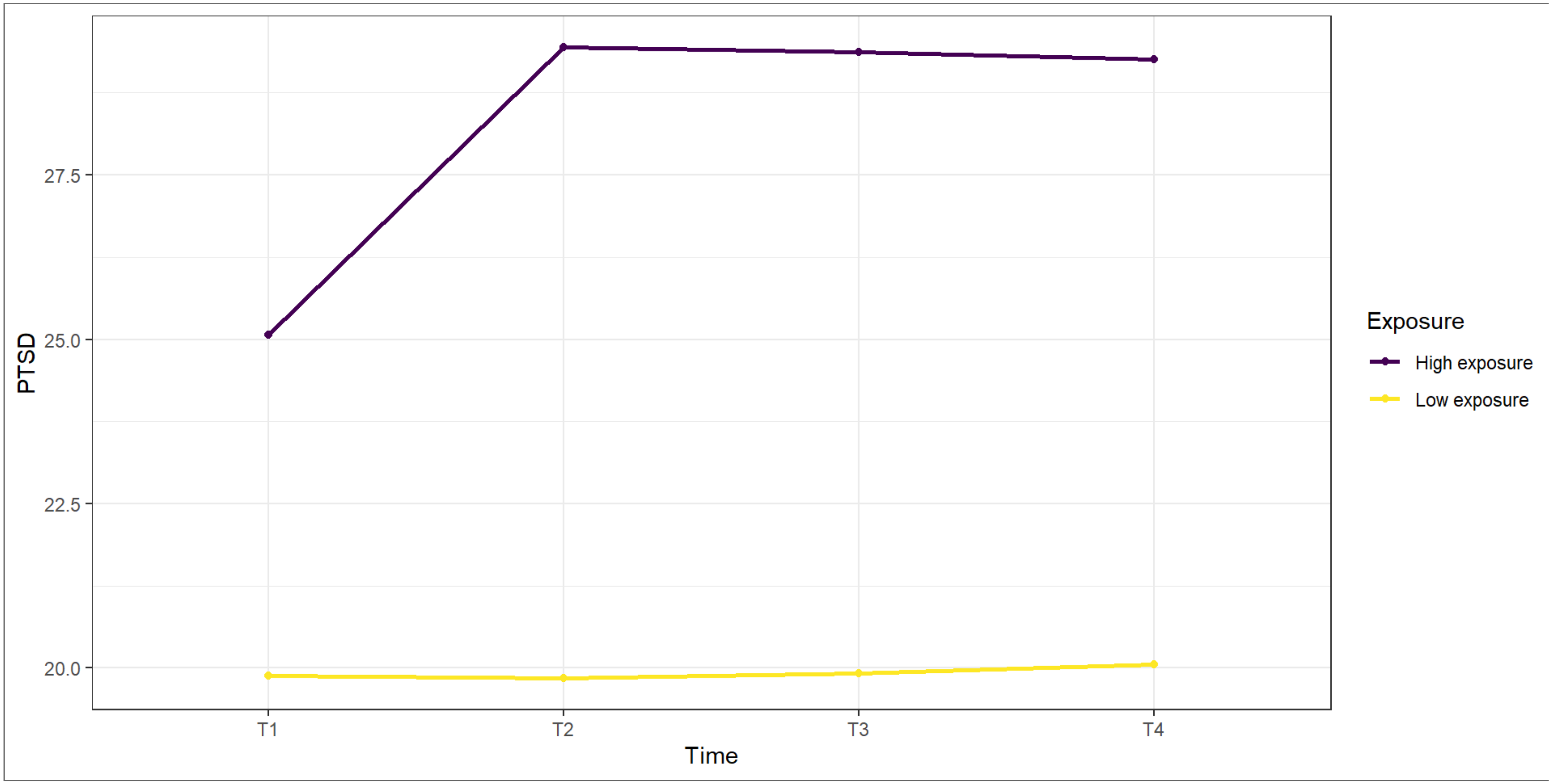
	T1	T2	T3	T4
Low exposure	19.89	19.84	19.92	20.06
High exposure	25.07	29.44	29.37	29.25

Interactions

This requires testing an interaction effect between time and exposure

- If there is a significant interaction, you can use simple effects analysis:
  - Test whether the within-participants factor has an effect for each level of the between-participants factor.
  - Test whether the between-participants factor has an effect for each level of the within-participants factor.

Graphical display



Test for interaction

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
time	Sphericity Assumed	783.088	3	261.029	69.637	<.001
	Greenhouse-Geisser	783.088	2.948	265.593	69.637	<.001
	Huynh-Feldt	783.088	2.961	264.432	69.637	<.001
	Lower-bound	783.088	1.000	783.088	69.637	<.001
time * Exposure	Sphericity Assumed	750.910	3	250.303	66.776	<.001
	Greenhouse-Geisser	750.910	2.948	254.679	66.776	<.001
	Huynh-Feldt	750.910	2.961	253.566	66.776	<.001
	Lower-bound	750.910	1.000	750.910	66.776	<.001
Error(time)	Sphericity Assumed	10975.407	2928	3.748		
	Greenhouse-Geisser	10975.407	2877.694	3.814		
	Huynh-Feldt	10975.407	2890.323	3.797		
	Lower-bound	10975.407	976.000	11.245		

Significant interaction, we could perform simple effects tests

Follow-up: Simple Effect of Time

Multivariate Tests

Exposure		Value	F	Hypothesis df	Error df	Sig.
Low	Pillai's trace	.007	2.148[a]	3.000	974.000	.093
	Wilks' lambda	.993	2.148[a]	3.000	974.000	.093
	Hotelling's trace	.007	2.148[a]	3.000	974.000	.093
	Roy's largest root	.007	2.148[a]	3.000	974.000	.093
High	Pillai's trace	.172	67.614[a]	3.000	974.000	<.001
	Wilks' lambda	.828	67.614[a]	3.000	974.000	<.001
	Hotelling's trace	.208	67.614[a]	3.000	974.000	<.001
	Roy's largest root	.208	67.614[a]	3.000	974.000	<.001

- Not significant in low exposure group
- Significant in high exposure group

Follow-up: Simple Effect of Exposure

Univariate Tests

Measure: MEASURE_1

time		Sum of Squares	df	Mean Square	F	Sig.
1	Contrast	1488.785	1	1488.785	368.540	<.001
	Error	3942.732	976	4.040		
2	Contrast	5108.932	1	5108.932	718.764	<.001
	Error	6937.348	976	7.108		
3	Contrast	4959.123	1	4959.123	755.419	<.001
	Error	6407.176	976	6.565		
4	Contrast	4691.217	1	4691.217	722.939	<.001
	Error	6333.356	976	6.489		

Each F tests the simple effects of Exposure within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

- Significant difference at each time
- We could apply Bonferroni correction, but these p-values are very small (< .001)

Error

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