Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page (http://vision.stanford.edu/teaching/cs231n/assignments.html)](http://vision.stanford.edu/teaching/cs231n/assignments.html) on the course website.

In this exercise you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

In [1]:

```
# Run some setup code for this notebook.

import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules;
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-ipyt
hon
%load_ext autoreload
%autoreload 2
```

CIFAR-10 Data Loading and Preprocessing

In [2]:

```
# Load the raw CIFAR-10 data.
cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

In [3]:

```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'shi
p', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```

In [4]:

```python
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

In [5]:

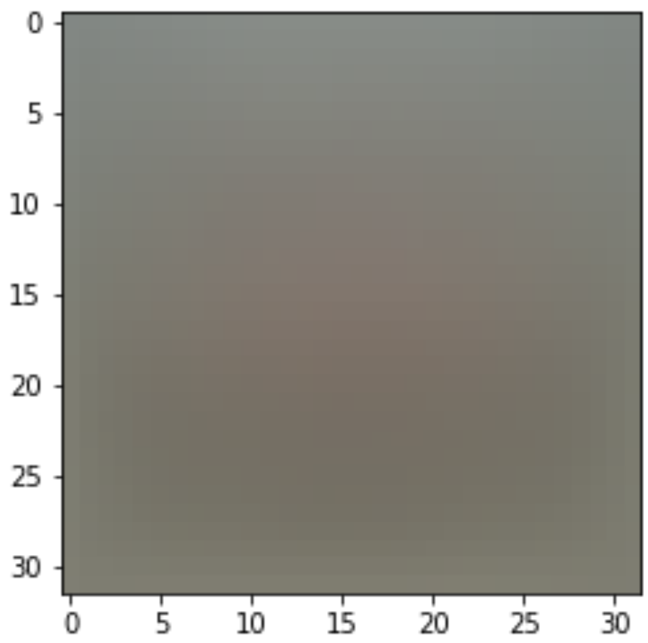
```
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
print('dev data shape: ', X_dev.shape)
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

In [6]:

```
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean
image
plt.show()
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```



In [7]:

```
# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image
```

In [8]:

```
# third: append the bias dimension of ones (i.e. bias trick) so that our SVM
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
```

```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

SVM Classifier

Your code for this section will all be written inside **cs231n/classifiers/linear_svm.py**.

As you can see, we have prefilled the function `compute_loss_naive` which uses for loops to evaluate the multiclass SVM loss function.

In [15]:

```
# Evaluate the naive implementation of the loss we provided for you:
from cs231n.classifiers.linear_svm import svm_loss_naive
import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))
```

```
loss: 9.309398
```

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

In [16]:

```
# Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, a
nd
# compare them with your analytically computed gradient. The numbers should ma
tch
# almost exactly along all dimensions.
from cs231n.gradient_check import grad_check_sparse
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = svm_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

```
numerical: 15.978575 analytic: 15.978575, relative error: 2.679892
e-12
numerical: 1.192792 analytic: 1.192792, relative error: 2.837703e-
10
numerical: 10.196176 analytic: 10.196176, relative error: 4.551366
e-11
numerical: 3.172156 analytic: 3.172156, relative error: 8.977846e-
11
numerical: -5.340760 analytic: -5.340760, relative error: 3.898166
e-11
numerical: -1.698587 analytic: -1.698587, relative error: 2.324467
e-11
numerical: 27.401141 analytic: 27.401141, relative error: 4.099129
e-12
numerical: 5.696905 analytic: 5.696905, relative error: 1.156655e-
11
numerical: -7.815756 analytic: -7.815756, relative error: 4.824848
e-12
numerical: 4.624755 analytic: 4.624755, relative error: 4.442760e-
11
numerical: -6.451287 analytic: -6.451287, relative error: 2.643944
e-11
numerical: -2.091683 analytic: -2.091683, relative error: 1.105307
e-11
numerical: 18.316954 analytic: 18.316954, relative error: 1.353427
e-11
numerical: 1.854728 analytic: 1.854728, relative error: 2.874498e-
11
numerical: 38.956685 analytic: 38.956685, relative error: 4.295082
e-12
numerical: -4.943086 analytic: -4.943086, relative error: 6.702459
e-12
numerical: -22.534658 analytic: -22.534658, relative error: 1.4985
79e-11
numerical: 2.880195 analytic: 2.880195, relative error: 1.107867e-
10
numerical: -12.502013 analytic: -12.502013, relative error: 4.1086
56e-11
numerical: 32.012838 analytic: 32.012838, relative error: 9.233906
e-12
```

Inline Question 1:

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? *Hint: the SVM loss function is not strictly speaking differentiable*

Your Answer: It is possible since the SVM loss function is not differentiable at 0. As such, it has a step near 0 so there are values of W near where L(W)=0 where the function is very different. In other words, the derivative is not continuous in W.

In [47]:

```
# Next implement the function svm_loss_vectorized; for now only compute the lo
ss;
# we will implement the gradient in a moment.
tic = time.time()
loss_naive, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from cs231n.classifiers.linear_svm import svm_loss_vectorized
tic = time.time()
loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# The losses should match but your vectorized implementation should be much fa
ster.
print('difference: %f' % (loss_naive - loss_vectorized))
```

```
Naive loss: 9.309398e+00 computed in 0.106013s
Vectorized loss: 9.309398e+00 computed in 0.085901s
difference: 0.000000
```

In [58]:

```
# Complete the implementation of svm_loss_vectorized, and compute the gradient
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match, but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss and gradient: computed in %fs' % (toc - tic))

tic = time.time()
_, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, so
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('difference: %f' % difference)
```

```
Naive loss and gradient: computed in 0.101540s
Vectorized loss and gradient: computed in 0.007132s
difference: 0.000000
```

Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

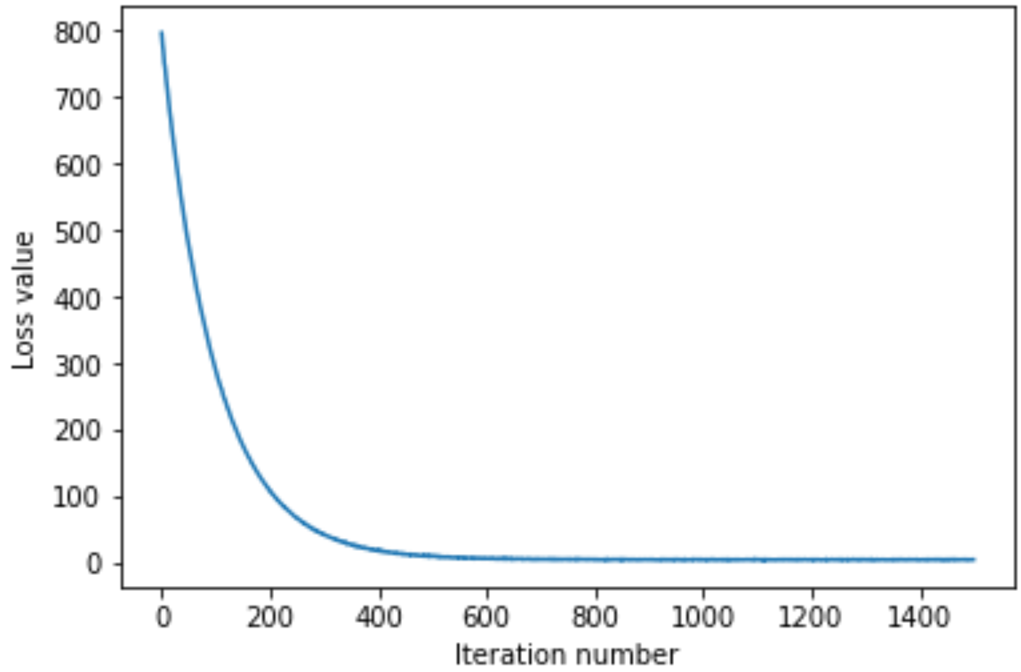
In [59]:

```
# In the file linear_classifier.py, implement SGD in the function
# LinearClassifier.train() and then run it with the code below.
from cs231n.classifiers import LinearSVM
svm = LinearSVM()
tic = time.time()
loss_hist = svm.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 796.870435
iteration 100 / 1500: loss 290.053241
iteration 200 / 1500: loss 109.270816
iteration 300 / 1500: loss 42.326102
iteration 400 / 1500: loss 18.329774
iteration 500 / 1500: loss 10.746536
iteration 600 / 1500: loss 7.233366
iteration 700 / 1500: loss 5.898094
iteration 800 / 1500: loss 5.558225
iteration 900 / 1500: loss 5.429921
iteration 1000 / 1500: loss 5.543017
iteration 1100 / 1500: loss 5.335327
iteration 1200 / 1500: loss 5.585750
iteration 1300 / 1500: loss 5.468736
iteration 1400 / 1500: loss 5.304937
That took 7.225673s
```

In [60]:

```
# A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```



In [71]:

```
# Write the LinearSVM.predict function and evaluate the performance on both th
e
# training and validation set
y_train_pred = svm.predict(X_train)
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = svm.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

```
training accuracy: 0.366408
validation accuracy: 0.373000
```

In [75]:

```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
# get a classification accuracy of about 0.4 on the validation set.
#learning_rates = [1e-7, 5e-5]
#regularization_strengths = [2.5e4, 5e4]
learning_rates = [2e-7, 1e-7, 5e-8]
regularization_strengths = [2e4, 5e4, 1e5]
# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fractio
n
# of data points that are correctly classified.
results = {}
best_val = -1   # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation ra
te
```

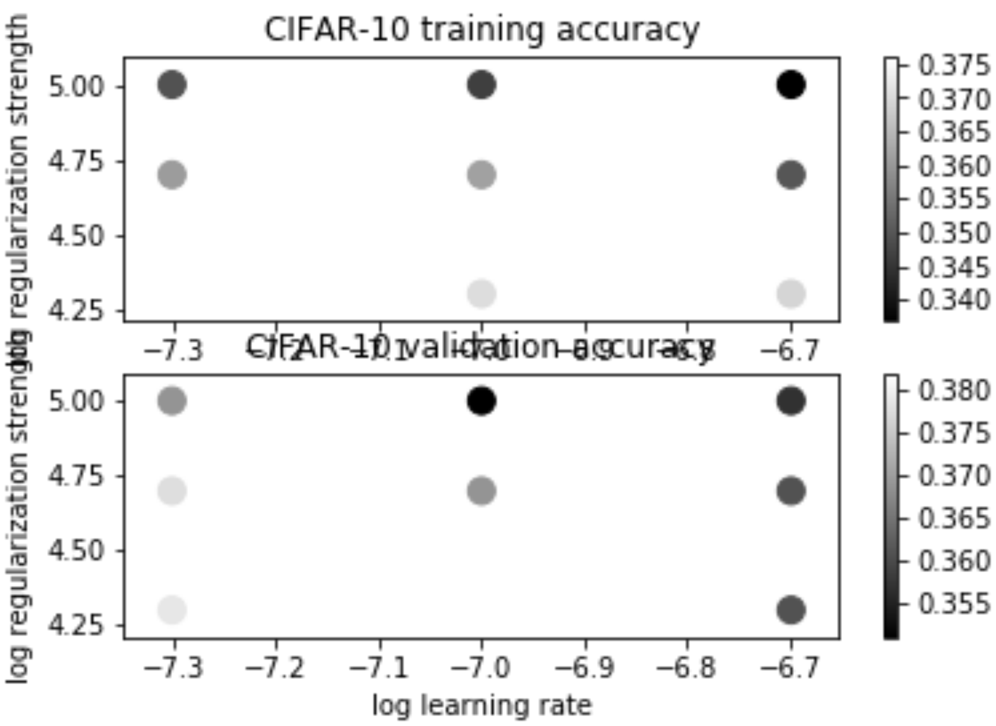
```
te.

################################################################################
##
# TODO:
#
# Write code that chooses the best hyperparameters by tuning on the validation
#
# set. For each combination of hyperparameters, train a linear SVM on the
#
# training set, compute its accuracy on the training and validation sets, and
#
# store these numbers in the results dictionary. In addition, store the best
#
# validation accuracy in best_val and the LinearSVM object that achieves this
#
# accuracy in best_svm.
#
#
#
# Hint: You should use a small value for num_iters as you develop your
#
# validation code so that the SVMs don't take much time to train; once you are
#
# confident that your validation code works, you should rerun the validation
#
# code with a larger value for num_iters.
#
################################################################################
##
for lr in learning_rates:
    for rs in regularization_strengths:
        svm = LinearSVM()
        svm.train(X_train, y_train, learning_rate=lr, reg=rs, num_iters=1500,
verbose=False)
        y_train_pred = svm.predict(X_train)
        training_accuracy = np.mean(y_train == y_train_pred)
        y_val_pred = svm.predict(X_val)
        validation_accuracy = np.mean(y_val == y_val_pred)
        results[(lr,rs)]= (training_accuracy, validation_accuracy)
        if validation_accuracy > best_val:
            best_val = validation_accuracy
            best_svm = svm

################################################################################
##
#                              END OF YOUR CODE
#
################################################################################
##

# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
                lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_v
```

```
al)
```

```
lr 5.000000e-08 reg 2.000000e+04 train accuracy: 0.375980 val accu
racy: 0.379000
lr 5.000000e-08 reg 5.000000e+04 train accuracy: 0.360735 val accu
racy: 0.378000
lr 5.000000e-08 reg 1.000000e+05 train accuracy: 0.349408 val accu
racy: 0.369000
lr 1.000000e-07 reg 2.000000e+04 train accuracy: 0.370735 val accu
racy: 0.382000
lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.361510 val accu
racy: 0.369000
lr 1.000000e-07 reg 1.000000e+05 train accuracy: 0.346592 val accu
racy: 0.351000
lr 2.000000e-07 reg 2.000000e+04 train accuracy: 0.369510 val accu
racy: 0.361000
lr 2.000000e-07 reg 5.000000e+04 train accuracy: 0.350061 val accu
racy: 0.361000
lr 2.000000e-07 reg 1.000000e+05 train accuracy: 0.336857 val accu
racy: 0.357000
best validation accuracy achieved during cross-validation: 0.38200
0
```

In [89]:

```
# Visualize the cross-validation results
import math
x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')

# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()
```



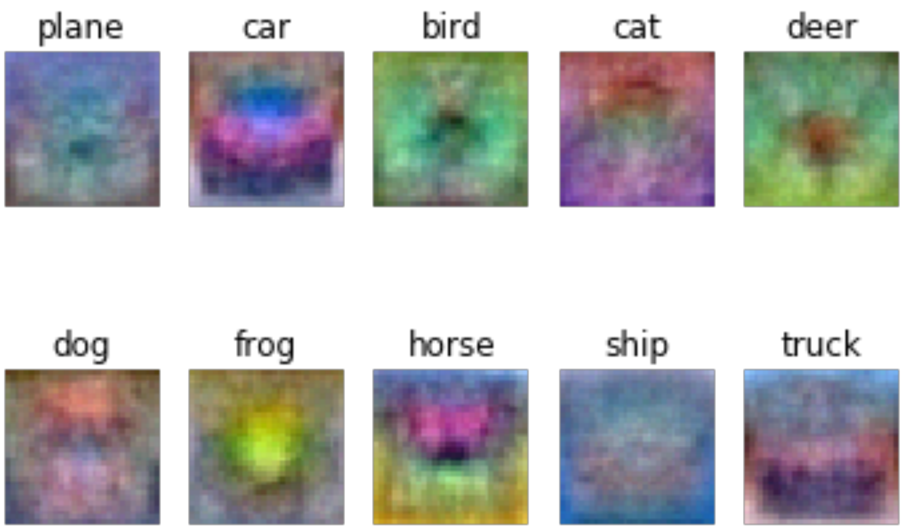
In [90]:

```
# Evaluate the best svm on test set
y_test_pred = best_svm.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

```
linear SVM on raw pixels final test set accuracy: 0.364000
```

In [91]:

```
# Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these
may
# or may not be nice to look at.
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'shi
p', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```



Inline question 2:

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look they way that they do.

Your answer: My visualized SVM weights look like some sort of averaging on photos of their corresponding classes. I like to think of it as described in the course's notes. Interpretating the linear classifiers as template matching.

This interpretation for the weights W is that each row of W corresponds to a template for one of the classes. The score of each class for an image is then obtained by comparing each template with the image using an inner product one by one to find the one that “fits” best. With this terminology, the linear classifier is doing template matching, where the templates are learned. Another way to think of it is that we are still effectively doing Nearest Neighbor, but instead of having thousands of training images we are only using a single image per class, and we use the (negative) inner product as the distance instead of the L1 or L2 distance.

Additionally, note that the horse template seems to contain a two-headed horse, which is due to both left and right facing horses in the dataset. The linear classifier merges these two modes of horses in the data into a single template. Similarly, the car classifier seems to have merged several modes into a single template which has to identify cars from all sides, and of all colors. In particular, this template ended up being red, which hints that there are more red cars in the CIFAR-10 dataset than of any other color.