

# k-Nearest Neighbor (kNN) exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the [assignments page \(http://vision.stanford.edu/teaching/cs231n/assignments.html\)](http://vision.stanford.edu/teaching/cs231n/assignments.html) on the course website.

The kNN classifier consists of two stages:

- During training, the classifier takes the training data and simply remembers it
- During testing, kNN classifies every test image by comparing to all training images and transferring the labels of the k most similar training examples
- The value of k is cross-validated

In this exercise you will implement these steps and understand the basic Image Classification pipeline, cross-validation, and gain proficiency in writing efficient, vectorized code.

```
In [280]: # Run some setup code for this notebook.

import random
import numpy as np
from cs231n.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

from __future__ import print_function

# This is a bit of magic to make matplotlib figures appear inline in
# rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

# Some more magic so that the notebook will reload external python modules
# see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in-jupyter
%load_ext autoreload
%autoreload 2
```

The autoreload extension is already loaded. To reload it, use:

```
%reload_ext autoreload
```

```
In [281]: # Load the raw CIFAR-10 data.
cifar10_dir = 'cs231n/datasets/cifar-10-batches-py'
X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

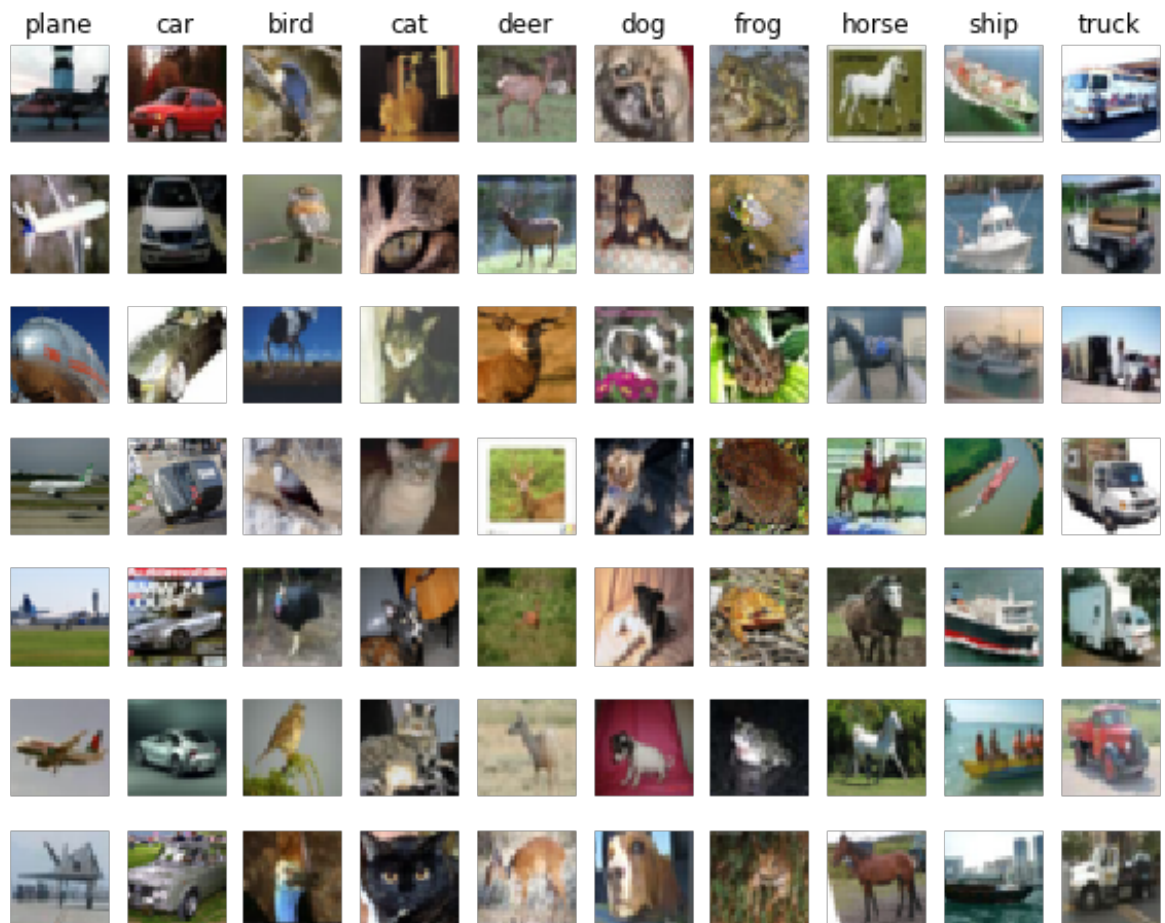
# As a sanity check, we print out the size of the training and test data
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)

Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)
```

```

In [282]: # Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()

```



```
In [283]: # Subsample the data for more efficient code execution in this exerci
num_training = 5000
mask = list(range(num_training))
X_train = X_train[mask]
y_train = y_train[mask]

num_test = 500
mask = list(range(num_test))
X_test = X_test[mask]
y_test = y_test[mask]
```

```
In [284]: # Reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
print(X_train.shape, X_test.shape)

(5000, 3072) (500, 3072)
```

```
In [285]: from cs231n.classifiers import KNearestNeighbor

# Create a kNN classifier instance.
# Remember that training a kNN classifier is a noop:
# the Classifier simply remembers the data and does no further proces
classifier = KNearestNeighbor()
classifier.train(X_train, y_train)
```

We would now like to classify the test data with the kNN classifier. Recall that we can break down this process into two steps:

1. First we must compute the distances between all test examples and all train examples.
2. Given these distances, for each test example we find the  $k$  nearest examples and have them vote for the label

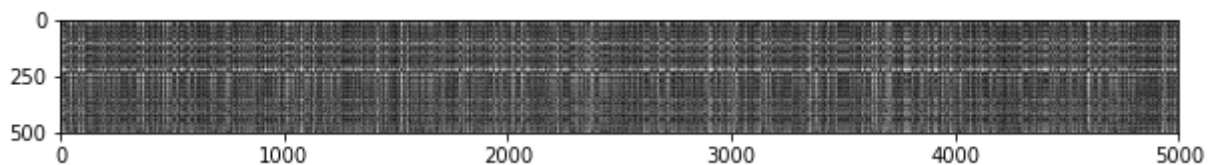
Lets begin with computing the distance matrix between all training and test examples. For example, if there are  $N_{tr}$  training examples and  $N_{te}$  test examples, this stage should result in a  $N_{te} \times N_{tr}$  matrix where each element  $(i,j)$  is the distance between the  $i$ -th test and  $j$ -th train example.

First, open `cs231n/classifiers/k_nearest_neighbor.py` and implement the function `compute_distances_two_loops` that uses a (very inefficient) double loop over all pairs of (test, train) examples and computes the distance matrix one element at a time.

```
In [286]: # Open cs231n/classifiers/k_nearest_neighbor.py and implement
# compute_distances_two_loops.
# Test your implementation:
dists = classifier.compute_distances_two_loops(X_test)
print(dists.shape)
```

(500, 5000)

```
In [287]: # We can visualize the distance matrix: each row is a single test exa
# its distances to training examples
plt.imshow(dists, interpolation='none')
plt.show()
```



**Inline Question #1:** Notice the structured patterns in the distance matrix, where some rows or columns are visible brighter. (Note that with the default color scheme black indicates low distances while white indicates high distances.)

- What in the data is the cause behind the distinctly bright rows?
- What causes the columns?

**\*\*Your Answer\*\*:**

The white rows indicate test points (observations) that have no training examples near them. This means that either they are observations that are not in any of the classes, or look very different from the training examples.

The white columns indicate training points that have no test points near them.

**Expansion:**

This can be explained by the the underline distribution of the data (although this is synthetic data). Meaning, since we look only at a small number of example points, they do not represent the entire distribution, hence some test points don't have the amount of examples near them proportional to their volume in the distribution. Another option is that the test points themselves are low probability instances in the distribution.

```
In [288]: # Now implement the function predict_labels and run the code below:
# We use k = 1 (which is Nearest Neighbor).
y_test_pred = classifier.predict_labels(dists, k=1)

# Compute and print the fraction of correctly predicted examples
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test,

Got 137 / 500 correct => accuracy: 0.274000
```

You should expect to see approximately 27% accuracy. Now lets try out a larger  $k$ , say  $k = 5$ :

```
In [289]: y_test_pred = classifier.predict_labels(dists, k=5)
num_correct = np.sum(y_test_pred == y_test)
accuracy = float(num_correct) / num_test
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test,

Got 139 / 500 correct => accuracy: 0.278000
```

You should expect to see a slightly better performance than with  $k = 1$ .

```
In [290]: # Now lets speed up distance matrix computation by using partial vect
# with one loop. Implement the function compute_distances_one_loop an
# code below:
dists_one = classifier.compute_distances_one_loop(X_test)

# To ensure that our vectorized implementation is correct, we make su
# agrees with the naive implementation. There are many ways to decide
# two matrices are similar; one of the simplest is the Frobenius norm
# you haven't seen it before, the Frobenius norm of two matrices is t
# root of the squared sum of differences of all elements; in other wo
# the matrices into vectors and compute the Euclidean distance betwee
difference = np.linalg.norm(dists - dists_one, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')

Difference was: 0.000000
Good! The distance matrices are the same
```

```
In [291]: # Now implement the fully vectorized version inside compute_distances
# and run the code
dists_two = classifier.compute_distances_no_loops(X_test)

# check that the distance matrix agrees with the one we computed before
difference = np.linalg.norm(dists - dists_two, ord='fro')
print('Difference was: %f' % (difference, ))
if difference < 0.001:
    print('Good! The distance matrices are the same')
else:
    print('Uh-oh! The distance matrices are different')
```

```
Difference was: 0.000000
Good! The distance matrices are the same
```

```
In [292]: # Let's compare how fast the implementations are
def time_function(f, *args):
    """
    Call a function f with args and return the time (in seconds) that
    """
    import time
    tic = time.time()
    f(*args)
    toc = time.time()
    return toc - tic

two_loop_time = time_function(classifier.compute_distances_two_loops, X_test)
print('Two loop version took %f seconds' % two_loop_time)

one_loop_time = time_function(classifier.compute_distances_one_loop, X_test)
print('One loop version took %f seconds' % one_loop_time)

no_loop_time = time_function(classifier.compute_distances_no_loops, X_test)
print('No loop version took %f seconds' % no_loop_time)

# you should see significantly faster performance with the fully vectorized version
```

```
Two loop version took 45.120206 seconds
One loop version took 39.336675 seconds
No loop version took 0.397479 seconds
```

## Cross-validation

We have implemented the k-Nearest Neighbor classifier but we set the value  $k = 5$  arbitrarily. We will now determine the best value of this hyperparameter with cross-validation.

```
In [293]: num_folds = 5
k_choices = [1, 3, 5, 8, 10, 12, 15, 20, 50, 100]

X_train_folds = []
```

```

y_train_folds = []
#####
# TODO:
# Split up the training data into folds. After splitting, X_train_fol
# y_train_folds should each be lists of length num_folds, where
# y_train_folds[i] is the label vector for the points in X_train_fold
# Hint: Look up the numpy array_split function.
#####
X_train_folds = np.array_split(X_train, num_folds)
y_train_folds = np.array_split(y_train, num_folds)
#####
#                                     END OF YOUR CODE
#####

# A dictionary holding the accuracies for different values of k that
# when running cross-validation. After running cross-validation,
# k_to_accuracies[k] should be a list of length num_folds giving the
# accuracy values that we found when using that value of k.
k_to_accuracies = {}
#####
# TODO:
# Perform k-fold cross validation to find the best value of k. For ea
# possible value of k, run the k-nearest-neighbor algorithm num_folds
# where in each case you use all but one of the folds as training dat
# last fold as a validation set. Store the accuracies for all fold an
# values of k in the k_to_accuracies dictionary.
#####
for k in k_choices:
    k_accuracy=[]
    for i in range(num_folds):
        X_validate = X_train_folds[i]
        y_validate = y_train_folds[i]

        X_train_partial = [X_train_folds[j] for j in range(num_folds)
        X_train_partial = np.concatenate(X_train_partial, axis=0)

        y_train_partial = [y_train_folds[j] for j in range(num_folds)
        y_train_partial = np.concatenate(y_train_partial, axis=0)

        classifier = KNearestNeighbor()
        classifier.train(X_train_partial, y_train_partial)

        dists = classifier.compute_distances_no_loops(X_validate)
        y_test_pred = classifier.predict_labels(dists,k)
        # Compute and print the fraction of correctly predicted examp
        num_correct = np.sum(y_test_pred == y_validate)
        num_test2 = y_test_partial.shape[0]
        accuracy = float(num_correct) / num_test2
        k_accuracy.append(accuracy)
    k_to_accuracies[k] = k_accuracy

#####
#                                     END OF YOUR CODE
#####

```



```
# Print out the computed accuracies
for k in sorted(k_to_accuracies):
    for accuracy in k_to_accuracies[k]:
        print('k = %d, accuracy = %f' % (k, accuracy))
```

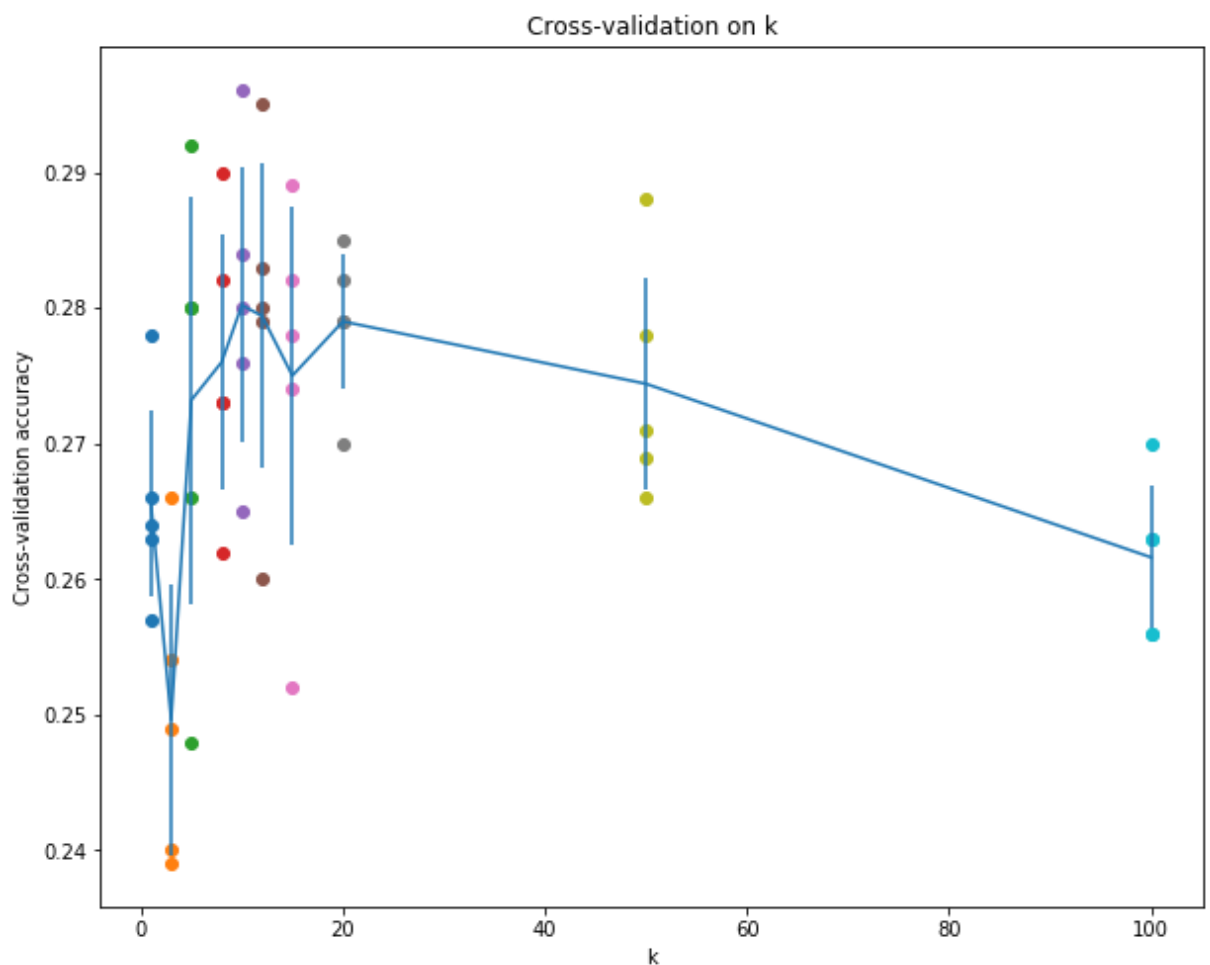
```
k = 1, accuracy = 0.263000
k = 1, accuracy = 0.257000
k = 1, accuracy = 0.264000
k = 1, accuracy = 0.278000
k = 1, accuracy = 0.266000
k = 3, accuracy = 0.239000
k = 3, accuracy = 0.249000
k = 3, accuracy = 0.240000
k = 3, accuracy = 0.266000
k = 3, accuracy = 0.254000
k = 5, accuracy = 0.248000
k = 5, accuracy = 0.266000
k = 5, accuracy = 0.280000
k = 5, accuracy = 0.292000
k = 5, accuracy = 0.280000
k = 8, accuracy = 0.262000
k = 8, accuracy = 0.282000
k = 8, accuracy = 0.273000
k = 8, accuracy = 0.290000
k = 8, accuracy = 0.273000
k = 10, accuracy = 0.265000
k = 10, accuracy = 0.296000
k = 10, accuracy = 0.276000
k = 10, accuracy = 0.284000
k = 10, accuracy = 0.280000
k = 12, accuracy = 0.260000
k = 12, accuracy = 0.295000
k = 12, accuracy = 0.279000
k = 12, accuracy = 0.283000
k = 12, accuracy = 0.280000
k = 15, accuracy = 0.252000
k = 15, accuracy = 0.289000
k = 15, accuracy = 0.278000
k = 15, accuracy = 0.282000
k = 15, accuracy = 0.274000
k = 20, accuracy = 0.270000
k = 20, accuracy = 0.279000
k = 20, accuracy = 0.279000
k = 20, accuracy = 0.282000
k = 20, accuracy = 0.285000
k = 50, accuracy = 0.271000
k = 50, accuracy = 0.288000
k = 50, accuracy = 0.278000
k = 50, accuracy = 0.269000
k = 50, accuracy = 0.266000
k = 100, accuracy = 0.256000
k = 100, accuracy = 0.270000
k = 100, accuracy = 0.263000
```

k = 100, accuracy = 0.256000

k = 100, accuracy = 0.263000

```
In [294]: # plot the raw observations
for k in k_choices:
    accuracies = k_to_accuracies[k]
    plt.scatter([k] * len(accuracies), accuracies)

# plot the trend line with error bars that correspond to standard dev
accuracies_mean = np.array([np.mean(v) for k,v in sorted(k_to_accurac
accuracies_std = np.array([np.std(v) for k,v in sorted(k_to_accuracie
plt.errorbar(k_choices, accuracies_mean, yerr=accuracies_std)
plt.title('Cross-validation on k')
plt.xlabel('k')
plt.ylabel('Cross-validation accuracy')
plt.show()
```



```
In [295]: # Based on the cross-validation results above, choose the best value  
# retrain the classifier using all the training data, and test it on  
# data. You should be able to get above 28% accuracy on the test data  
  
max_acc_indx = np.argmax(accuracies_mean)  
best_k = k_choices[max_acc_indx]  
  
classifier = KNearestNeighbor()  
classifier.train(X_train, y_train)  
y_test_pred = classifier.predict(X_test, k=best_k)  
  
# Compute and display the accuracy  
num_correct = np.sum(y_test_pred == y_test)  
accuracy = float(num_correct) / num_test  
print('Got %d / %d correct => accuracy: %f' % (num_correct, num_test,  
Got 141 / 500 correct => accuracy: 0.282000
```