Lecture 6: Call Admission Control

From "Broadband Integrated Networks" by M. Schwartz.

- Without Access Buffer
- With Access Buffer

Call Admission Control (CAC)

CAC: How much traffic can a network handle if a prescribed quality of service (QoS) for each traffic class is to be maintained?

We focus on on-off models of traffic. Fig. 4-1

Assuming that there are k traffic classes, each with its own traffic descriptors, to be multiplexed onto one network access link of capacity C_L cells/sec. Fig. 4-2

Note that there must be a maximum number of connections or calls of each class for a given scheduling strategy. These maximum numbers represent an admissible region for this system. Fig. 4-3

The admission region depends on the scheduling strategy, the access capacity C_L, the number of classes, and the QoS requirement of each class.

Given the admission region, in principle a call is admitted if the system, with the call present, still operates within the admission region. A call is blocked if its acceptance would take the system outside of the admission region.

Example: Consider N homogeneous sources. Fig. 4-4

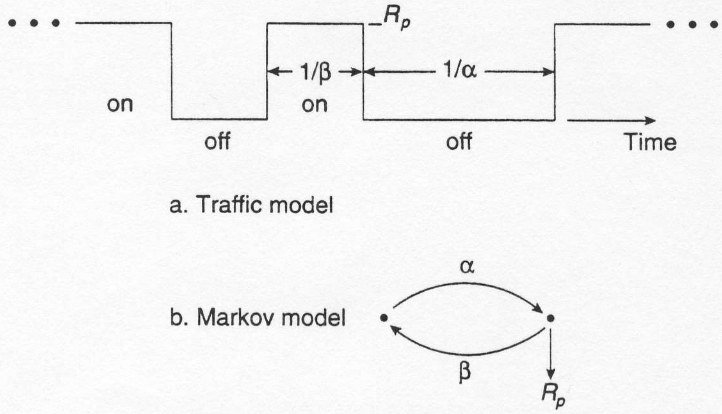

FIGURE 4–1 ■ Basic on-off traffic source.

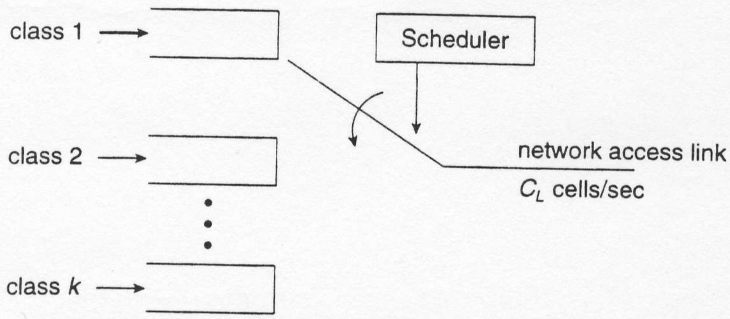

FIGURE 4–2 ■ Network access node with access scheduler.

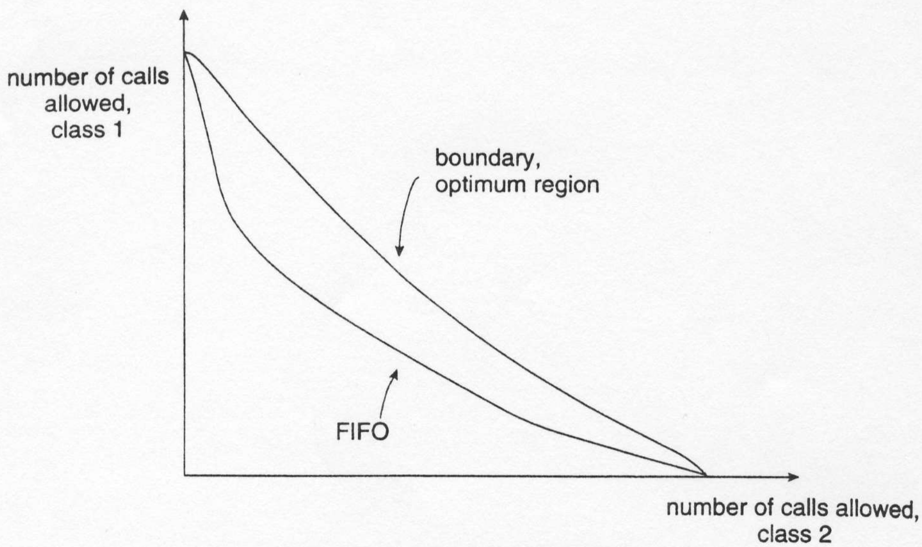

FIGURE 4–3 ■ Two-class admissible regions.

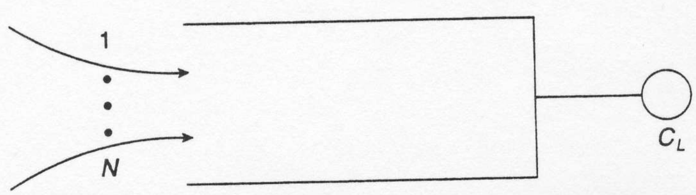

FIGURE 4–4 ■ Statistical multiplexing of homogeneous sources.

The probability that a source is in the "on" state is $p = \frac{\alpha}{\alpha+\beta}$. The average rate of transmission from each source is pR_p

The utilization is $\rho = NpR_p/C_L$.

Let $\rho = 1$, then the maximum N can be obtained: $N = \lfloor \frac{C_L}{pR_p} \rfloor$

For k traffic classes:

$$\sum_{i=1}^{k} n_i p_i R_{pi} = C_L$$

Tradeoffs are possible among the maximum number of calls allowed per user class.

For homogeneous case (Fig. 4-5):

Peak bandwidth assignment: $p = 1$, or $NR_p = C_L$. No statistical multiplexing gain. No cell loss.

Average bandwidth assignment: $NpR_p = C_L$. Maximum multiplexing gain. Maybe unacceptable in terms of cell loss.

Question: for a given QoS (cell loss) and C_L, how to find the maximum N?

Define $m \equiv pN$ as the mean (average) number of sources "on", and σ as the standard deviation of the number of "on" sources. The mean traffic generation rate inputted by these m connections is mR_p. $\sigma^2 = Np(1-p) = m(1-p)$. Apparently, $C_L \geq mR_p$.

Let

$$C_L = (m + K\sigma)R_p$$

with K a constant to be determined.

Let

$$C = C_L/R_p = m + K\sigma = Np + K\sqrt{Np(1-p)}$$

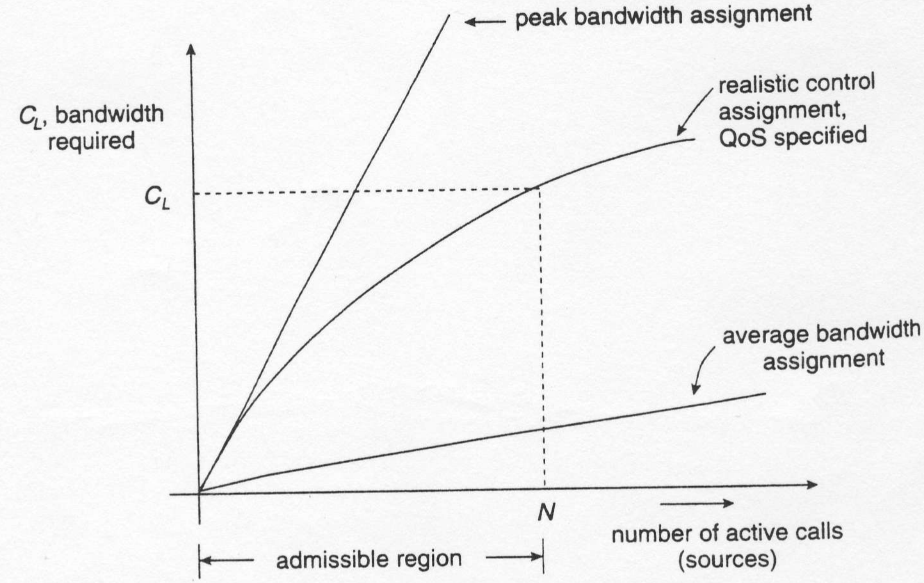

FIGURE 4–5 ■ Admission control, homogeneous on-off sources.

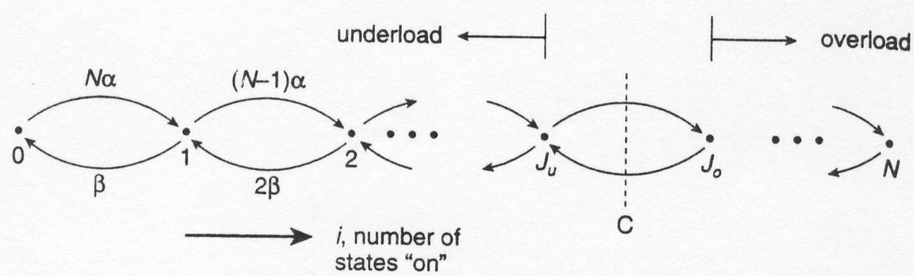

FIGURE 4–6 ■ State diagram, N on-off sources.

For the Markov chain (Fig. 4-6) of the multiplexed N on-off sources, state i means the number of "on" sources is i.

$$J_o = \lceil C \rceil, J_u = \lfloor C \rfloor$$
$$\rho = m/C < 1$$

Without Access Buffer

In overload states, all cells arriving beyond the capacity C will be lost. Then in overload state i, the cell loss rate is $(i - C)R_p$. The overall cell loss rate over all the overload states is

$$\sum_{i=J_o}^{N} (i - C) R_p \pi_i$$

with π_i being the steady state probability of state i in the preceding Markov chain, given by

$$\pi_i = \binom{N}{i} p^i (1-p)^{N-i}.$$

The total cell flow is given by mR_p. Then the cell loss probability

$$P_L = \frac{\sum_{i=J_o}^{N} (i - C) R_p \pi_i}{mR_p} = \sum_{i=J_o}^{N} \frac{(i - C)\pi_i}{m}$$

Another measure of loss probability is simply the *probability of the system being in overload states*:

$$\varepsilon = \sum_{i=J_o}^{N} \pi_i.$$

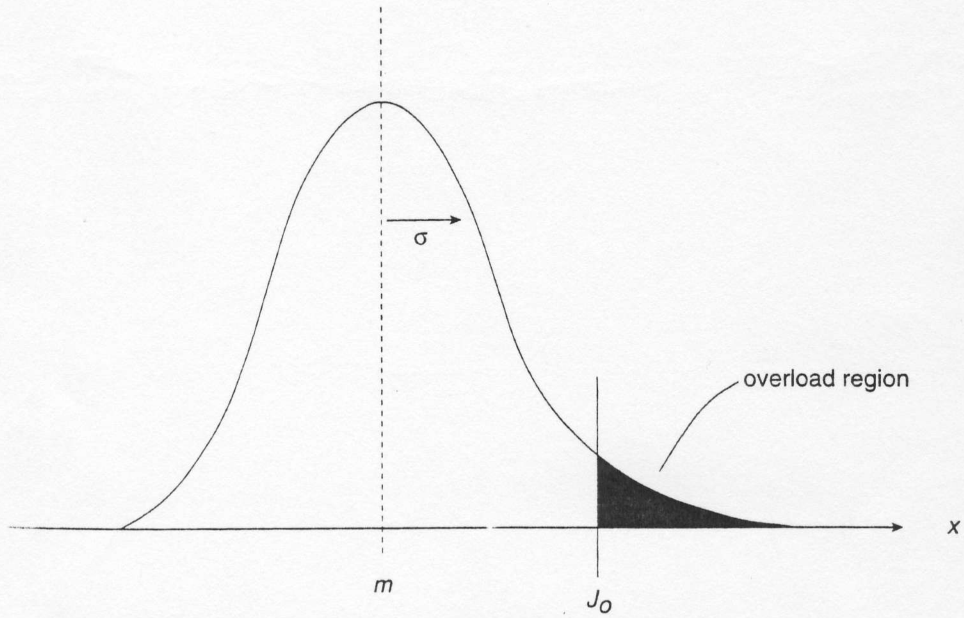

FIGURE 4–7 ▪ Gaussian approximation to binomial distribution: $m = Np$; $\sigma^2 = m(1 - p)$.

Assume $N \gg 1$ and $p \ll 1$. Then the binomial distribution solution for π_i,

$$\pi_i = \binom{N}{i} p^i (1-p)^{N-i}$$

is approximated quite closely by the normal distribution with the same mean value $m = Np$ and variance $\sigma^2 = Np(1-p)$. And the overload region corresponds to the tail of the distribution.

$$\begin{aligned}
P_L &= \sum_{i=J_o}^{N} \pi_i \frac{i-C}{m} \\
&\doteq \frac{1}{m} \int_{J_o}^{\infty} \frac{e^{-(x-m)^2/2\sigma^2}(x-C)}{\sqrt{2\pi\sigma^2}} dx \\
& \quad \text{by replacing } J_o \text{ by } C \text{ and writing} (x-C) = (x-m) - (C-m) \\
&\doteq \frac{1}{m} \left[\int_{C}^{\infty} \frac{(x-m)e^{-(x-m)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} dx - (C-m)\varepsilon \right] \\
&= \frac{\sigma}{m} \frac{e^{-(C-m)^2/2\sigma^2}}{\sqrt{2\pi}} - \frac{(C-m)\varepsilon}{m} \\
\varepsilon &\doteq \int_{J_o}^{\infty} \frac{e^{-(x-m)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}} dx
\end{aligned}$$

If $(C-m) > 3\sqrt{2}\sigma$, after some algebra

$$\begin{aligned}
\varepsilon &\doteq \frac{\sigma e^{-(C-m)^2/2\sigma^2}}{\sqrt{2\pi}(C-m)} \\
P_L &\doteq \frac{(1-p)\sigma e^{-(C-m)^2/2\sigma^2}}{\sqrt{2\pi}(C-m)^2} = \frac{1-p}{C-m}\varepsilon
\end{aligned}$$

Consider the approximation for ε. Taking natural logs, we get

$$\ln(\sqrt{2\pi}\varepsilon) = \ln\left(\frac{\sigma}{C-m}\right) - \frac{(C-m)^2}{2\sigma^2}$$

Neglecting the first term on the right-hand side:

$$C \doteq m + \sigma\sqrt{-\ln(2\pi) - 2\ln\varepsilon}$$

Recall that $C = m + K\sigma$. We have

$$K = \sqrt{-\ln(2\pi) - 2\ln\varepsilon}$$

Consider the approximation for P_L. Similarly we have

$$C \doteq m + \sigma\sqrt{-\ln(2\pi) - 2\ln P_L}$$

For example, if we set $\varepsilon = 10^{-5}$, we get $K = 4.6$. So

$$C = m + 4.6\sigma$$

or

$$C = Np + 4.6\sqrt{Np(1-p)}$$

If $p = 0.02$: Fig. 4-8

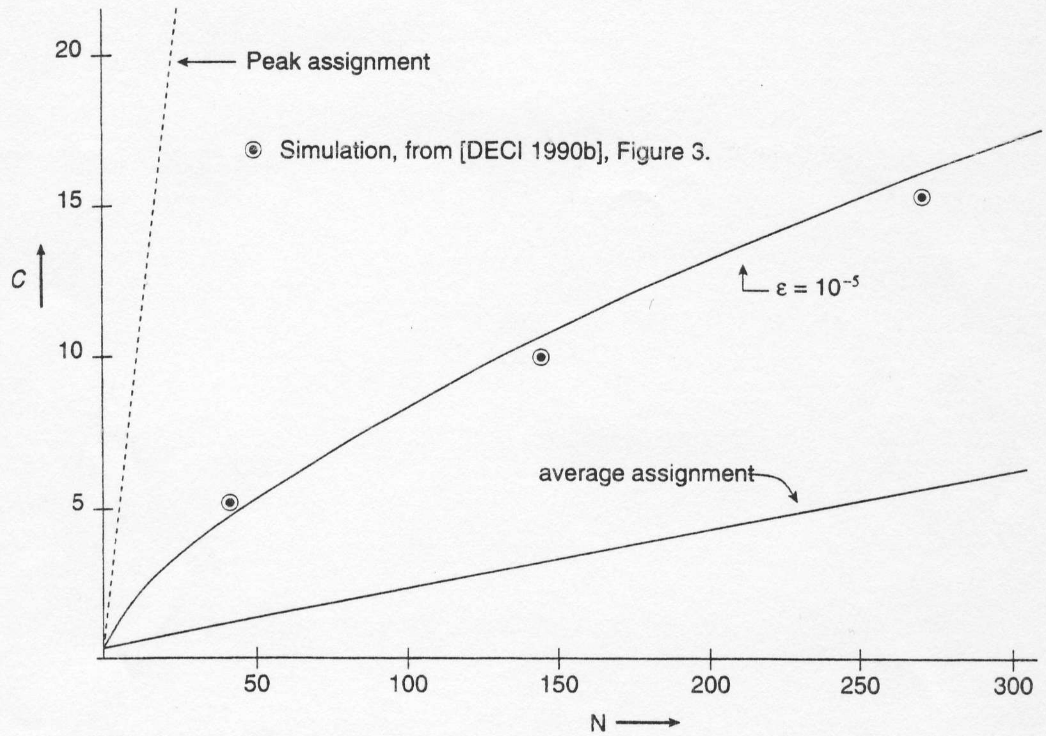

FIGURE 4–8 ■ Admission control, *N* homogeneous sources, $p = 0.02$.

In general

$$C = Np + K\sqrt{Np(1-p)}$$

Then

$$N = \frac{C}{p} - \frac{1}{p}\left[\sqrt{4\omega(C+\omega)} - 2\omega\right]$$

where $\omega = K^2(1-p)/4$

With Access Buffer

The fluid-flow analysis approach is used.

Assume there are M on-off minisources statistically multiplexed at the access buffer:

- M is replaced by N
- KA bits/sec ($K = 7.5 \times 10^6$ pixels/sec) is replaced by R_p
- KC bits/sec is replaced by C_L

The survivor function $G(x)$ for the M-minisource model (the probability that the buffer occupancy exceeds x bits) can be approximated by

$$G(x) \sim A_N \rho^N e^{-\beta r x/R_p}$$

where

$$r = (1-\rho)\left(1+\frac{\alpha}{\beta}\right) / \left(1 - \frac{C_L}{NR_p}\right)$$

$$\rho = NpR_p/C_L < 1$$

Assume traffic loss happens when the buffer occupancy exceeds x. Then $G(x)$ will be an approximation to P_L.

Using the exponential part $e^{-\beta r x/R_p}$ in $G(x)$ only:

$$P_L \doteq e^{-\beta r x/R_p}$$

The solution is

$$\frac{C_L}{R_p N} = \frac{1-k}{2} + \sqrt{\left(\frac{1-k}{2}\right)^2 + kp}$$

where $k = \frac{\beta x}{R_p(1-p)\ln(1/P_L)}$

Example

N on-off sources are to be multiplexed together at an access server. Each source has exponentially distributed on- and off-durations, with average values of 1 sec and 10 sec, respectively. When "on", a source transmits at its peak rate of 5 Mbps. The outgoing link capacity of the multiplexer is 100 Mbps.

a) Find the number of sources that may be accommodated if (1) peak rate allocation is used; (2) average rate allocation is used. What is the probability of loss with peak rate allocation?

b) Find the number of sources that may be multiplexed if the probability of loss is $P_L = 10^{-6}$. P_L is approximated by the average time portion the multiplexer is in the overload region.

c) Repeat b) if the approximate fluid-flow analysis is used, with the probability of loss defined as $P_L = P[\text{buffer occupancy} > x]$, where x is chosen such that the maximum buffer delay is 1 sec.

Solution:

$R_p = 5$ Mbps, $C_L = 100$ Mbps, $1/\beta = 1$ sec, $1/\alpha = 10$ sec.

a) For peak rate allocation:

$$N = C_L/R_p = 100/5 = 20$$

For average rate allocation:

$$N = C_L/R_p p = \frac{100}{5 \cdot \frac{1}{10+1}} = 220$$

The probability of loss with peak rate allocation is $P_L = 0$.

b) $P_L = 10^{-6}$, $P_L = \varepsilon$

$C = C_L/R_p = Np + K\sqrt{Np(1-p)}$

$K = \sqrt{-\ln(2\pi) - 2\ln(P_L)} = 5.1$

$\Rightarrow \frac{100}{5} = N \times \frac{1}{11} + 5.1\sqrt{N \times \frac{1}{11} \times \frac{10}{11}}$

$\Rightarrow N = 77$

c) $P_L = P$[buffer occupancy $> x$].

The maximum buffer delay is 1 sec, i.e., $\frac{x}{C_L} = 1$ sec $\Rightarrow$ x =1 sec $\times$ 100 Mbps = 100 Mbits.

Since $\frac{C_L}{R_p N} = \frac{1-k}{2} + \sqrt{\left(\frac{1-k}{2}\right)^2 + kp}$

$$k = \frac{\beta x}{R_p(1-p)\ln(1/P_L)} = 1.59$$

$\Rightarrow N = 107$

This result lies between the peak rate assignment and average rate assignment.