Lecture #3

Chapter 5 Fundamentals of Cellular Communications

From ”Wireless Communications and Networking” by J. W. Mark and W. Zhuang

- Cellular concept and frequency reuse
- Cochannel and adjacent channel interference
- Trunking and grade of service
- Mechanisms for capacity increase

5.1 Cellular Concept

- Given a propagation environment, increasing transmitted power will increase the service coverage area.

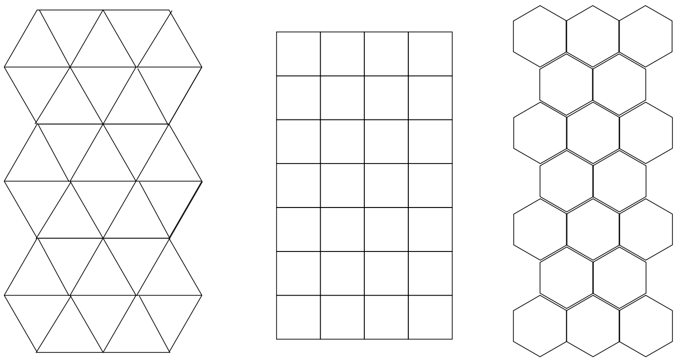
  $\Longrightarrow$ The coverage area can be controlled by using a proper transmitted power level.

- In cellular systems, the total service area is divided into a number of smaller areas, each of which is a radio cell.

  $\Longrightarrow$ Advantages:

  - Low transmitted power
  - Frequency reuse possible.

- Regular polygons may be used to represent the cell coverage.



(a) (b) (c)

- Hexagonal cells are popular because

  - closest to a circle
  - tight cellular packing
  - perfect partitioning of the service area.

- Frequency reuse is limited by co-channel interference. Cells which use the same frequency channels are called co-channel cells.

- Frequency is reused from cell cluster to cell cluster. No frequency channel is reused among cells in the same cell cluster.

  $\Longrightarrow$ Cells in each cell cluster use unique frequency channels.

Let

K - total number of channels in the system without frequency reuse

N - the number of cells in each cell cluster

J - total number of channels in each cell

then

$$K = JN \quad \text{or} \quad J = K/N.$$

Let

M - total number of cell clusters in the system

C - total number of channels in the system with frequency reuse

then

$$C = MK = MJN.$$

$\Longrightarrow$ cluster size $N \downarrow$

$\longrightarrow M \uparrow$ to cover the same area

$\longrightarrow C \uparrow$ for a given K value

That is, given K, C is maximized when N is minimized. However, the minimum N depends on the requirement on the co-channel interference level.

The cell cluster size N is also called the frequency reuse factor.

5.2 Frequency Reuse Factor

Consider hexagonal cells

- A hexagonal cells has exactly 6 equidistant neighbours.
- The lines joining the centers of any cell and each of its neighbours are separated by multiples of 60 degrees.

Nearest co-channel neighbours

To find the nearest co-channel neighbour of a particular cell, execute the folllowing two steps:

- move i cells along any chain of hexagons
- turn 60 degrees counterclockwise and move j cells

where the integers i and j are parameters for determining co-channel cells and for determining the size of the cell cluster (N)

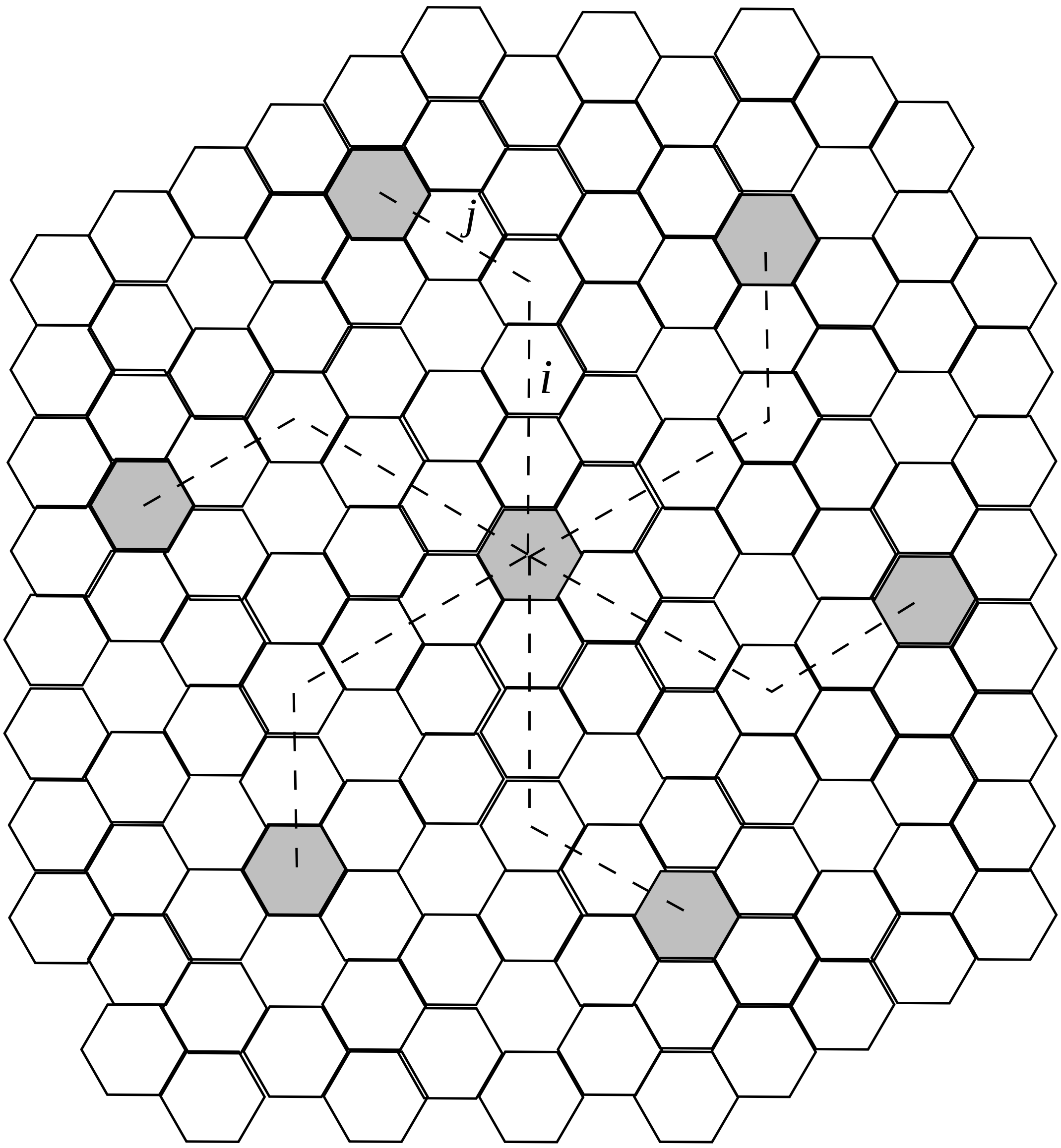
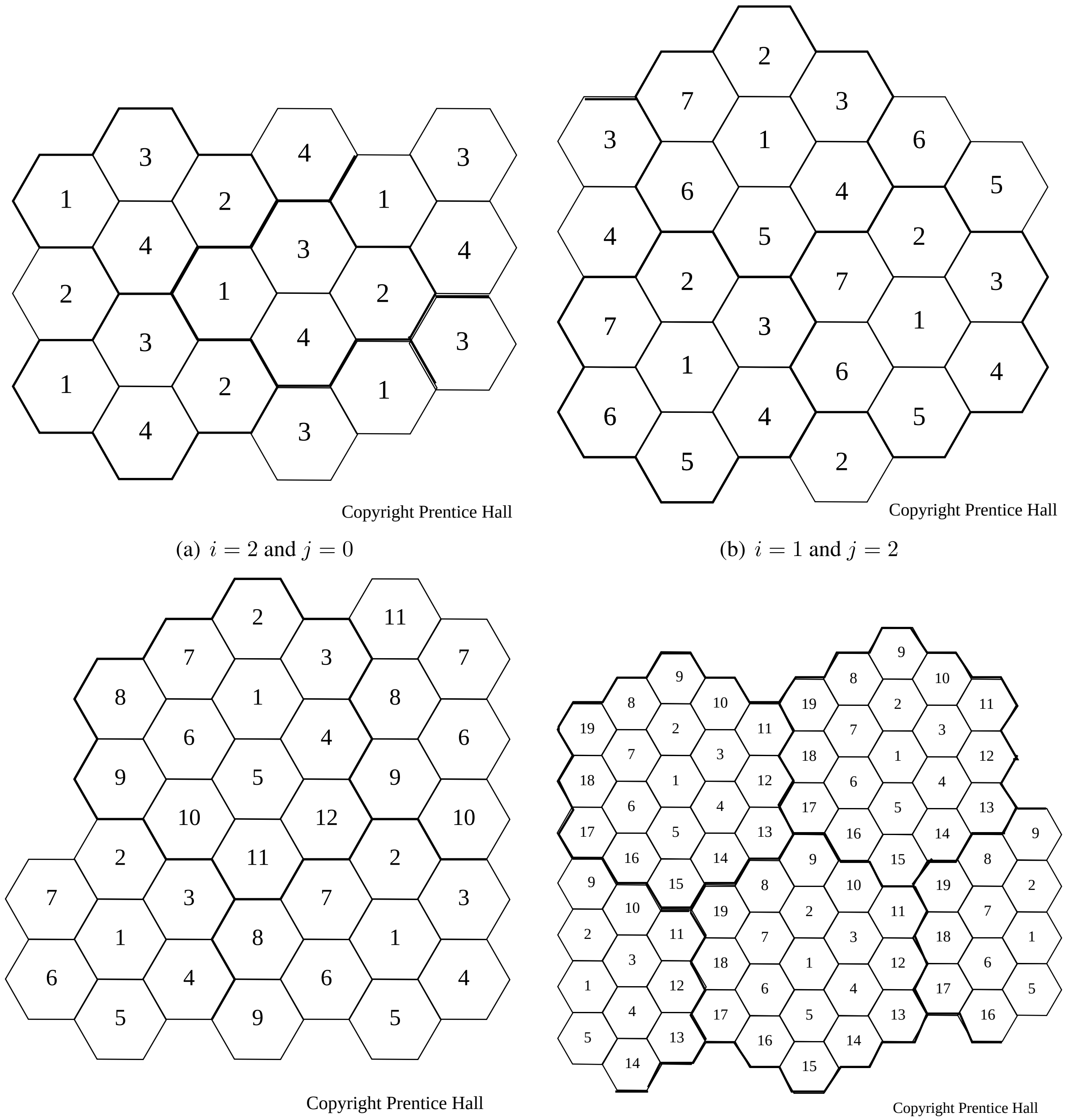


Figure 1: Locating cochannel cells in a cellular system



(a) $i = 2$ and $j = 0$

(b) $i = 1$ and $j = 2$

(c) $i = 2$ and $j = 2$

(d) $i = 2$ and $j = 3$

Figure 2: Cell clusters

Geometry of hexagonal cells

Let

R - radius of the cell (from center to vertex)

D - distance from the center of the candidate cell to the cell of the nearest co-channel cell

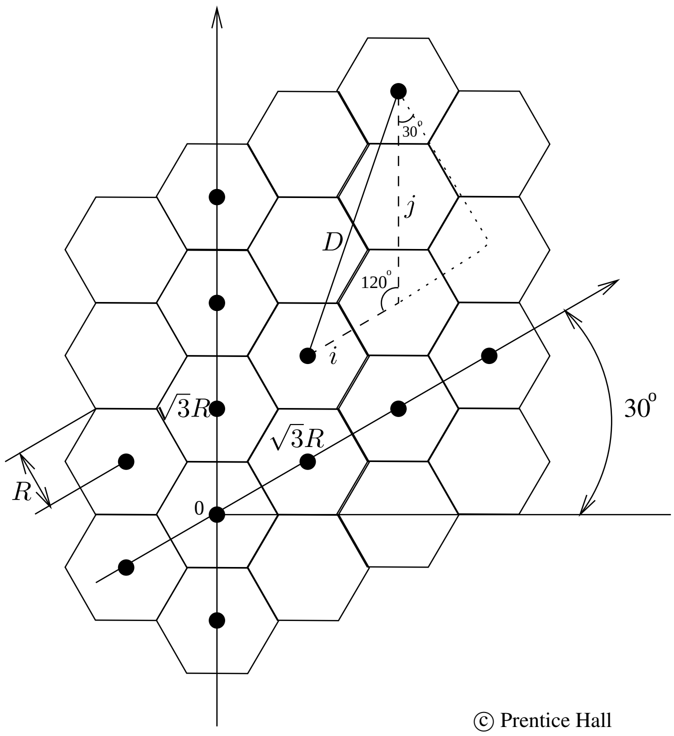


Figure 3: Distance between nearest cochannel cells

The actual distance between the centers of two adjacent cells is

$$2R\cos 30^0 = 2R\frac{\sqrt{3}}{2} = \sqrt{3}R.$$

$\Longrightarrow$

$$\begin{aligned} D^2 &= (j \cdot \sqrt{3}R\cos 30^0)^2 + (i \cdot \sqrt{3}R + j \cdot \sqrt{3}R\sin 30^0)^2 \\ &= (i^2 + j^2 + ij)3R^2 \end{aligned}$$

or,

$$\begin{aligned} D^2 &= (i \cdot \sqrt{3}R)^2 + (j \cdot \sqrt{3}R)^2 - 2(i \cdot \sqrt{3}R)(j \cdot \sqrt{3}R)\cos 120^0 \\ &= (i^2 + j^2 + ij)3R^2 \end{aligned}$$

It can be shown that the cell cluster size is given by

$$N = i^2 + j^2 + ij.$$

$\Longrightarrow$

$$D^2 = 3NR^2, \quad \text{or} \quad D = \sqrt{3N}R.$$

$$\Longrightarrow \quad D_{\text{norm}} \triangleq \frac{D}{\sqrt{3}R} = \sqrt{N}.$$

Frequency reuse ratio q

$$q \triangleq \frac{D}{R} = \frac{\sqrt{3N}R}{R} = \sqrt{3N}.$$

- The frequency reuse ratio q and the frequency reuse factor N carry the same information:

$$\begin{aligned} q \text{ (or } N) \uparrow &\implies \text{ cochannel interference} \downarrow \\ &\implies \text{ frequency reuse less often and system capacity} \downarrow \end{aligned}$$

- We should choose the minimum q (or N) subject to the constraint on the signal to cochannel interference ratio requirement.

Table 1: Frequency reuse ratio and cluster size

Frequency Reuse Pattern (i, j)	Cluster Size N	Frequency Reuse Ratio q
$(1, 1)$	3	3.00
$(2, 0)$	4	3.46
$(2, 1)$	7	4.58
$(3, 0)$	9	5.20
$(2, 2)$	12	6.00
$(3, 1)$	13	6.24
$(4, 0)$	16	6.93
$(3, 2)$	19	7.55
$(4, 1)$	21	7.94
$(3, 3)$	27	9.00
$(4, 2)$	28	9.17
$(4, 3)$	37	10.54

5.3 Cochannel and Adjacent Channel Interference

Cochannel interference

Let

N_I - the number of co-channel interfering cells

I_i - cochannel interference from the ith co-channel cell

S - the received power of the desired signal

The signal-to-cochannel interference ratio (S/I), also referred to as carrier-to-co-channel interference ratio (CIR), is

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{N_I} I_i}.$$

Consider only distance-dependent path loss. From chapter 2, we have

$$P_r(d) = P_0(d/d_0)^{-\kappa}$$

where

$P_r(d)$ – the received power at distance d $(\geq d_0)$

P_0 – the received power at distance d_0

κ - the path loss exponent

d - the distance between the transmitter and receiver.

Consider the forward link and assume that the transmitted power levels from all the BSs are the same, then

$$I_i \propto D_i^{-\kappa}$$

where D_i is the distance from the ith cochannel cell BS to the mobile.

When the mobile is at the cell boundary (the worst case),

$$S \propto R^{-\kappa}$$

$\Longrightarrow$

$$\frac{S}{I} = \frac{R^{-\kappa}}{\sum_{i=1}^{N_I} D_i^{-\kappa}}$$

which which is not a function of the transmitted power!

If we neglect cochannel interference from the second and other higher tiers, this means that $N_I = 6$. ~~In the case that~~ $\cancel{r = R}$ ~~and~~ using $D_i \approx D$ for $i = 1, 2, \ldots, N_I$,

$$\frac{S}{I} = \frac{(D/R)^{\kappa}}{N_I} = \frac{q^{\kappa}}{N_I} = \frac{(\sqrt{3N})^{\kappa}}{N_I}.$$

$\Longrightarrow$

$$q = \left(N_I \times \frac{S}{I}\right)^{1/\kappa} = \left(6 \times \frac{S}{I}\right)^{1/\kappa}.$$

Example 5.4 S/I ratio versus cluster size

Suppose the acceptable signal-to-cochannel interference ratio in a certain cellular communications situation is $S/I = 20$ dB or 100. Also, from measurements, it is determined that $\kappa = 4$. What is the minimum cluster size?

Solution:

The frequency reuse ratio can be calculated as

$$q = (6 \times 100)^{1/4} = 4.9492.$$

Then, the cluster size is given by

$$N = q^2/3 = 8.165 \simeq 9.$$

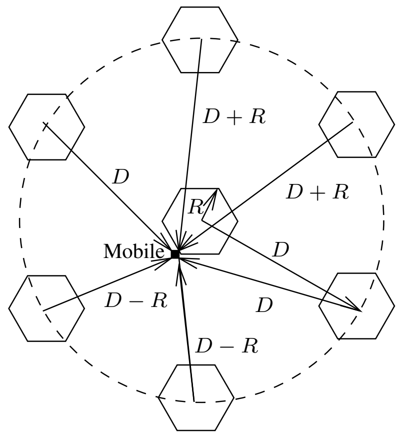
In this case, a 9-reuse pattern is needed for an S/I ratio of at least 20 dB. Since

$$q = D/R \quad \text{or} \quad D = qR,$$

D can be determined, given the cell radius R, and vice versa. Note that if N is less than 9, the S/I value would be below the acceptable level of 20 dB.

□

Consider a better approximation of the distances:



$$\frac{S}{I} \approx \frac{R^{-\kappa}}{2(D-R)^{-\kappa} + 2D^{-\kappa} + 2(D+R)^{-\kappa}}.$$

Substituting $D/R = q$, we have

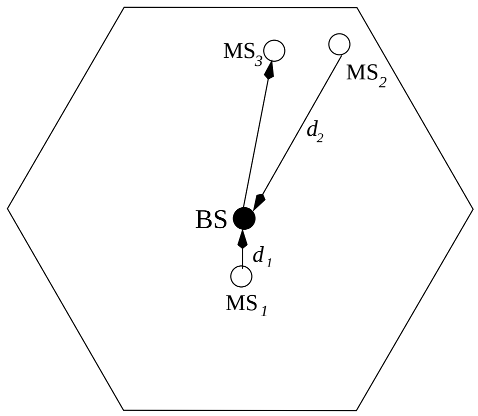
$$\frac{S}{I} = \frac{1}{2(q-1)^{-\kappa} + 2q^{-\kappa} + 2(q+1)^{-\kappa}}.$$

For $\kappa = 4$ and $N = 7$, $q = \sqrt{3N} = 4.6 \Longrightarrow S/I = 17.3$ dB

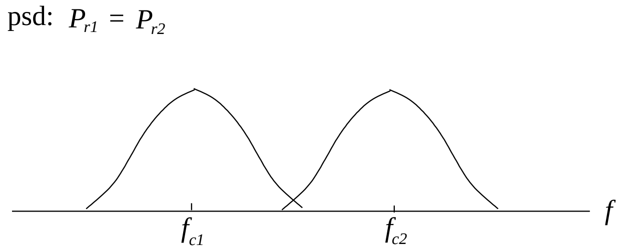
For $\kappa = 4$ and $N = 9$, $q = \sqrt{3N} = 5.2 \Longrightarrow S/I = 19.8$ dB

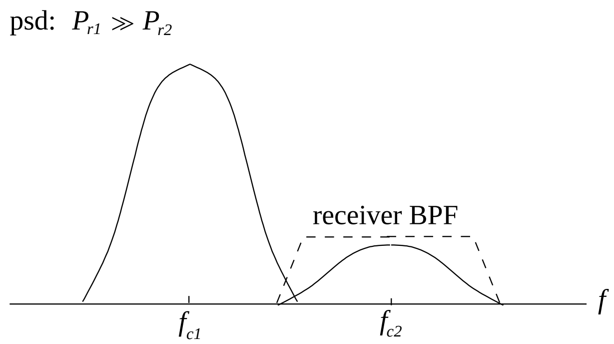
Design tradeoff: $N = 7$ or 9 if it requires $S/I = 18$ dB?

Adjacent channel interference



Near-far effect: $d_1 \ll d_2 \Longrightarrow P_{r1} \gg P_{r2}$ at the BS





For the signal from MS_2, the adjacent channel interference ↑↑ due to the near-far effect.

To reduce adjacent channel interference

- use modulation schemes which have small out-of-band radiation (e.g., MSK is better than QPSK)
- carefully design the receiver BPF
- use proper channel interleaving by assigning adjacent channels to different cells, e.g., for $N = 7$

Channel	1	2	3	4	5	6	7	8	9	10	11
Cell	A	B	C	D	E	F	G	A	B	C	D

f

- furthermore, do not use adjacent channels in adjacent cells, which is possible only when N is very large. For example, if N =7, adjacent channels must be used in adjacent cells
- use FDD or TDD to separate the forward link and reverse link.

5.4 Trunking and Grade of Service

- In cellular systems, a relatively small number of radio channels are used to serve a large population of mobile users, which is possible by frequency reuse and by trunking
- Trunking exploits the statistical behaviour of mobile users, so that a large number of mobile users can share the fixed radio channels in each cell on demand
- Based on traffic load, the number of radio channels in each cell should be determined in such a way that
  - all the channels are utilized efficienctly
  - call blocking rate is below a predetermined threshold
- Given a traffic load, number of channels $\uparrow \implies$ utilization efficiency $\downarrow$ and call blocking rate $\downarrow$
- The measure of traffic efficiency: 1 Erlang represents the amount of traffic intensity carried by a channel that is completely occupied, e.g., a radio channel that is occupied for 30 minutes during an hour carries 0.5 Erlangs of traffic
- The grade of service (GoS) is a measure of the ability of a user to access a trunked system during the busiest hour. GoS is typically given as
  - the likelihood that a call is blocked (for Erlang B systems), or
  - the likelihood that a call experiences a delay larger than a certain queueing delay (for Erlang C systems)

Definitions

- blocked call (lost call): call which cannot be completed at the time of request, due to congestion
- holding time (H): average duration of a typical call
- traffic intensity (ρ) to the system: measured in Erlangs
- request rate (λ_u): the average number of call requests per unit time per user

Relations

- The traffic intensity offered by each user is (in Erlangs)

$$\rho_u = \lambda_u \cdot H$$

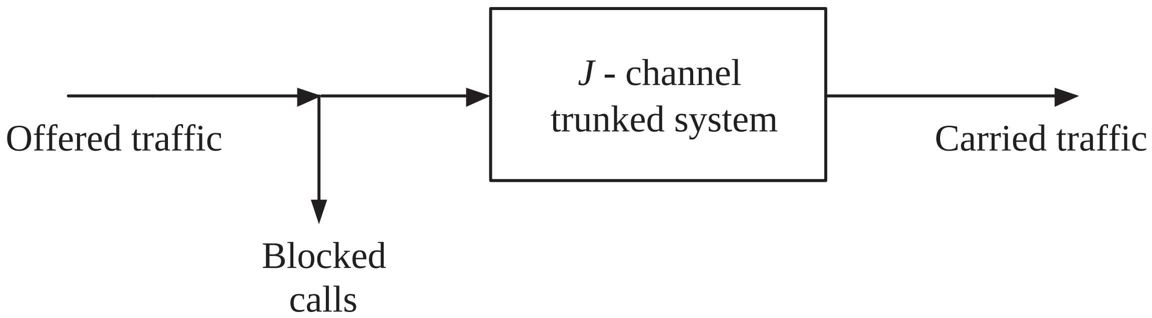
- For a system with u users and an unspecified number of channels, the total offered traffic intensity is (in Erlangs)

$$\rho = u \cdot \rho_u = u \cdot \lambda_u \cdot H$$

- In a Jchannel trunked system, if the traffic is equally distributed among the channels, then the traffic intensity per channel is

$$\rho_c = u \cdot \rho_u / J = u \cdot \lambda_u \cdot H/J \quad \text{(Erlangs)}$$

- Difference between offered traffic and carried traffic



where the intensity of the offered traffic is ρ, while the intensity of the carried traffic is $\rho(1 - P_B) \leq \rho$.

Types of trunked systems

Blocked calls cleared

- If no channels are available, the requesting user is blocked without access and is free to try again later;
- Call arrivals follows a Poisson distribution. Let X denote the number of calls arrivales per unit time, then

$$P(X = i) = e^{-\lambda} \cdot \frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \ldots$$

  where λ is the request rate (average number of call arrivals per unit time), and we have $E(X) = \lambda, V(X) = \lambda$;
- Channel holding time, Y, follows an exponential distribution

$$f_Y(y) = \begin{cases} \mu e^{-\mu x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

  with $E(Y) = H = 1/\mu$ and $V(Y) = 1/\mu^2$;
- There are a finite number of channels available in the trunked pool.

$\Longrightarrow$ The system can be modeled by an M/M/N/N queue ($N = J$).

$\Longrightarrow$ The Erlang-B formula (the blocked calls cleared formula):

$$P(\text{blocking}) = \frac{\rho^J/J!}{\sum_{j=0}^{J} \rho^j/j!} \quad \text{– call blocking probability}$$

Here $\rho = \lambda/\mu$, which is also consistent with the definition of ρ on the previous page.

Blocked calls delayed

- If a channel is not available immediately, the call request may be delayed until a channel becomes available;
- Other conditions (assumptions) are the same as those in the case of blocked calls cleared;
- The Erlang-C formula: The probability of a call not having immediate access to a channel is

$$P(\text{queueing}) = \frac{\frac{J\rho^J}{J!(J-\rho)}}{[\frac{J\rho^J}{J!(J-\rho)}] + \sum_{j=0}^{J-1}(\frac{\rho^j}{j!})}.$$

  Queueing gives rise to delay. The probability of nonzero delay is given by

$$P(\text{delay} > 0) = P(\text{queueing}) = \frac{\rho^J}{\rho^J + J!(1 - \frac{\rho}{J})\sum_{j=0}^{J-1}\frac{\rho^j}{j!}}.$$

- If no channels are immediately available the call is delayed, and the probability that the delayed call is forced to wait more than t seconds is

$$\begin{aligned} P(\text{delay} > t) &= P(\text{delay} > 0) \times P(\text{delay} > t|\text{delay} > 0) \\ &= P(\text{delay} > 0)\exp[-(J-\rho)\mu t]. \end{aligned}$$

- The average delay, $\bar{D}$, for all calls in the queueing system (calls that are queued, and calls that receive service immediately) is given by

$$\bar{D} = P(\text{delay} > 0) \times \frac{1}{\mu(J-\rho)}.$$

- The average delay for those calls which are queued is $H/(J-\rho)$.

5.5 Capacity Enhancement in Cellular Systems

The capacity can be improved by

- cell splitting
- antenna sectoring
- dynamic channel assignment.

Cell splitting

- Subdivide a congested cell into smaller cells, each with its own base station and a corresponding reduction in antenna height and transmitted power.
- Cell splitting increases the capacity of a cellular system since it increases the number of times that channels are reused ($M \uparrow$).
- Reducing cell size increases handoffs, the number of base stations needed, and may result in a difficulty in finding a proper site for the base station.
- Old base station should be kept in some splitting cells.

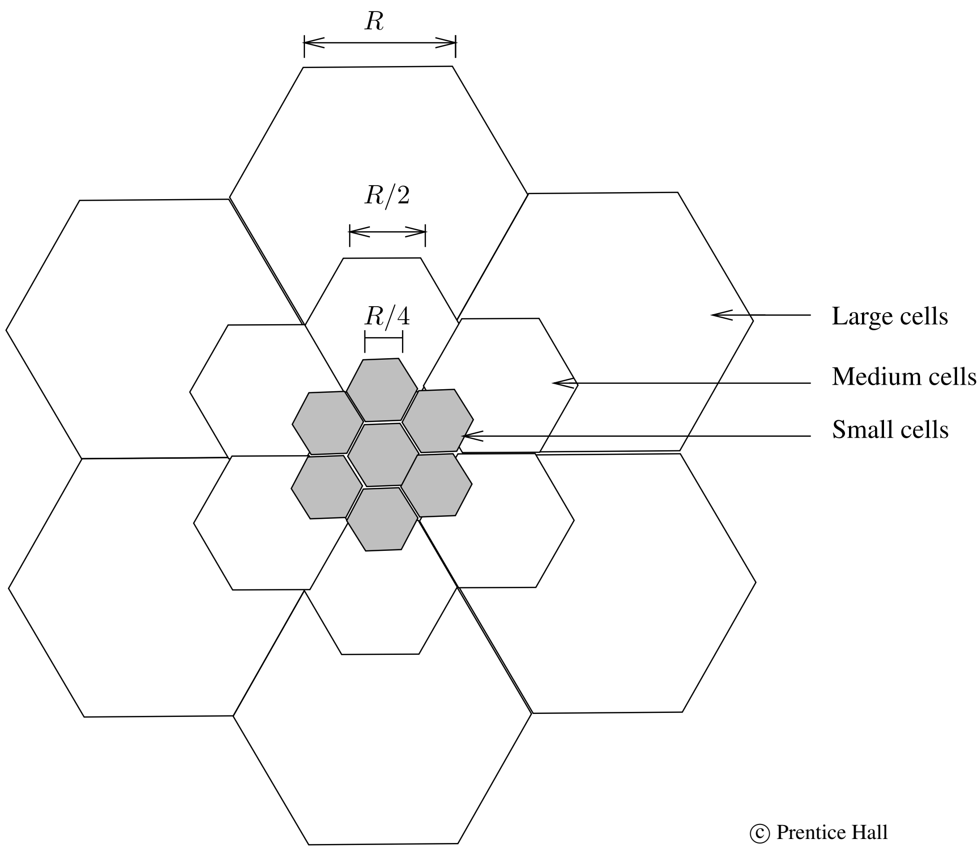


Figure 4: Illustrate of cell splitting from radius R to $R/2$ and to $R/4$

Let

P_{t1} the transmitted power of large cell BS

P_{t2} - the transmitted power of ~~small~~ medium cell BS

P_r - the received power at cell boundary

Then

P_r (large cell) $\propto P_{t1} \cdot R^{-\kappa}$

P_r (~~small~~ medium cell) $\propto P_{t2} \cdot (R/2)^{-\kappa}$

On the basis of equal received power, we have

$$P_{t1} \cdot R^{-\kappa} = P_{t2} \cdot (R/2)^{-\kappa}$$

$\Longrightarrow$

$$P_{t1}/P_{t2} = 2^{\kappa}$$

$\Longrightarrow$

$$10 \log_{10}(P_{t1}/P_{t2}) = 10\kappa \log_{10} 2 \approx 3\kappa \text{ dB}.$$

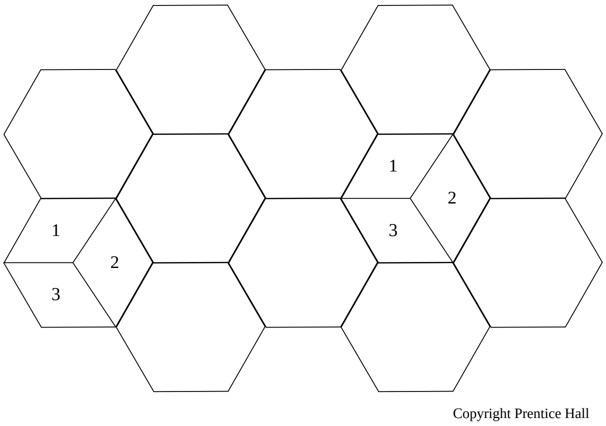
For $\kappa = 4, \;\; P_{t1}/P_{t2} = 12$ dB. In general

$R \longrightarrow R/2$ in cell splitting

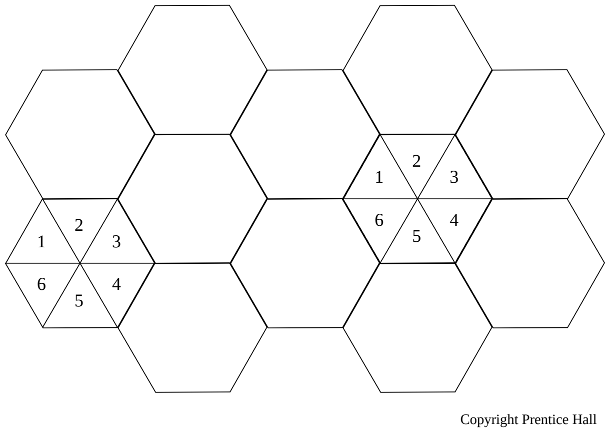
$\Longrightarrow$ Cell area $\longrightarrow$(1/4) cell area

$\Longrightarrow$ Capacity is increased by 3 times (or 4 times in total).

Cell sectoring



(a) 3 sectors of 120° each



(b) 6 sectors of 60° each

Assuming 7-cell reuse pattern, for the 3-sector case, the number of interferers in the first tier is reduced from 6 to 2.

The CIR is given by

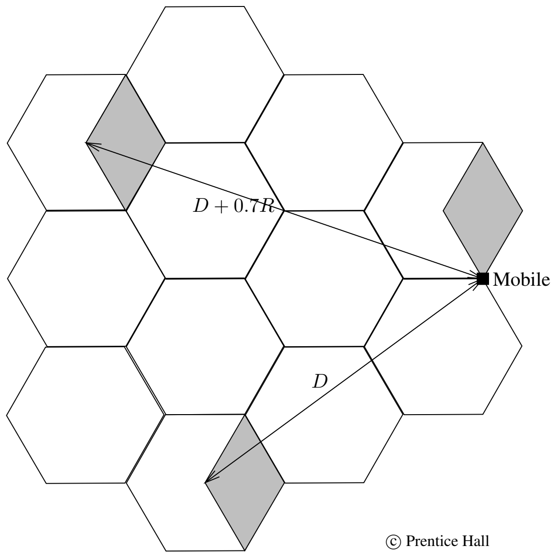
$$\frac{S}{I} = \frac{R^{-\kappa}}{\sum_{i=1}^{N_I}(D_i)^{-\kappa}} = \frac{R^{-\kappa}}{\sum_{i=1}^{2}(D_i)^{-\kappa}}$$

which is larger than the omnidirectional case where $N_I = 6$.

With $D_i \approx D$,

$$(\frac{S}{I})_{omni} = \frac{1}{6} \times q^{\kappa} \quad \text{and} \quad (\frac{S}{I})_{120^\circ} = \frac{1}{2} \times q^{\kappa}.$$

Worst-case scenario in a $120°$ sectoring:



$$\left(\frac{S}{I}\right)_{120°} = \frac{R^{-\kappa}}{D^{-\kappa} + (D + 0.7R)^{-\kappa}} = \frac{1}{q^{-\kappa} + (q + 0.7)^{-\kappa}}.$$

For $\kappa = 4$ and $N = 7$ ($q = 4.6$), we have $(S/I)_{120°} = 24.5$ dB > 18 dB

$\Longrightarrow$ The 3-sector worst case for 7-cell reuse is acceptable for (S/I) requirement of 18 dB

Example 5.8 Cochannel interference with sectoring
In Example 5.6, it is shown that, with a frequency reuse factor of 7, base stations using omnidirectional antennas cannot satisfy the 18 dB signal-to-cochannel interference ratio requirement. Determine whether the use of $120°$ sectoring and 7-cell frequency reuse would satisfy the 18 dB requirement.

Solution:

For a 7-cell reuse, we have $q = \sqrt{3 \times 7} = 4.6 \Longrightarrow$

$$\left(\frac{S}{I}\right)_{120°} = 285 \text{ or } 24.5\ dB.$$

Since this is greater than 18 dB, the 3-sector worst case for a 7-cell reuse is acceptable.

□

Channel Assignment Strategies

Fixed channel assignment (FCA)

- Each cell is allocated a predetermined set of voice channels.
- Any call attempt within the cell can only be served by the unused channels in that particular cell Channel allocation cannot adapt to traffic load dynamics.
- Borrowing option:
    - A cell is allowed to borrow channels from a neighbouring cell if all of its channels are already occupied.
    - Borrowing is supervised by the MSC to satisfy constraints on co-channel and adjacent channel interference.

Dynamic channel assignment (DCA)

- Voice channels are not allocated to different cells on a permanent basis.
- Each time a call request is made, the home BS requests a channel from the MSC.
- The MSC determines (dynamically) the availability of a channel and executes its allocation procedure accordingly.
- The MSC only allocates a given channel if the channel is not presently in use in the cell or any other cell which falls within the minimum restricted distance of the frequency reuse to avoid co-channel interference (co-channel reuse locking).
- DCA reduces the likelihood of call blocking, which increases the trunking capacity of the system, since all available channels under the control of the MSC are accessible to all the calls.
- DCA strategies require the MSC to collect real-time data on channel occupancy, traffic load distribution and radio signal strength indications of all the channel on a continuous basis.