Assignment #1

ECE 686 (Wireless Communication Networks) Dr. Hai Jiang
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1. Consider an M/M/1 system. The average inter-arrival time is 20 minutes, and the average service time of a customer is 15 minutes. What is the probability of the system having an empty queue? For a customer, please calculate the average queueing time. **(6 points)**

2. Consider the M/M/m/m system discussed in the last three pages of our Lecture 2. There are m=2 servers. The arrival rate to the system is 15 per hour, while the service rate of each server is 12 per hour. For a server, what is the probability of the server being idle? **(5 points)**

3. Consider a router in a network. Packet arrivals to the router follow a Poisson process with average arrival rate being 1000 packets per second. The router has a processing unit, which processes packets (i.e., the processing unit transmits the packets to other routers). The processing time of a packet is a random variable following an exponential distribution with mean value 0.0008 second. The buffer of the router can store 5 packets (not including the packet being processed by the processing unit of the router). When a new packet arrives, if the buffer is full, then the new packet will be dropped by the router. Please calculate packet dropping probability at the router. **(5 points)**

1. $E[\tau] = \frac{1}{\lambda} = 20 \text{ min} \Rightarrow \lambda = 3$ per hour
service Time $= \frac{1}{\mu} = 15 \text{ min} \Rightarrow \mu = 4$ per hour
$\rho = \frac{3}{4}$

Empty Queue $\equiv$ 1 person in the system (at most)

$$P(n \leq 1) = (1-\rho) + (1-\rho)\rho = 1 - \rho^2 = \frac{7}{16} = 0.4375$$

$$E[T_q] = \int_0^\infty \tau f_{T_q}(\tau)\, d\tau = \int_0^\infty \tau\, \rho\mu(1-\rho) e^{-(1-\rho)\mu\tau}\, d\tau = \frac{\rho}{\mu(1-\rho)} = \frac{\rho}{1-\rho}\frac{1}{\mu} = 45 \text{ min}$$

2. P_{idle} := probability of That server being idle

$P_{idle} = P_0 + P_1\left(\frac{1}{2}\right)$

↳ since system is symetrical, the customer is either in this server or that server with equal probability $= 1/2$

$$\longrightarrow P_{idle} = \frac{1}{\sum_{k=0}^{2} \rho^k/k!}\left[1 + \left(\frac{1}{2}\right)\left(\frac{\rho}{1!}\right)\right] = \frac{1 + \rho/2}{1 + \rho + \frac{\rho^2}{2}} = \frac{1 + \frac{1}{2}\left(\frac{15}{12}\right)}{1 + \frac{15}{12} + \frac{1}{2}\left(\frac{15}{12}\right)^2} = 0.536$$

3. $\lambda = 1000 \ \frac{\text{Packets}}{s}$

$\mu = \frac{1}{0.0008} = 1250 \ \frac{\text{Packets}}{s}$

system: $M/M/1/5$

$$\begin{tikzcd}
0 \arrow[r, shift left, "\lambda"] & 1 \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & 2 \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & 3 \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & 4 \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & 5 \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & 6 \arrow[l, shift left, "\mu"]
\end{tikzcd}$$

$$\left.\begin{aligned} \lambda P_0 &= \mu P_1 \\ \lambda P_1 &= \mu P_2 \\ &\vdots \\ \lambda P_5 &= \mu P_6 \end{aligned}\right\} \quad \begin{aligned} &\sum P_n = 1 \rightarrow P_0(1+\rho+\rho^2+\rho^3+ \dots \rho^6) = 1 \\ &\rightarrow P_0 = \frac{1-\rho}{1-\rho^7}, \quad \rho = \frac{\lambda}{\mu} = \frac{1000}{1250} \rightarrow P_0 \simeq 0.253 \end{aligned}$$

$$\begin{tikzcd}
{} \arrow[r, "\lambda"] \arrow[d] & {} \arrow[r, "\lambda(1-P_B)"] & \boxed{M/M/1/m} \arrow[r, "\mu(1-P_0)"] & {} \\
\lambda P_B := \text{Blocking pakets} & & &
\end{tikzcd}$$

$$\lambda(1-P_B) = \mu(1-P_0)$$

$$1 - P_B = \frac{1-P_0}{\rho} \rightarrow P_B = 1 - \frac{1-P_0}{\rho} \boxed{= 0.06634}$$