

## Assignment #1

ECE 686 (Wireless Communication Networks)

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1. Consider an M/M/1 system. The average inter-arrival time is 20 minutes, and the average service time of a customer is 15 minutes. What is the probability of the system having an empty queue? For a customer, please calculate the average queueing time. (6 points)

2. Consider the M/M/m/m system discussed in the last three pages of our Lecture 2. There are m=2 servers. The arrival rate to the system is 15 per hour, while the service rate of each server is 12 per hour. For a server, what is the probability of the server being idle? (5 points)

3. Consider a router in a network. Packet arrivals to the router follow a Poisson process with average arrival rate being 1000 packets per second. The router has a processing unit, which processes packets (i.e., the processing unit transmits the packets to other routers). The processing time of a packet is a random variable following an exponential distribution with mean value 0.0008 second. The buffer of the router can store 5 packets (not including the packet being processed by the processing unit of the router). When a new packet arrives, if the buffer is full, then the new packet will be dropped by the router. Please calculate packet dropping probability at the router. (5 points)

$$1. E[\tau] = \frac{1}{\lambda} = 20 \text{ min} \Rightarrow \lambda = 3 \text{ per hour} \quad \left. \rho = \frac{\lambda}{\mu} \right\}$$

$$\text{Service Time} = \frac{1}{\mu} = 15 \text{ min} \Rightarrow \mu = 4 \text{ per hour}$$

Empty Queue  $\equiv$  1 person in the system (at most)

$$P(n \leq 1) = (1-\rho) + (1-\rho)\rho = 1 - \rho^2 = \frac{7}{16} = 0.4375$$

$$E[\tau_q] = \int_0^\infty \tau f_{\tau_q}(\tau) d\tau = \int_0^\infty \rho \mu t \cdot \rho e^{-(1-\rho)\mu t} dt = \frac{\rho}{\mu(1-\rho)} = \frac{\rho}{1-\rho} \frac{1}{\mu} = 45 \text{ min}$$

2.  $P_{idle} :=$  probability of that server being idle

$$P_{idle} = P_0 + P_1 \left( \frac{1}{2} \right)$$

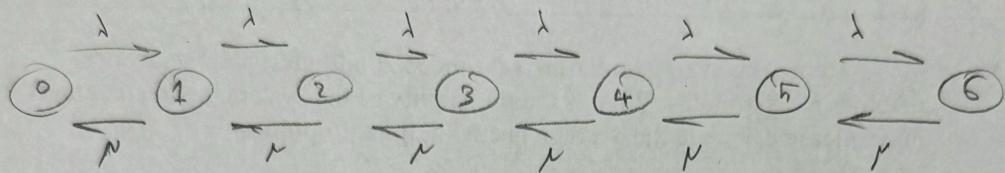
$\hookrightarrow$  since system is symmetrical, the customer is either in this server or that server with equal probability  $\Rightarrow \frac{1}{2}$

$$\rightarrow P_{idle} = \sum_{k=0}^{\infty} \frac{1}{k!} \rho^k \left[ 1 + \left( \frac{1}{2} \right) \left( \frac{\rho}{1!} \right) \right] = \frac{1 + \frac{\rho}{2}}{1 + \rho + \frac{\rho^2}{2}} = \frac{1 + \frac{1}{2} \left( \frac{15}{12} \right)}{1 + \frac{15}{12} + \frac{1}{2} \left( \frac{15}{12} \right)^2} = 0.536$$

$$3. \quad \lambda = 1000 \quad \frac{\text{Packets}}{\text{s}}$$

$$\mu = \frac{1}{0.0008} = 1250 \quad \frac{\text{Packets}}{\text{s}}$$

System: M/M/1/5



$$\left. \begin{array}{l} \lambda P_0 = \mu P_1 \\ \lambda P_1 = \mu P_2 \\ \vdots \\ \lambda P_5 = \mu P_6 \end{array} \right\} \quad \sum P_n = 1 \rightarrow P_0 (1 + \rho + \rho^2 + \rho^3 + \rho^4) = 1$$

$$\rightarrow P_0 = \frac{1 - \rho}{1 - \rho^5}, \quad \rho = \frac{\lambda}{\mu} = \frac{1000}{1250} \rightarrow P_0 \approx 0.253$$

$$\lambda (1 - P_B) = \mu (1 - P_0)$$

$$1 - P_B = \frac{1 - P_0}{\rho} \rightarrow P_B = 1 - \frac{1 - P_0}{\rho} = 0.06634$$