Simulation 2 Adaptive Control

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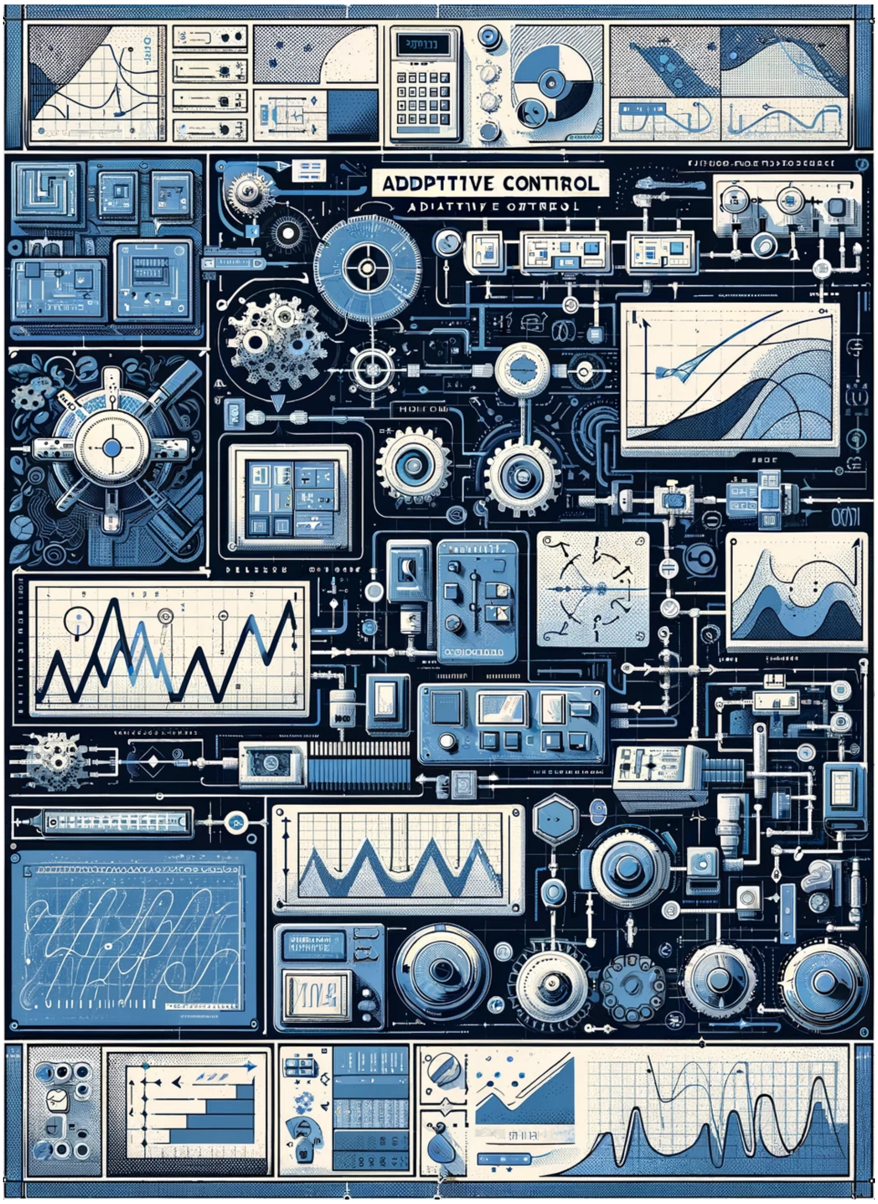

June 10, 2024

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1.1 Dynamic System Transfer Function Derivation

A mass, spring, and damper system is given as follows:

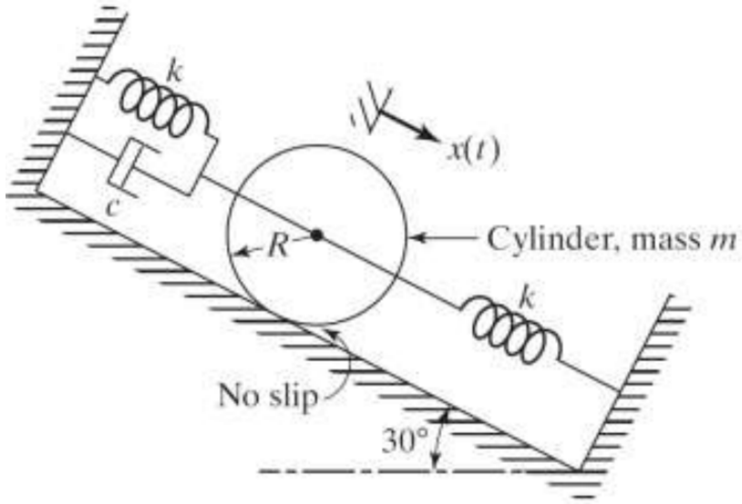

Figure 1: The dynamic system.

Applying Newton's second law along the incline direction:

$$m\frac{d^2x(t)}{dt^2} = -kx(t) - c\frac{dx(t)}{dt} + \frac{mg}{2}$$

The torque caused by the no-slip condition:

- The rolling condition implies $x(t) = R\theta(t)$.
- Thus, the angular acceleration $\alpha = \frac{d^2\theta(t)}{dt^2} = \frac{1}{R}\frac{d^2x(t)}{dt^2}$.

  The moment of inertia of the cylinder: $I = \frac{1}{2}mR^2$.

  The torque equation:

$$\tau = I\alpha = \frac{1}{2}mR^2\left(\frac{1}{R}\frac{d^2x(t)}{dt^2}\right) = \frac{1}{2}mR\frac{d^2x(t)}{dt^2}$$

Combining the translational and rotational equations and solving for $\frac{d^2x(t)}{dt^2}$:

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + 2kx(t) = \frac{mg}{2}$$

$$\left(m + \frac{1}{2}m\right)\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + 2kx(t) = \frac{mg}{2}$$

$$\frac{3m}{2}\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + 2kx(t) = \frac{mg}{2}$$

$$\frac{3m}{2}\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + 2kx(t) = \frac{mg}{2}$$

To find the transfer function $G(s) = \frac{X(s)}{F(s)}$:

We Take the Laplace transform of the differential equation (assuming zero initial conditions):

$$\frac{3m}{2}s^2X(s) + csX(s) + 2kX(s) = \frac{mg}{2s}$$

$$\implies G(s) = \frac{mg}{2(s + c/k)\left(\frac{3m}{2}s^2 + cs + 2k\right)}$$

We will choose the parameters: $\left\{m = 10 \quad g = 9.81 \quad c = 5 \quad k = 100\right\}$ so the transfer function will be:

$$G(s) = \frac{98.1}{30s^3 + 11.5s^2 + 400.5s + 20}$$

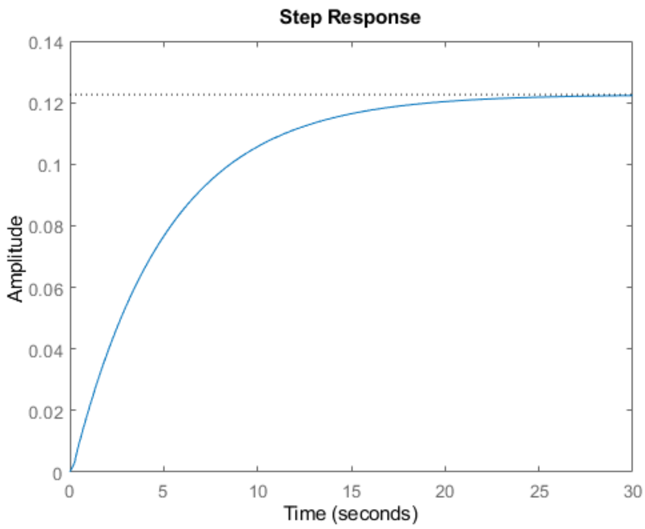

Figure 2: The dynamic system step response.

As it is shown in the figure above the system is stable.

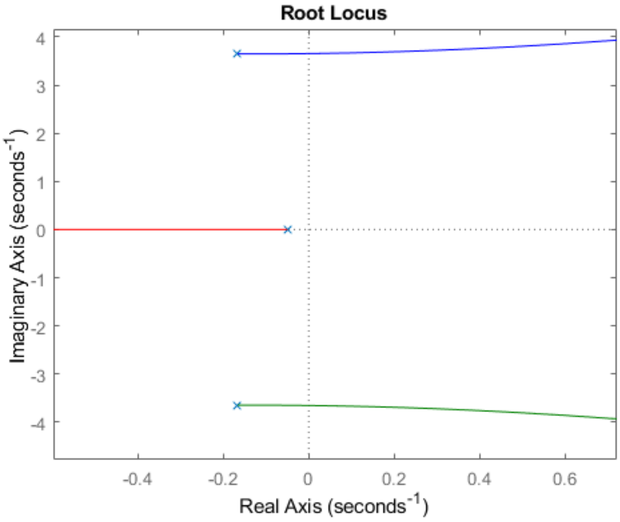

Figure 3: The dynamic system root locus diagram.

As it is illustrated in the figure above the system is minimum-phase.

We will create the discrete system using c2d() command in MATLAB and we take the sampling time equal to 0.01 seconds and proceed through the controller design.

```
%% Discretization of the Model
s = tf('s');
m = 10;
g = 9.81;
c = 5;
k = 100;
Gc = (m*g)/(2*(s+c/k)*(3*m/2*s^2+c*s+2*k)); % Continuous-time transfer function

z = tf('z', 0.1); % Discrete-time transfer function with sampling time 0.1 seconds
sysDiscrete = (5.401*10^(-7)*z-3.7807*10^(-7))/((2.141*10^(-6)*z^2-2.0554*10^(-6)*z+ 2.7405*10^(-7)));
[numDiscrete, denDiscrete] = tfdata(sysDiscrete, 'v'); % Extract numerator and denominator
samplingTime = 0.1; % Sampling time

```

1.2 Indirect STR with zero cancellation (without noise)

The code and the algorithm is same as the previous simulation set and we will use a dynamic desired system with the same order as our system and we choose a desired specifications as follows:

```
%% Desired System Parameters
desiredOvershoot = 0.15;
desiredRiseTime = 0.1;

cRatio = cos(atan2(pi,-log(desiredOvershoot))); % c ratio
naturalFrequency = 1.8/(desiredRiseTime); % Natural frequency

desiredZero = -20; % Zero of the desired system
gainFactor = -1/desiredZero;

```

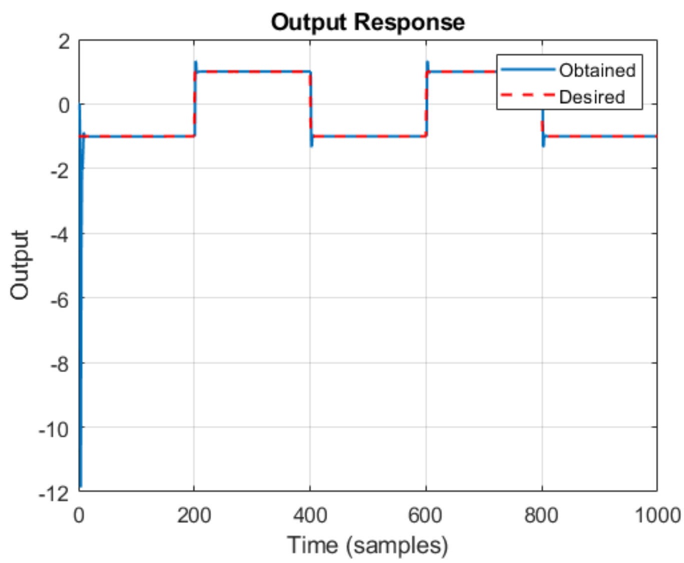

Figure 4: System tracking (indirect STR without noise).

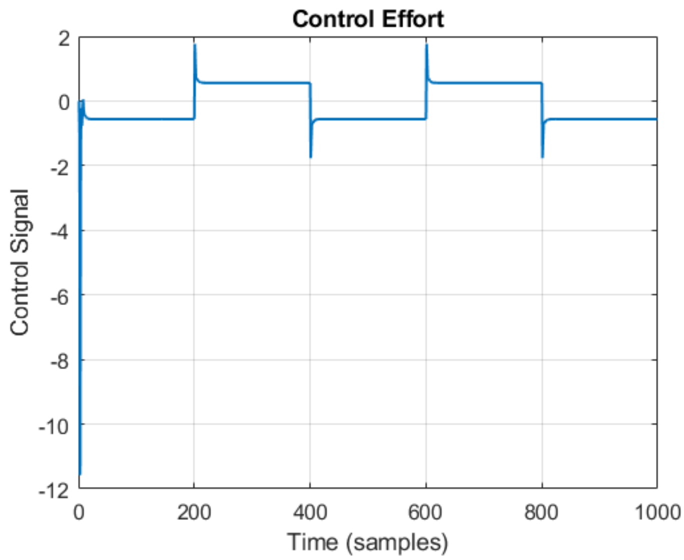

Figure 5: System control effort (indirect STR without noise).

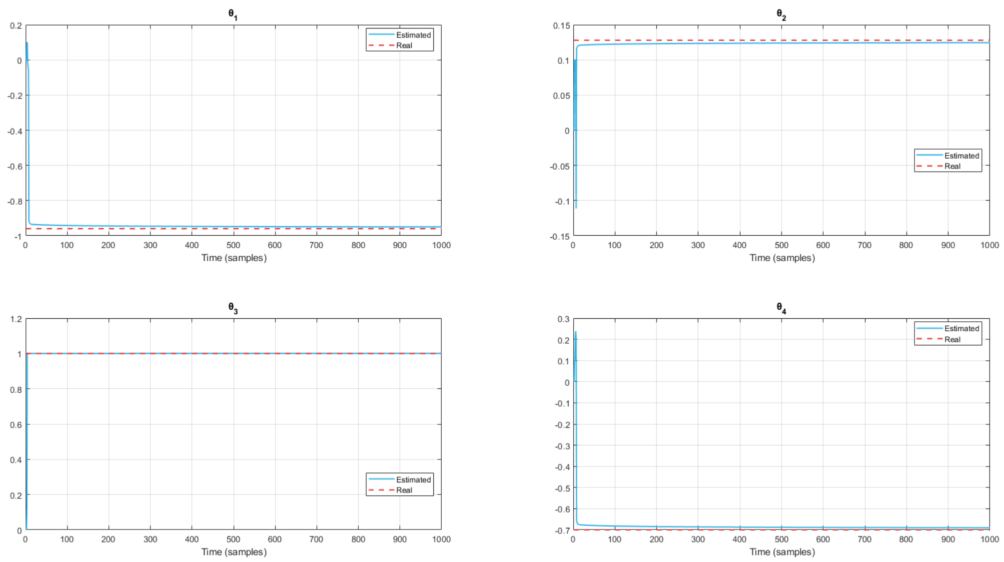

Figure 6: System parameter estimation (indirect STR without noise).

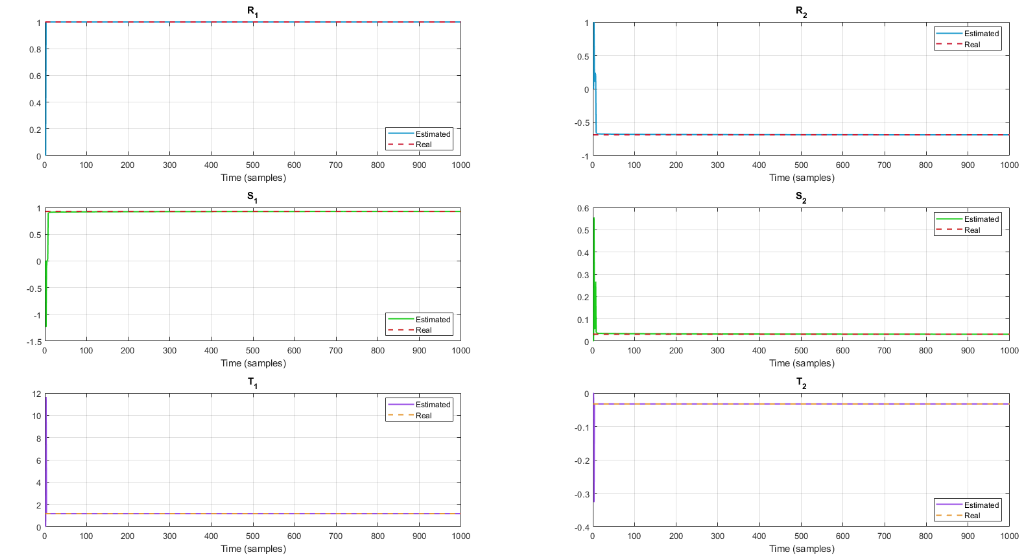

Figure 7: System S, T, R polynomials estimation (indirect STR without noise).

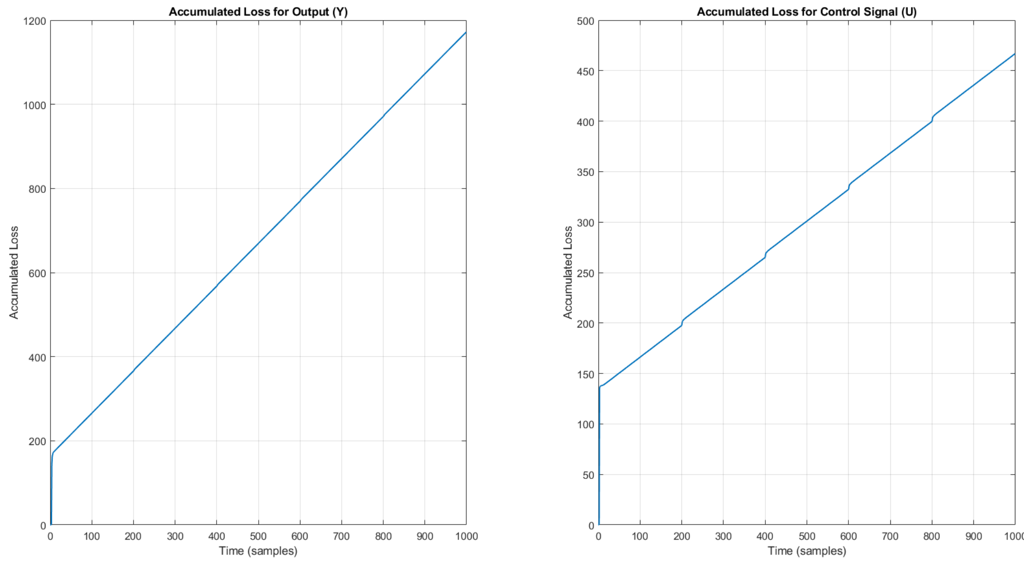

Figure 8: System accumulated errors for out put and control effort (indirect STR without noise).

1.2.1 Studying Results (Indirect STR, No Noise)

In the case where there is no input and output noise in the system and the system input is a square wave function, the system output and control effort are obtained as shown in figure 4. Additionally, the system parameters were estimated as shown in figure 6. (the ELS estimator, which is not very suitable here, Hence RLS algorithm is performed).

As observed, in the absence of noise, the identification signal is not sufficiently rich, and the parameters are not estimated accurately. Since we are dealing with a signal tracking problem, the accumulated loss is not a particularly suitable metric for comparing algorithms. However, given that the overall mean should be zero, this chart is included, although it is not the best for evaluation purposes.

- When there is no noise in the input and output of the system, and the input is a square wave function, we obtain the system output and control effort as shown.
- Without noise, the identification signal lacks sufficient richness, leading to poor parameter estimation.
- In signal tracking problems, accumulated loss is not the ideal metric for comparison. Nevertheless, since the overall mean should be zero, the accumulated loss chart is provided but should be interpreted with caution.

1.3 Indirect STR with zero cancellation (with white noise)

The code and the algorithm is same as the previous simulation set and we will use a dynamic desired system with the same order as our system and we choose a desired specifications as follows:

```
%% Desired System Parameters
desiredOvershoot = 0.15;
desiredRiseTime = 0.1;

cRatio = cos(atan2(pi,-log(desiredOvershoot))); % c ratio
naturalFrequency = 1.8/(desiredRiseTime); % Natural frequency

desiredZero = -20; % Zero of the desired system
gainFactor = -1/desiredZero;

```

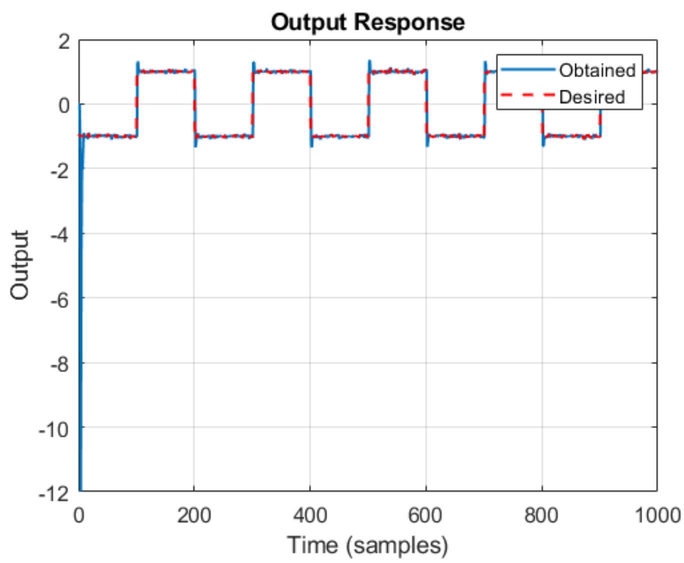

Figure 9: System tracking (indirect STR with white noise).

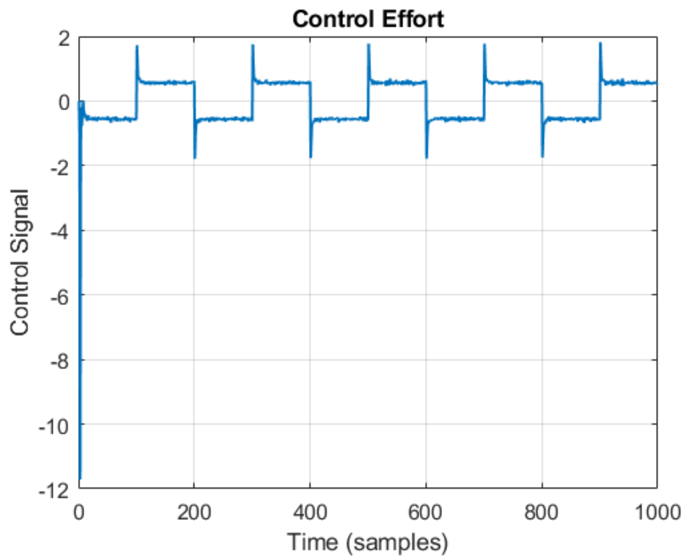

Figure 10: System control effort (indirect STR with white noise).

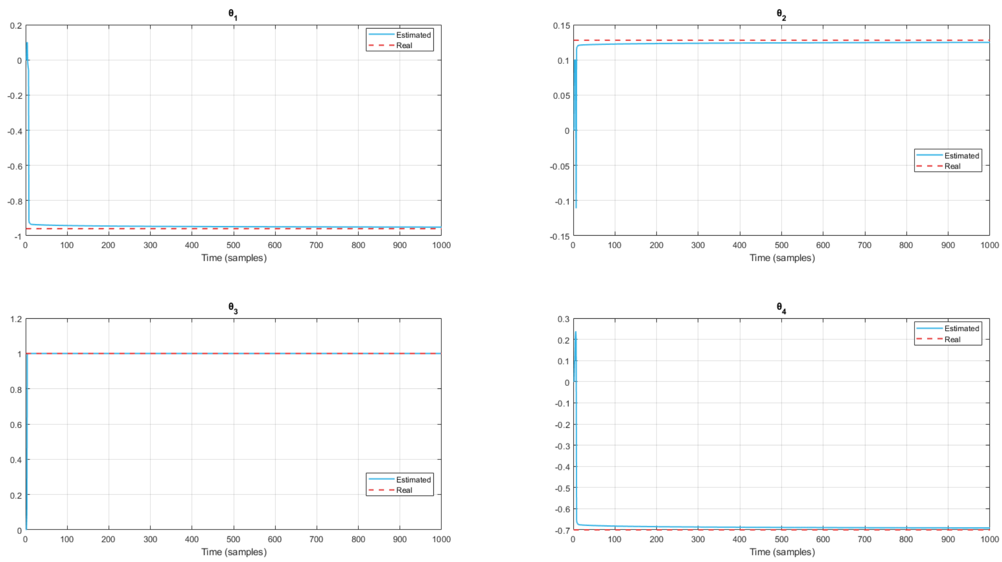

Figure 11: System parameter estimation (indirect STR with white noise).

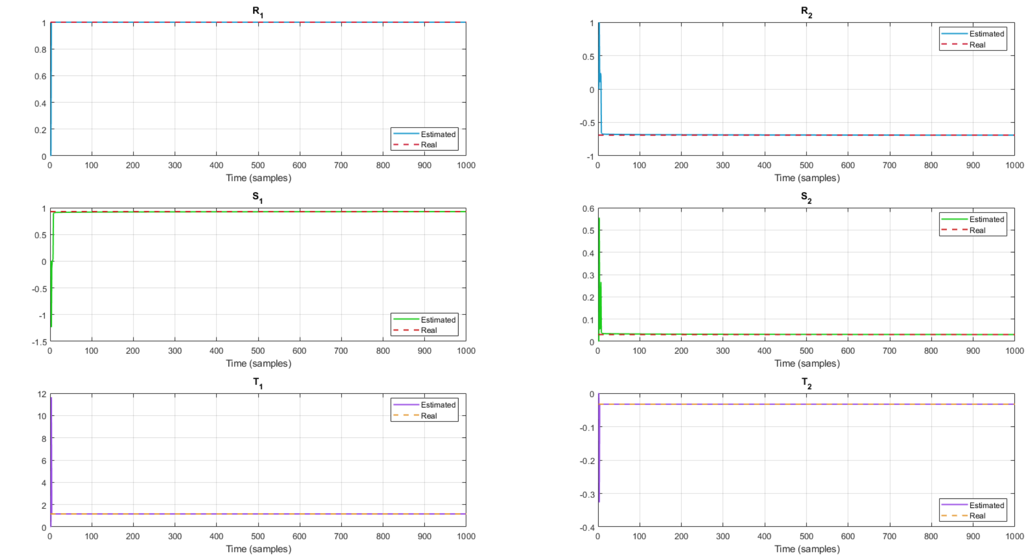

Figure 12: System S, T, R polynomials estimation (indirect STR with white noise).

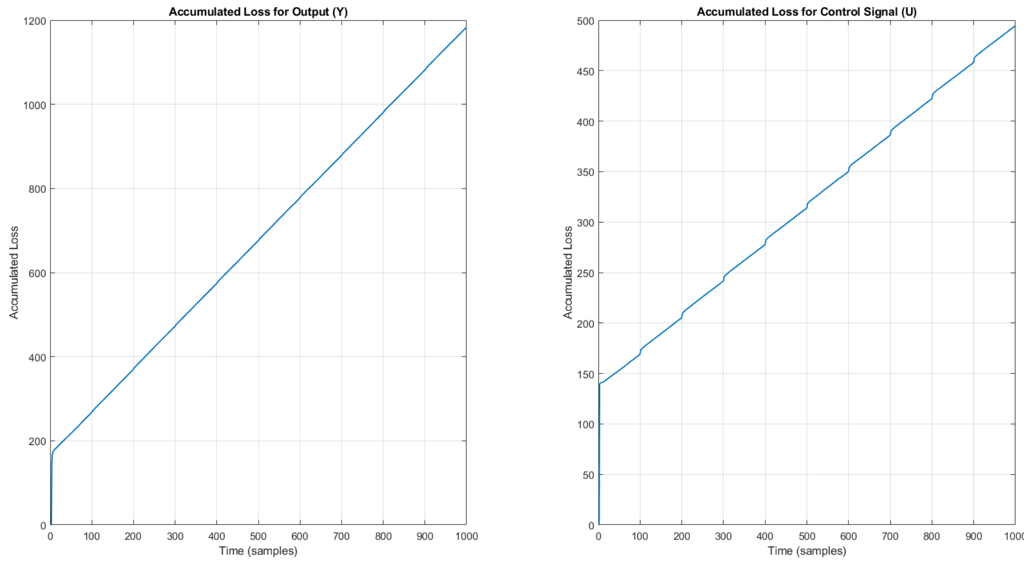

Figure 13: System accumulated errors for out put and control effort (indirect STR with white noise).

1.3.1 Studying Results (Indirect STR, White Noise)

In the case where white measurement noise with a variance of 0.0001 is present in the system, the system output is obtained as shown below. Additionally, to increase the order of the PE (persistent excitation) of the identification signal, noise with a variance of 0.001 was also added to the input. It is important to note that the estimator used is an ELS (Extended Least Squares), which performs better in the presence of colored noise. While an RLS (Recursive Least Squares) estimator could also be used in this scenario, the change would not be significant. Essentially, the white noise is treated as colored noise, but these parameters will tend to zero in the long run.

The presence of noise is expected to make the identification signal richer, resulting in better parameter estimation. The accumulated error chart is also shown in figure 11.

- When white measurement noise with a variance of 0.0001 is present, the system output is influenced accordingly. To enhance the richness of the identification signal, an additional noise with a variance of 0.001 is introduced into the input.

- The ELS estimator is chosen for its superior performance with colored noise. Though an RLS estimator could be employed, it would not significantly alter the results.

- The white noise is effectively treated as colored noise by the estimator, and these noise parameters will eventually diminish over time.

- The introduction of noise is beneficial as it enriches the identification signal, leading to more accurate parameter estimation.

- The accumulated error plot, which reflects these dynamics, is provided below.

1.4 Direct STR with zero cancellation (without noise)

The code and the algorithm is same as the previous simulation set and we will use a dynamic desired system with the same order as our system and we choose a desired specifications as follows:

```
%% Desired System Parameters
desiredOvershoot = 0.15;
desiredRiseTime = 0.1;

cRatio = cos(atan2(pi,-log(desiredOvershoot))); % c ratio
naturalFrequency = 1.8/(desiredRiseTime); % Natural frequency

desiredZero = -20; % Zero of the desired system
gainFactor = -1/desiredZero;
```

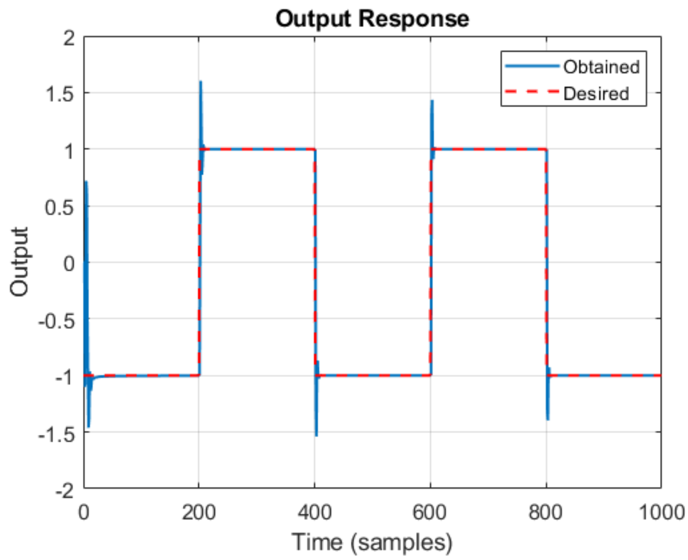

Figure 14: System tracking (direct STR without noise).

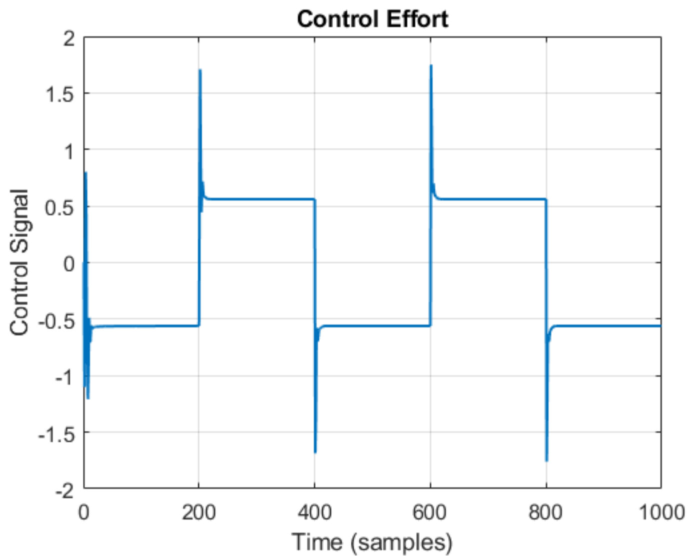

Figure 15: System control effort (direct STR without noise).

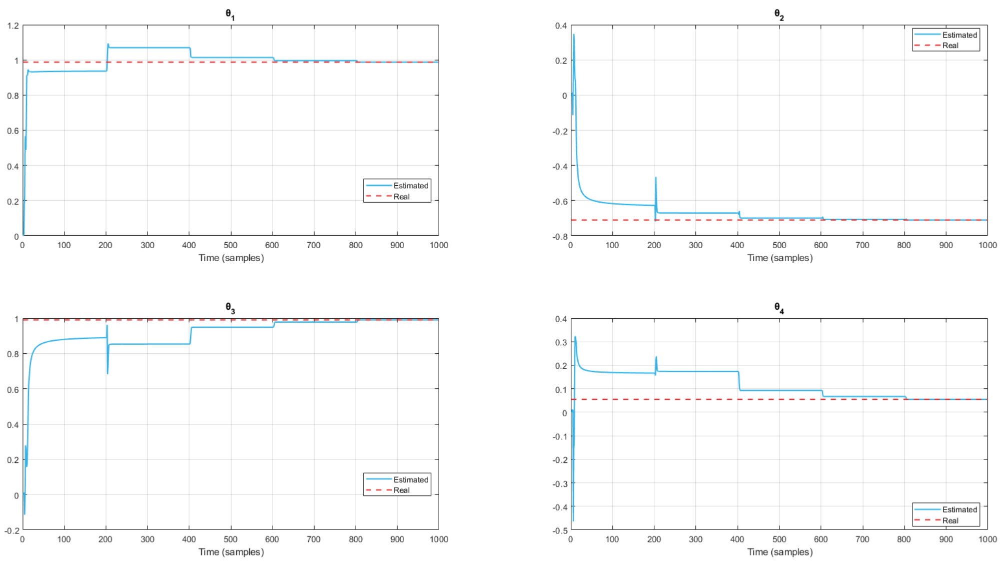

Figure 16: System parameter estimation (direct STR without noise).

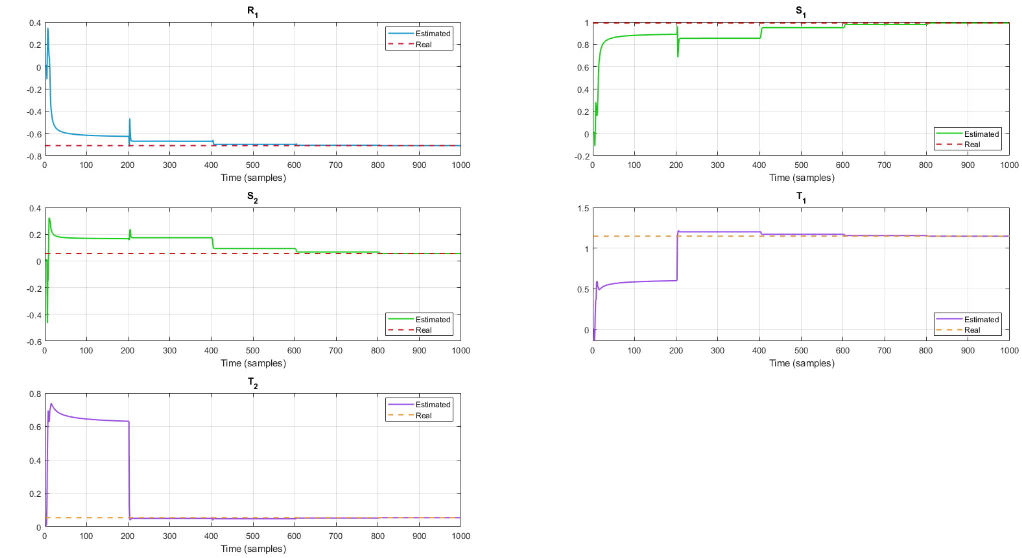

Figure 17: System S, T, R polynomials estimation (direct STR without noise).

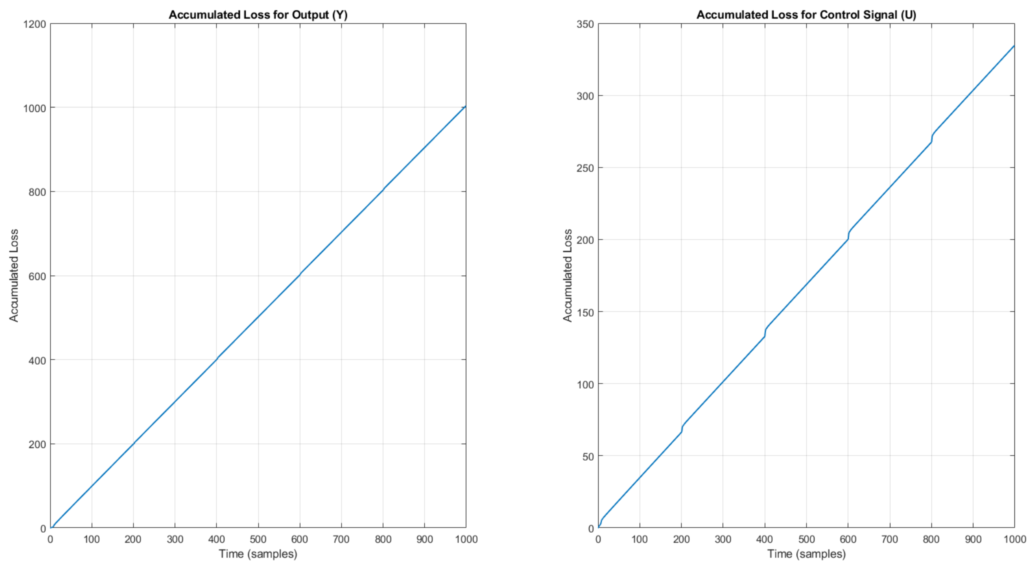

Figure 18: System accumulated errors for out put and control effort (direct STR without noise).

1.4.1 Studying Results (direct STR, No Noise)

As shown in Figure 14, the system output is desirable, and the control input has not increased significantly, which is a positive indication. However, as expected, the parameter identification in the direct algorithm is not as accurate as in the indirect algorithm, and it can be observed that the identification process is significantly slower.

This statement also holds true for the identification of the S, T, and R polynomials. The identification process is slow, but ultimately, the controller performs well and correctly tracks the control input.

- Figure 14 demonstrates that the system output meets the desired performance criteria, and the control input remains within acceptable limits, indicating a well-functioning control system.
- Despite the positive performance, the direct algorithm's parameter identification is less precise compared to the indirect algorithm. The identification process is notably slower, which affects the overall efficiency.
- The same issue of slow identification is observed in the S, T, and R polynomial estimations. However, despite the slower identification process, the controller eventually stabilizes and accurately follows the desired control input.
- The ultimate goal of maintaining system stability and performance is achieved, but the efficiency of the identification process in the direct algorithm could be improved.

1.5 Direct STR with zero cancellation (with white noise)

The code and the algorithm is same as the previous simulation set and we will use a dynamic desired system with the same order as our system and we choose a desired specifications as follows:

```
%% Desired System Parameters
desiredOvershoot = 0.15;
desiredRiseTime = 0.1;

cRatio = cos(atan2(pi,-log(desiredOvershoot))); % c ratio
naturalFrequency = 1.8/(desiredRiseTime); % Natural frequency

desiredZero = -20; % Zero of the desired system
gainFactor = -1/desiredZero;
```

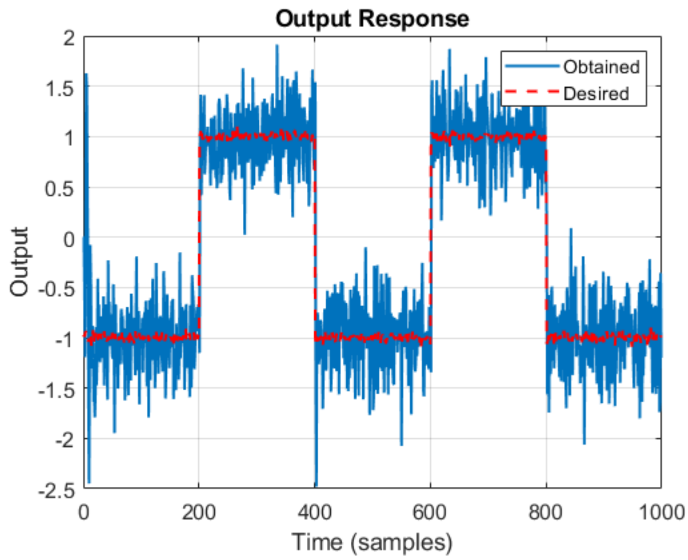

Figure 19: System tracking (direct STR with white noise).

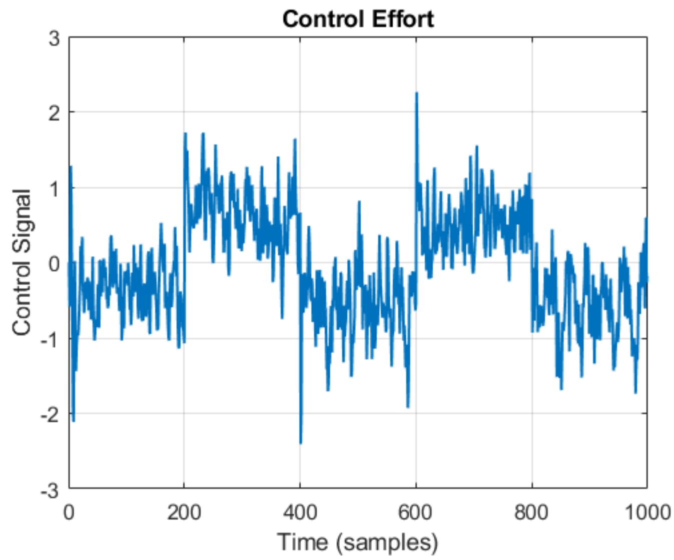

Figure 20: System control effort (direct STR with white noise).

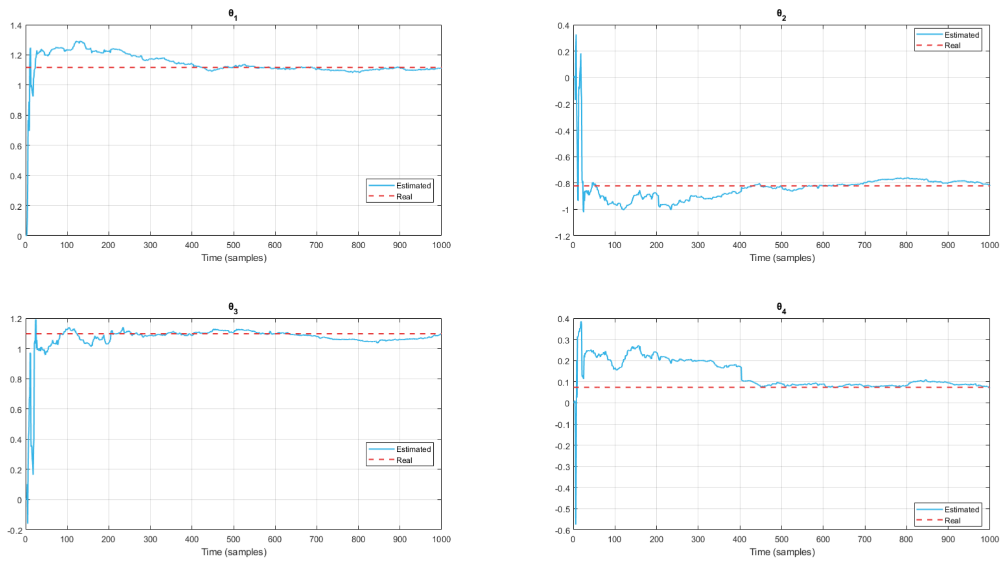

Figure 21: System parameter estimation (direct STR with white noise).

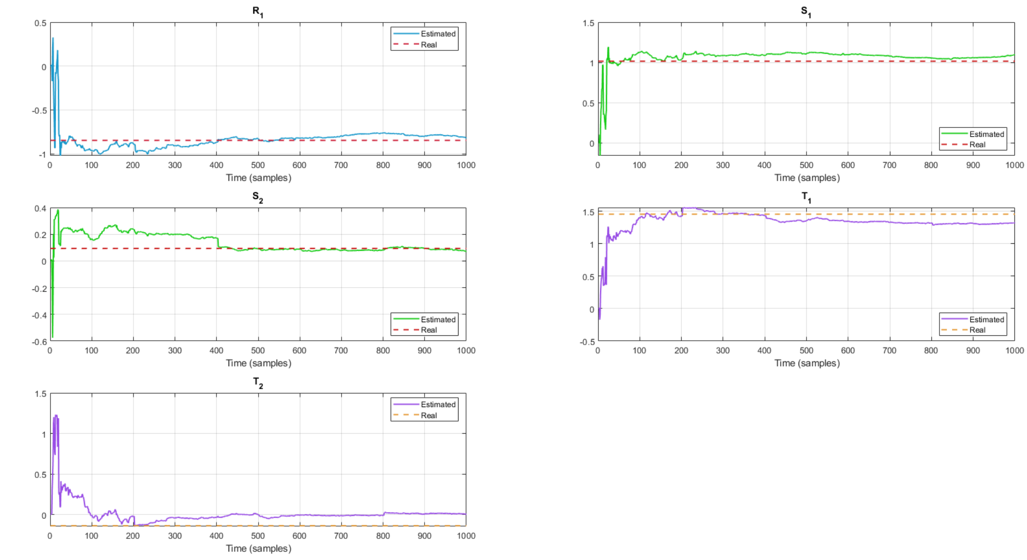

Figure 22: System S, T, R polynomials estimation (direct STR with white noise).

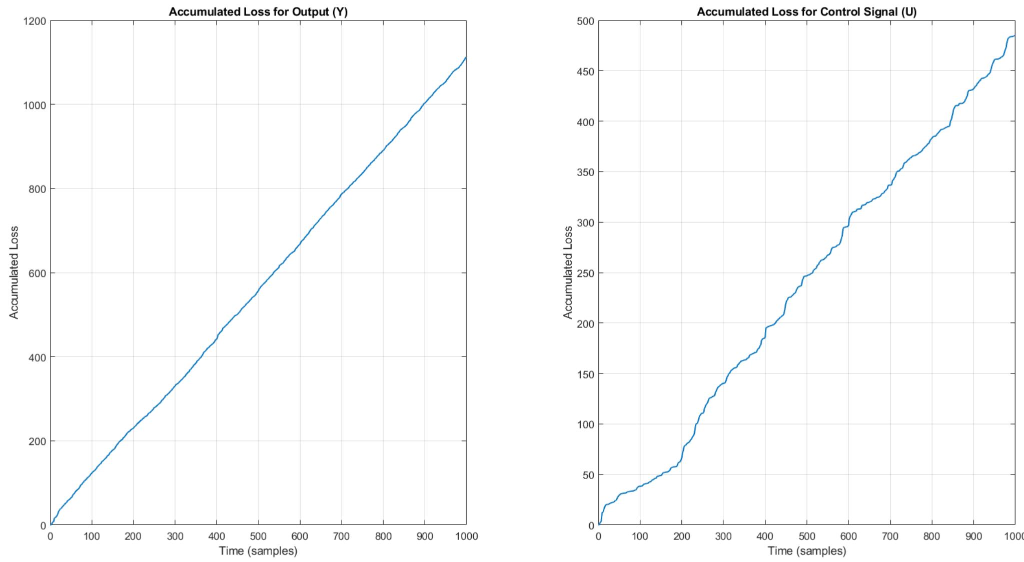

Figure 23: System accumulated errors for out put and control effort (direct STR with white noise).

1.5.1 Studying Results (Direct STR, White Noise)

As observed in Figure 19, when we run the direct STR algorithm in the presence of white noise, the error in the control input tracking problem is significantly high and unacceptable. Additionally, the control signal is highly oscillatory, which could potentially damage the actuators and is not desirable.

Regarding the system parameter identification, as expected, the presence of noise results in a richer identification process and faster convergence. However, the final estimate is biased, but overall, the identification performance is satisfactory.

The same observation applies to the estimation of the S, T, and R polynomials. The estimation process is quicker and converges more rapidly to the near-final values. Nevertheless, there is a residual error, and the estimates do not fully converge, which was anticipated.

- Figure 19 highlights the challenges posed by running the direct STR algorithm with white noise. The control input tracking error is unacceptably high, compromising the system's performance.

- The control signal's oscillatory nature is problematic, as it could lead to wear and tear on the actuators, making the system less reliable.

- Despite the noise, the parameter identification process benefits from a richer dataset, leading to faster convergence. However, the final parameter estimates exhibit bias, indicating room for improvement in accuracy.

- The S, T, and R polynomial estimates also converge faster in the presence of noise. The estimates approach their final values more quickly, but a small error remains, preventing complete convergence. This residual error was anticipated given the noise conditions.

1.6 Comparing Direct and Indirect STR (With and Without Noise)

Method	$\bar{u}$	$\bar{y}$	σ_u	σ_y
Indirect	-0.2871	-0.4313	2.7423	5.3036
Direct	-25.6982	-53.7354	2.314E05	9.321E05

Table 1: Comparing the values of variance and mean in input and output in different methods (No noise).

Method	$\bar{u}$	$\bar{y}$	σ_u	σ_y
Indirect	-0.2321	-0.4132	2.2417	4.9172
Direct	-3.5546	-5.3461	8.821E03	0.987E04

Table 2: Comparing the values of variance and mean in input and output in different methods (White noise).

In general, it can be said that in direct methods, the initial error introduced into the system is higher, and the system experiences more oscillations to reach the desired value. When noise is present in the system, compared to the case without noise, the system exhibits more oscillations. However, on the other hand, parameter estimation is faster and more accurate, and the initial error is reduced in some cases. Nonetheless, in both scenarios, the system is not very robust against colored noise, and the controller does not account for the dynamics of the noise. This results in the variance of the system output being close to or greater than the measurement noise variance. The results for the accumulated error and output variance for six different cases are presented in the table below.

Since the input is a square wave and the output oscillates between positive and negative values with a mean of zero, variance alone cannot adequately capture the changes we are interested in. Therefore, in one of the cases, the absolute value of the output was taken, and then the variance was calculated. In this way, only the variations around the point 1 are measured, which better reflects the impact of the noise.

As mentioned earlier, the performance in the direct method is somewhat inferior compared to the indirect method, with the system output exhibiting more oscillations.

1.7 Non-Adaptive Minimum-Variance (Indirect,Colored Noise)

In a minimum variance controller, the aim is to minimize the variance of y. Ideally, in a regulation problem, the value of y would equal the noise, resulting in the minimum variance, which is the variance of the noise itself. The control command at time t should be chosen in such a way that the output at time $d+t$ has the least variance.

$$u(t) = -\frac{G(z)}{B(z)F(z)}y(t)$$

Now we can write a Diophantine equation as follows:

$$z^{d_0-1}CB = AR + BS$$

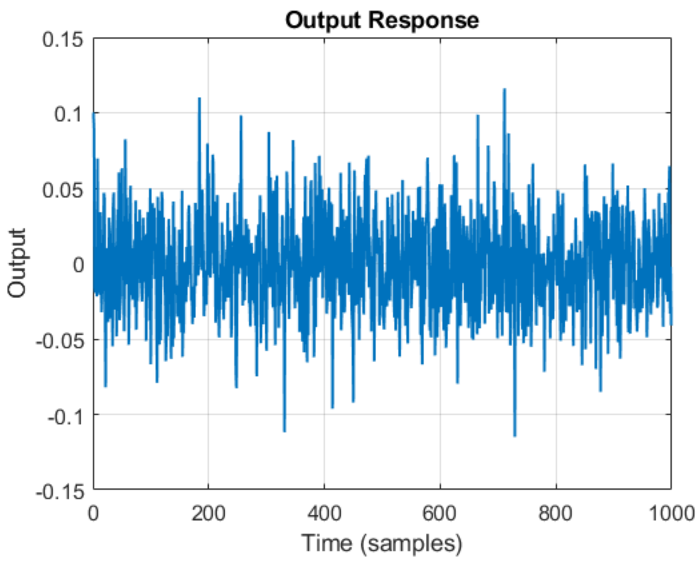

Figure 24: System tracking (Indirect Non-Adaptive Minimum Variance colored noise).

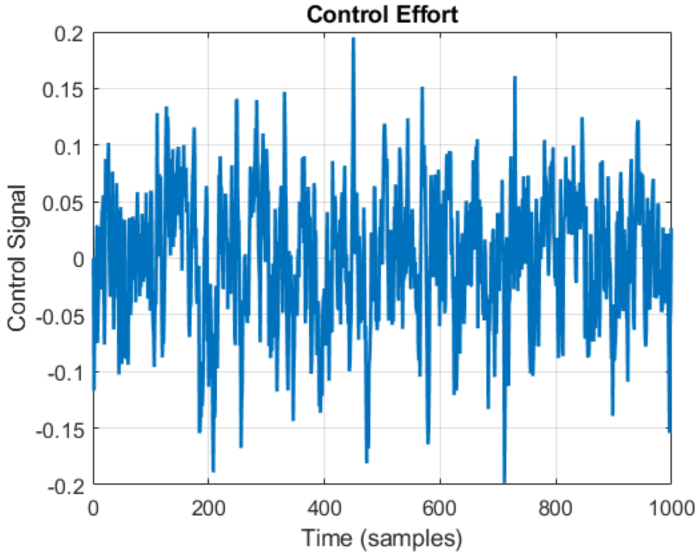

Figure 25: System control effort (Indirect Non-Adaptive Minimum Variance colored noise).

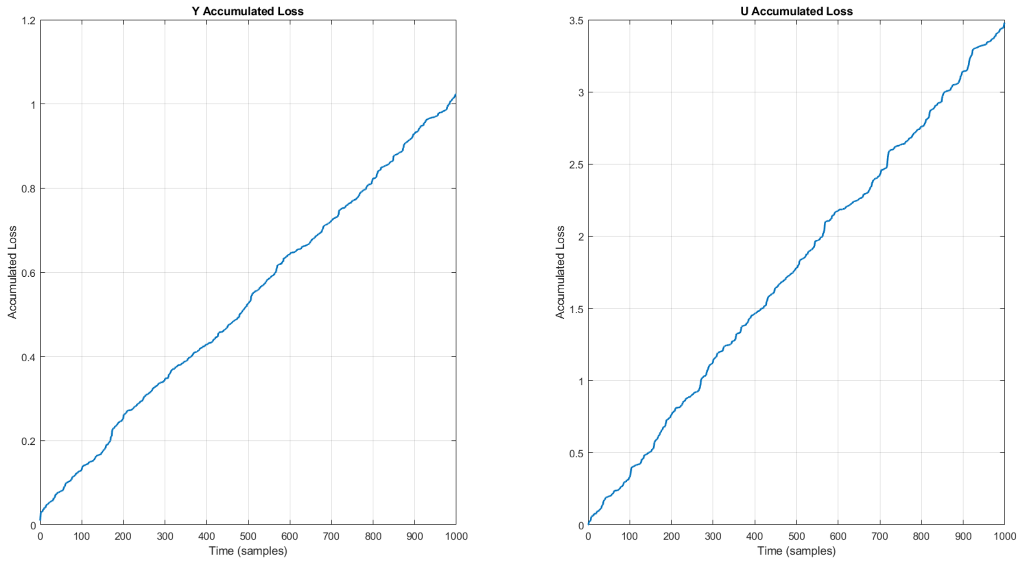

Figure 26: System accumulated errors for out put and control effort (Indirect Non-Adaptive Minimum Variance colored noise).

1.8 Adaptive Minimum-Variance (Indirect,Colored Noise)

In this case, it is first necessary to estimate the system parameters using an estimator. Then, the estimated parameters will be used to solve the Diophantine equation in each iteration. The following code snippet is developed for this purpose. The noise matrix C is assumed to be monic, and its other two parameters are estimated. It is also necessary to check in each iteration whether the obtained C has an unstable zero. If an unstable zero is encountered, the roots can be inverted to obtain a stable equation.

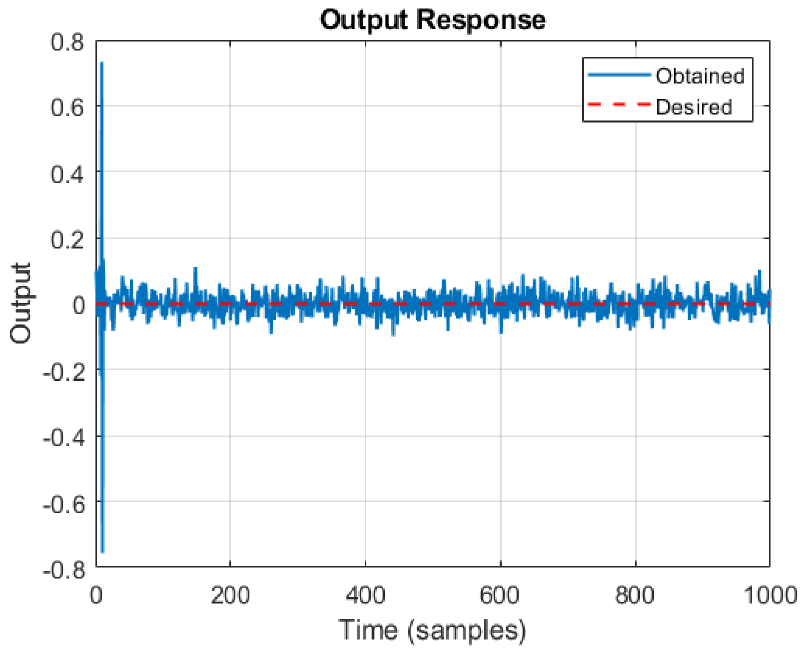

Figure 27: System tracking (Indirect Adaptive Minimum Variance colored noise).

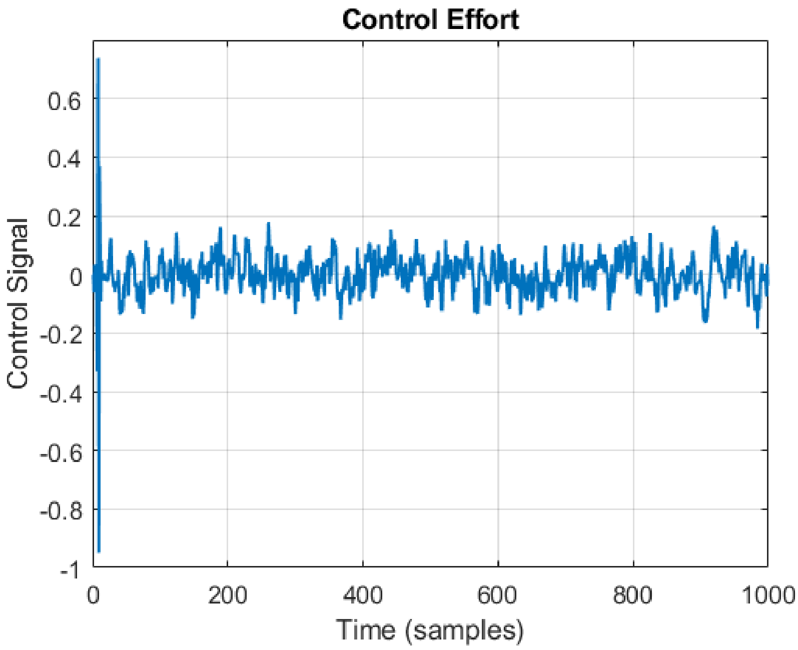

Figure 28: System control effort (Indirect Adaptive Minimum Variance colored noise).

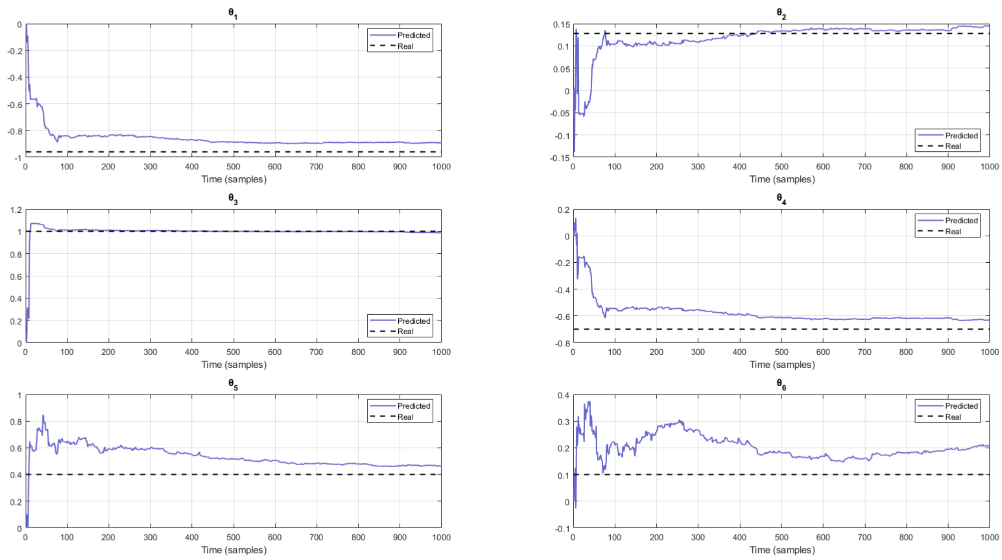

Figure 29: System parameters estimation (Indirect Adaptive Minimum Variance colored noise).

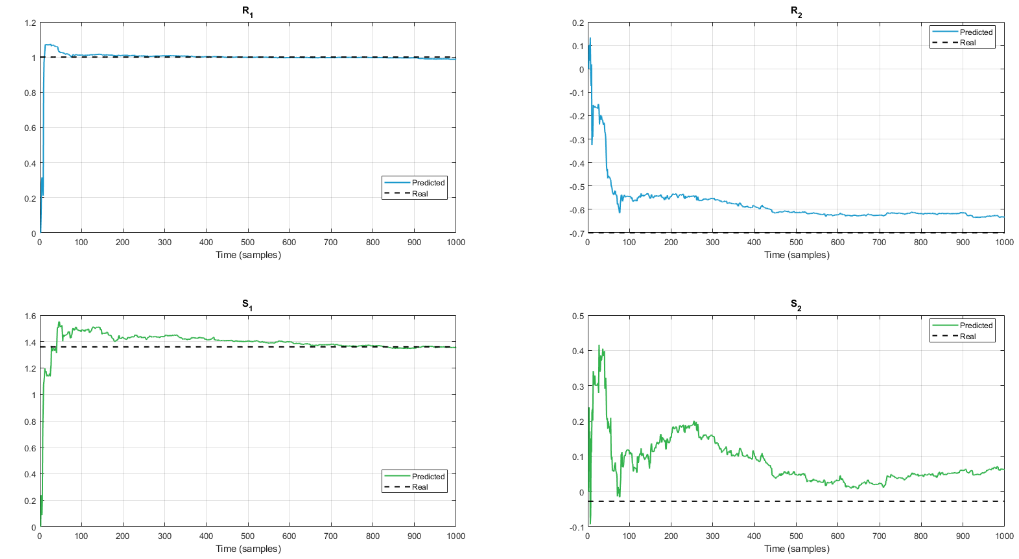

Figure 30: System S, R estimation (Indirect Adaptive Minimum Variance colored noise).

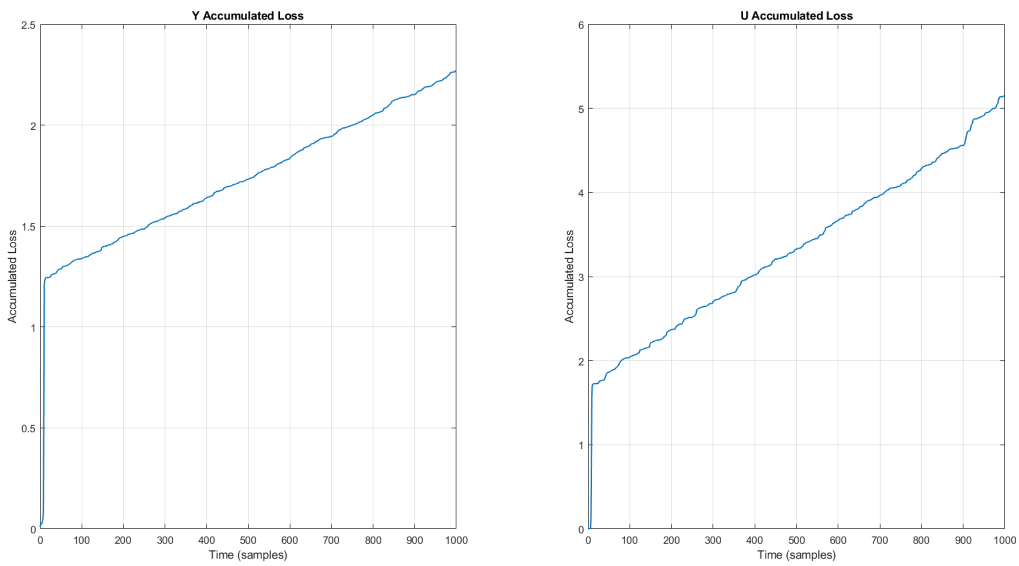

Figure 31: System accumulated errors for out put and control effort (Indirect Adaptive Minimum Variance colored noise).

1.9 Studying The Results of Indirect Adaptive and Non-Adaptive Minimum Variance

As observed, there is an initial spike in the variance, which is related to the lack of parameter identification in the system. After some time, the slope of the graph becomes almost identical to the adaptive case, and eventually, we reach a final accumulated error value close to the non-adaptive scenario. The key point is that the final difference in the accumulated error value is approximately equal to the initial spike in the accumulated error.

The first four parameters correspond to the A and B parameters, and the last two parameters relate to the noise. It can be said that all six parameters are well estimated using the ELS method. Additionally, since the problem is also solved in the non-adaptive case, we can compare the calculated R and S values with those in the non-adaptive case, as shown in the graphs below.

As observed, there is initially a significant difference from the actual values, but after some time, the system converges to the true values.

- Initially, the variance experiences a sharp increase due to the system's parameters not being identified yet. As the estimation process continues, the variance slope aligns with that of the adaptive scenario.

- Ultimately, the final accumulated error value in the adaptive scenario closely matches the non-adaptive scenario, with the initial spike being the primary difference.

- The first four parameters are associated with the A and B polynomials, while the last two parameters are related to noise. The ELS method effectively estimates all six parameters.

- The comparison of the R and S parameters between the adaptive and non-adaptive cases shows that initially, there is a notable discrepancy. However, the system parameters gradually converge to their true values over time.

- The convergence process demonstrates the effectiveness of the ELS method in both adaptive and non-adaptive scenarios, ultimately leading to accurate parameter estimation and system performance.

1.10 Non-Adaptive Moving-Average (Indirect,Colored Noise)

The most important difference between MA (Moving Average) and the minimum variance case is the value of d_0. The value of d_0 actually represents the number of zeros eliminated from the system, which are replaced by a zero at the origin. The parameter d indicates the number of eliminated zeros in the MA controller. If $d_0 = d$ is chosen, all the system's zeros are eliminated, and the controller will be the same as the minimum variance controller. If $d = n$, no zeros are eliminated. This case is useful in non-minimum phase systems where zeros cannot be eliminated.

Here, we assume that the system zeros are not eliminated and set $d = 2$. Additionally, B^+ is set to 1. By solving the Diophantine equation as shown below, the control commands can be calculated. The system output and control commands are obtained as shown in the figure below.

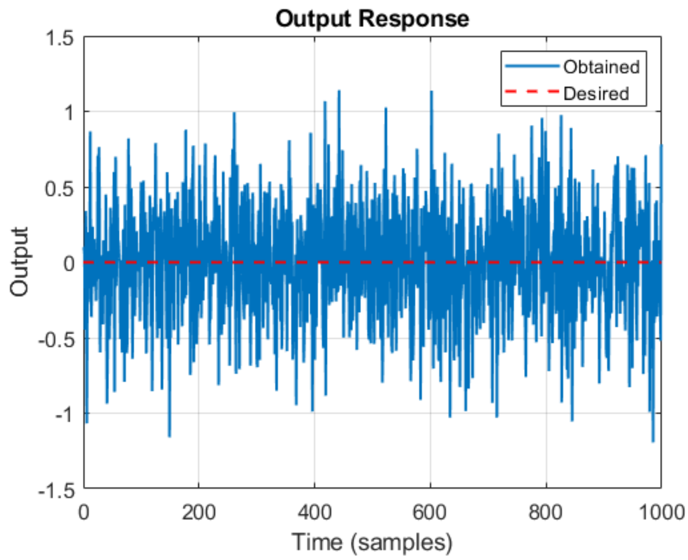

Figure 32: System tracking (Indirect Non-Adaptive Moving-Average colored noise).

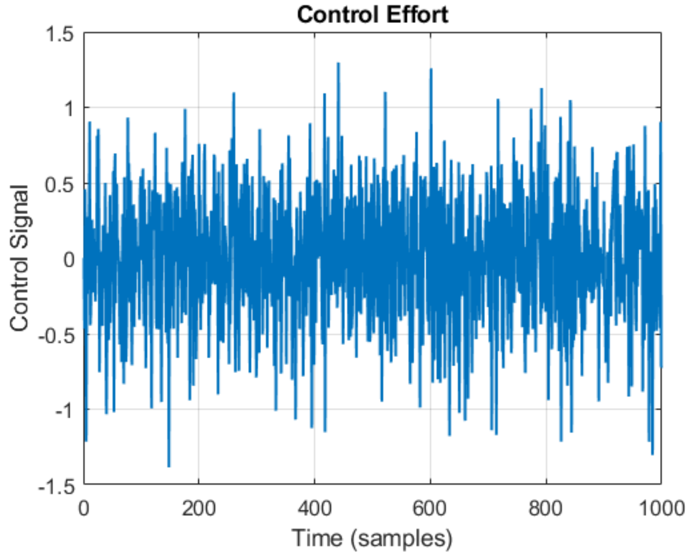

Figure 33: System control effort (Indirect Non-Adaptive Moving-Average colored noise).

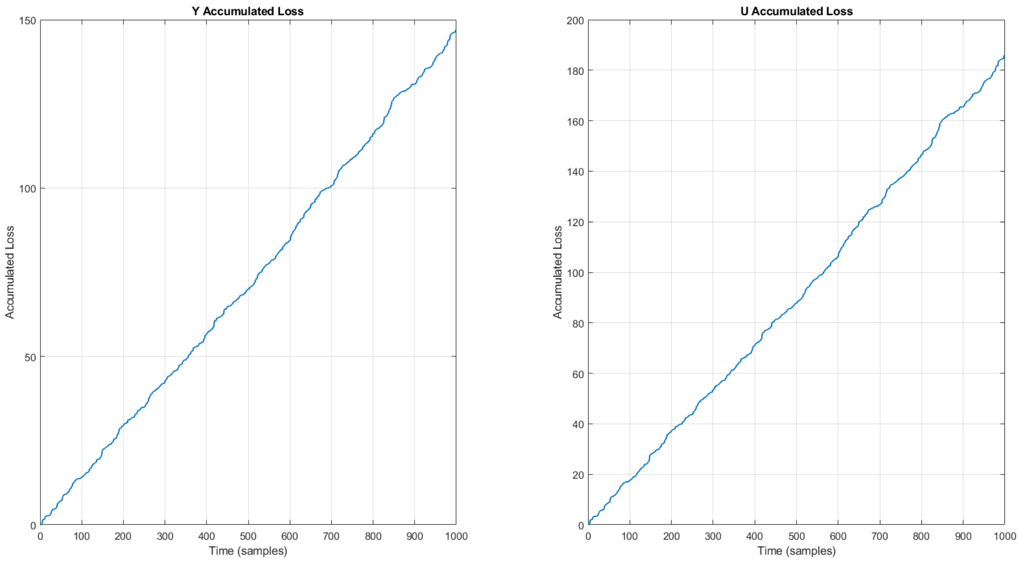

Figure 34: System accumulated errors for out put and control effort (Indirect Non-Adaptive Moving-Average colored noise).

1.11 Adaptive Moving-Average (Indirect,Colored Noise)

In this case, it is first necessary to estimate the system parameters using an estimator. Then, the estimated parameters will be used to solve the Diophantine equation in each iteration. The following code snippet is developed for this purpose. The noise matrix C is assumed to be monic, and its other two parameters are estimated. It is also necessary to check in each iteration whether the obtained C has an unstable zero. If an unstable zero is encountered, the roots can be inverted to obtain a stable equation. The adaptive part is basically the same as minimum-variance.

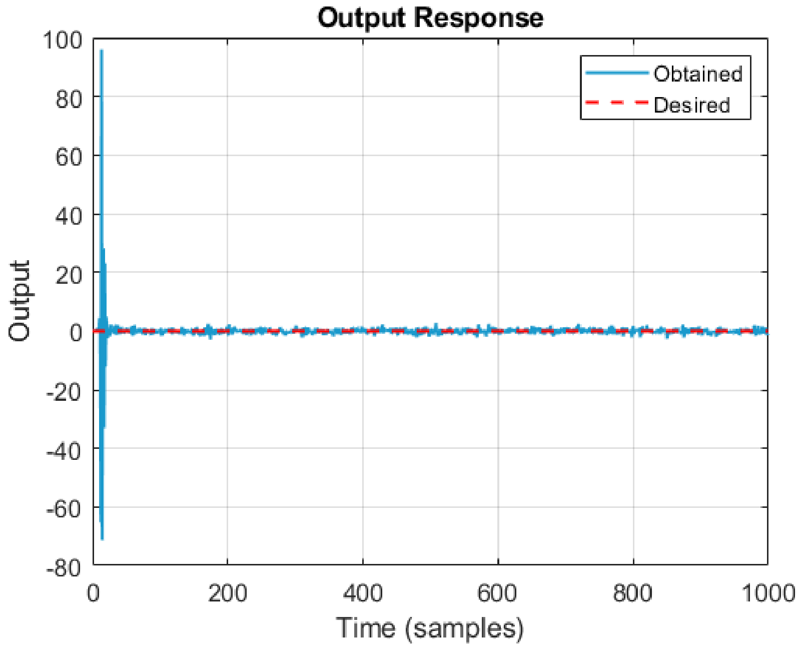

Figure 35: System tracking (Indirect Adaptive Moving-Average colored noise).

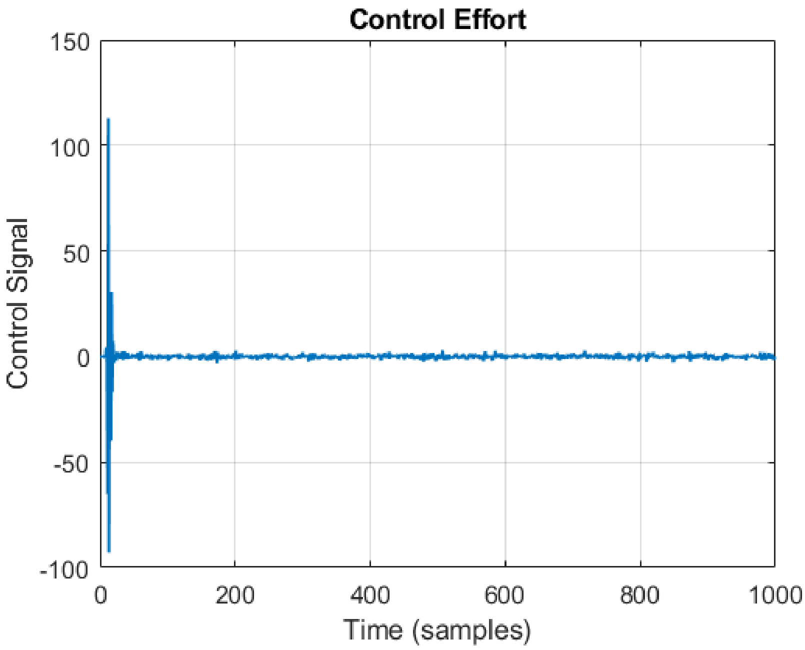

Figure 36: System control effort (Indirect Adaptive Moving-Average colored noise).

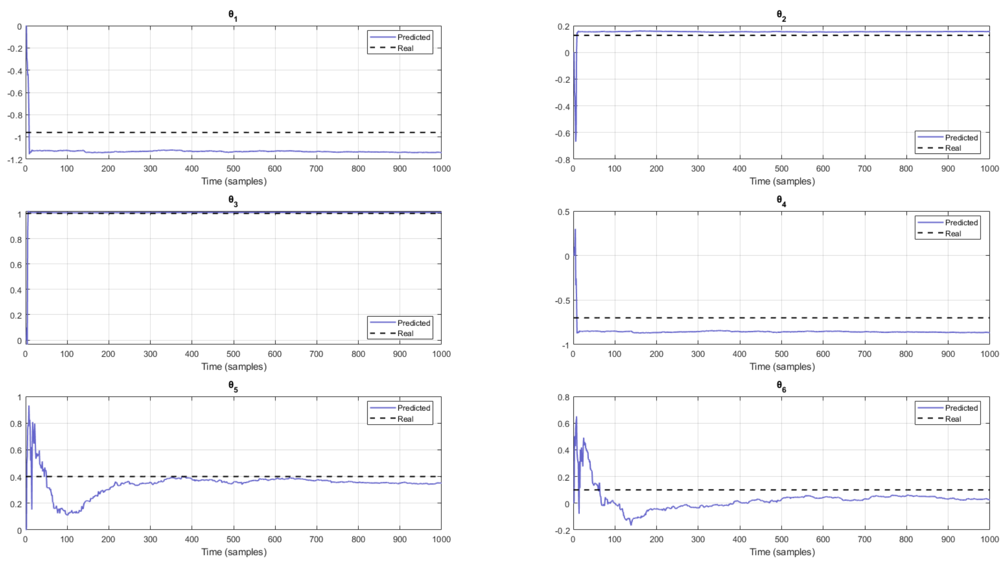

Figure 37: System parameters estimation (Indirect Adaptive Moving-Average colored noise).

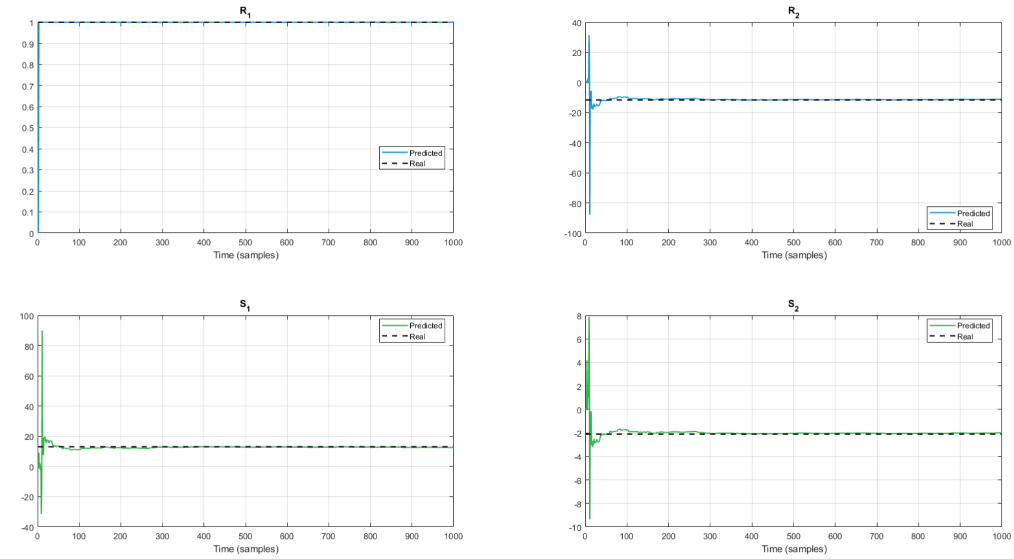

Figure 38: System S, R estimation (Indirect Adaptive Moving-Average colored noise).

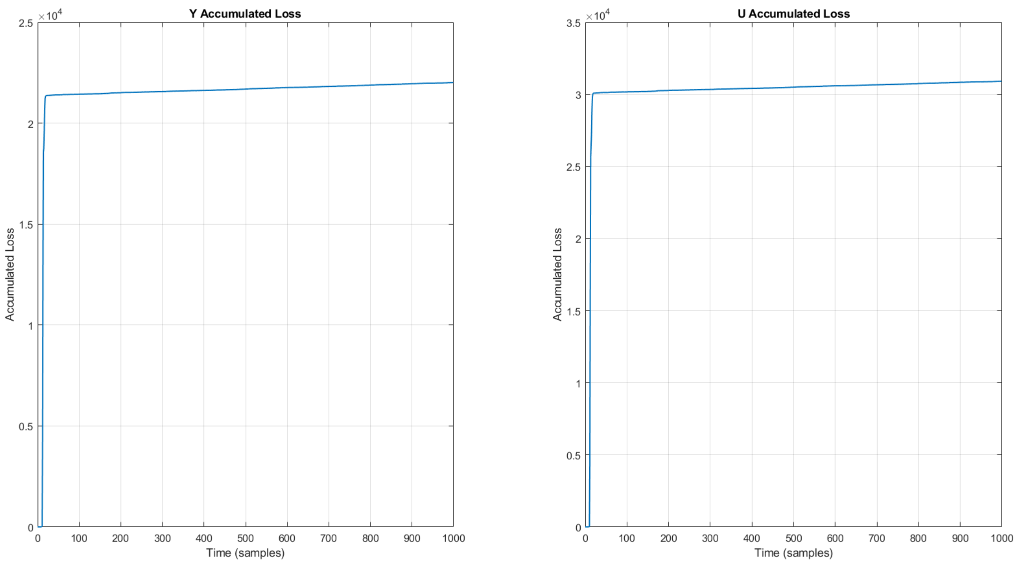

Figure 39: System accumulated errors for out put and control effort (Indirect Adaptive Moving-Average colored noise).

1.12 Studying The results of Indirect Adaptive and Non-Adaptive Moving-Average

As observed, without eliminating the system zeros, the system response is not satisfactory, and the desired output is not achieved. In this scenario, a zero at infinity is introduced into the system (which becomes a pole in the closed-loop system), creating a lag in the system. This degrades the system performance, but the response to a step input might improve, and the steady-state error might decrease. The MA (Moving Average) controller is suitable when the system is non-minimum phase and a minimum phase controller cannot be used.

It is evident that the system exhibits significant oscillations initially. Additionally, we have not eliminated the stable zero of the system, and a pole at the origin has been added, leading to considerable oscillations before parameter identification is achieved.

As shown, after some time, the system parameters are estimated, but there is substantial error initially. Ultimately, considering the non-adaptive solution, we can compare the calculated R and S values with the non-adaptive case, as illustrated in the figure below.

Initially, there is a significant discrepancy due to the incorrect estimation of the system parameters. However, as the identification process progresses, the parameters converge to their true values, and the system performance stabilizes.

- Without eliminating the system zeros, the response is inadequate, leading to poor performance.
- A zero at infinity introduces a lag and converts to a pole in the closed-loop system, causing an initial increase in oscillations.
- The MA controller is particularly beneficial for non-minimum phase systems where a minimum phase controller is not feasible.
- Initial system oscillations are high, and the presence of a pole at the origin, without eliminating the stable zero, contributes to this instability.
- Over time, the system parameters are accurately estimated, reducing the error and stabilizing the system.
- Comparing the R and S parameters between adaptive and non-adaptive cases reveals that initial discrepancies reduce as parameter estimation improves.

1.13 Non-Adaptive Minimum-Variance (Direct,Colored Noise)

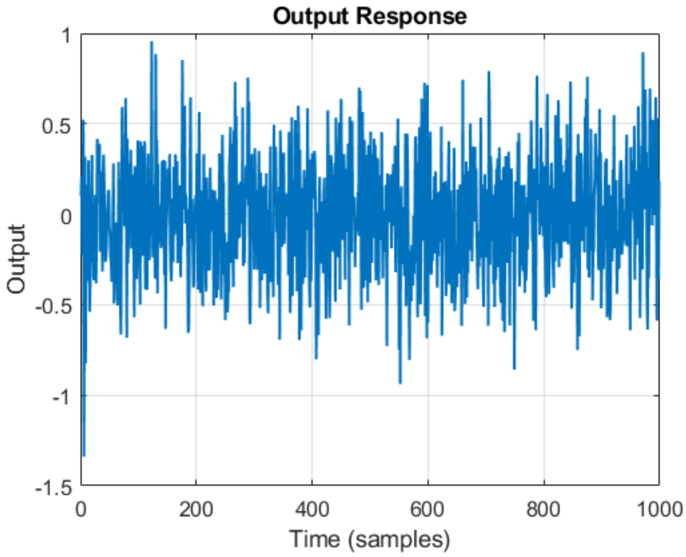

Figure 40: System tracking (direct Non-Adaptive Minimum Variance colored noise).

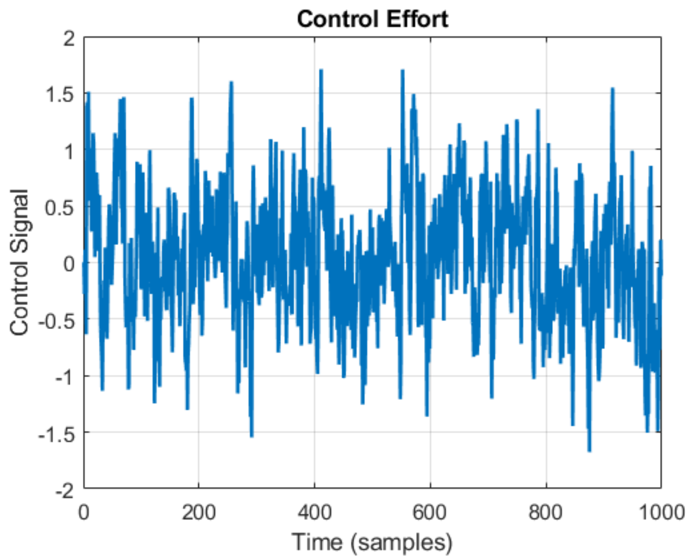

Figure 41: System control effort (direct Non-Adaptive Minimum Variance colored noise).

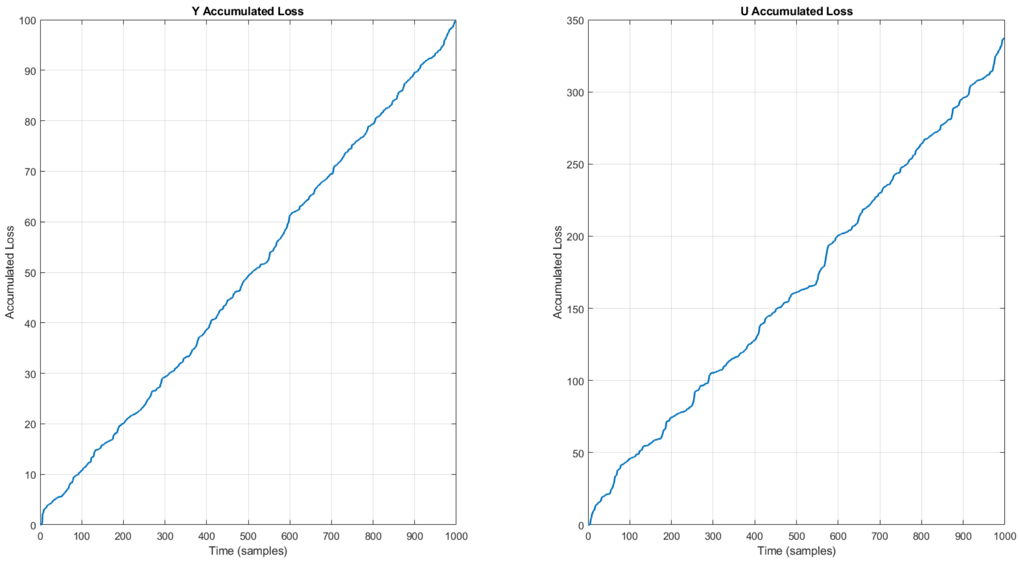

Figure 42: System accumulated errors for out put and control effort (direct Non-Adaptive Minimum Variance colored noise).

1.14 Adaptive Minimum-Variance (Direct,Colored Noise)

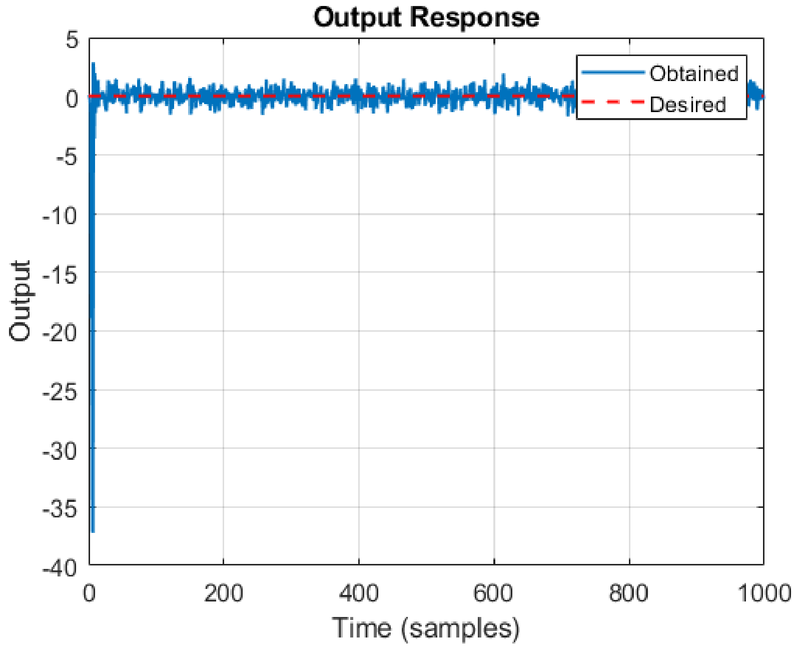

Figure 43: System tracking (direct Adaptive Minimum Variance colored noise).

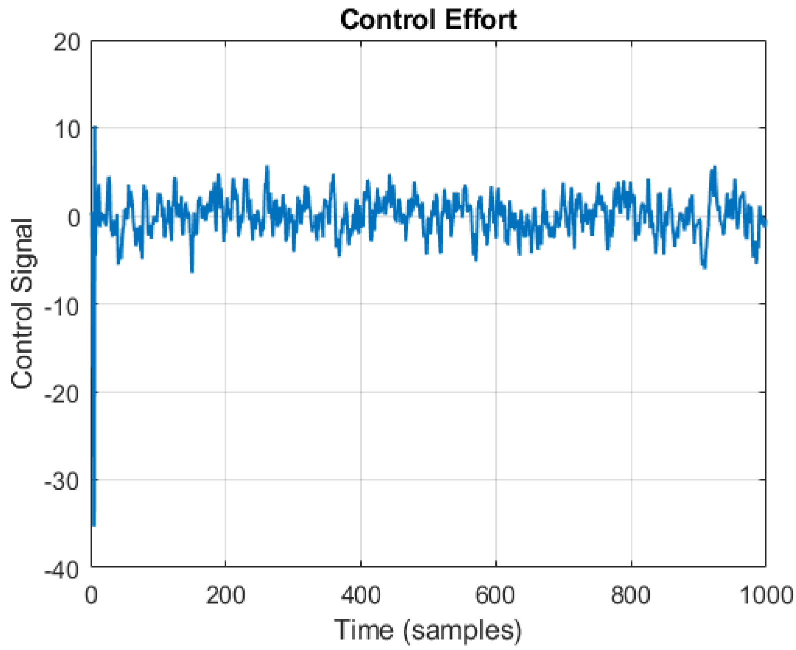

Figure 44: System control effort (direct Adaptive Minimum Variance colored noise).

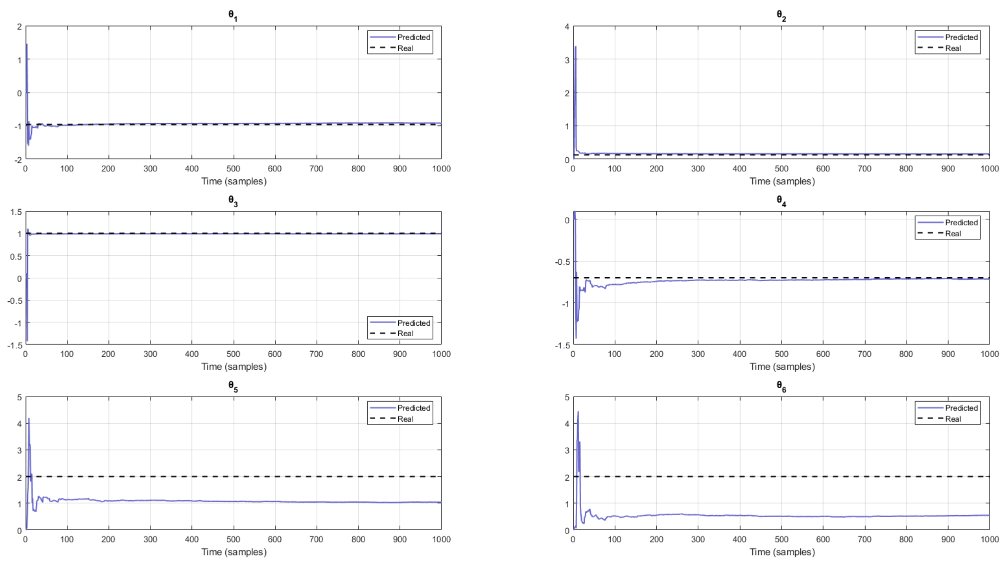

Figure 45: System parameters estimation (direct Adaptive Minimum Variance colored noise).

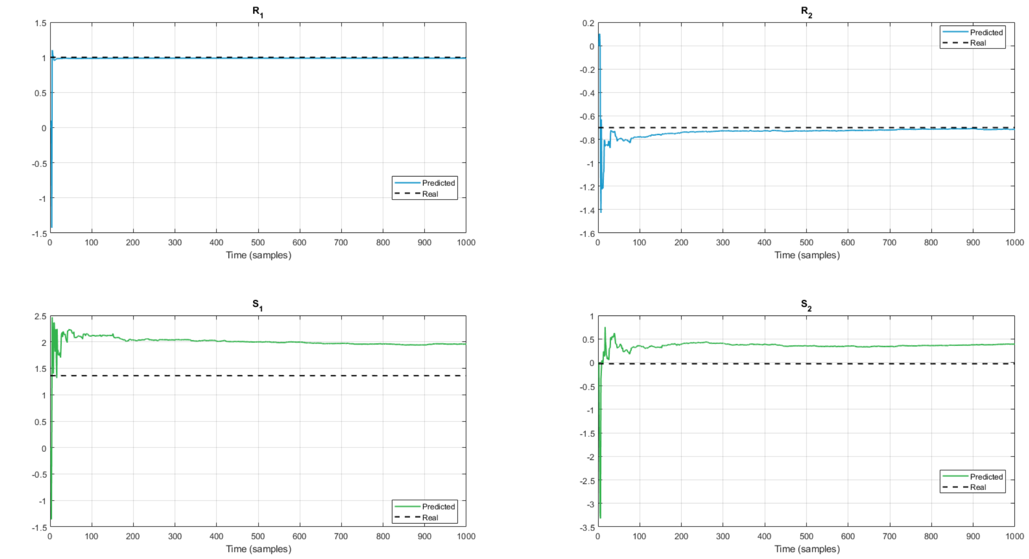

Figure 46: System S, R estimation (direct Adaptive Minimum Variance colored noise).

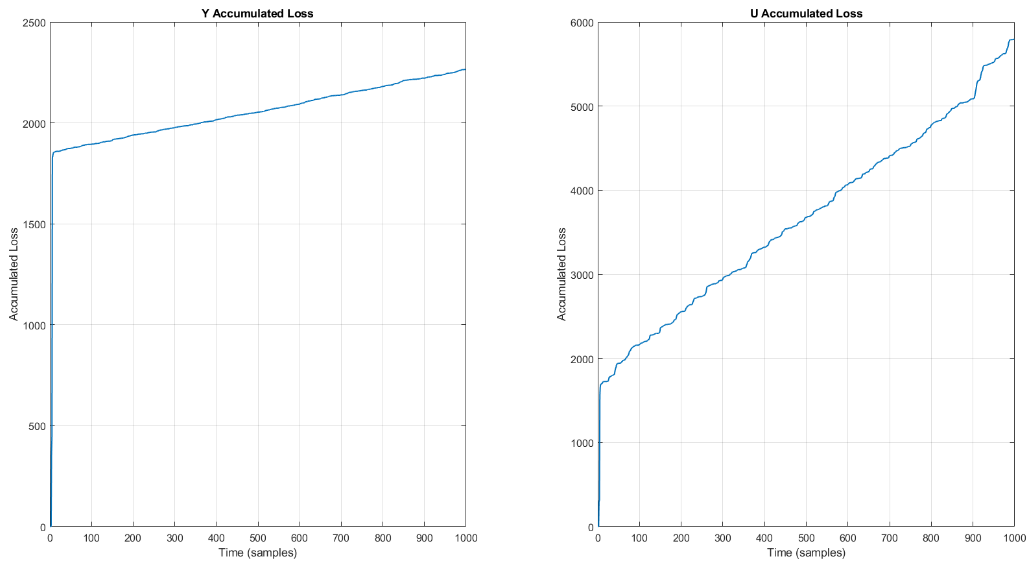

Figure 47: System accumulated errors for out put and control effort (direct Adaptive Minimum Variance colored noise).

1.15 Studying The Results of Direct Adaptive and Non-Adaptive Minimum Variance

As expected, when using the direct method, the results deteriorate, similar to what was observed with the basic STR method. Both the adaptive and non-adaptive minimum variance methods reveal that the output error is significantly high during the initial samples, resulting in a substantial accumulated loss. Additionally, the control effort is considerably higher when employing the direct method. Consequently, the indirect method proves to be more effective in minimum variance mode.

- In the initial stages, the output error is notably high, which contributes to a large accumulated loss.
- The control effort required with the direct method is substantially greater, indicating inefficiencies and potential stress on the system.
- The indirect method, on the other hand, demonstrates better performance in minimizing variance, providing a more stable and accurate response.
- This performance difference underscores the indirect method's robustness and efficiency, making it the preferred choice for achieving minimum variance in control systems.

1.16 Non-Adaptive Moving-Average (Direct,Colored Noise)

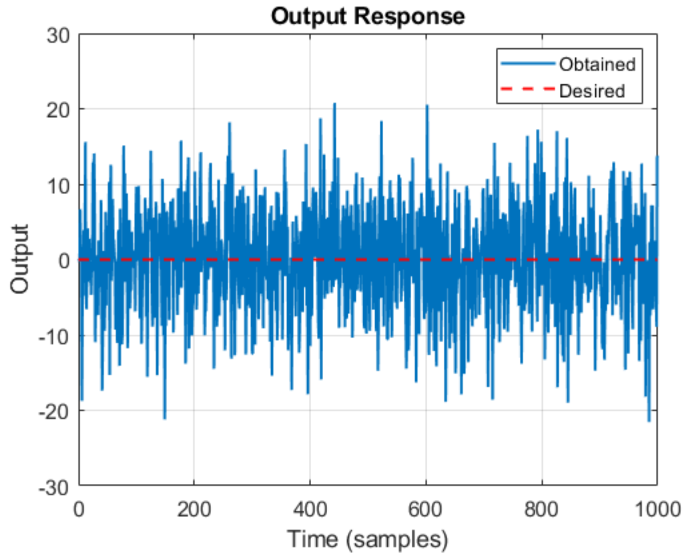

Figure 48: System tracking (direct Non-Adaptive Moving-Average colored noise).

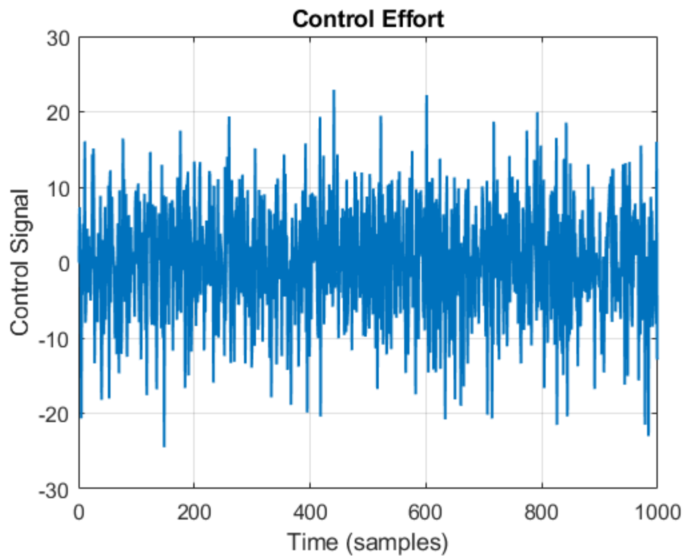

Figure 49: System control effort (direct Non-Adaptive Moving-Average colored noise).

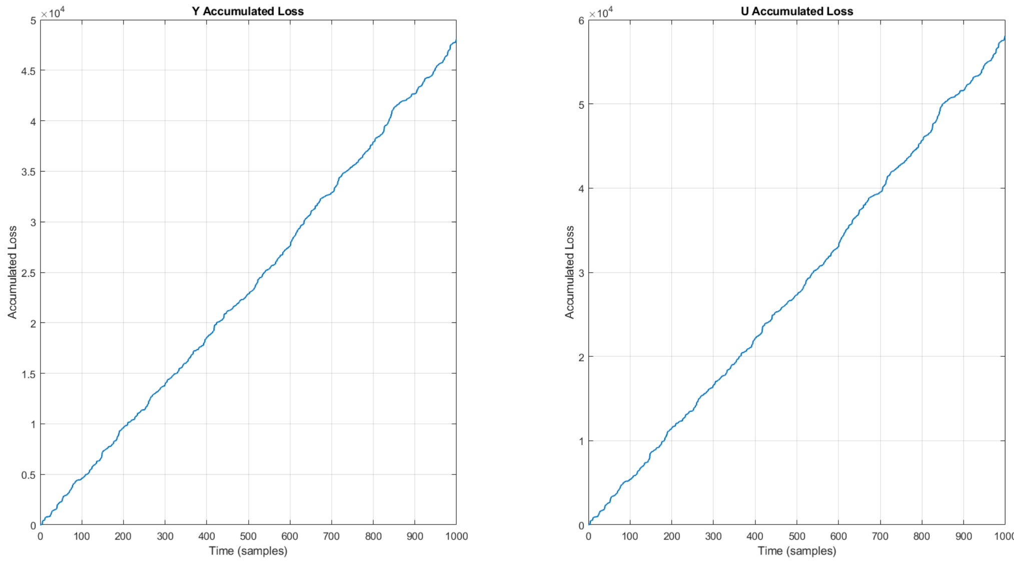

Figure 50: System accumulated errors for out put and control effort (direct Non-Adaptive Moving-Average colored noise).

1.17 Adaptive Moving-Average (Direct,Colored Noise)

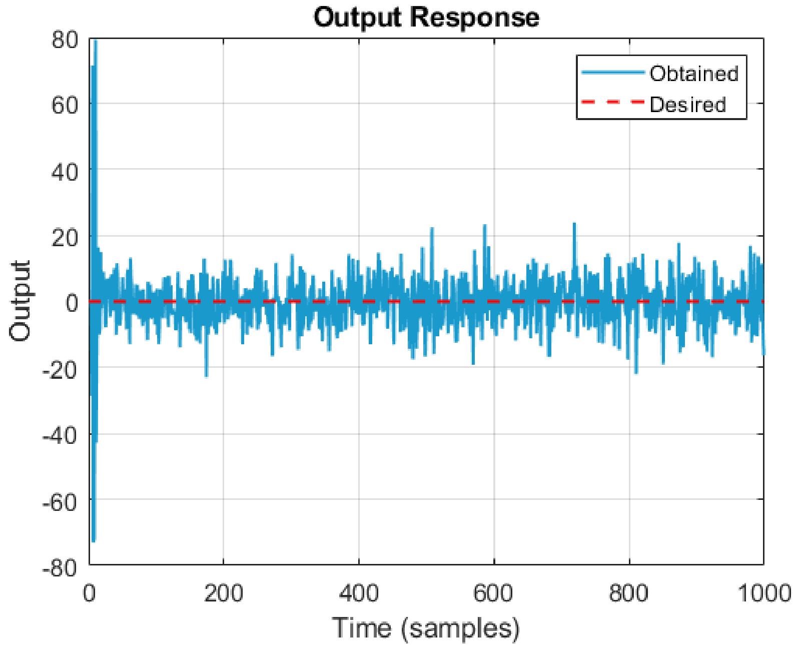

Figure 51: System tracking (direct Adaptive Moving-Average colored noise).

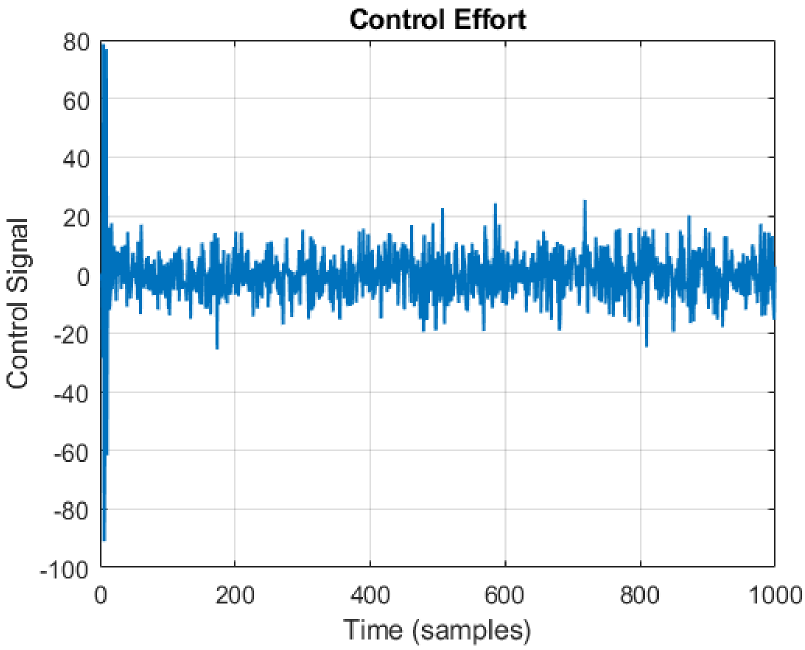

Figure 52: System control effort (direct Adaptive Moving-Average colored noise).

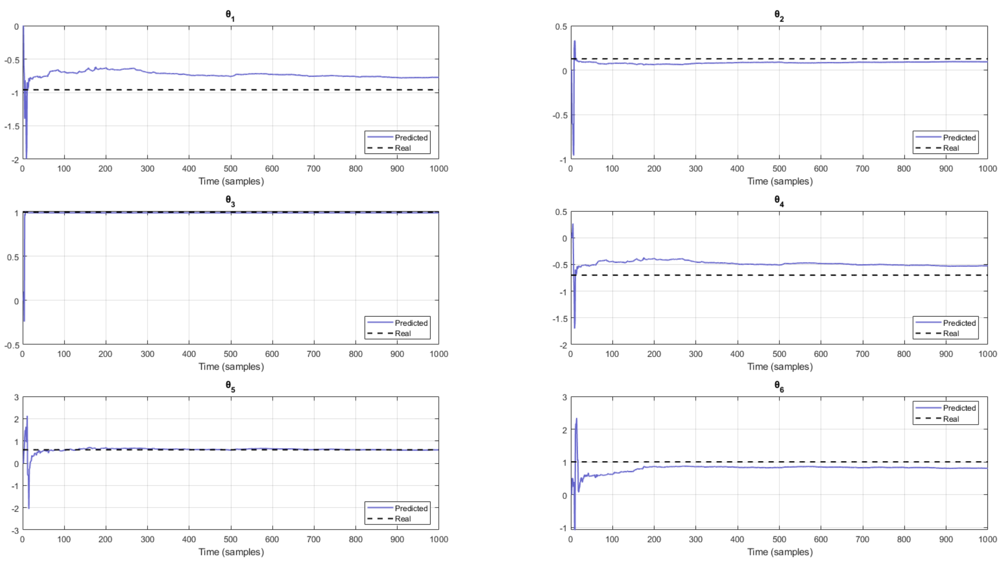

Figure 53: System parameters estimation (direct Adaptive Moving-Average colored noise).

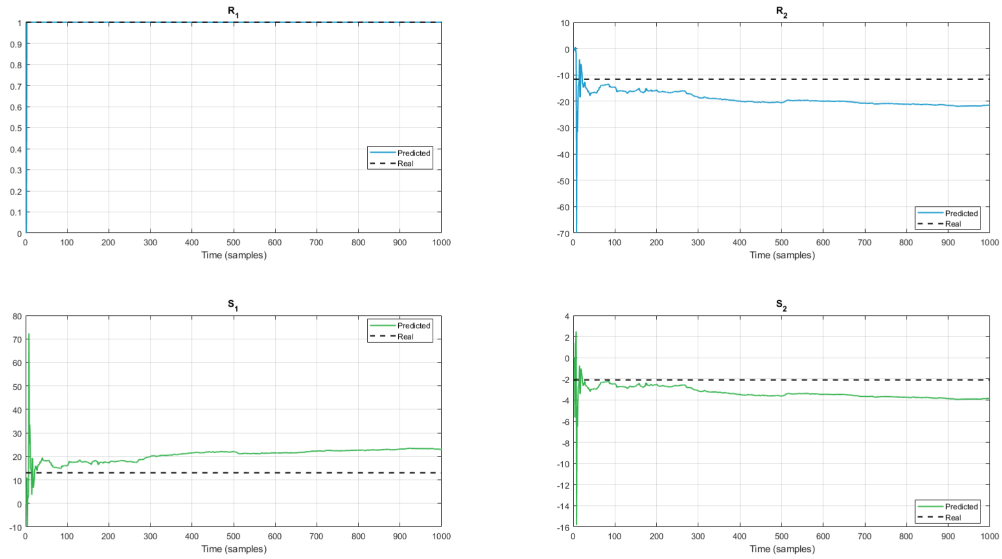

Figure 54: System S, R estimation (direct Adaptive Moving-Average colored noise).

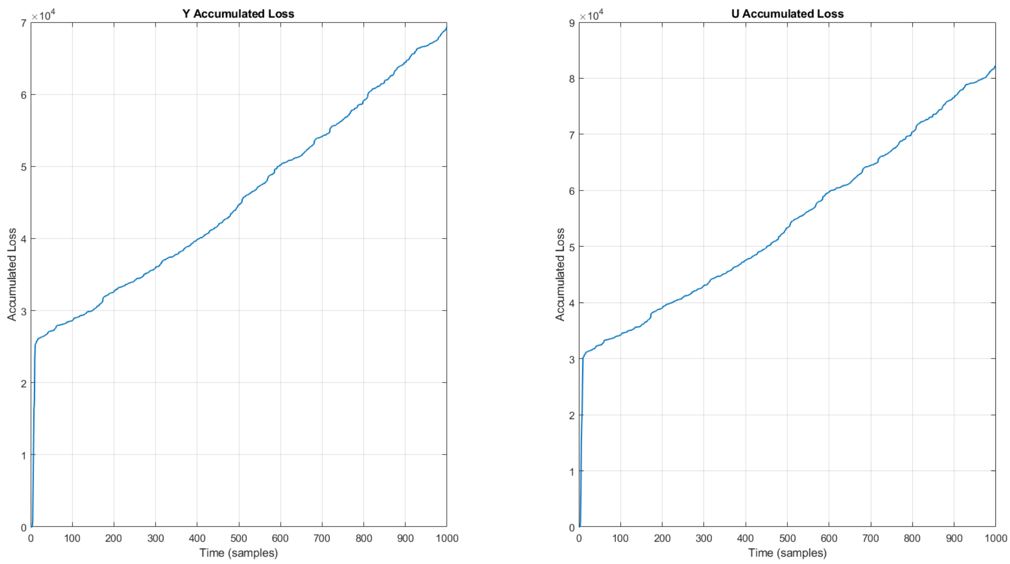

Figure 55: System accumulated errors for out put and control effort (direct Adaptive Moving-Average colored noise).

1.18 Studying The Results of Direct Adaptive and Non-Adaptive Moving-Average

As expected, when using the direct method, the results deteriorate, similar to what was observed with the basic STR method. Both the adaptive and non-adaptive moving average methods reveal that the output error is significantly high during the initial samples, resulting in a substantial accumulated loss. Additionally, the control effort is considerably higher when employing the direct method. Consequently, the indirect method proves to be more effective in moving average mode.

- In the initial stages, the output error is notably high, which contributes to a large accumulated loss.
- The control effort required with the direct method is substantially greater, indicating inefficiencies and potential stress on the system.
- The indirect method, on the other hand, demonstrates better performance, providing a more stable and accurate response.
- This performance difference underscores the indirect method's robustness and efficiency, making it the preferred choice for achieving minimum variance in control systems.
- The results are similar in moving average mode, with a slight improvement, but overall, the direct method is still worse than the indirect method.

1.19 Making the System Non-Minimum Phase

The codes used for this part are exactly the same, but when I attempt to make my system non-minimum phase by reversing the zero for the non-adaptive part, I encounter very poor and unacceptable results. The primary reason is that my real zeros are very small, and reversing them results in a very high value, which is far away from the unit circle. This causes disastrous results in non-adaptive methods. Furthermore, in adaptive methods, the estimation process faces serious problems, and most matrices reach NaN values during the calculations. This leads to the code execution stopping due to calculation errors.

To circumvent these issues, I have opted to use a non-minimum phase system that is manually created and maintains the same order as my original system. The transfer function of this manually created system is as follows:

$$G(z) = \frac{z + 2.4}{z^2 - 0.8z + 0.1}$$

This approach ensures that the system remains stable and avoids the complications associated with reversing the zeros of the original system. By designing a non-minimum phase system manually, we can better control the placement of zeros and poles, thereby maintaining a more predictable and manageable behavior during the estimation and control processes. This adjustment allows for a more accurate comparison and analysis of the adaptive and non-adaptive methods in handling non-minimum phase systems.

1.20 Non-Adaptive Minimum-Variance (Indirect,Colored Noise, NMP System)

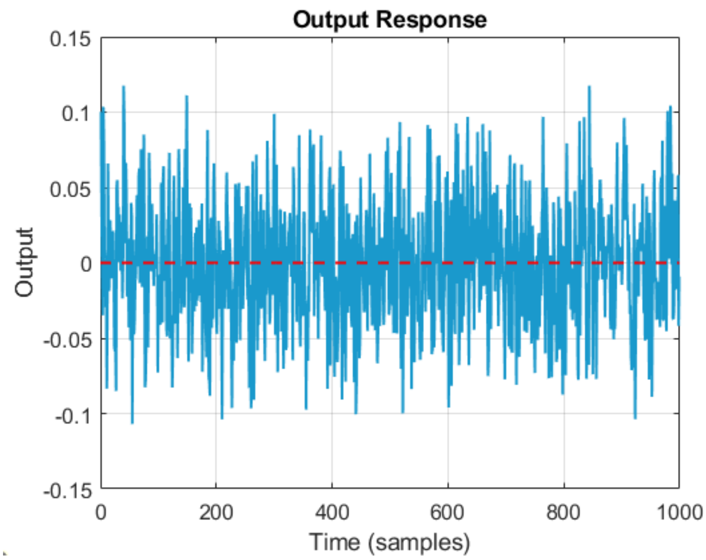

Figure 56: System tracking (Indirect Non-Adaptive Minimum Variance colored noise, NMP System).

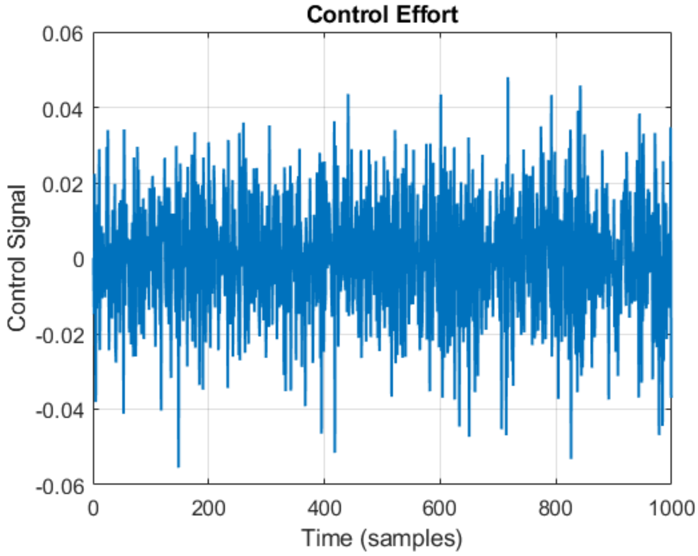

Figure 57: System control effort (Indirect Non-Adaptive Minimum Variance colored noise, NMP System).

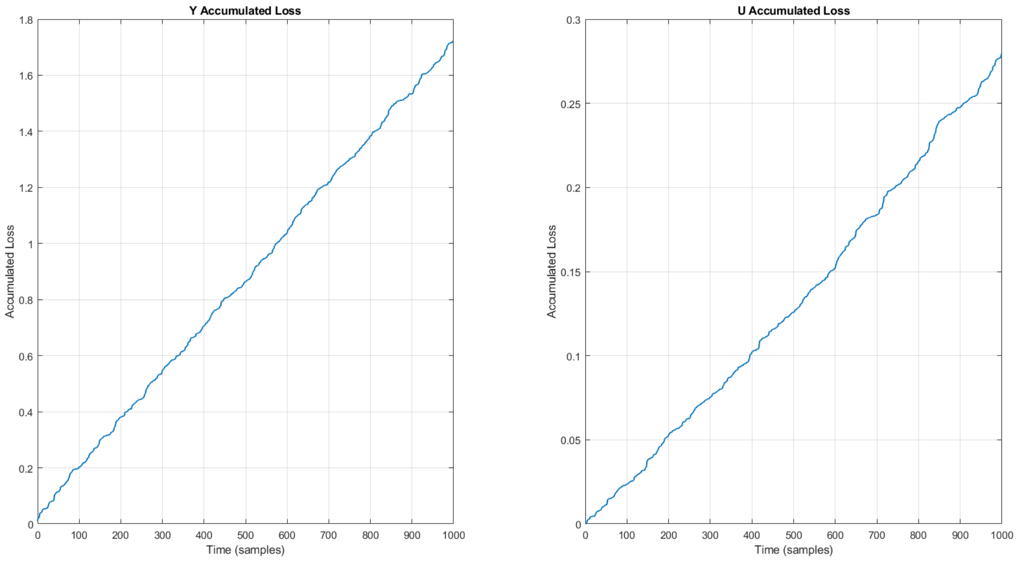

Figure 58: System accumulated errors for out put and control effort (Indirect Non-Adaptive Minimum Variance colored noise, NMP System).

1.21 Adaptive Minimum-Variance (Indirect,Colored Noise, NMP System)

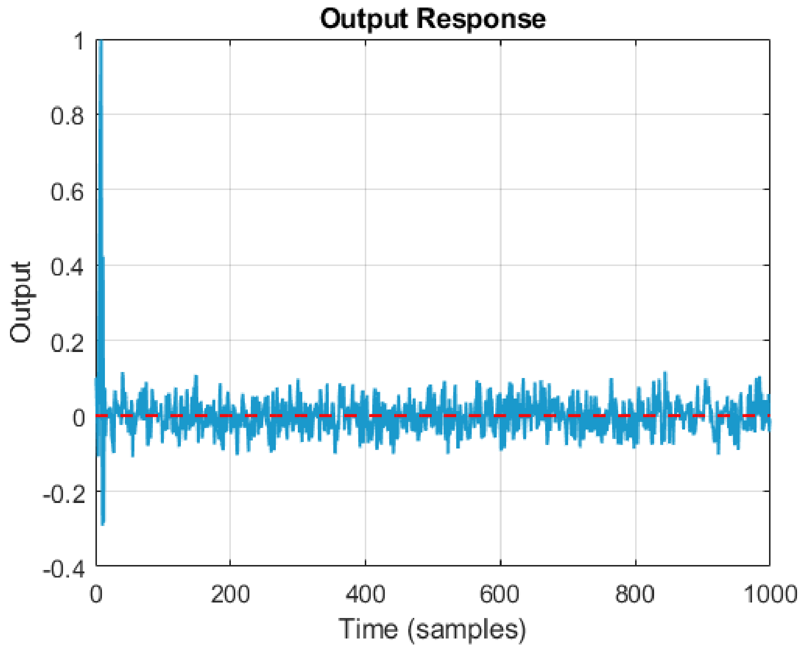

Figure 59: System tracking (Indirect Adaptive Minimum Variance colored noise, NMP System).

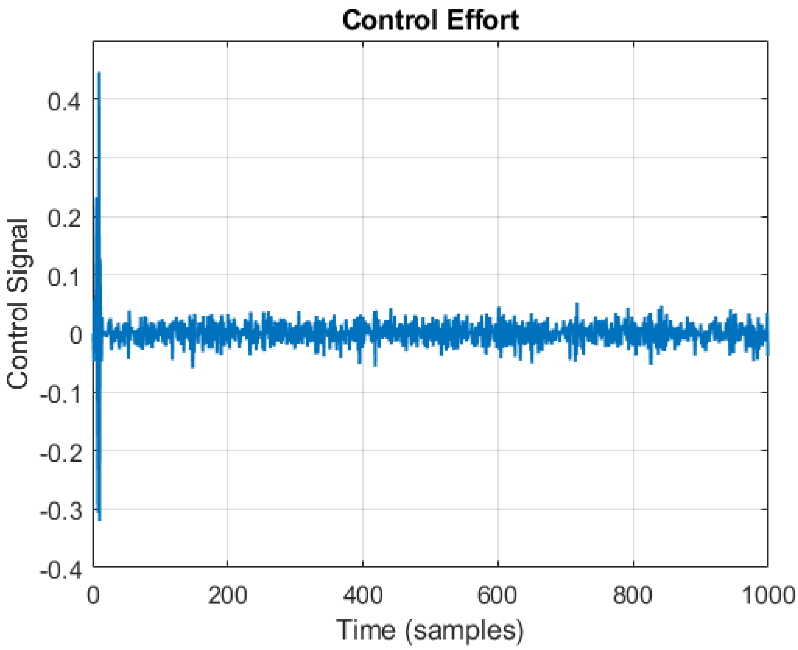

Figure 60: System control effort (Indirect Adaptive Minimum Variance colored noise, NMP System).

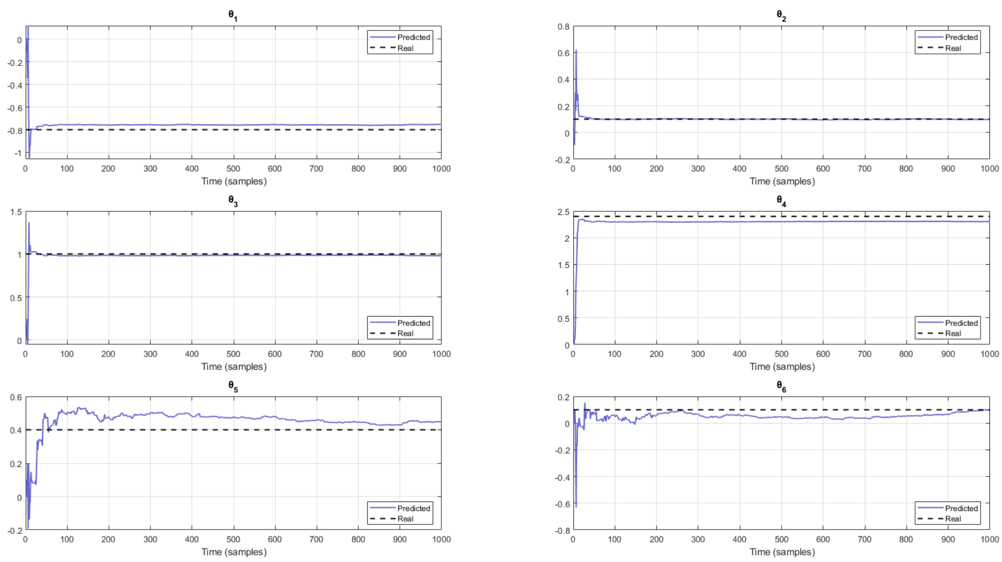

Figure 61: System parameters estimation (Indirect Adaptive Minimum Variance colored noise, NMP System).

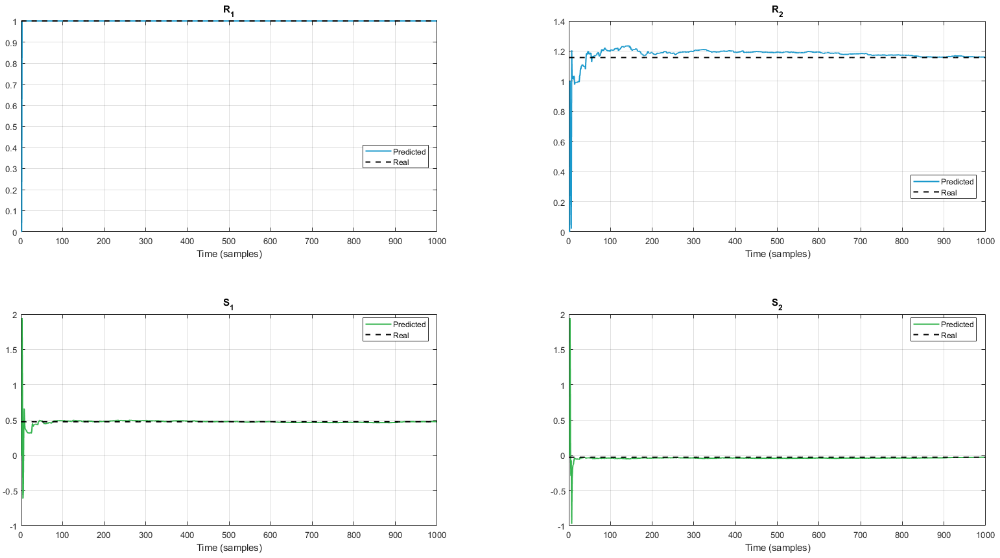

Figure 62: System S, R estimation (Indirect Adaptive Minimum Variance colored noise, NMP System).

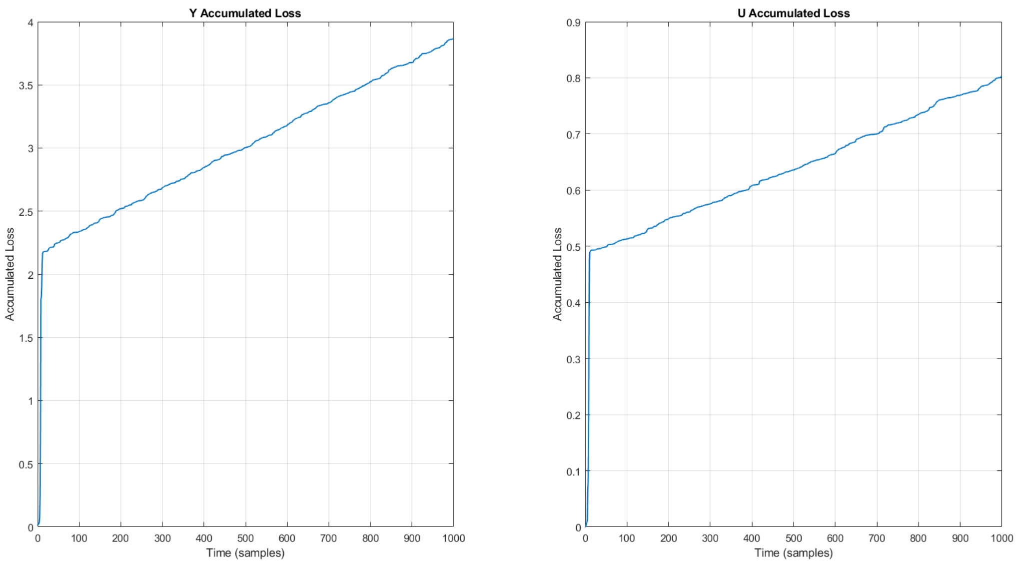

Figure 63: System accumulated errors for out put and control effort (Indirect Adaptive Minimum Variance colored noise, NMP System).

1.22 Non-Adaptive Moving-Average (Indirect,Colored Noise, NMP System)

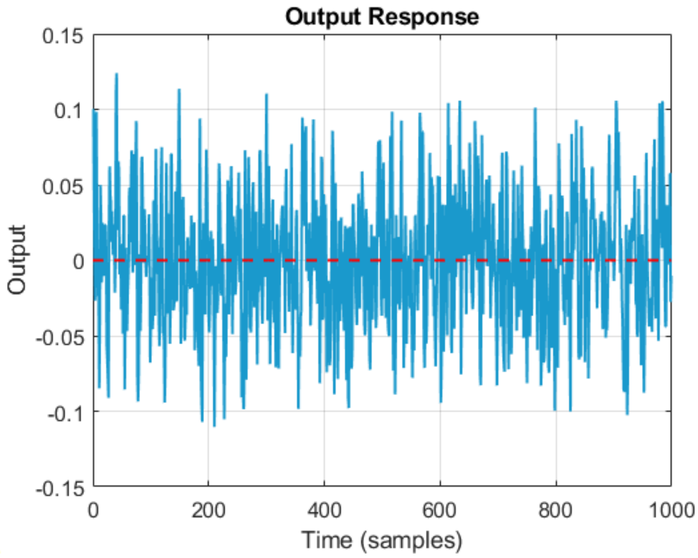

Figure 64: System tracking (Indirect Non-Adaptive Moving-Average colored noise, NMP System).

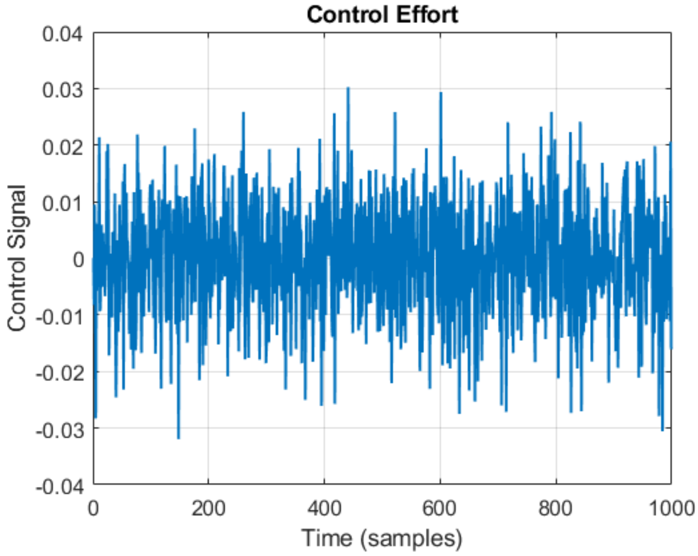

Figure 65: System control effort (Indirect Non-Adaptive Moving-Average colored noise, NMP System).

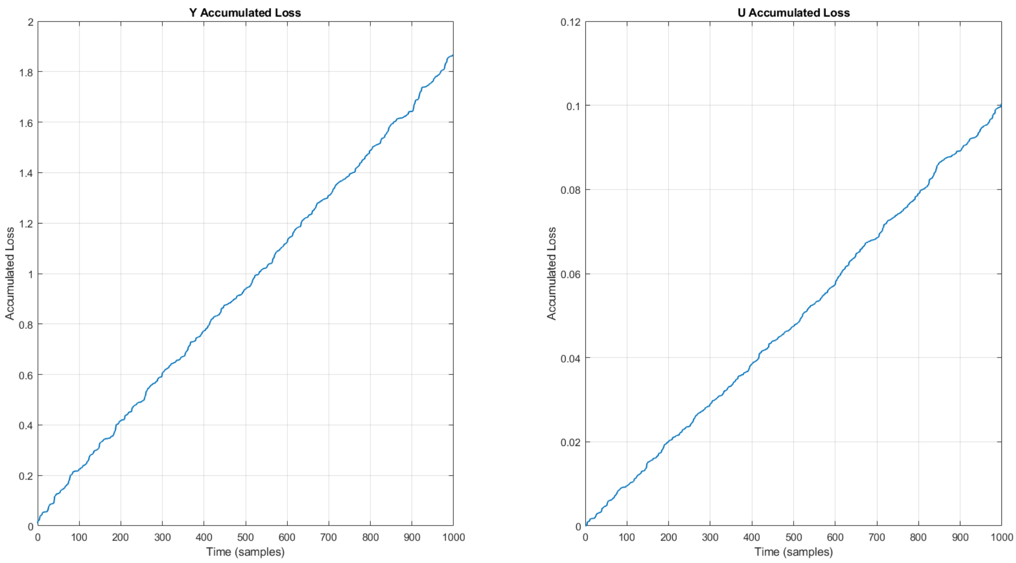

Figure 66: System accumulated errors for out put and control effort (Indirect Non-Adaptive Moving-Average colored noise, NMP System).

1.23 Adaptive Moving-Average (Indirect,Colored Noise, NMP System)

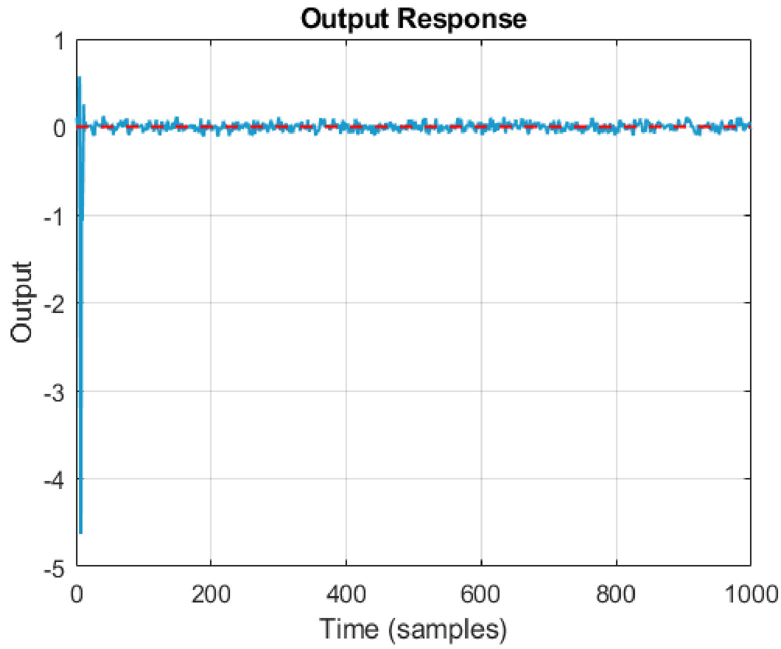

Figure 67: System tracking (Indirect Adaptive Moving-Average colored noise, NMP System).

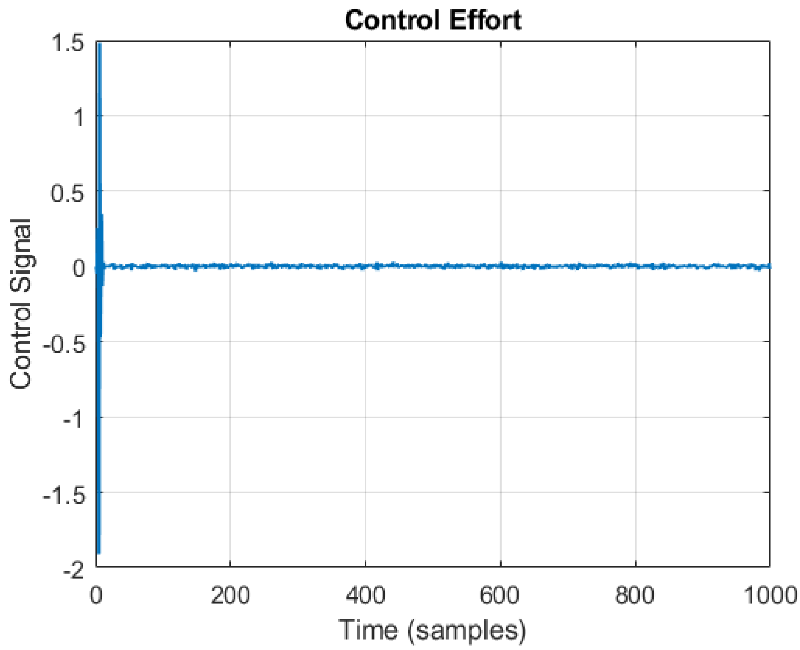

Figure 68: System control effort (Indirect Adaptive Moving-Average colored noise, NMP System).

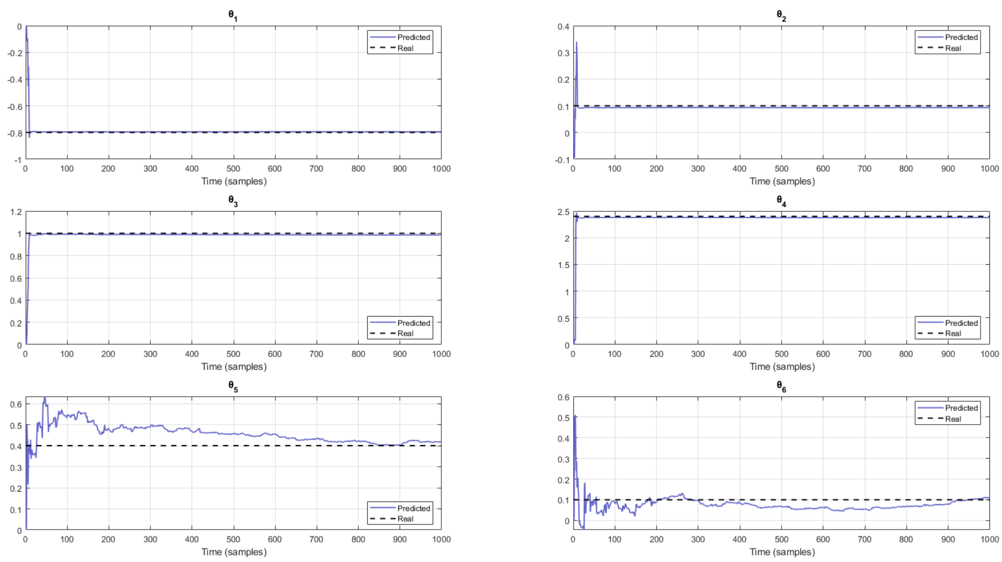

Figure 69: System parameters estimation (Indirect Adaptive Moving-Average colored noise, NMP System).

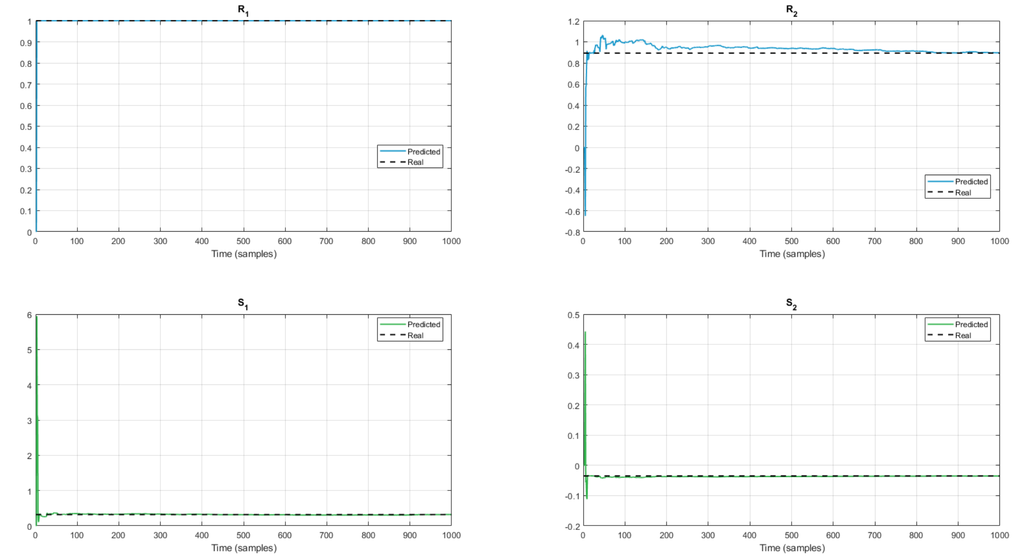

Figure 70: System S, R estimation (Indirect Adaptive Moving-Average colored noise, NMP System).

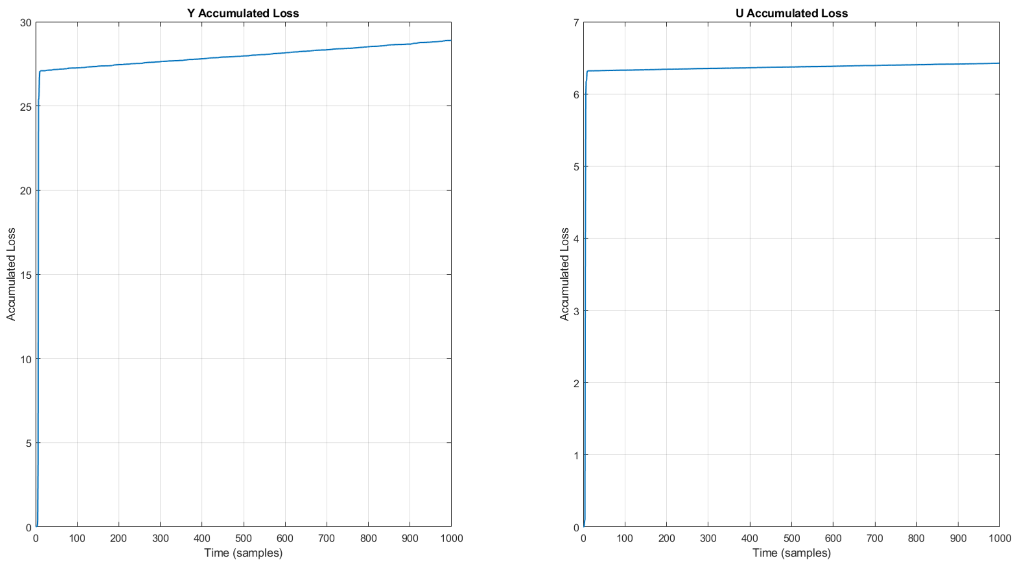

Figure 71: System accumulated errors for out put and control effort (Indirect Adaptive Moving-Average colored noise, NMP System).

1.24 Conclusion On Comparing All methods

The minimum variance controller performs well for minimum phase systems, while the moving average (MA) controller does not perform as effectively in these cases. Adding an extra zero at the origin can degrade system performance and introduce additional delay. However, for non-minimum phase systems, where there is an unstable zero, the addition of this zero at the origin by the MA controller helps stabilize the system and reduces the error. On the other hand, although the minimum variance controller produces a stable response in such cases, its performance is not as satisfactory. The MA controller, by addressing the instability, ensures a more reliable and consistent control performance, making it a better choice for non-minimum phase systems.

Method	$\bar{u}$	$\bar{y}$	σ_u	σ_y
Minimum-Variance	-0.0012	-0.0018	0.0044	0.00098
Moving-Average	0.00132	0.00217	0.241	0.182

Table 3: Comparing the values of variance and mean in input and output in MA and MV method (Non-Adaptive, Minimum-Phase system).

Method	$\bar{u}$	$\bar{y}$	σ_u	σ_y
Minimum-Variance	7.12E-04	3.12E-04	0.00581	0.00251
Moving-Average	-0.0264	-0.0361	1.002	0.961

Table 4: Comparing the values of variance and mean in input and output in MA and MV method (Adaptive, Minimum-Phase system).

Method	$\bar{u}$	$\bar{y}$	σ_u	σ_y
Minimum-Variance	5.42E-04	2.31E-04	0.0021	0.00099
Moving-Average	0.00211	0.0124	0.0231	0.0662

Table 5: Comparing the values of variance and mean in input and output in MA and MV method (Non-Adaptive, Non Minimum-Phase system).

Method	$\bar{u}$	$\bar{y}$	σ_u	σ_y
Minimum-Variance	8.98E-04	3.93E-04	0.00721	0.00481
Moving-Average	-0.00996	-0.0267	0.0713	0.0795

Table 6: Comparing the values of variance and mean in input and output in MA and MV method (Adaptive, Non Minimum-Phase system).

As observed, for the non-minimum phase case, the MA controller achieves the lowest output variance, while for the minimum phase case, the minimum variance controller performs best in terms of output variance. The pole placement controller (R) is suited for solving tracking problems, and the minimum variance controller is tailored for regulation issues, making direct comparisons of mean values somewhat misleading.

In both the minimum variance and MA controllers, noise dynamics are incorporated into the controller design, allowing for a more effective reduction in output variance and overall system fluctuations in the presence of noise. This is evident from the calculated variance values. It is worth noting that for STR pole placement, evaluating the absolute variance might provide better insights.

On the other hand, pole placement allows us to achieve a desired system response and improve transient performance, which is not possible with the minimum variance controller. Hence, we can conclude that the minimum variance controller is better suited for scenarios where reducing the effect of measurement noise is crucial.

It is important to note that the minimum variance and MA controllers can be considered subsets of the STR pole placement method, where the focus is on minimizing output noise. In these controllers, Am and Ao are designed with this objective in mind.

Overall, while the minimum variance controller excels in minimizing noise impact, the pole placement method offers better control over transient response, highlighting the trade-offs between these approaches in control system design.