Simulation 2 Adaptive Control

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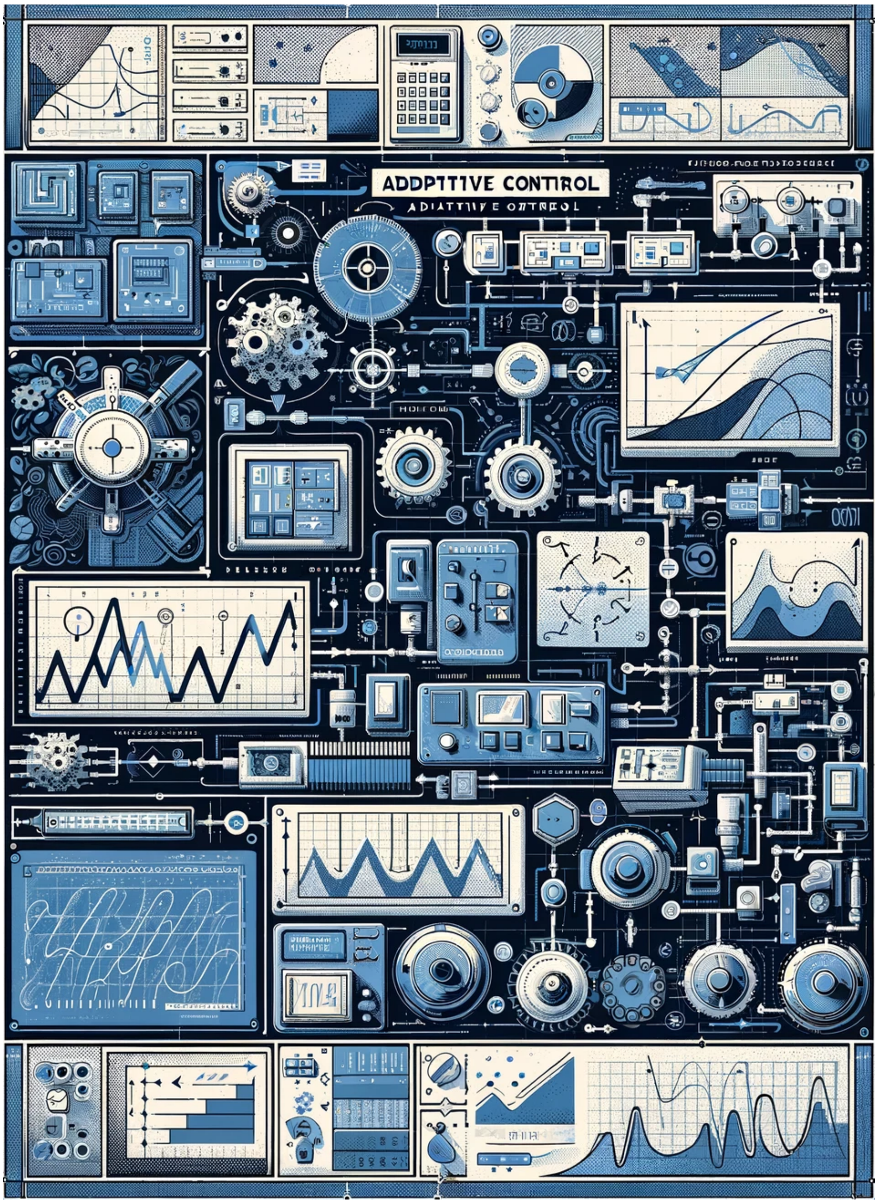

October 26, 2024

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1 MDPP Using Dynamic Feedback

The continuous system is given as follows:

$$G(s) = \frac{(s+0.5)(s+7)}{(s+1)(s+4.1)(s-2)}$$

We will find the discrete system as follows:

```matlab
%% Discretization of the Model

% Define the transfer function variable
s = tf('s');

% Continuous-time system transfer function
sysC = (s+0.5)*(s+7)/((s+1)*(s+4.1)*(s-2));

% Calculate the bandwidth of the continuous-time system
BW = bandwidth(sysC);

% Discretization ratio and sampling frequency
disc_ratio = 10;
fs = BW * disc_ratio/(2*pi);
Ts = 1/fs; % Sampling time

% Discretize the continuous-time system using zero-order hold
sysD = c2d(sysC, Ts, 'zoh');
[numD, denD] = tfdata(sysD, 'v'); % Get numerator and denominator

B = numD;
A = denD;

```

$$\rightarrow H(z) = \frac{0.2052z^2 - 0.2557z + 0.06087}{z^3 - 2.75z^2 + 2.345z - 0.6199}$$

Now we will find the desired system output as follows:

```
%% Desired System Specifications

% Desired overshoot and settling time
overshoot = 10;
settling_time = 3;

% Calculate damping ratio and natural frequency
zeta = cos(atan2(pi,-log(overshoot/100)));
wn = 4/(zeta*settling_time);
```

With placing the other pole at least 3.5 times away from the imaginary axis we get the desired system.

1.1 MDPP Without Zero Cancellation

$$H_m(z) = \frac{12.5(z^2 - 0.5203z + 0.09421)}{z(0.002401)(z^2 - 1.7748z + 0.78937)}$$

Now that the chosen system is in place, we can proceed to solve the Diophantine equation and find the controller values. To do so, it is necessary to first define the variables B^+ and B^-. Since zero elimination does not take place, $B^+ = 1$ and $B^- = 1$ are chosen. Now we can calculate the Diophantine equation, which is as follows:

$$AR' + B^-S = A_0A_m$$

For this purpose, a function called Diophantine is used. In the first part of the code, the degrees related to A_0, R, and S are calculated and compared with the given inputs to avoid errors. Then, the left-hand side (LHS) and right-hand side (RHS) of the equation are computed symbolically, and the coefficients of the polynomials are extracted to form the system of equations. By solving the system of equations, the values corresponding to S, R, and T are obtained. The advantage of this method is that by changing the degrees of R and S, there is no need to amend this part of the code.

R	S	T
1	27.4925	5.2142
-4.4999	-40.4313	0
1.2695	12.9274	0

Table 1: Calculated STR MDPP

The results are plotted as follows:

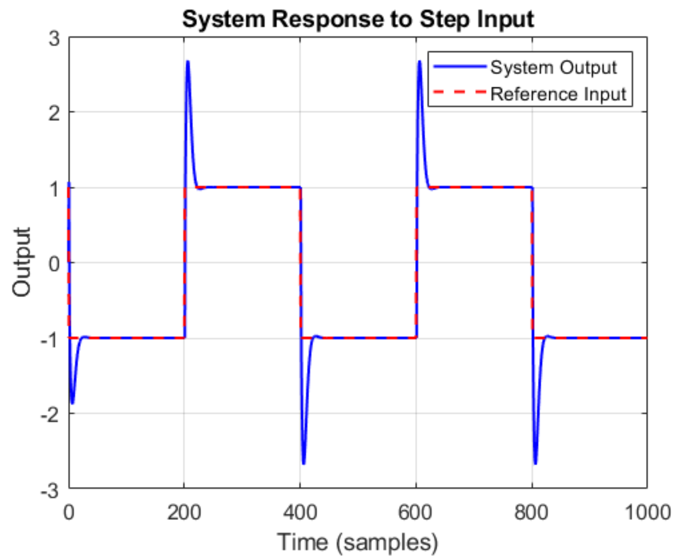

Figure 1: System output vs. Reference input (With zero cancellation)

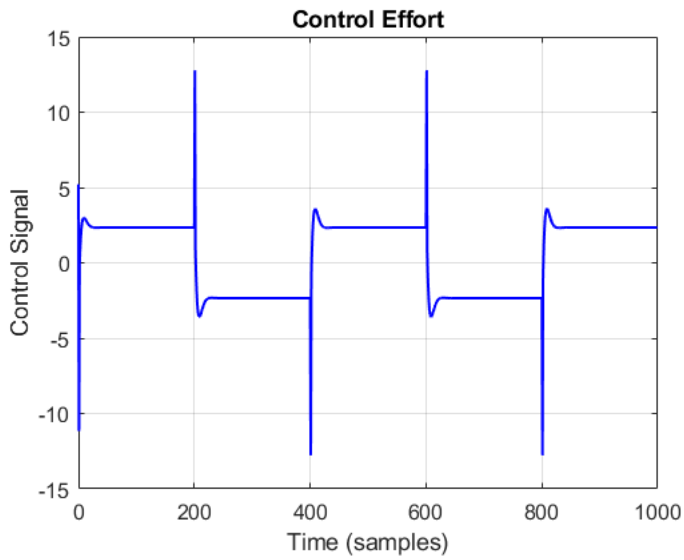

Figure 2: Control effort for MDPP with zero cancellation

1.2 With Zero Cancellation

In the case where zero elimination is allowed, we have more flexibility to achieve the desired response for the system. Therefore, we can also use the zeros of the new system. Considering the parameters given in Figure 1, and taking a new pole at -40 and new zeros at -20 and -30, we can derive the desired transfer function of the system as follows.

$$H_m(z) = \frac{0.04324(z^2 - 0.21543z + 0.024431)}{(z - 0.02867)(z^2 - 1.75421z + 0.78921)}$$

Now that the chosen system is in place, we can proceed to solve the Diophantine equation

R	S	T
1	5.7665	1.4857
-1.2463	-8.1852	-1.1446
0.2967	3.0145	0.1367

Table 2: Calculated STR MDPP

and find the controller values. To do so, it is necessary to first define the variables B^+ and B^-. Since zero elimination does not take place, $B^+ = 1$ and $B^- = 1$ are chosen. Now we can calculate the Diophantine equation, which is as follows:

$$AR' + B^- S = A_0 A_m$$

For this purpose, a function called Diophantine is used. In the first part of the code, the degrees related to A_0, R, and S are calculated and compared with the given inputs to avoid errors. Then, the left-hand side (LHS) and right-hand side (RHS) of the equation are computed symbolically, and the coefficients of the polynomials are extracted to form the system of equations. By solving the system of equations, the values corresponding to S, R, and T are obtained. The advantage of this method is that by changing the degrees of R and S, there is no need to amend this part of the code.

The results are plotted as follows:

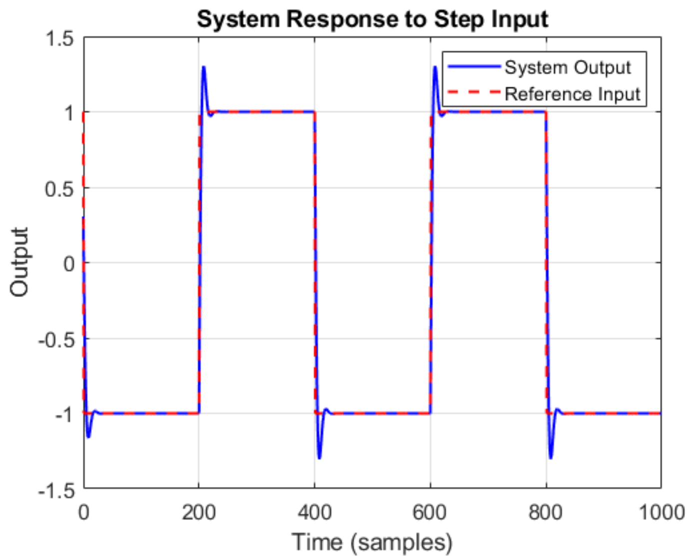

Figure 3: System output vs. Reference input (With zero cancellation)

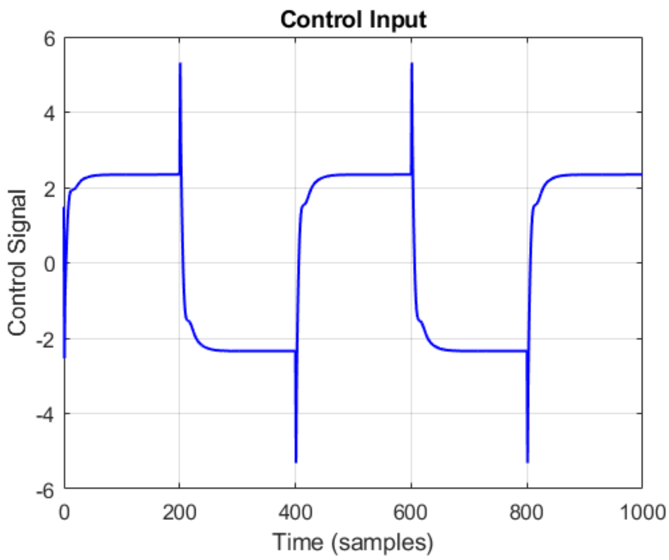

Figure 4: Control effort for MDPP with zero cancellation

1.3 Conclusion on MDPP With and Without Zero cancellation

When we don't cancel out the zeros in the system, the system response differs slightly from the actual response, as illustrated in Figure 1. This discrepancy occurs because the presence of zeros impacts the transient behavior of the system. If we desire a response with less overshoot, we would need to choose poles that are positioned further away from the imaginary axis. While this approach reduces overshoot, it results in very large control efforts, which are not acceptable for our application due to practical constraints such as actuator limitations and energy consumption.

On the other hand, when we do cancel out the zeros, the most significant difference observed is in the T polynomial. The parameters of the T polynomial must be chosen carefully to ensure that new zeros are introduced into the system at desired locations. This adjustment is crucial because the placement of these zeros can significantly affect the system's dynamic response, including its stability and performance characteristics. By strategically positioning the new zeros, we can achieve a more desirable balance between system performance and control effort, ensuring that the system operates efficiently while meeting the required specifications. Thus, the ability to cancel out zeros provides us with greater flexibility in designing the control system to meet both performance and practical constraints.

2 STR Using MDPP For Minimum Phase System

In this section we will design STR controllers using MDPP for a non-minimum phase system and without noise.

2.1 Indirect STR Without Pole-Zero Cancellation

In indirect methods, during each program execution, the aim is to first estimate the parameters A and B of the system. After these estimations are made, the controller polynomials are then calculated. Initially, the code was implemented using the Recursive Least Squares (RLS) method.

The STR Parameters are obtained as follows:

R	S	T
1	18.2185	3.3711
-2.6253	-23.7952	0
0.6498	6.7085	0

Table 3: Calculated indirect STR without zero cancellation.

The output of the system under these conditions is shown in the figure below.

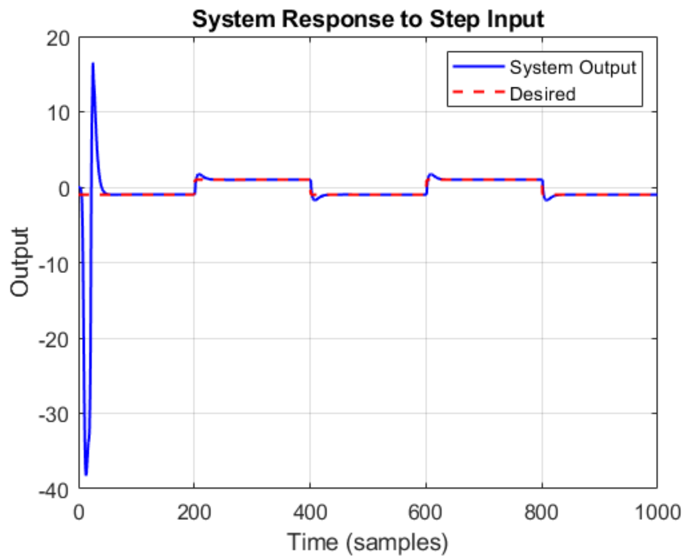

Figure 5: System output vs. Reference input (Indirect STR-Without zero cancellation)

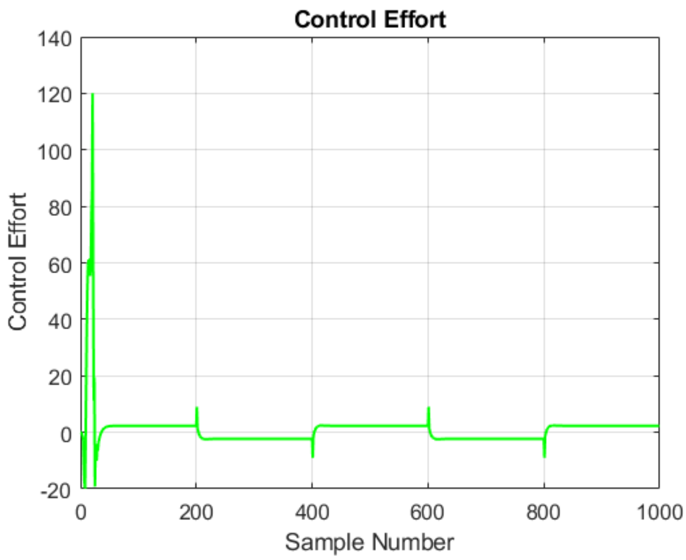

Figure 6: Control effort for Indirect STR without zero cancellation

The parameters for A and B polynomials are estimated as shown in the figure below..

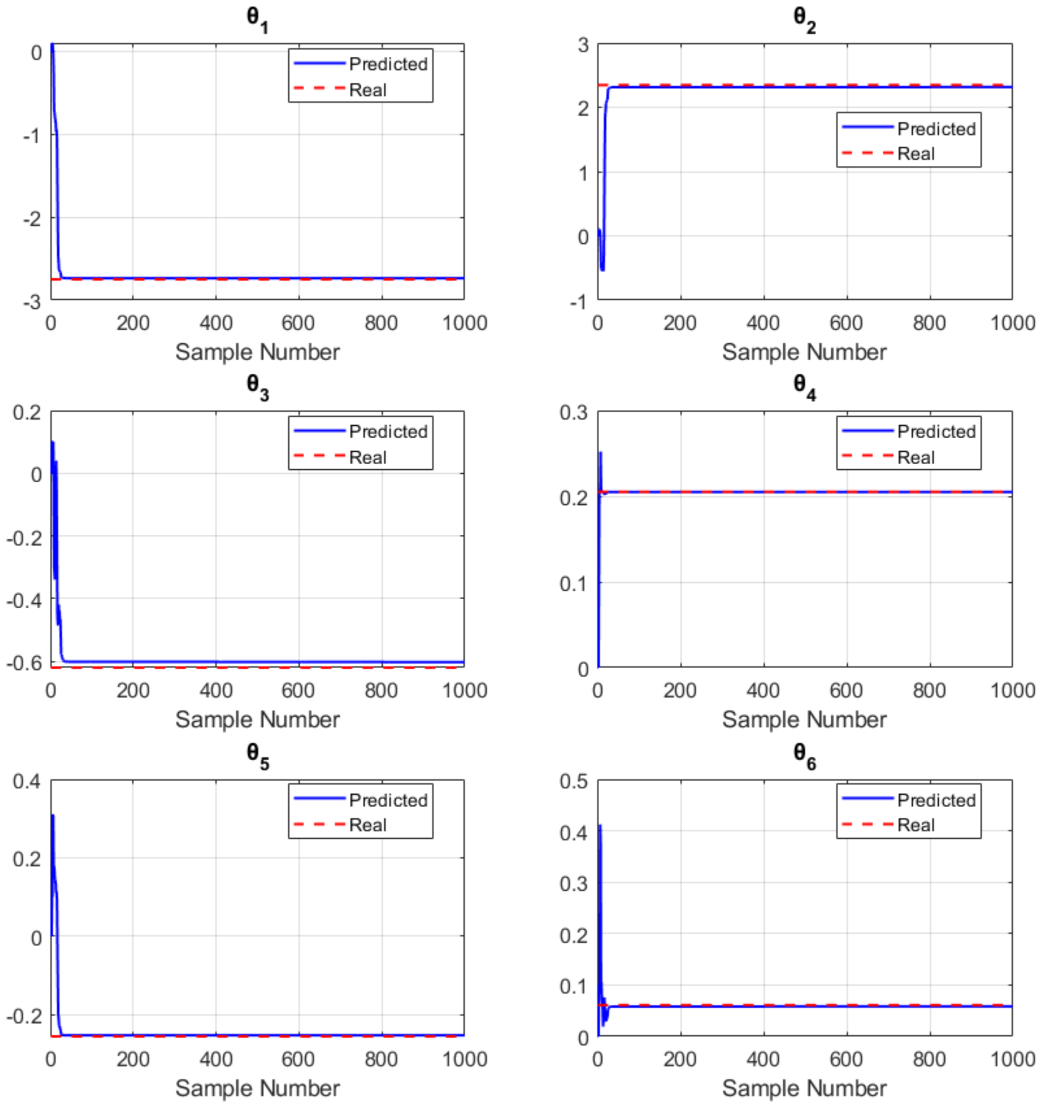

Figure 7: Estimated A and B polynomials. (Indirect STR Without zero cancellation).

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

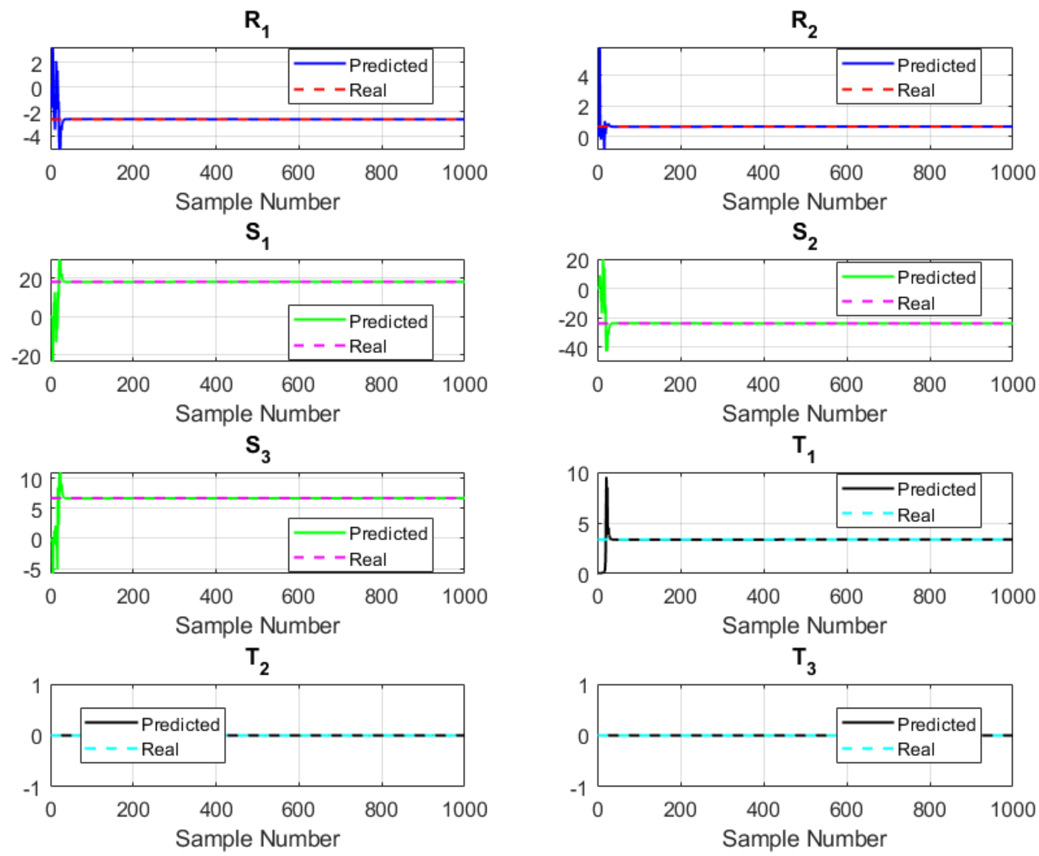

Figure 8: Estimated S, T and R polynomials. (Indirect STR Without zero cancellation).

2.2 Direct STR Without Pole-Zero Cancellation

In this case, instead of estimating the main system parameters, the values related to R, S, and T are estimated directly. Also, as recommended by the literature, when zero cancellation is not performed, T should be computed directly and the parameters of T should also be estimated directly.

When zero cancellation does not occur, the system equation is obtained as follows:

$$A_0 A_m y(t) = B^{-}(Ru(t) + Sy(t))$$

Similarly, it is seen that a common term B^{-} appears in both S and R estimates, which means that this common term needs to be eliminated during the identification process. First, we will discuss how to calculate R and S, and then explain how to calculate T. For estimating R and S, the coefficients of these values must be calculated. These filtered values $y(t)$ and $u(t)$ are provided by the polynomial $A_0 A_m$.

The filter f is a polynomial obtained using the `conv(Am,Ao)` function. However, for estimating T, we need to use the parameters of the system instead of using the S parameter as follows. The value of e is defined as:

$$e(t) = R^{*} u_f(t - d_0) + S^{*} y_f(t - d_0) - T^{*} u_{cf}(t - d_0)$$

where u_{cf} is the filtered input value. The way to filter this parameter is similar to that of u and y, which has been explained previously. Finally, we can define the vector $\phi(t)$ at each moment using the stated piece of the code.

```
% Filtering
uf(i) = u(i) - f_filter(2:end) * uf(i-1:-1:i-(length(f_filter)-1)).';
yf(i) = y(i) - f_filter(2:end) * yf(i-1:-1:i-(length(f_filter)-1)).';
ucf(i) = uc(i) - f_filter(2:end) * ucf(i-1:-1:i-(length(f_filter)-1)).';

phi_d0_filtered = [uf(i-d0:-1:i-d0-deg_BR), yf(i-d0:-1:i-d0-deg_BS), -
  ucf(i-d0:-1:i-d0-deg_T+1)].';
phi_t_m = [-y(i-1:-1:i-(deg_desA - 1)), uc(i-1:-1:i-deg_desB)].';
ym(i) = phi_t_m.' * theta_desired;
```

To update the output of ELS, the actual output $e(t)$ and the vector ϕ are used. The actual value e is obtained using the following relation:

$$y_m(t) = \frac{B_m}{A_m} u_c(t) = \frac{TB}{A_0 A_m} u_c(t)$$

$$e(t) = y(t) - y_m(t)$$

As previously mentioned, instead of directly estimating the values of R and S, the values of BR and BS are calculated as the output estimate. Therefore, we must remove this common factor from the estimated output using the `remove_common_roots` function. First, the polynomial is expanded, and then the estimated roots are identified and removed. If the roots are very close to each other, they are considered as the same and removed. However, it is possible that no common term is found at the beginning. It should be noted that initially there might be no common terms, but as the parameters converge to their true values, the common terms can be identified and removed more accurately. This approach allows for better and more precise parameter estimation.

The STR Parameters are obtained as follows:

R	S	T
-0.2996	-0.2402	-0.1473
0.8489	0.3583	0.3021
-0.9232	-0.1272	-0.1550

Table 4: Calculated direct STR without zero cancellation.

The output of the system under these conditions is shown in the figure below.

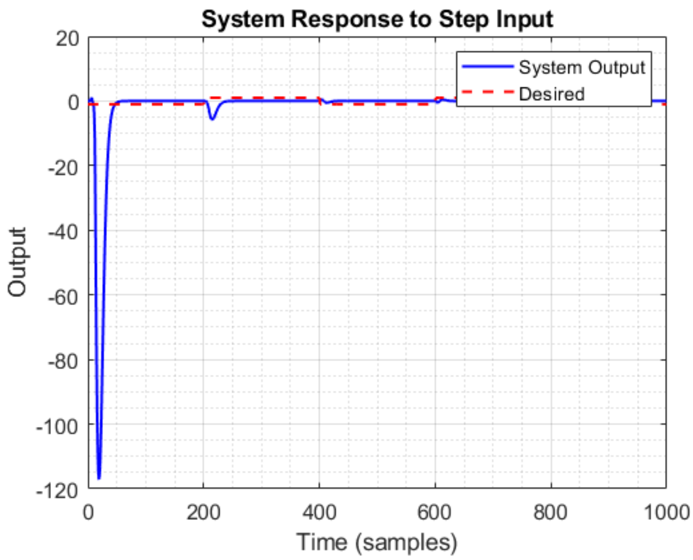

Figure 9: System output vs. Reference input (Direct STR-Without zero cancellation)

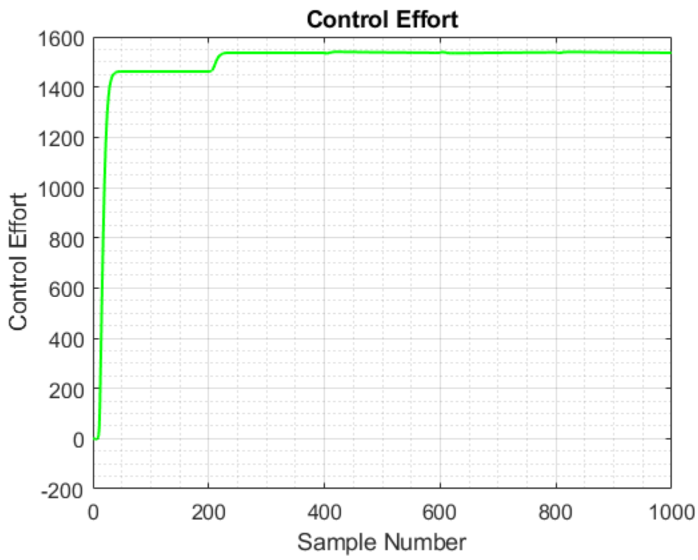

Figure 10: Control effort for Direct STR without zero cancellation

The parameters for A and B polynomials are estimated as shown in the figure below..

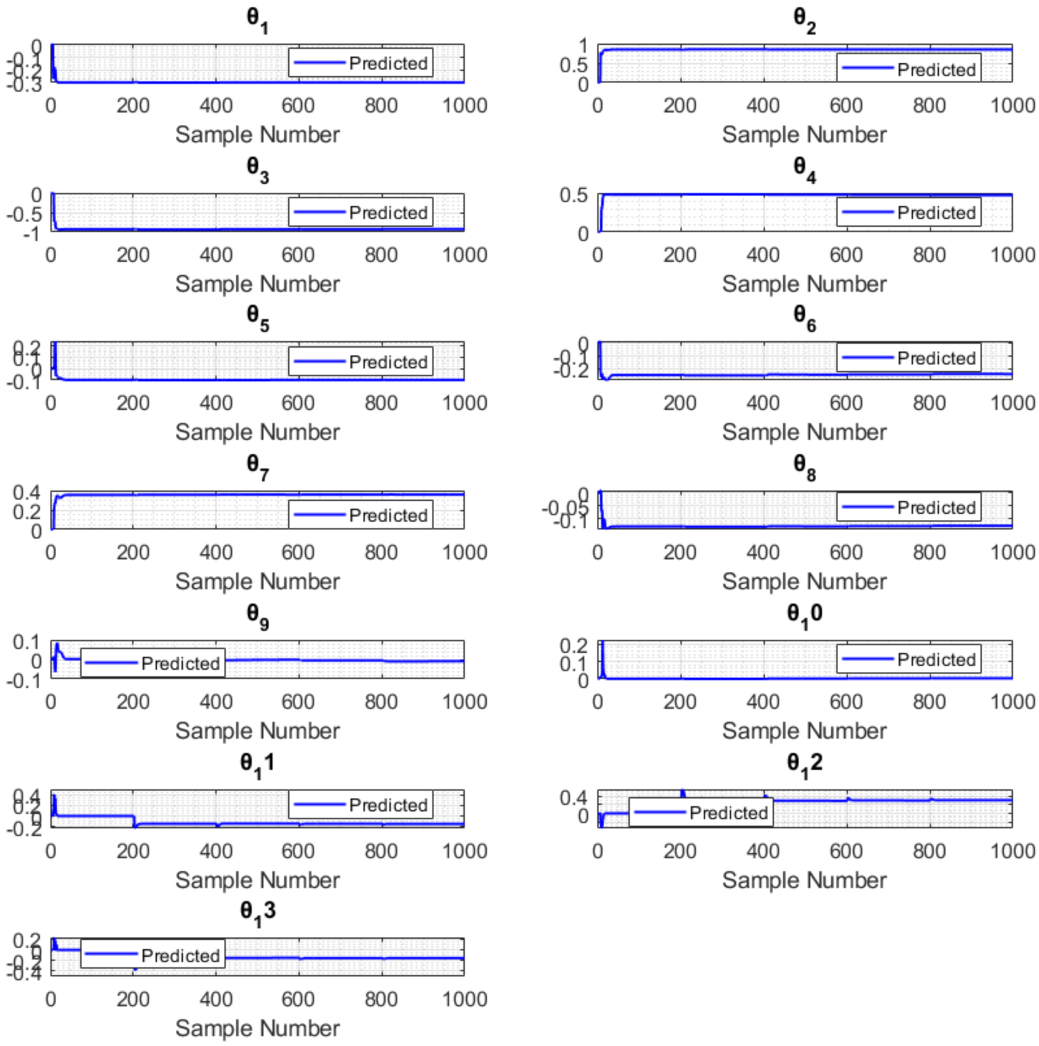

Figure 11: Estimated A and B polynomials. (Direct STR Without zero cancellation).

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

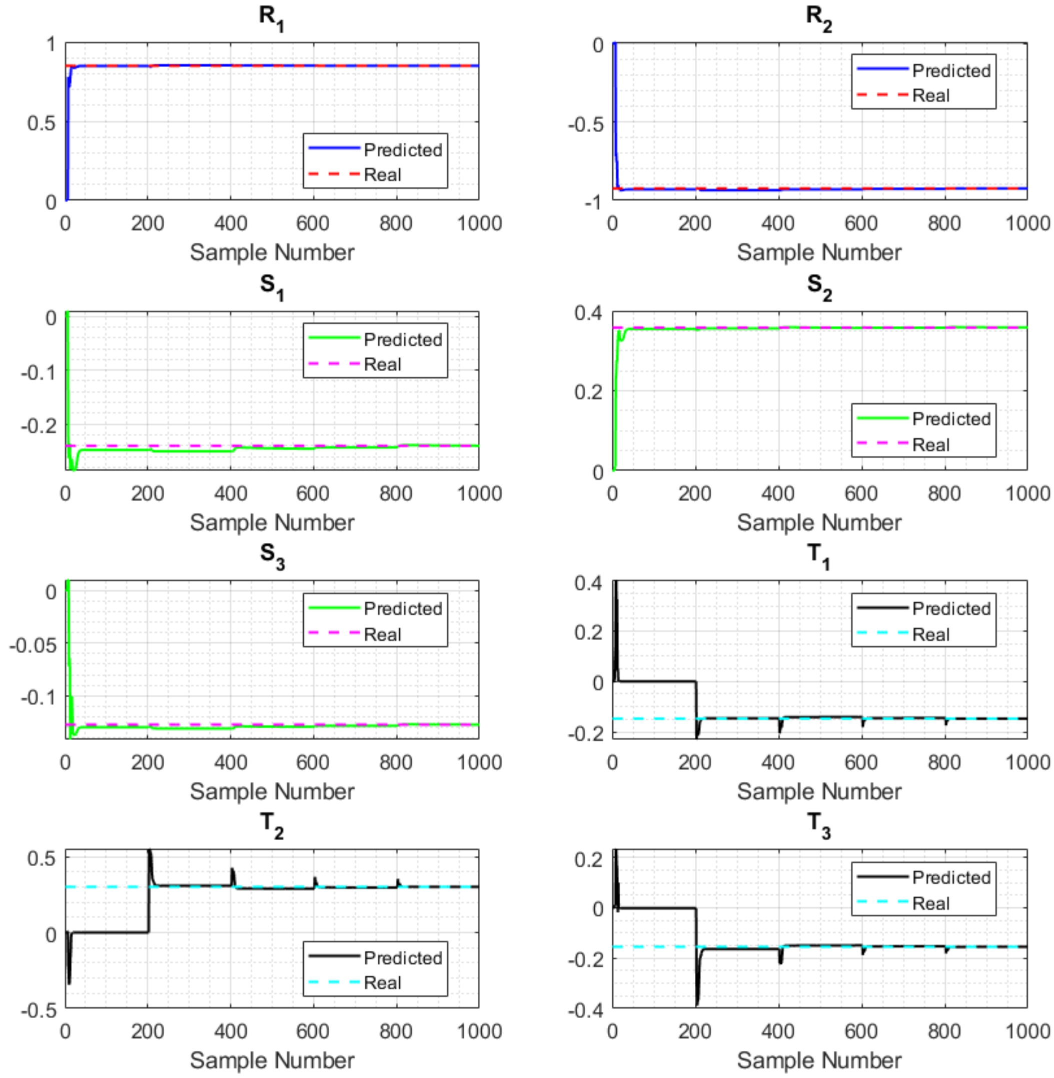

Figure 12: Estimated S, T and R polynomials. (Direct STR Without zero cancellation).

2.3 Indirect STR With Pole-Zero Cancellation

The only difference of this section with section 2.1 is the definition of B^- and B^+ polynomials which are defined as follows:

```
% Initial B
Bplus = [B_estimated/B_estimated(1)];
Bminus = B_estimated(1);
BmPrime = BmPrime_main/B_estimated(1);
```

The STR Parameters are obtained as follows:

R	S	T
1	5.3238	1.2053
-1.1888	-7.3002	-0.7324
0.2396	2.5526	-0.0144

Table 5: Calculated Indirect STR with zero cancellation.

The output of the system under these conditions is shown in the figure below.

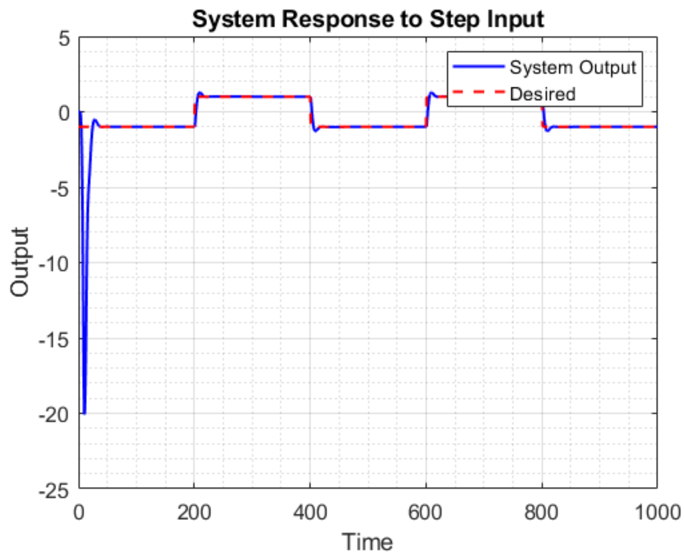

Figure 13: System output vs. Reference input (Indirect STR-With zero cancellation)

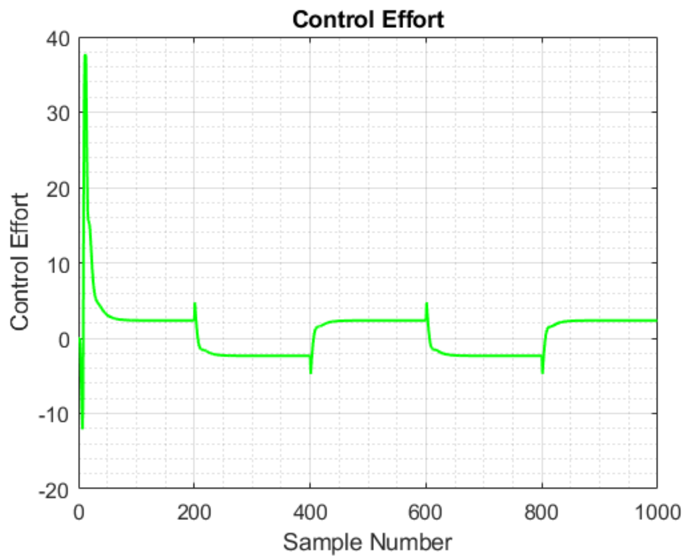

Figure 14: Control effort for Indirect STR with zero cancellation

The parameters for A and B polynomials are estimated as shown in the figure below..

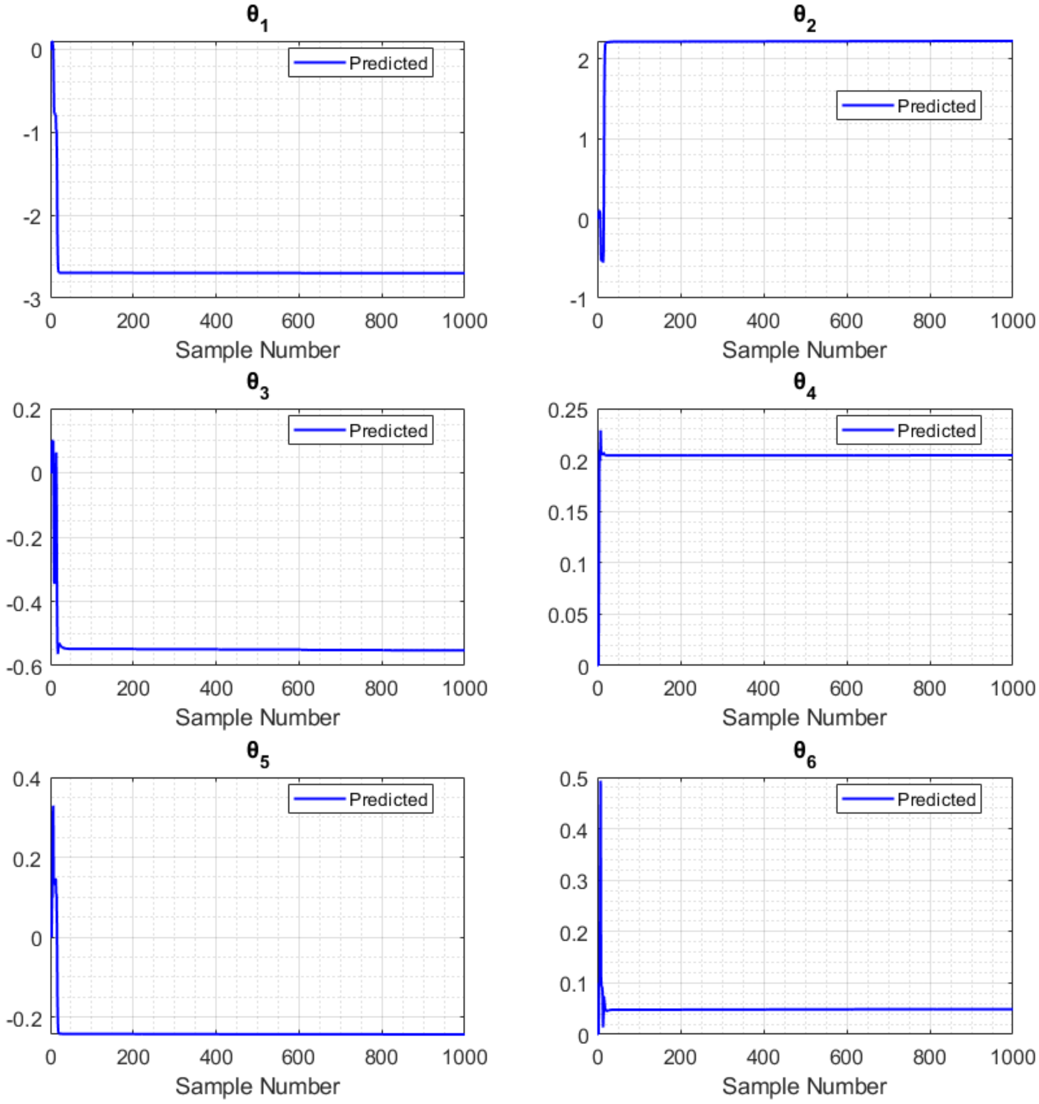

Figure 15: Estimated A and B polynomials. (Indirect STR With zero cancellation).

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

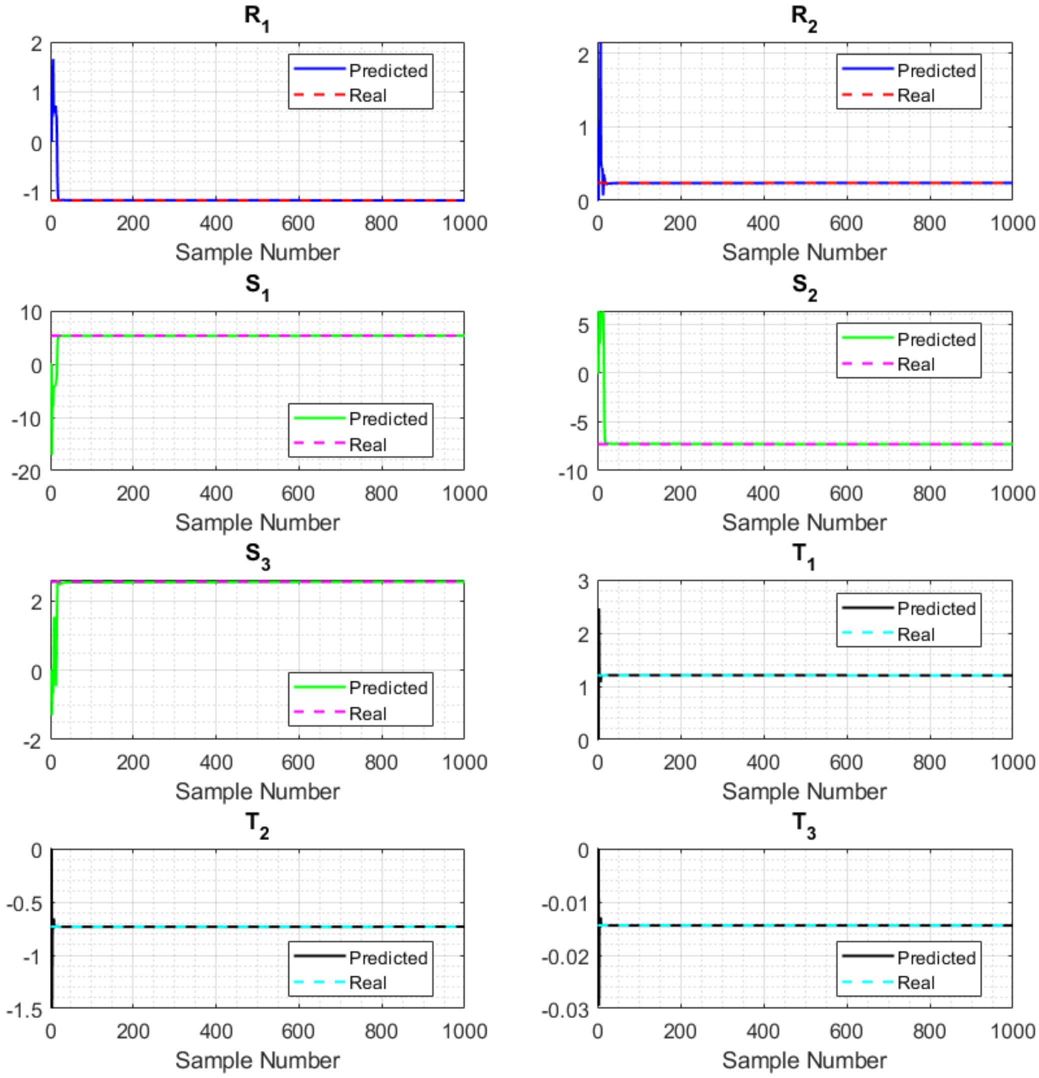

Figure 16: Estimated S, T and R polynomials. (Indirect STR With zero cancellation).

Direct STR With Pole-Zero Cancellation

This section is also same as section 2.2 but the definition of the polynomial B^- and B^+ is different.

```
% Desired B
Bplus = [BmPrime_main/BmPrime_main(1)];
Bminus = BmPrime_main(1);
Bm = conv(Bplus, Bminus);
BmPrime = BmPrime_main/BmPrime_main(1);
```

The STR Parameters are obtained as follows:

R	S	T
0.1743	0.4818	0.1842
-0.3439	-0.8662	-0.3212
0.1689	0.3925	0.1467

Table 6: Calculated Direct STR with zero cancellation.

The output of the system under these conditions is shown in the figure below.

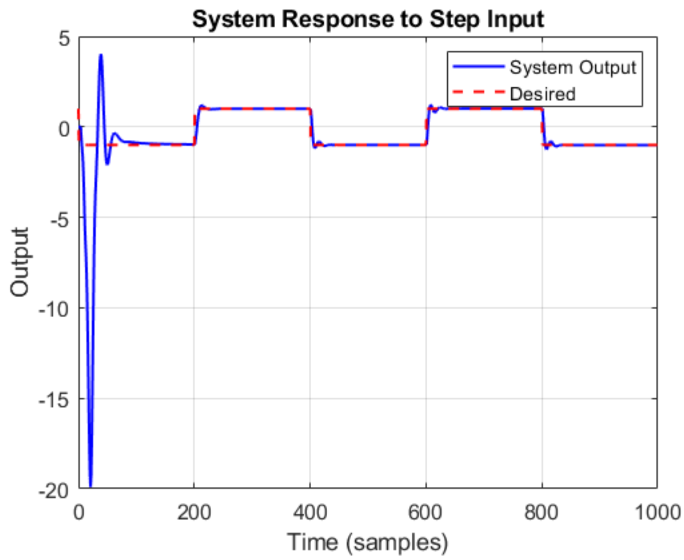

Figure 17: System output vs. Reference input (Direct STR-With zero cancellation)

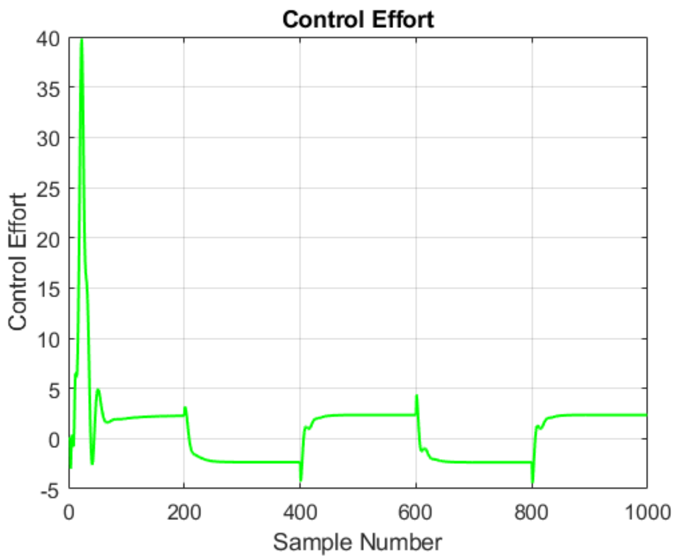

Figure 18: Control effort for Direct STR with zero cancellation

The parameters for A and B polynomials are estimated as shown in the figure below.

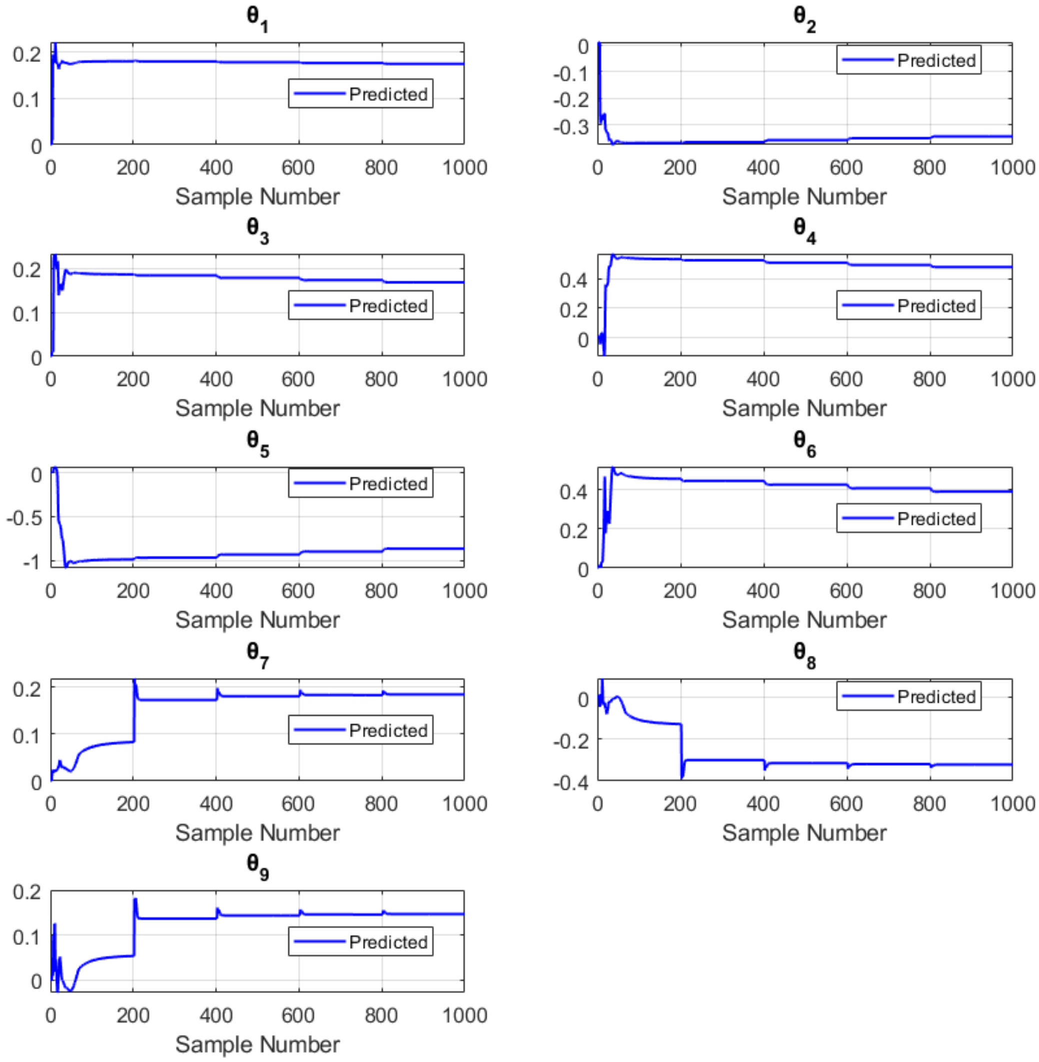

Figure 19: Estimated A and B polynomials. (Direct STR With zero cancellation).

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

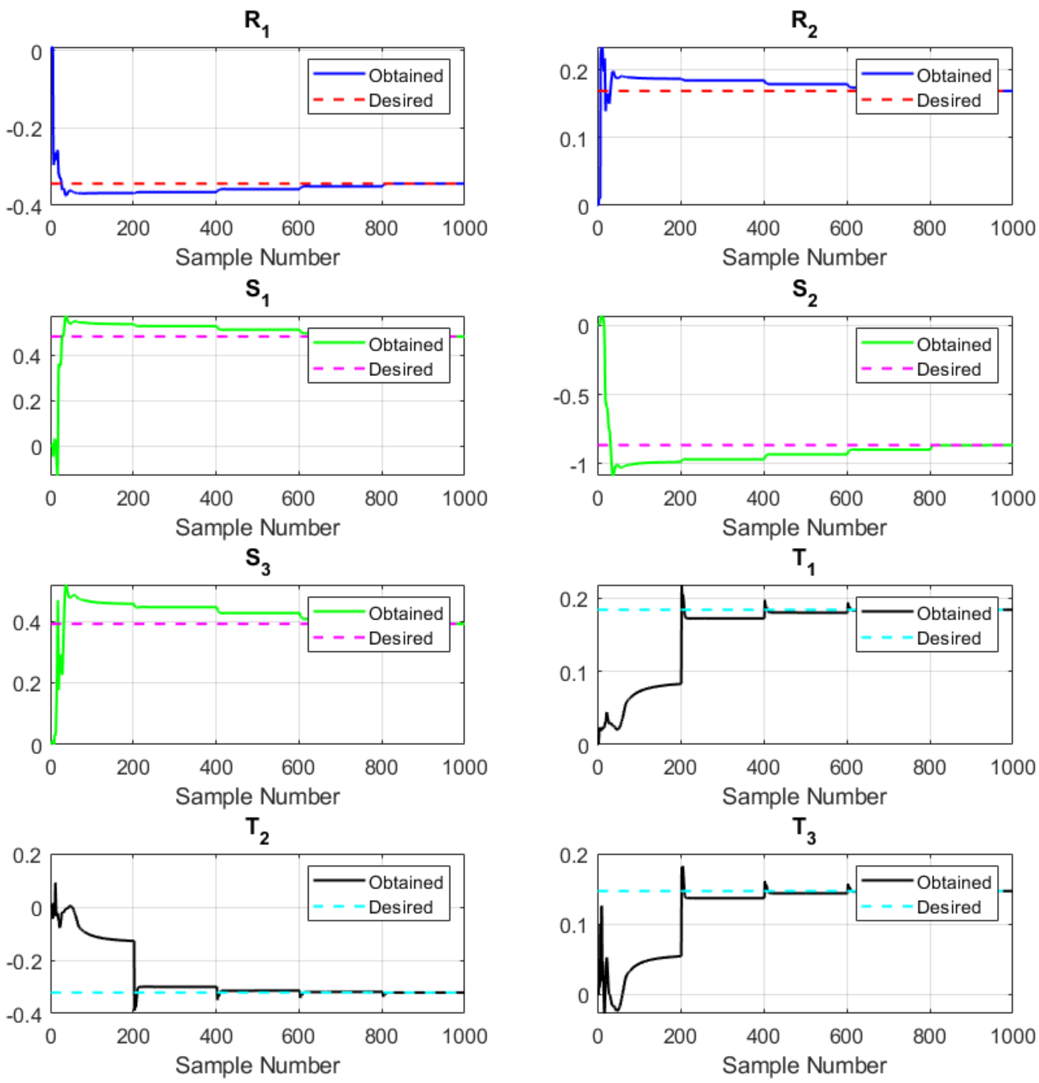

Figure 20: Estimated S, T and R polynomials. (Direct STR With zero cancellation).

2.4 Conclusion on Different STR Controllers

When we don't have zero cancellation and use the indirect method, the system output tracks the desired reference very well. However, there is a slight undershoot and overshoot at the corners. The control effort is substantial when the algorithm starts, which is not ideal, but it improves over time. All system parameters were estimated with very little error, and the polynomials S, R, and T were also estimated accurately.

When we don't have zero cancellation and use the direct method, the system output experiences more overshoot, which is undesirable. However, it eventually achieves good tracking over time. The initial control effort is excessive, and the final effort remains too large to be acceptable. All parameters were estimated well, and the polynomials S, T, and R were properly estimated.

When we cancel the zeros and use the indirect method, we obtain the best system output so far. This approach results in less undershoot and overshoot compared to previous methods. The control effort is also the best, starting large but being almost three times less than when using the indirect method without zero cancellation. System parameters were estimated accurately, and the polynomials S, T, and R were also properly estimated.

When we cancel the zeros and use the direct method, the system output tracking is good but not better than the indirect method. However, it is much improved compared to when zeros were not canceled. The control effort is acceptable but not superior to the indirect method. The parameters were estimated more slowly, and there is a step-like offset in the first 200 samples due to zero cancellation. The polynomials S, T, and R were estimated well, but again, there is a noticeable step-like offset, especially in the T polynomial.

In conclusion, the indirect method with zero cancellation provides the best results for this non-minimum phase dynamic system. However, the choice of method ultimately depends on the specific application and its requirements.

2.5 Over Parameter Estimation Affect

In this section we will design the indirect STR controller with zero cancellation for an over parameter model. We will assume the system has 8 parameters instead of 6 parameters.The STR Parameters are obtained as follows:

R	S	T
1	1.9118	1.2027
-0.4992	2.0816	-0.7308
-0.6136	-5.4748	-0.0144
0.2010	2.1456	0

Table 7: Calculated Indirect STR with zero cancellation for over parameter model.

The output of the system under these conditions is shown in the figure below.

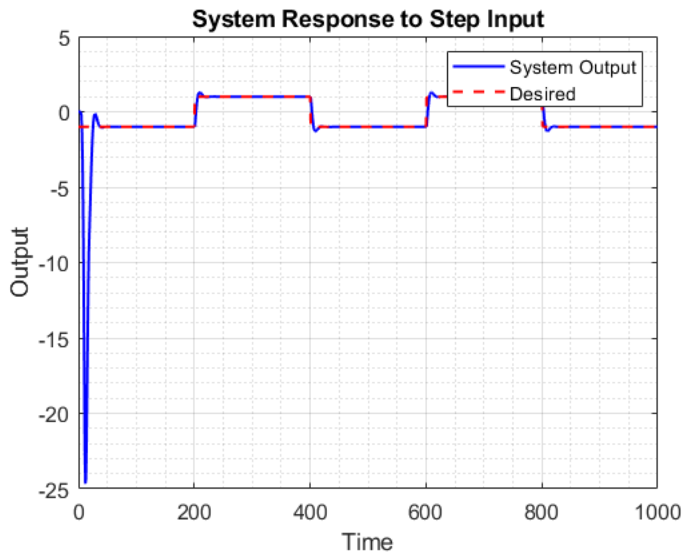

Figure 21: System output vs. Reference input (Indirect STR-With zero cancellation) for over parameter model

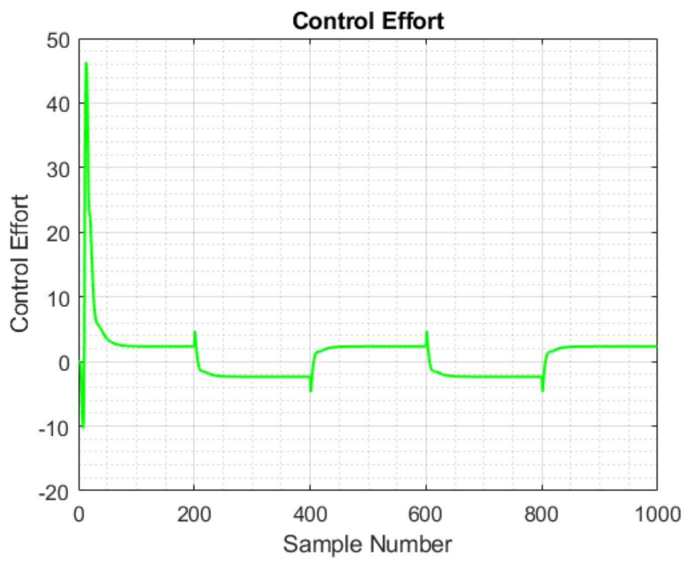

Figure 22: Control effort for Indirect STR with zero cancellation for over parameter model.

The parameters for A and B polynomials are estimated as shown in the figure below.

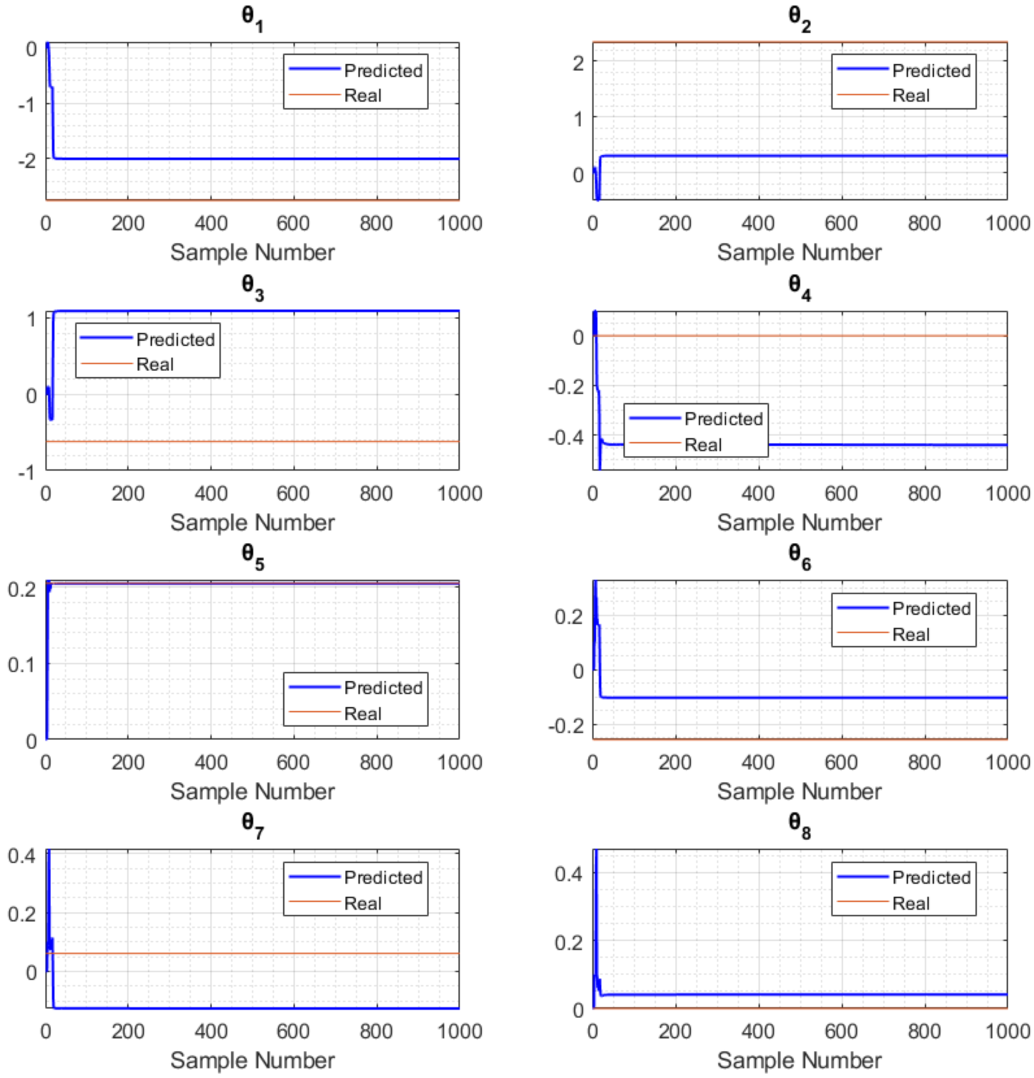

Figure 23: Estimated A and B polynomials. (Indirect STR With zero cancellation) for over parameter model.

As it is visible the parameters are not predicted properly.

In the figure below the STR parameters are estimated.

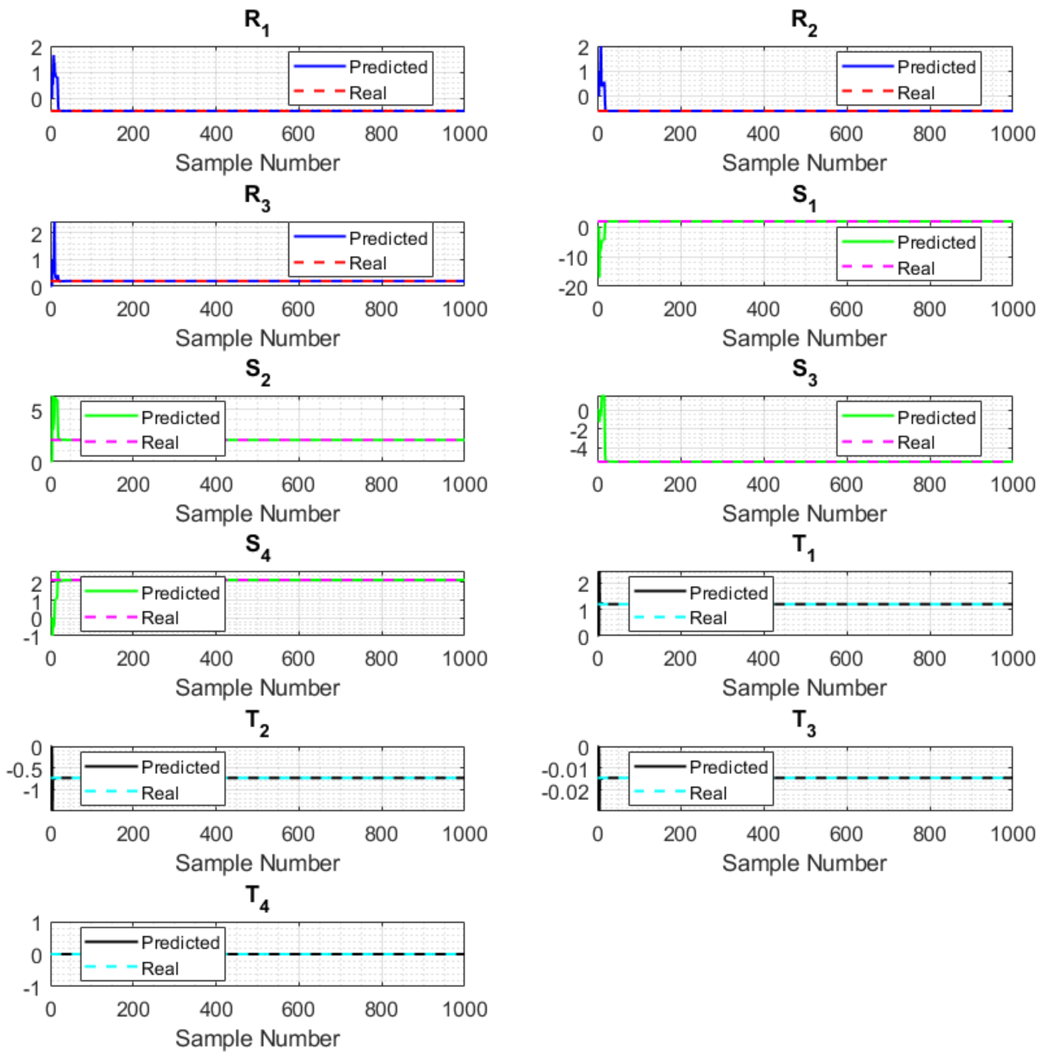

Figure 24: Estimated S, T and R polynomials. (Indirect STR With zero cancellation) for over parameter model.

2.6 Under Parameter Estimation Affect

In this section we will design the indirect STR controller with zero cancellation for an under parameter model. We will assume the system has 4 parameters instead of 6 parameters.The STR Parameters are obtained as follows:

R	S	T
1	3.5332	-83.4381
-0.9710	-3.0324	52.2274

Table 8: Calculated Indirect STR with zero cancellation for under parameter model.

The output of the system under these conditions is shown in the figure below.

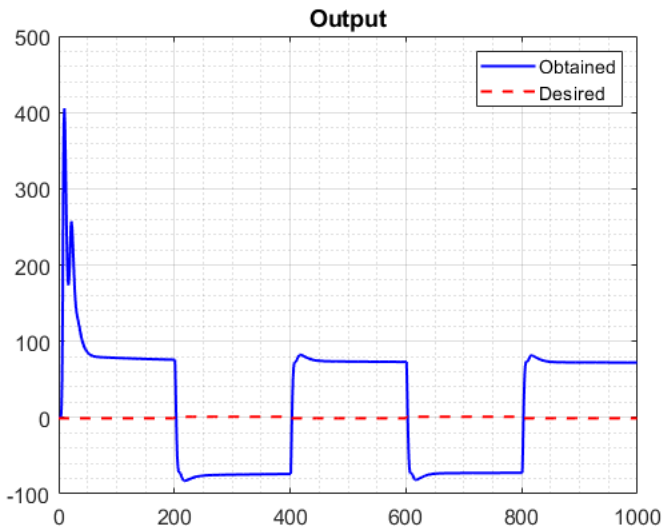

Figure 25: System output vs. Reference input (Indirect STR-With zero cancellation) for under parameter model

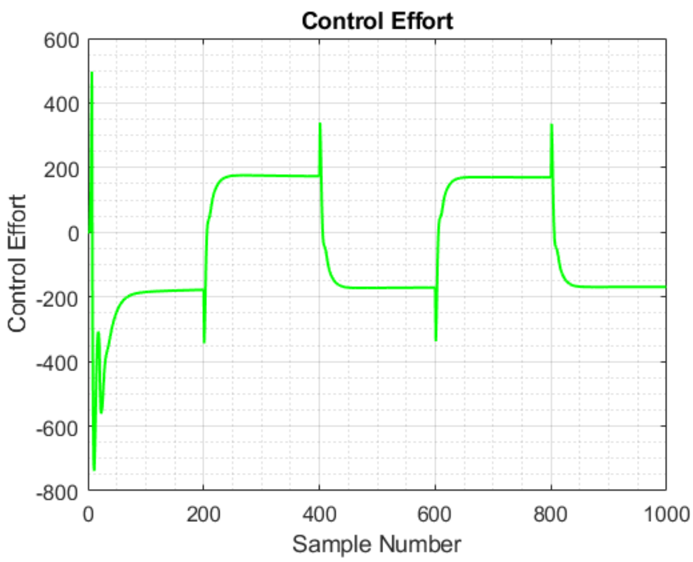

Figure 26: Control effort for Indirect STR with zero cancellation for under parameter model.

The parameters for A and B polynomials are estimated as shown in the figure below.

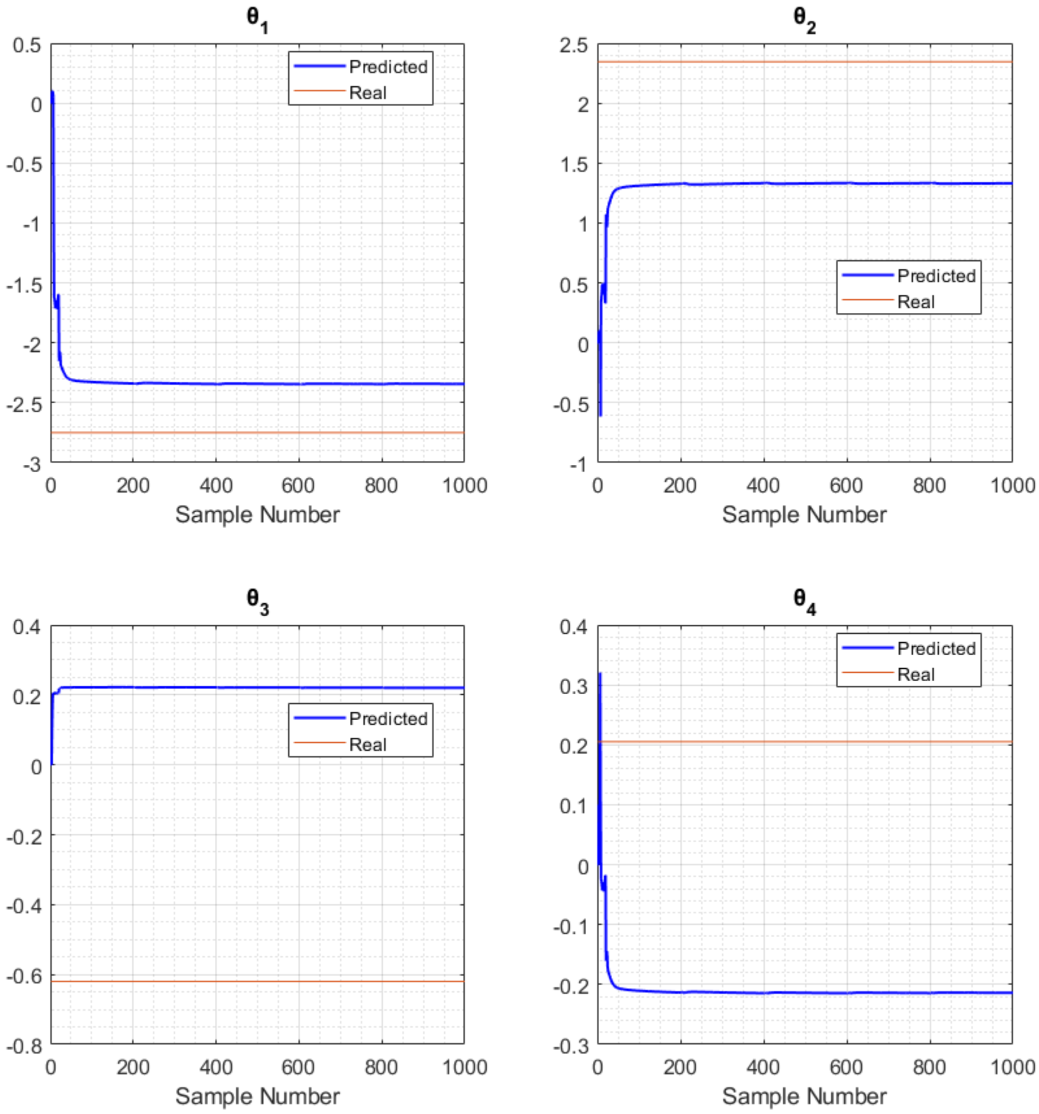

Figure 27: Estimated A and B polynomials. (Indirect STR With zero cancellation) for under parameter model.

As it is visible the parameters are not predicted properly.

In the figure below the STR parameters are estimated.

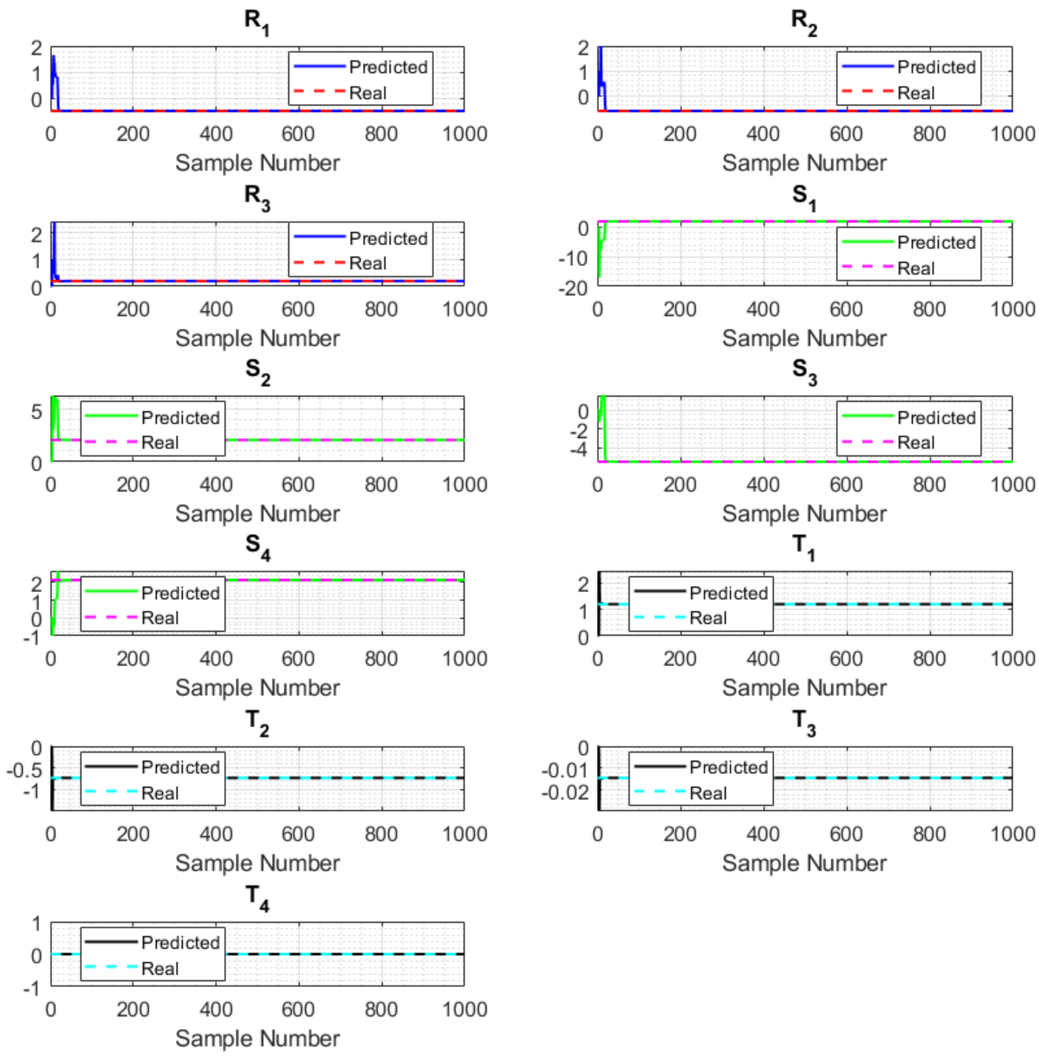

Figure 28: Estimated S, T and R polynomials. (Indirect STR With zero cancellation) for under parameter model.

2.7 Conclusion on Over and Under Parameter Estimation affect

As illustrated in the figures from the previous sections, using a higher-order system model results in good tracking, effective control effort, and fast convergence of the S, T, and R polynomials in simulations. However, the parameter estimation is not as accurate. When the primary objective is controlling the system, this discrepancy is not a major concern. It is important to note, though, that the simulations were conducted without any noise or disturbances. Introducing such factors may affect control quality if the model parameters are not estimated properly.

On the other hand, when controlling an under-parameterized model of the system, we observe poor tracking, unacceptably high control effort, and subpar parameter estimation. The only advantage is the rapid convergence of the S, T, and R polynomials, which is insignificant when the tracking performance is inadequate.

In conclusion, while using a higher-order model achieves good tracking and meets control objectives, it is not recommended to use a lower-order model. The tracking performance and control effort in such cases are not acceptable.

2.8 Designing Indirect STR With Zero Cancellation In Presence of Step Disturbance

In this section we will add a disturbance in step shape which is added to the system. The designing process is not different the only thing is that the input signal u for the process is now summed with a disturbance which is defined as follows:

```
% disturbance
v = [zeros([1,ceil(num_samples/2)]), 10*ones([1,ceil(num_samples/2)])];
```

The STR Parameters are obtained as follows:

R	S	T
1	5.7498	9.3926
-1.2480	-7.9550	-14.2578
0.2483	2.7077	5.3465

Table 9: Calculated Indirect STR with zero cancellation In Presence of disturbance.

The output of the system under these conditions is shown in the figure below.

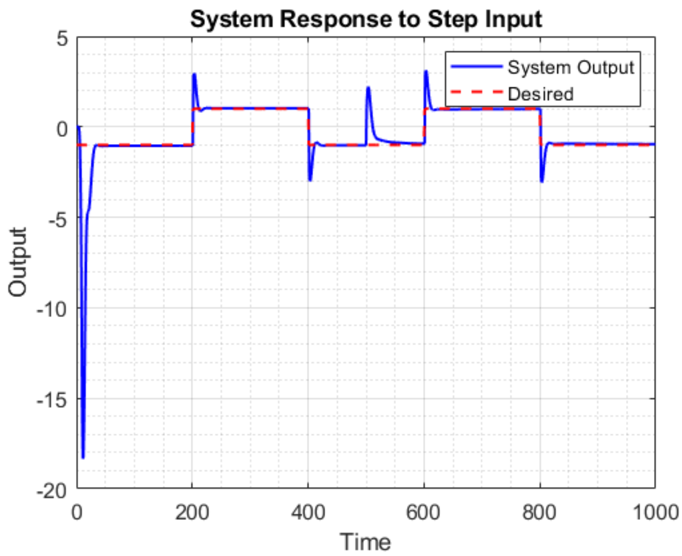

Figure 29: System output vs. Reference input (Indirect STR-With zero cancellation) In Presence of disturbance

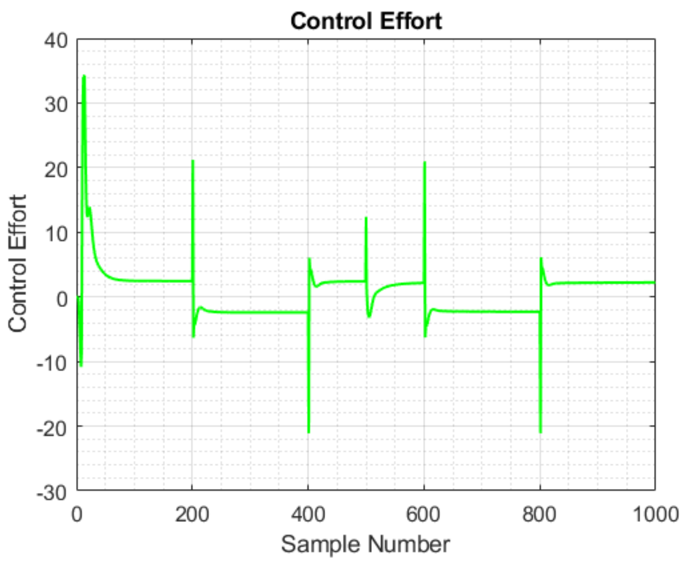

Figure 30: Control effort for Indirect STR with zero cancellation In Presence of disturbance.

The parameters for A and B polynomials are estimated as shown in the figure below.

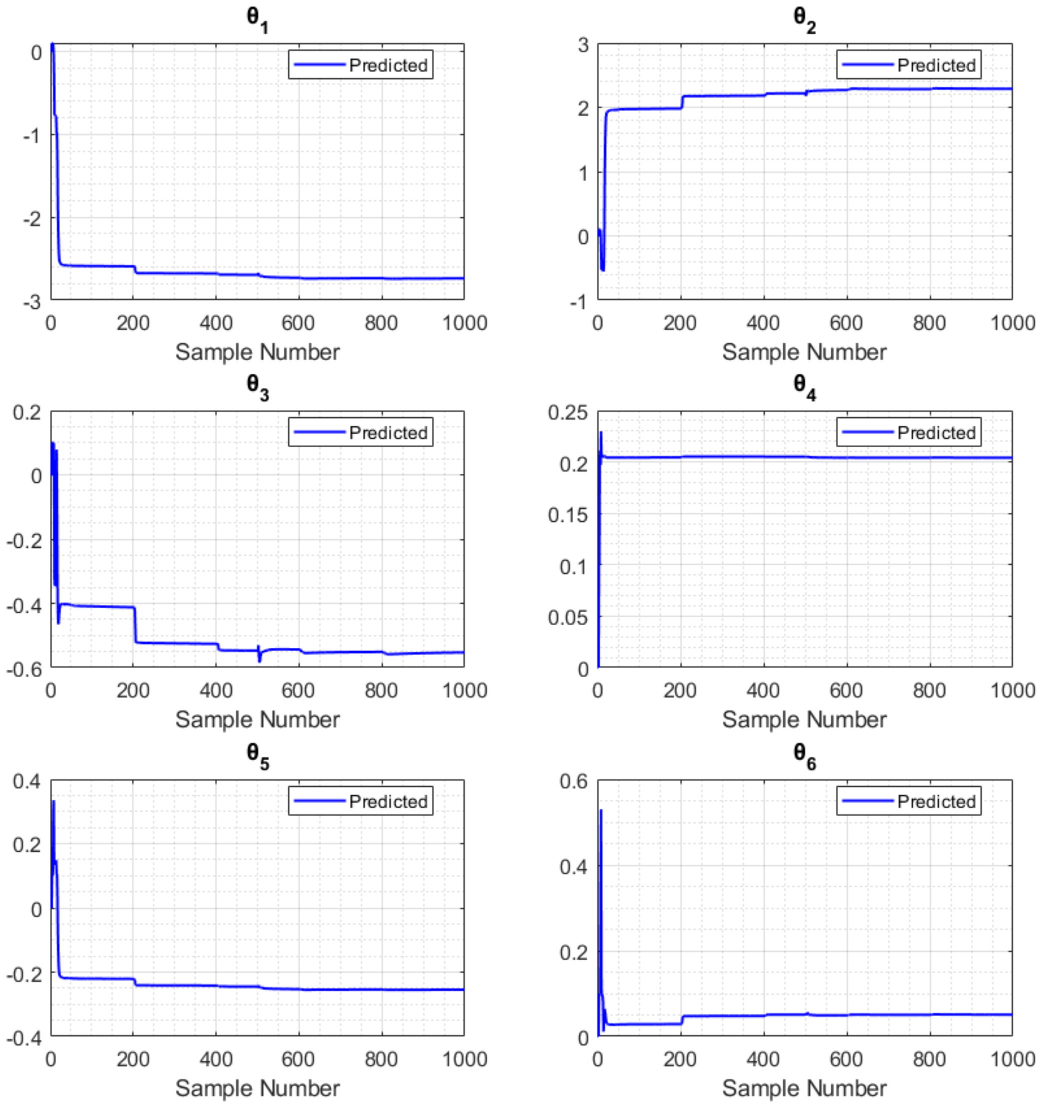

Figure 31: Estimated A and B polynomials. (Indirect STR With zero cancellation) In Presence of disturbance.

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

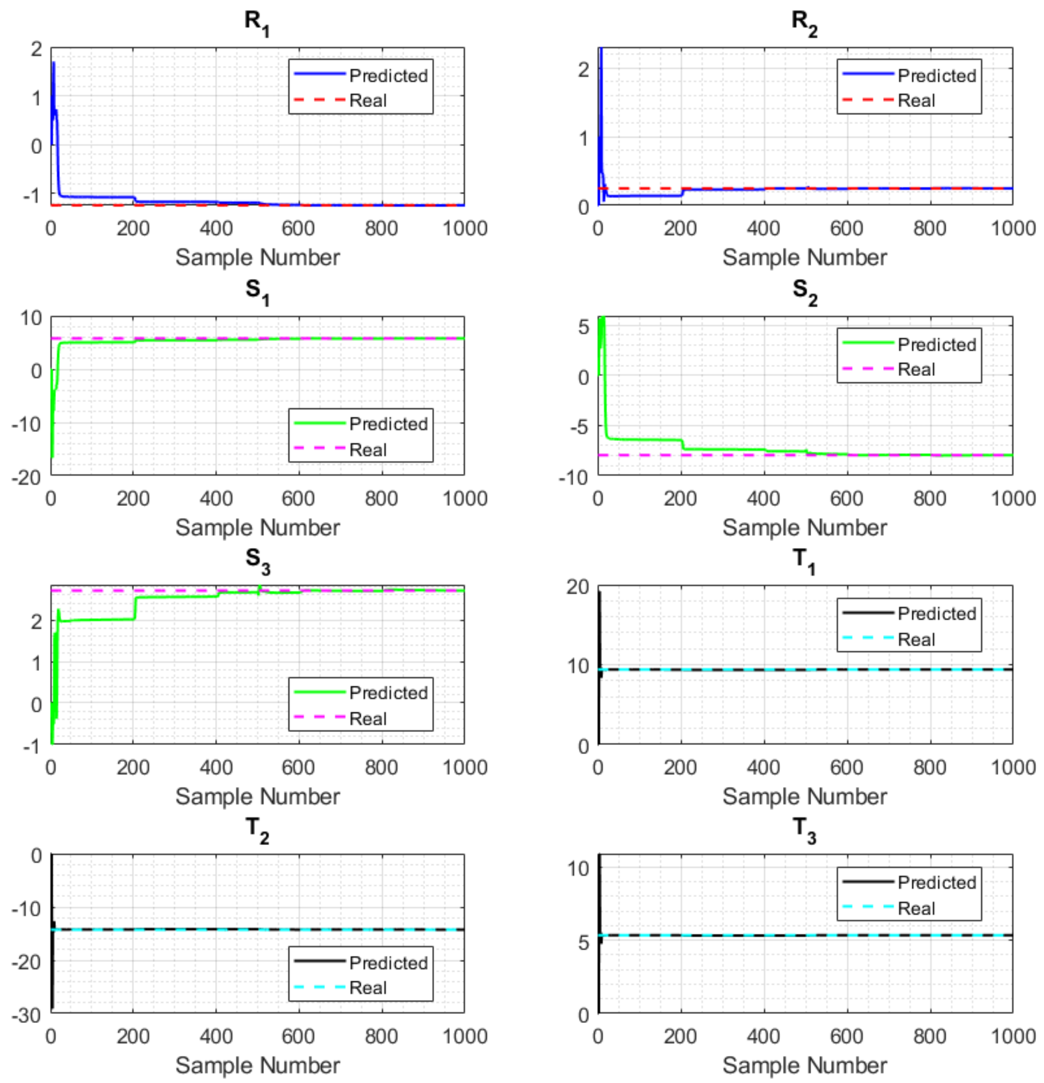

Figure 32: Estimated S, T and R polynomials. (Indirect STR With zero cancellation) In Presence of disturbance.

2.9 Designing Indirect STR With Zero Cancellation and Integrator In Presence of Step Disturbance

In this section we will add a disturbance in step shape which is added to the system. The designing process is not different the only thing is that the input signal u for the process is now summed with a disturbance and also an integrator is added to the system.

The disturbance is defined as follows:

```
% disturbance
v = [zeros([1,ceil(num_samples/2)]), 10*ones([1,ceil(num_samples/2)])];
```

The Integrator is defined as follows:

$$X = q + x_0$$

$$y_0 = -\frac{(1 + x_0)R^0(1)}{B(1)}$$

We get $x_0 = 0$ Therefore: $\rightarrow \begin{cases} R = XR^0 + YB \\ S = XS^0 - YA \end{cases}$

```
%integrator
X = [1, 0];

% S,R polynomials with integrator
R_estimated_withI = conv(X, R_solved) + [0,conv(Y, B_estimated)];
S_estimated_withI = conv(X, S_solved) - conv(Y, A_estimated);

```

The STR Parameters are obtained as follows:

R	S	T
1	5.5540	9.3684
-1.22062	-7.5999	-14.2210
0.2378	2.5986	5.3327

Table 10: Calculated Indirect STR with zero cancellation with Integrator In Presence of disturbance.

The output of the system under these conditions is shown in the figure below.

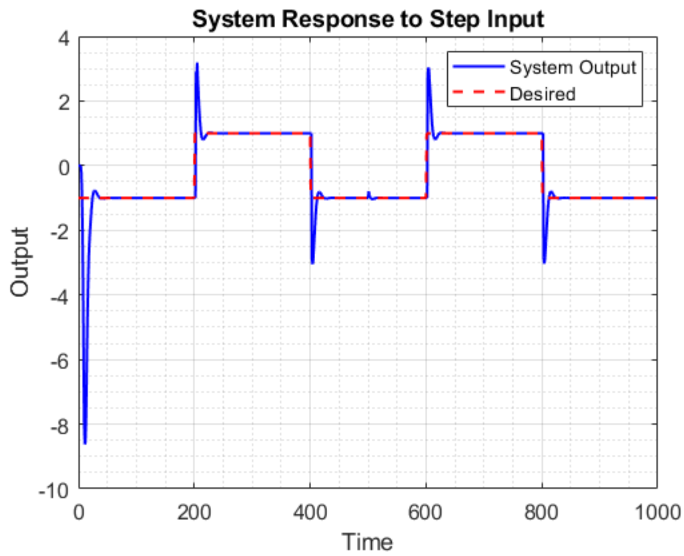

Figure 33: System output vs. Reference input (Indirect STR-With zero cancellation) with Integrator In Presence of disturbance

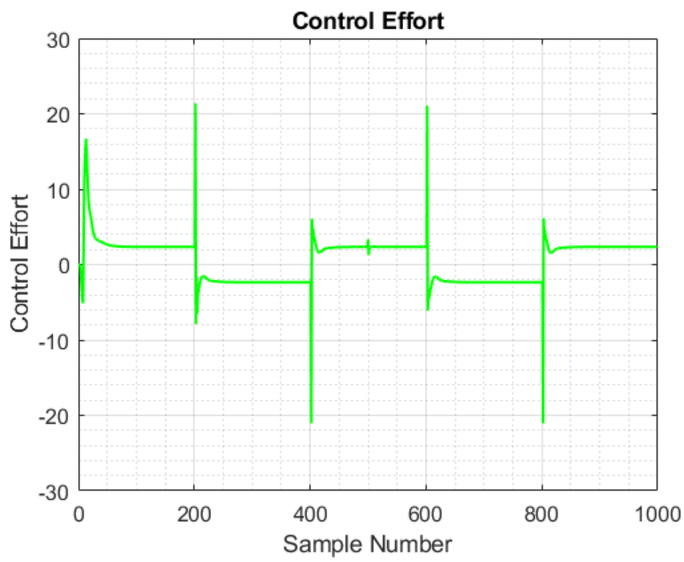

Figure 34: Control effort for Indirect STR with zero cancellation with Integrator In Presence of disturbance.

The parameters for A and B polynomials are estimated as shown in the figure below.

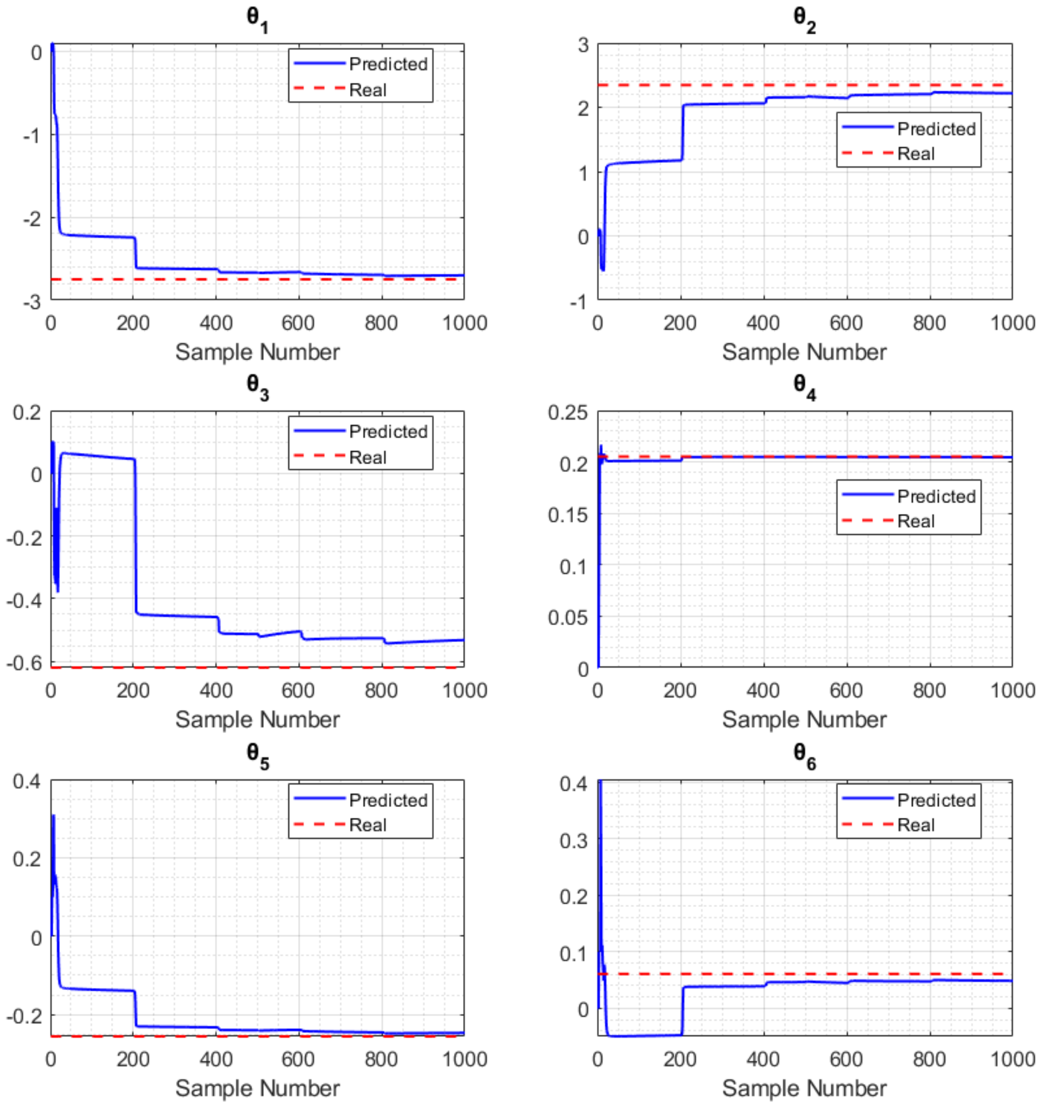

Figure 35: Estimated A and B polynomials. (Indirect STR With zero cancellation) with Integrator In Presence of disturbance.

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

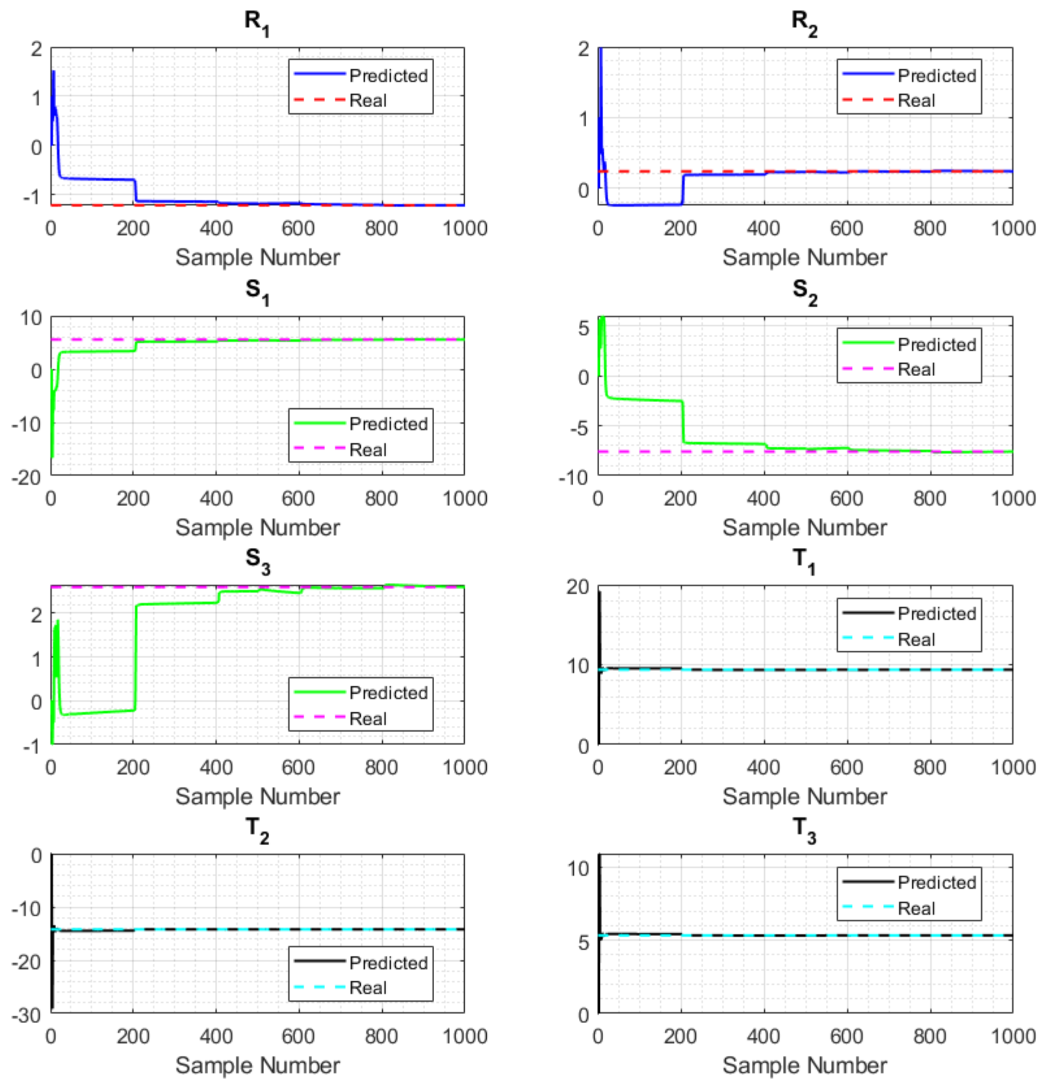

Figure 36: Estimated S, T and R polynomials. (Indirect STR With zero cancellation) with Integrator In Presence of disturbance.

2.10 Conclusion on Designing Indirect STR Controller With Zero Cancellation and With and Without Integrator In Presence of Step Shaped Disturbance

Examining Figure 29, which depicts the system output when the STR controller without an integrator is implemented, we observe an unwanted peak around the sample near 500. This indicates suboptimal tracking performance. Additionally, there is a significant undershoot at the beginning, and smaller overshoots and undershoots at other points, which, while acceptable, are not ideal. Notably, the sudden overshoot in the middle of the tracking process is unacceptable.

The control effort remains moderate and manageable, but it fluctuates significantly over time, which could be detrimental to our actuators. On a positive note, the parameters are estimated well, and the S, T, and R polynomials converge quickly and accurately.

When we incorporate an integrator into the STR controller, the system output improves considerably. The initial undershoot is reduced by half, and the sudden overshoot in the middle of the tracking process is almost eliminated, leading to much better overall tracking. The control effort is also improved, with the maximum control effort being lower than that of the non-integrator STR controller. However, both scenarios present fluctuations that could potentially harm the actuators.

The parameters are estimated correctly, albeit slightly slower, and they exhibit a step-like peak, which is a side effect of disturbances and is expected. The same can be said for the S, T, and R polynomials—they also converge more slowly and display a step-shaped offset at the beginning.

In summary, adding an integrator to the STR controller results in better system output and tracking performance. While both control efforts are acceptable, the integrator enhances stability and reduces undesired fluctuations. However, the impact on actuators due to changing values at the middle of the process needs to be considered.

2.11 Results For Long Time Simulations

In this section, we will extend the simulation duration to ten times longer than in the previous part to observe any changes in the control effort and analyze how these changes manifest over an extended period. By running the simulation for a longer duration, we aim to gain deeper insights into the behavior and stability of the control effort over time, identifying any potential long-term trends or fluctuations that may not have been apparent in the shorter simulation. This extended analysis will help us better understand the robustness and reliability of the control strategy under prolonged operation conditions.

2.11.1 Long Time Simulation Indirect STR Without Zero Cancellation

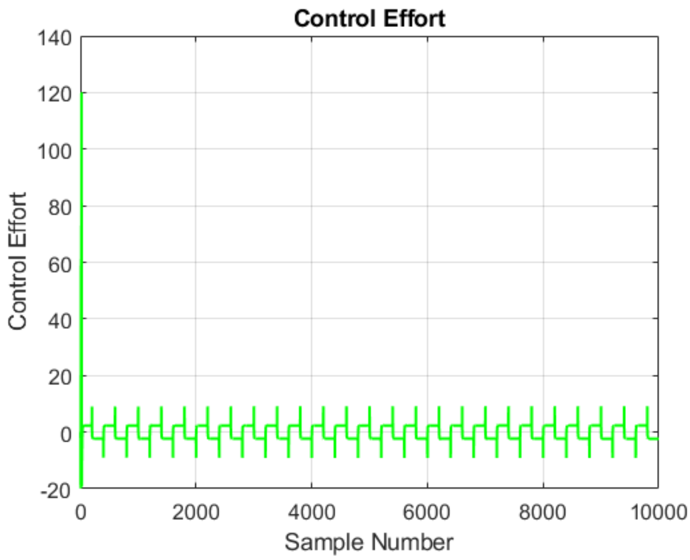

Figure 37: Control effort for Indirect STR without zero cancellation (Long time simulation).

As it is visible the control effort is not different and didn't get higher with the time.

Note: The simulation for this part took almost 18 minutes because the algorithm had to solve Diophantine equation 10000 times so i simulated the indirect only one time the results for the indirect method with zero cancellation will not be different.

2.11.2 Long Time Simulation Direct STR Without Zero Cancellation

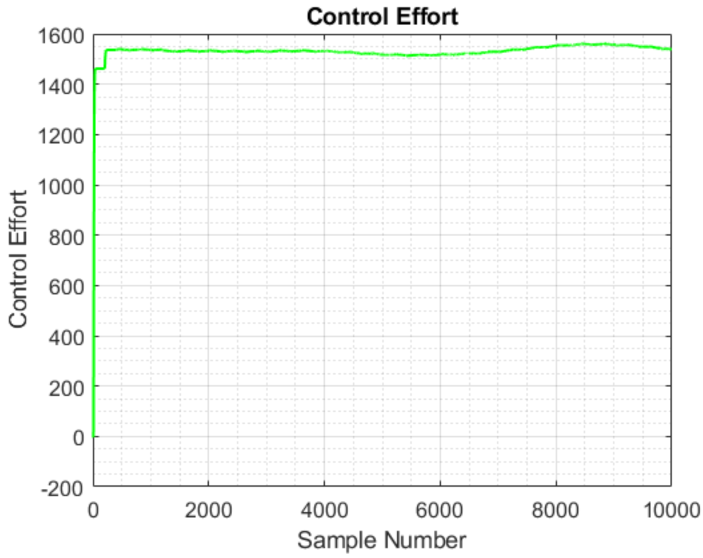

Figure 38: Control effort for Direct STR without zero cancellation (Long time simulation).

The Control effort is still very bad as it was for 1000 samples nothing has changed.

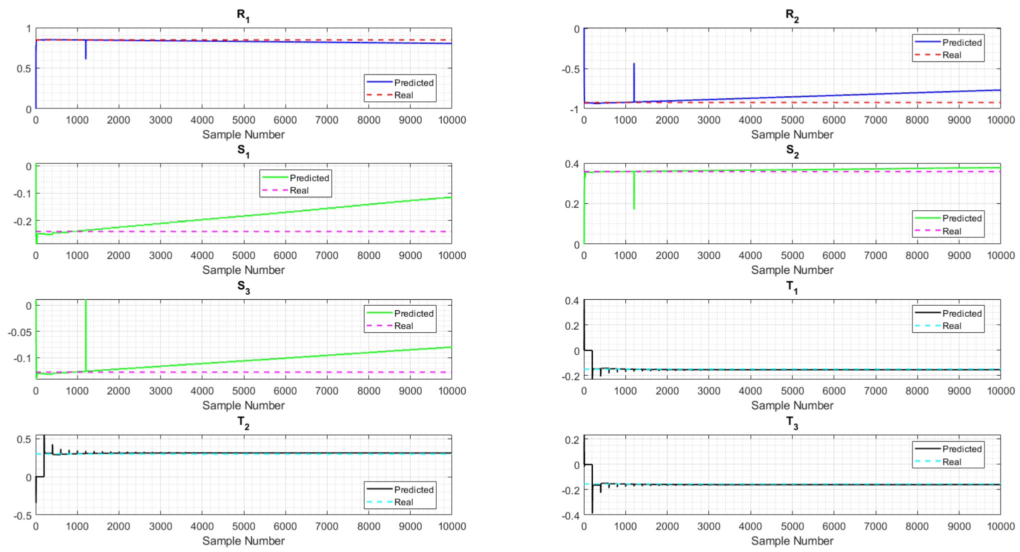

Figure 39: Estimated S, T and R polynomials. (Direct STR Without zero cancellation) For long time simulation.

Figure 39 is interesting we can understand that the polynomials will be unstable after some time and they wont converge to any value in long time simulations.

2.11.3 Long Time Simulation Direct STR With Zero Cancellation

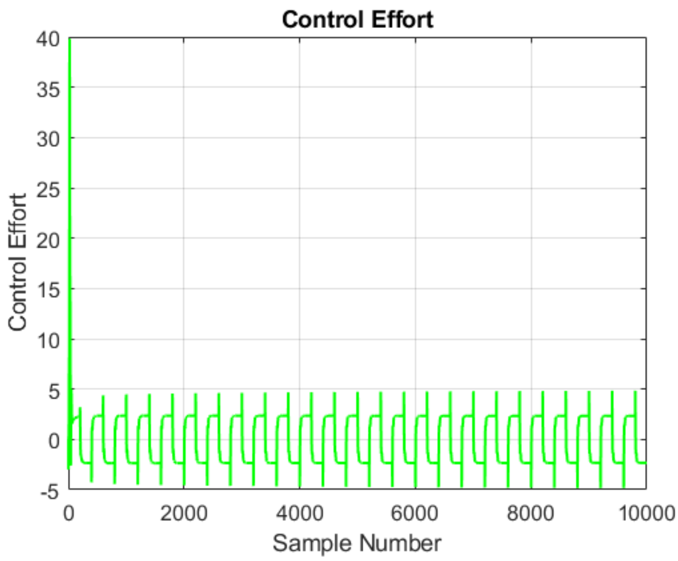

Figure 40: Control effort for Direct STR with zero cancellation (Long time simulation).

The Control effort is still good as it was for 1000 samples nothing has changed.

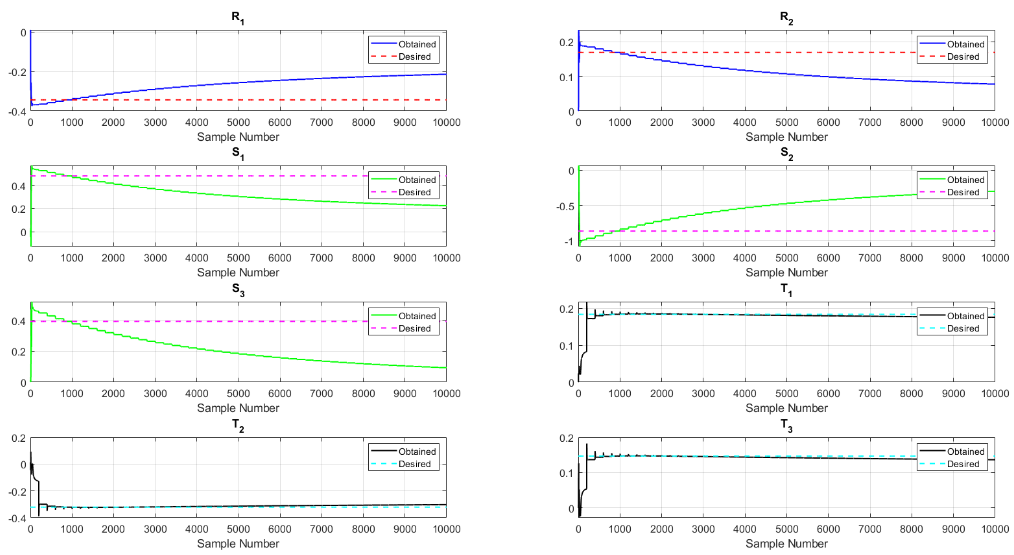

Figure 41: Estimated S, T and R polynomials. (Direct STR With zero cancellation) For long time simulation.

Figure 41 is interesting we can understand that the polynomials will be unstable after some time and they wont converge to any value in long time simulations.

3 STR Using MDPP For Non-Minimum Phase System

To get the non-minimum phase system we will simply put one of our zeros on the right side of the plane. So the Continuous transfer function will be:

$$G(s) = \frac{(s-0.5)(s+7)}{(s+1)(s+4.1)(s-2)}$$

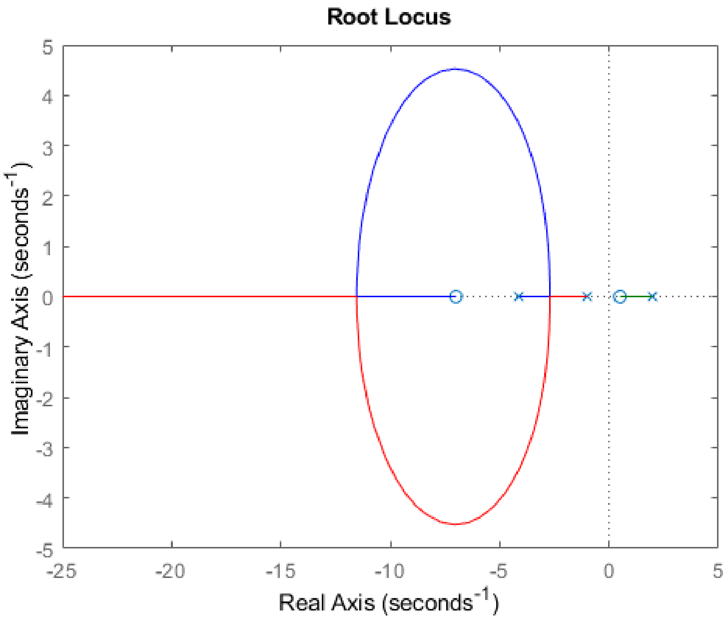

Figure 42: Root locus of the non-minimum phase system.

The discrete form of the non-minimum phase system will be as follows:

$$H(z) = \frac{0.191z^2 - 0.268z + 0.0666}{z^3 - 2.75z^2 + 2.345z - 0.6199}$$

The rest of the algorithm is pretty much the same.

3.1 Indirect STR Without Zero Cancellation (Non-Minimum Phase System)

In indirect methods, during each program execution, the aim is to first estimate the parameters A and B of the system. After these estimations are made, the controller polynomials are then calculated. Initially, the code was implemented using the Recursive Least Squares (RLS) method.

The STR Parameters are obtained as follows:

R	S	T
1	48.0597	-10.0237
-7.9882	-68.8044	0
2.3596	22.0352	0

Table 11: Calculated indirect STR without zero cancellation (Non-Minimum Phase System).

The output of the system under these conditions is shown in the figure below.

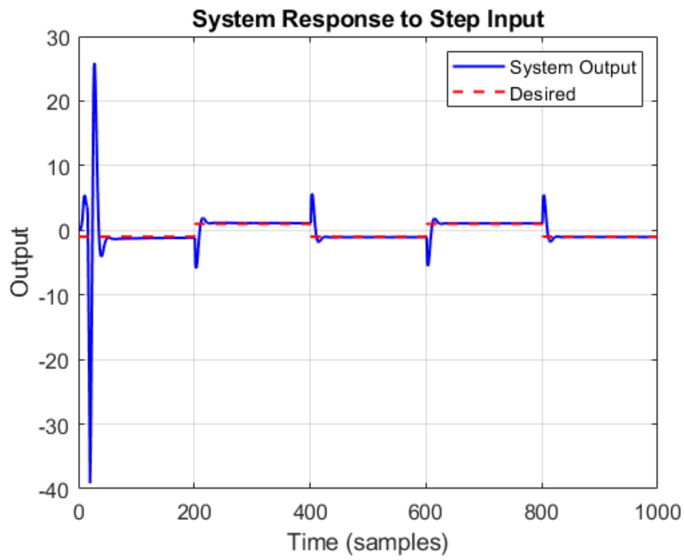

Figure 43: System output vs. Reference input (Indirect STR-Without zero cancellation)(Non-Minimum Phase System)

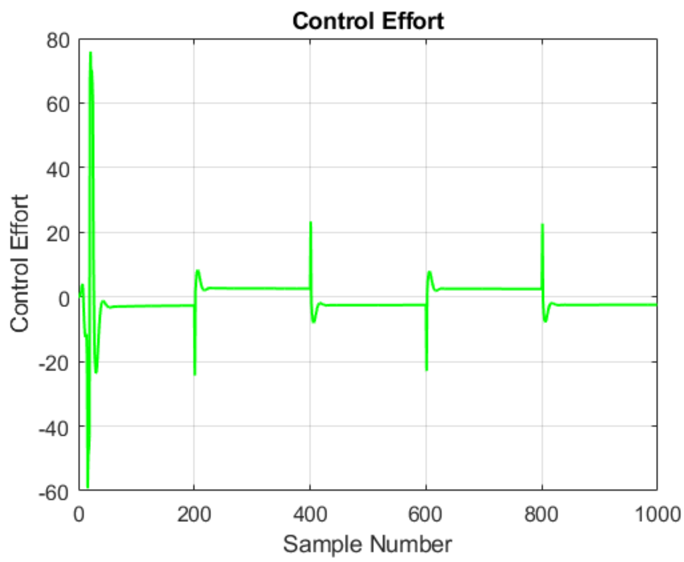

Figure 44: Control effort for Indirect STR without zero cancellation (Non-Minimum Phase System)

The parameters for A and B polynomials are estimated as shown in the figure below..

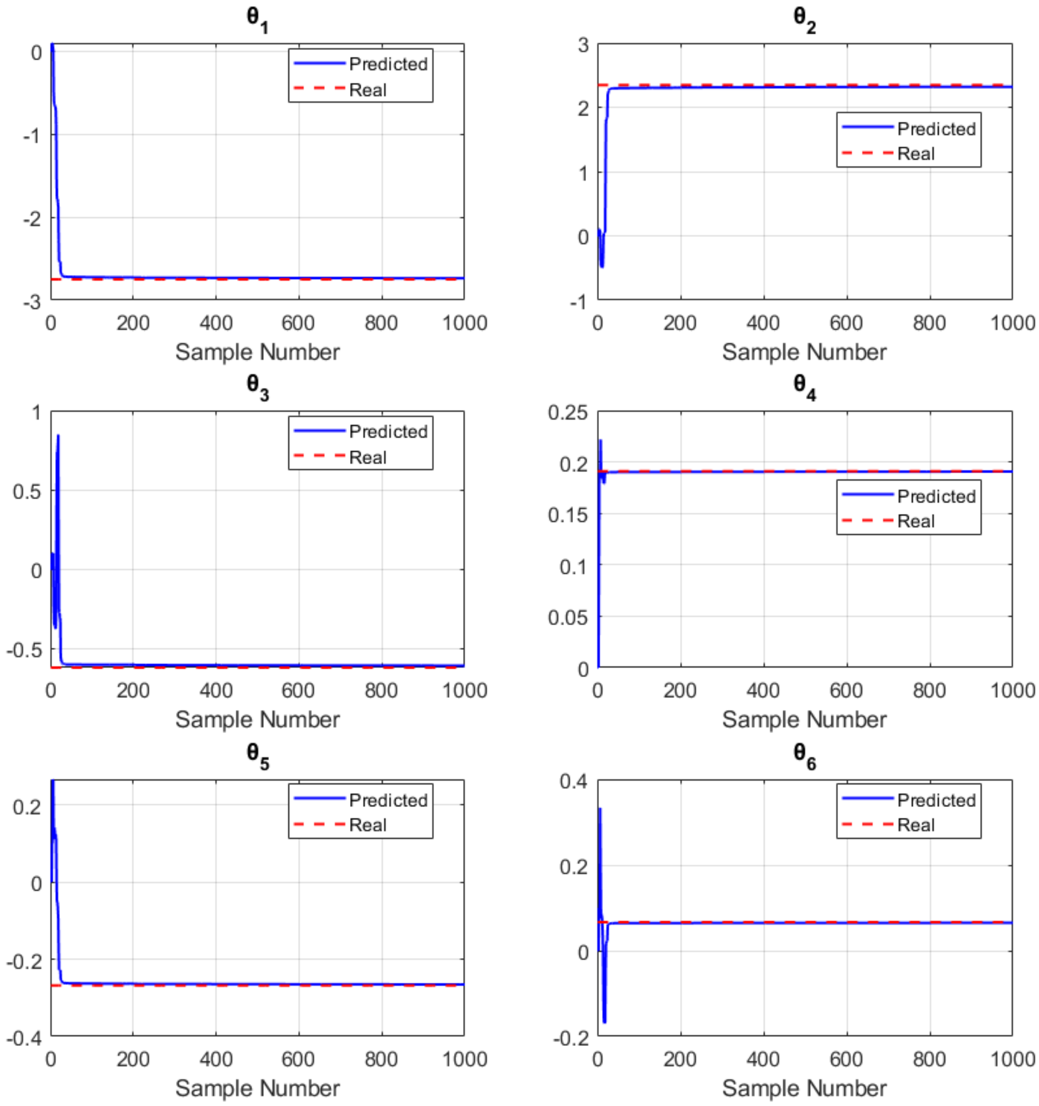

Figure 45: Estimated A and B polynomials. (Indirect STR Without zero cancellation) (Non-Minimum Phase System).

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

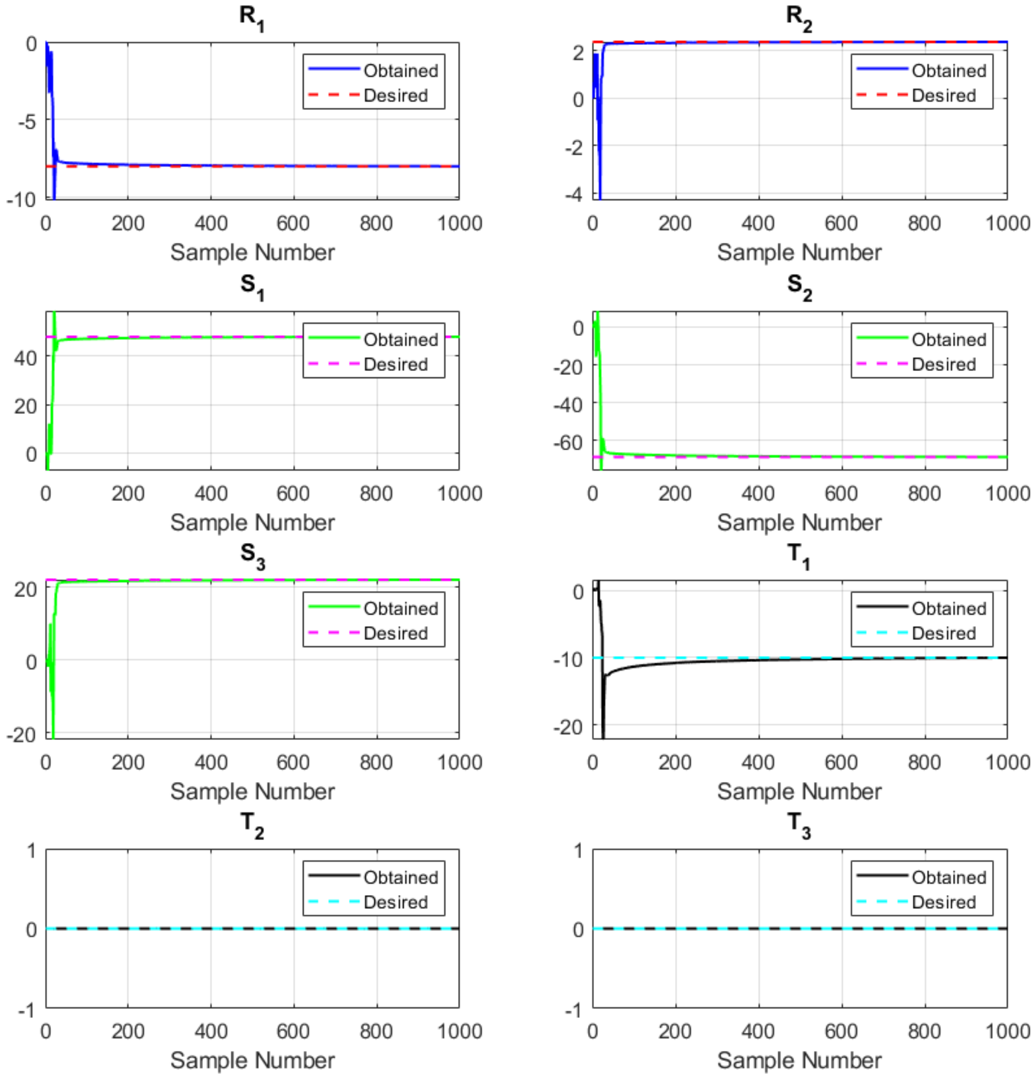

Figure 46: Estimated S, T and R polynomials. (Indirect STR Without zero cancellation) (Non-Minimum Phase System).

3.2 Indirect STR Without Zero Cancellation (Non-Minimum Phase System)

The STR Parameters are obtained as follows:

R	S	T
0.0015	-0.7057	0.8928
-0.0061	-0.03362	-2.0673
0.0150	0.0116	1.2832

Table 12: Calculated Direct STR without zero cancellation (Non-Minimum Phase System).

The output of the system under these conditions is shown in the figure below.

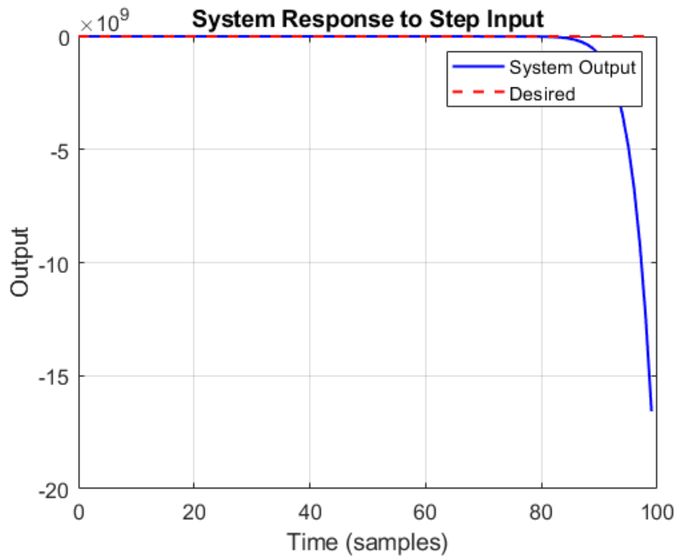

Figure 47: System output vs. Reference input (Direct STR-Without zero cancellation)(Non-Minimum Phase System)

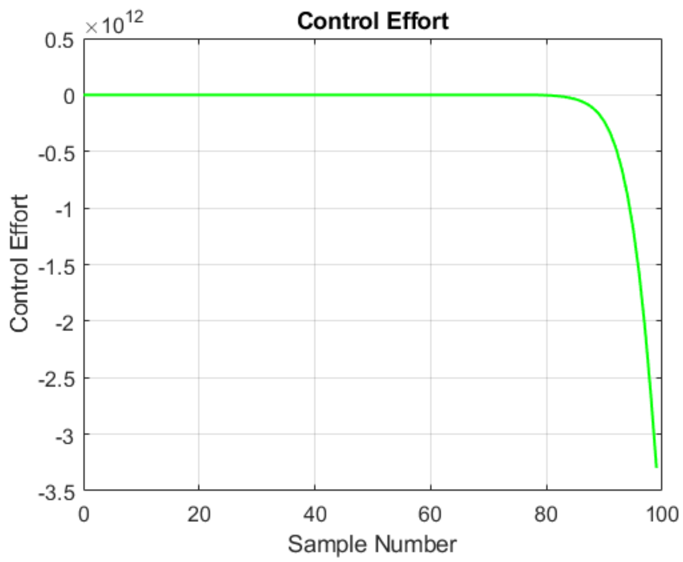

Figure 48: Control effort for Direct STR without zero cancellation (Non-Minimum Phase System)

The parameters for A and B polynomials are estimated as shown in the figure below.

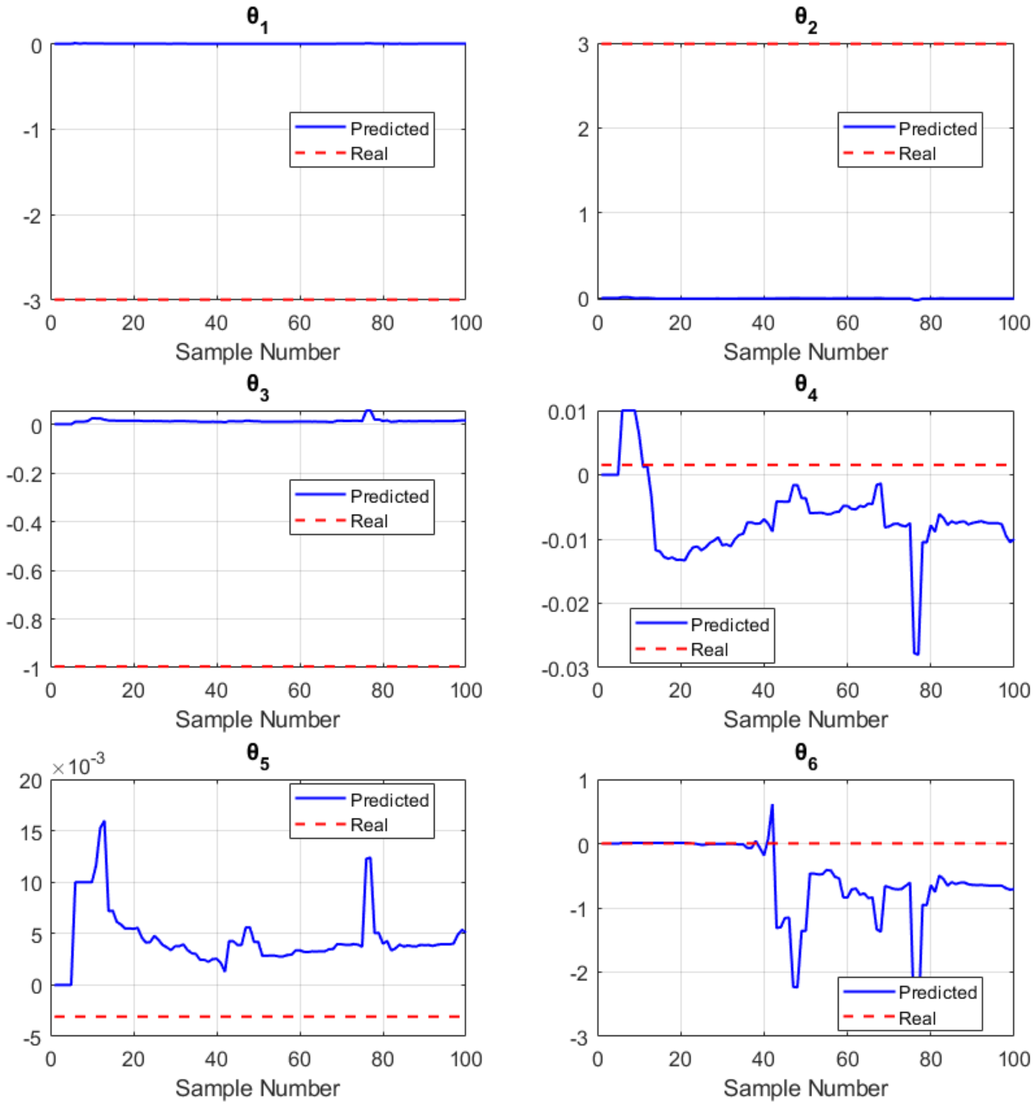

Figure 49: Estimated A and B polynomials. (Direct STR Without zero cancellation) (Non-Minimum Phase System).

As it is visible the parameters are predicted properly.

In the figure below the STR parameters are estimated.

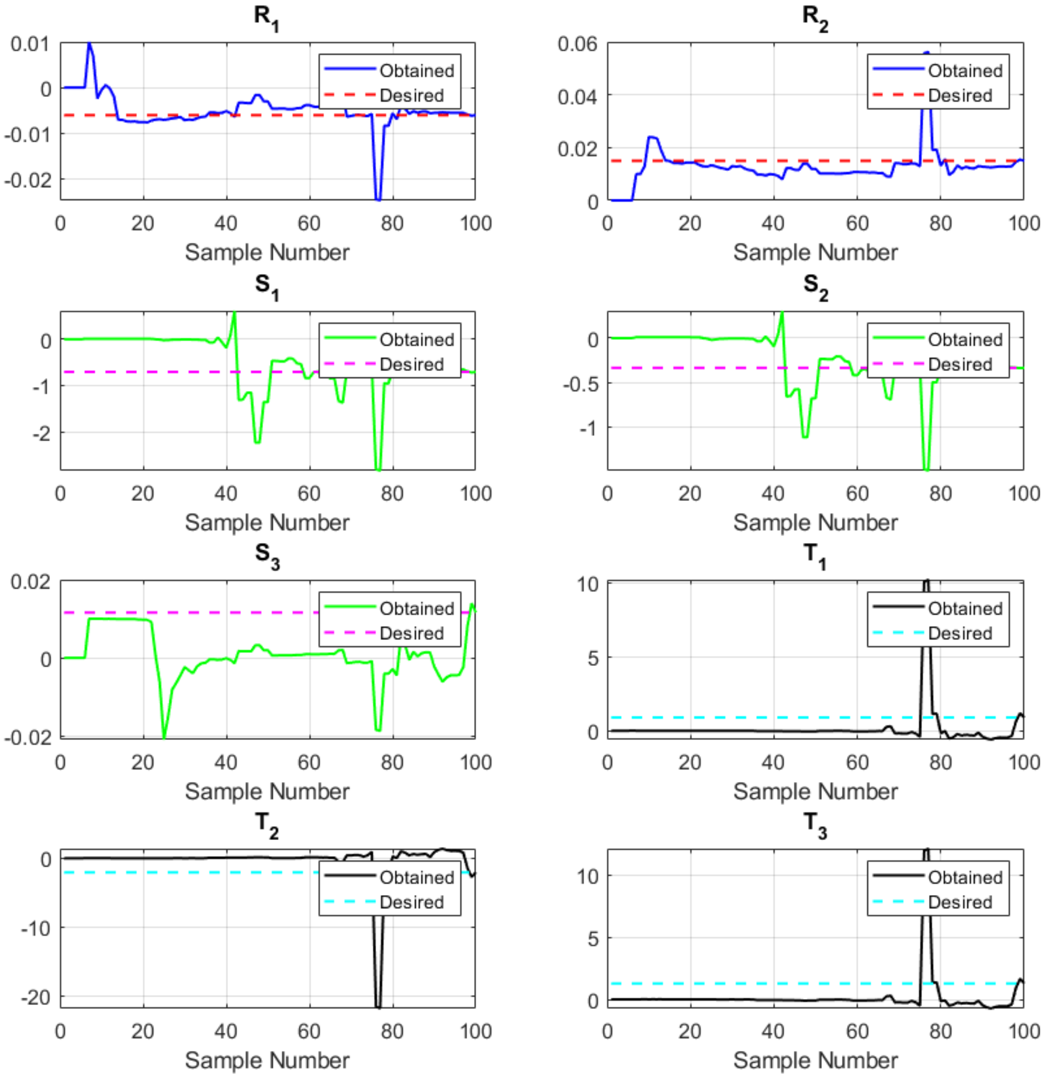

Figure 50: Estimated S, T and R polynomials. (Direct STR Without zero cancellation) (Non-Minimum Phase System).

3.3 Conclusion On Designing STR Without Zero Cancellation For Non-Minimum Phase System

As demonstrated in the previous sections, where both STR algorithms were implemented without zero cancellation, the indirect method yields superior results. Specifically, it provides good system tracking, effective control effort, accurate parameter estimation, and fast convergence of the S, T, and R polynomials.

However, using the direct method does not produce satisfactory results in terms of tracking, control effort, parameter estimation, and polynomial convergence. This is due to the presence of a zero on the right side of the imaginary axis in the root locus of the closed-loop system, which was also evident in the previous set of simulations on system identification.

In the direct method, since a controller was designed for the case where zeros are not canceled (and even when zeros are canceled, T is still estimated), we can use the same controller as in section 2-2. By utilizing the controller developed in section 2-2, where zeros are not canceled and the value of T is estimated, we can analyze the system's stability. Other scenarios were also examined, but stabilizing the system was not possible as the zeros were not correctly canceled in any case.

In summary, while the indirect method without zero cancellation achieves desirable outcomes, the direct method struggles due to the improper handling of zeros, affecting overall system performance and stability.

4 STR Using MDPP For Continuous System

In this section we want to design an STR using MDPP for a continuous system which is defined as follows:

$$G(s) = \frac{5}{s^2 + 7.5s + 3.5}$$

For continuous system as we have derivative operator we might have noise amplification. In order to avoid this we will use a low pass filter as the book has suggested. And the filter will be same as our desired system.

$$\begin{cases} y_f = G_m y \\ u_f = G_m u \end{cases}$$

The filtering and continuous estimation process of the plant is as follows:

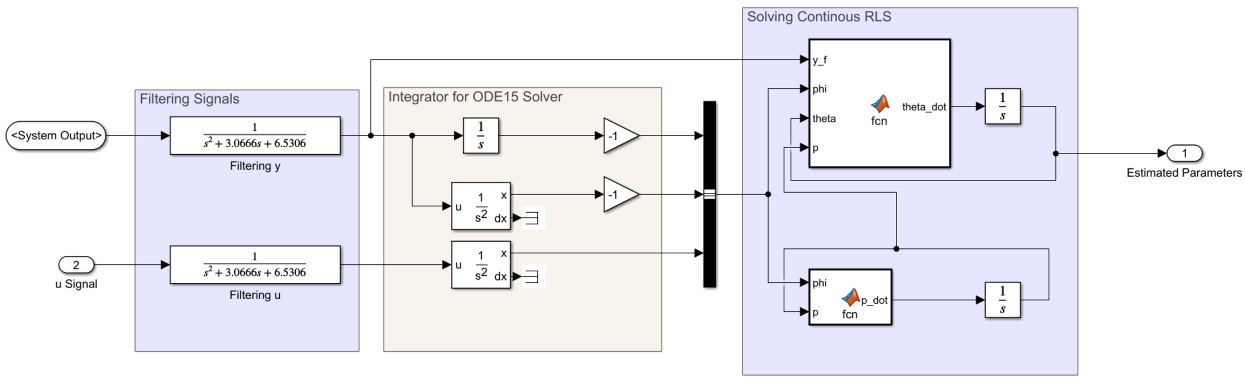

Figure 51: Filtering and Estimation block diagram.

The continuous RLS is performed by two functions bellow and is solved by ODE15.

```
function theta_dot = fcn(y_f,phi,theta,p)
theta_dot = (p)*(phi)*(y_f-(phi')*theta);

function p_dot = fcn(phi,p)
alpha = 1.0 ;
p_dot = alpha*p - p*(phi)*(phi)'*p ;
```

The S, T, and R polynomials are calculated as follows:

```
function [r1_real,s1_real,s2_real,t1_real,t2_real] = fcn(Wn,zeta,a0,
    a1_real,a2_real,b1_real)

% Solving Diphantine Equation by hand we get the following expressions
r1_real = 2*zeta*Wn+a0-a1_real ;
s1_real = (Wn^2+2*a0*zeta*Wn-a1_real*r1_real-a2_real)/b1_real ;
s2_real = (Wn^2*a0-a2_real*r1_real)/(b1_real) ;
t1_real = Wn^2/b1_real ;
t2_real = Wn^2*a0/b1_real ;
```

The whole block diagram of the process is as follows and ω_n and ζ where chosen like a good dynamic system and the parameter a_0 is obtained through try and error in the simulation process.

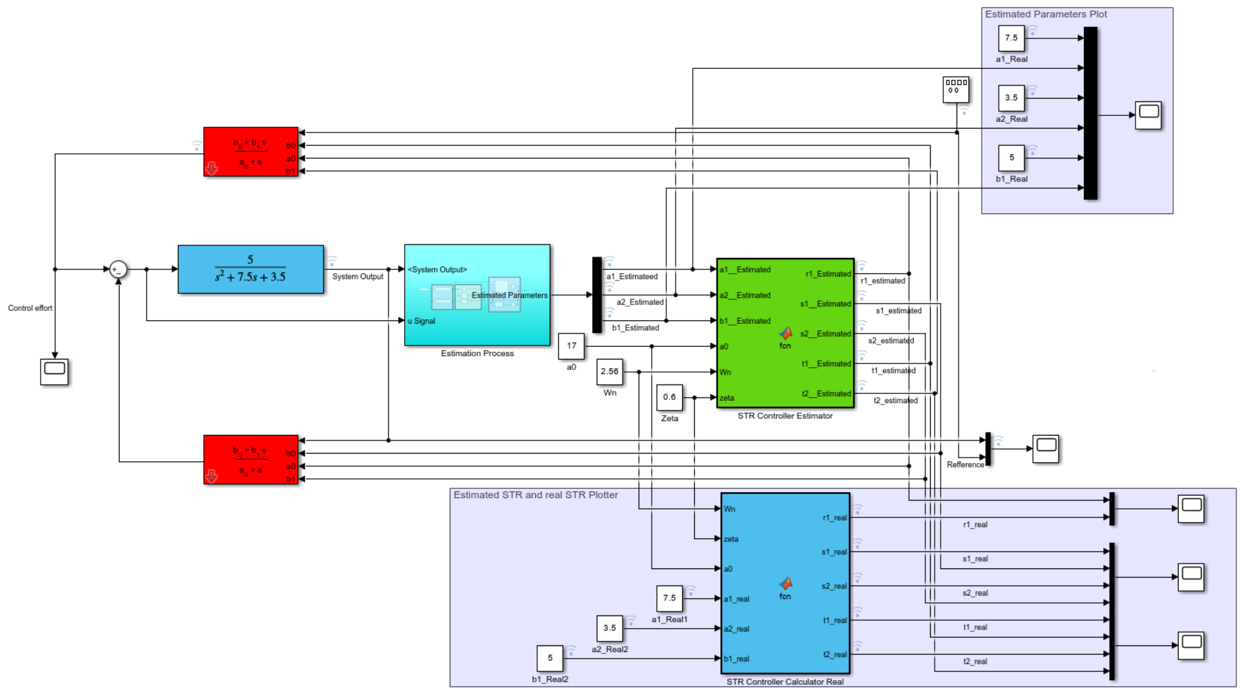

Figure 52: Whole block diagram of the process.

Note: in order to run the Simulink model there is no need to run any additional codes.

Now that the implementation of the model is complete we will see the results.

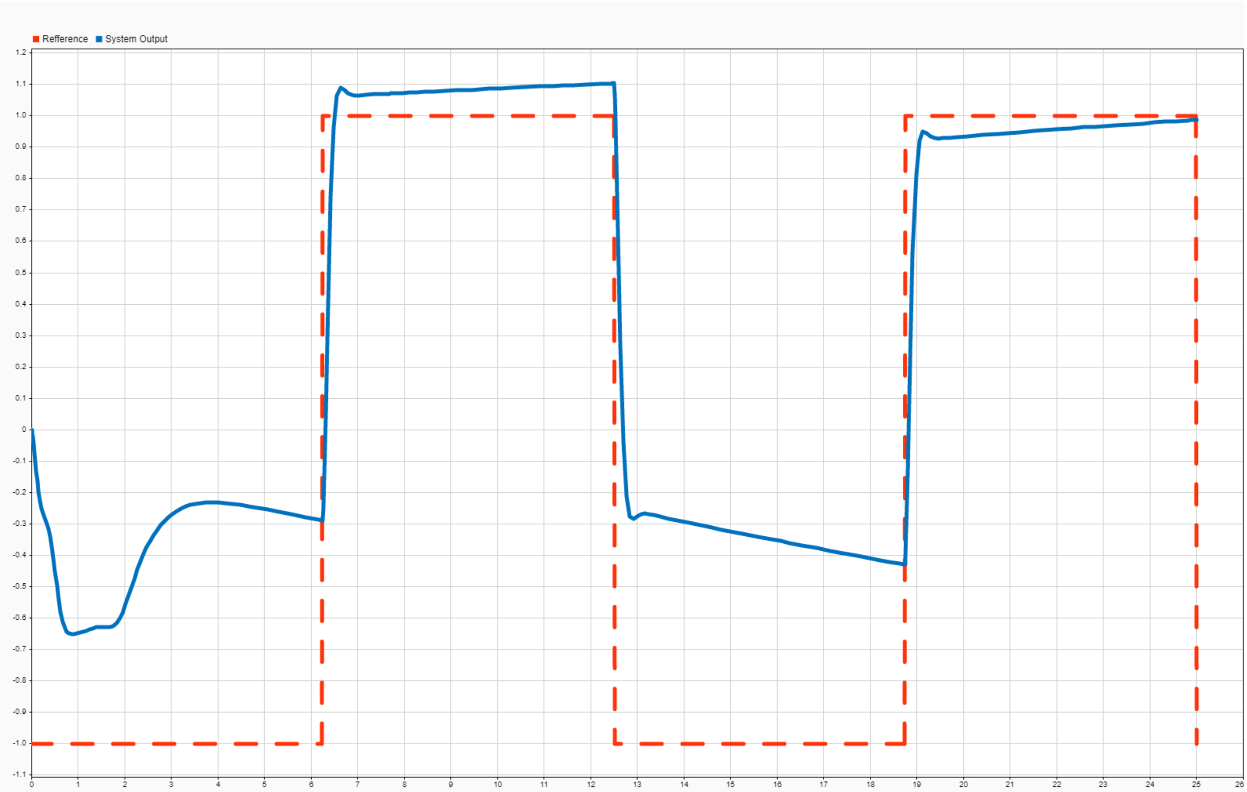

Figure 53: System Output vs. Reference signal (Continuous STR design).

As it is illustrated in the figure above the simulation is done for 25 seconds and as we can see the tracking is not very good, specially when the reference get low to the value of -1. But the tracking is good for high signals.

Figure 54: Control effort of the system (Continuous STR design).

As we can see the control effort is also not very good and gets high. But it is acceptable and if used in real case scenarios might not be a concern for our actuators.

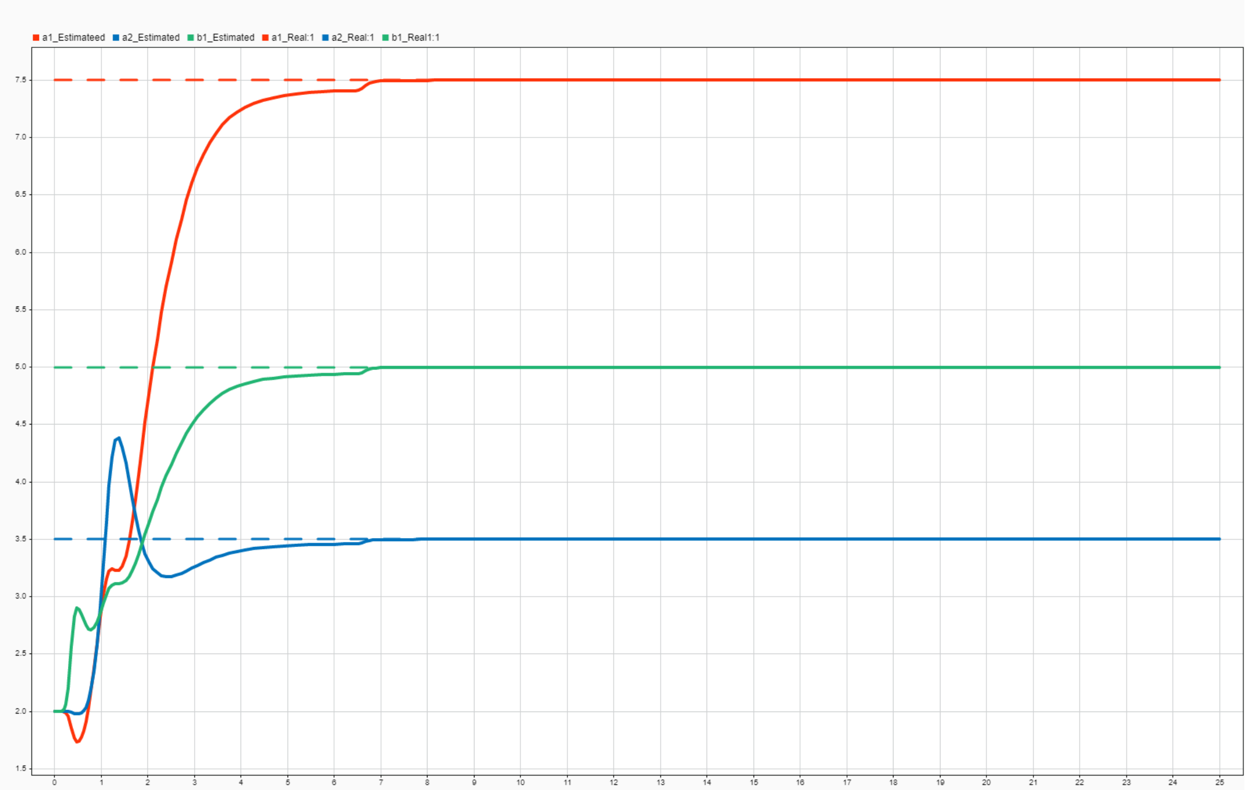

Figure 55: Estimated Parameters (Continuous STR design).

As it is shown in the figure above all parameters were estimated very good. We can see that the results of bad tracking and a little high control effort can't be blamed on the continuous RLS estimator.

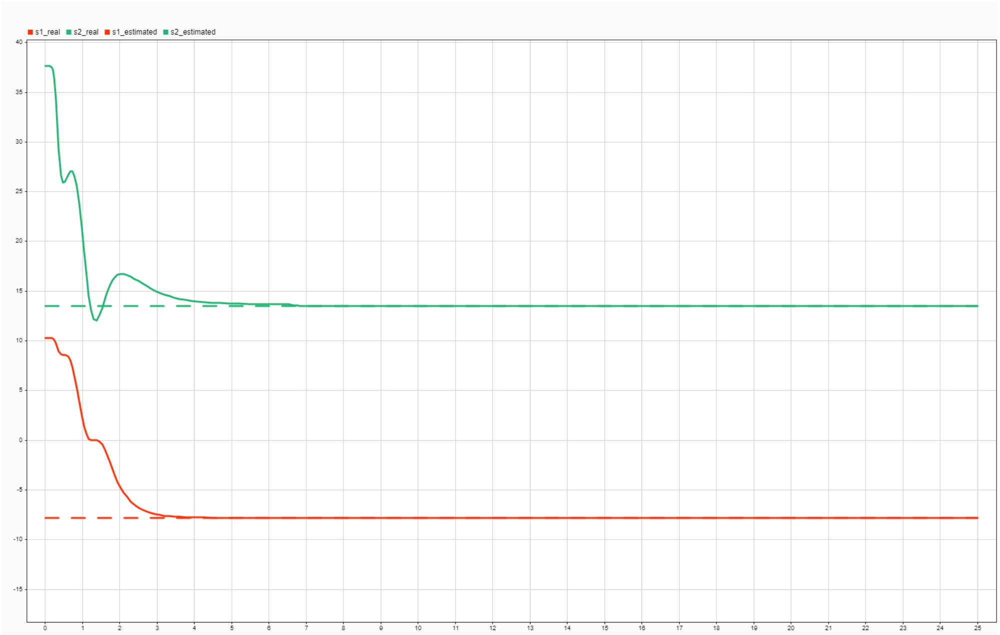

Figure 56: S polynomial Estimation (Continuous STR design).

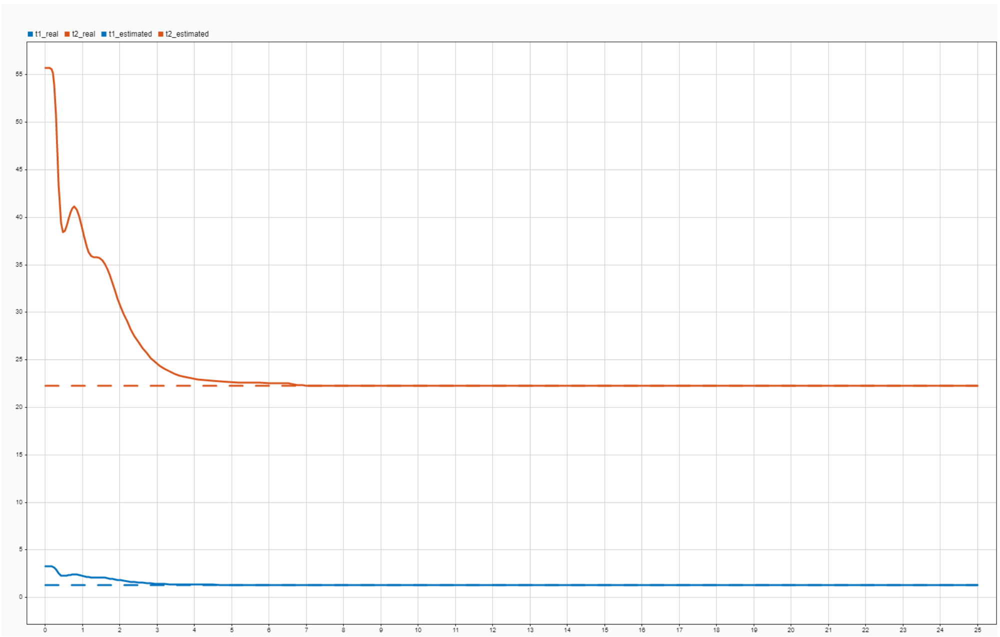

Figure 57: T polynomial Estimation (Continuous STR design).

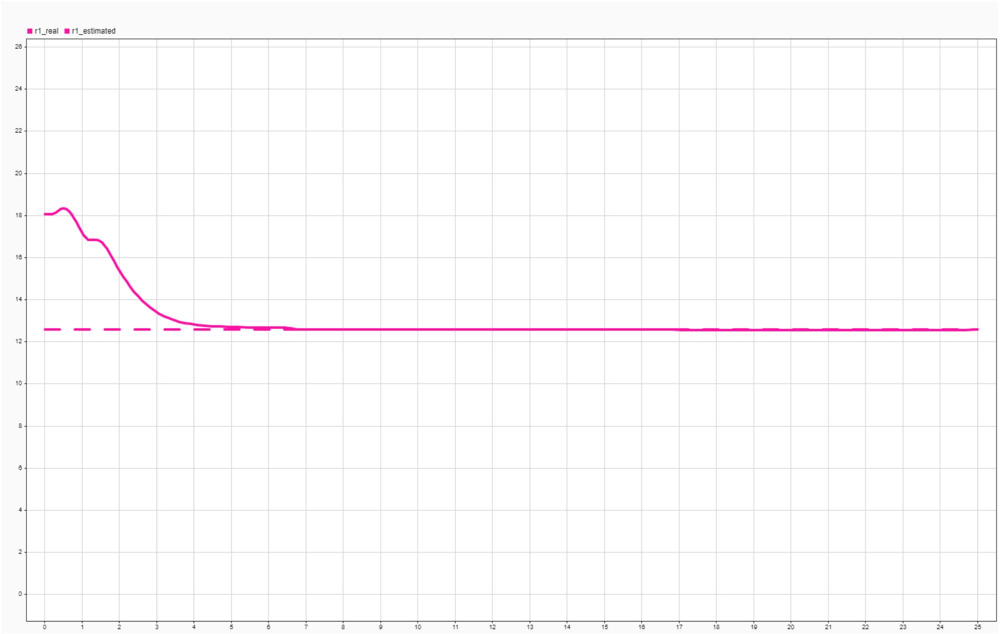

Figure 58: R polynomial Estimation (Continuous STR design).

As we can see in the figures 56 to 58 all S, T, and R parameters were estimated properly.