1. Wheel Travel vs. Camber, Toe and Track Alteration: Baseline Curves

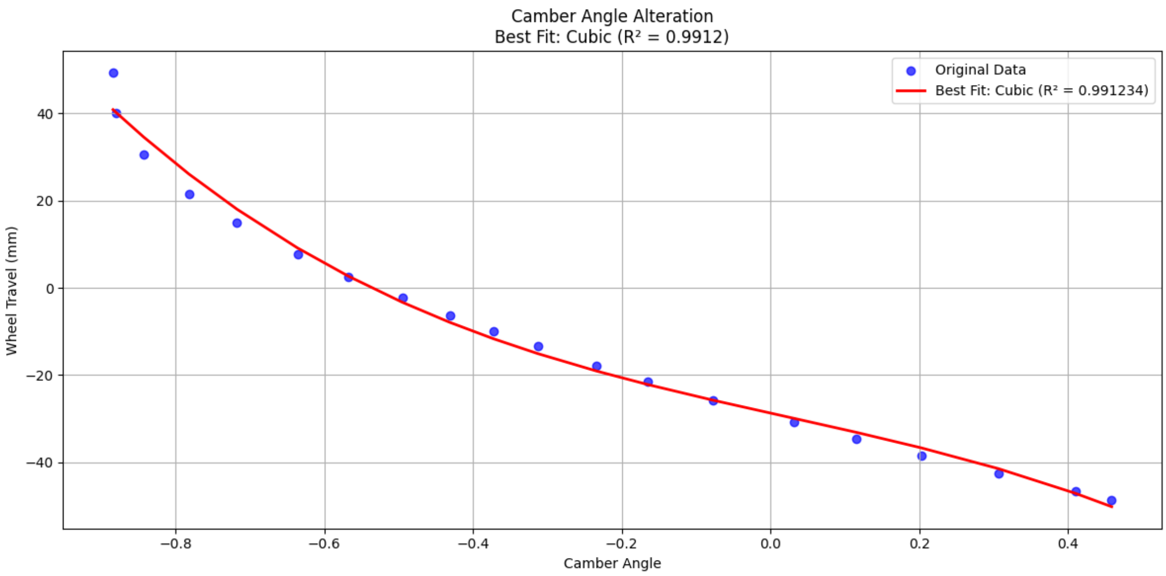
Best fitted functions are cubic as follows.

In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from sklearn.metrics import r2_score

# Define all possible fitting functions with bounds
def linear_func(x, a, b):
    return a * x + b

def quadratic_func(x, a, b, c):
    return a * x**2 + b * x + c

def cubic_func(x, a, b, c, d):
    return a * x**3 + b * x**2 + c * x + d

def power_func(x, a, b, c):
    with np.errstate(all='ignore'):
        return a * np.power(np.abs(x), b) + c  # Added abs() to handle ne

def exp_func(x, a, b, c):
    with np.errstate(all='ignore'):
        return a * np.exp(b * x) + c

def log_func(x, a, b, c):
    with np.errstate(all='ignore'):
        return a * np.log(np.abs(b) * np.abs(x) + 1e-10) + c  # Added sma

def sin_func(x, a, b, c, d):
    return a * np.sin(b * x + c) + d

# Store all functions with their names and parameter bounds
functions = [
    {'name': 'Linear', 'func': linear_func, 'bounds': (-np.inf, np.inf)},
    {'name': 'Quadratic', 'func': quadratic_func, 'bounds': (-np.inf, np.
    {'name': 'Cubic', 'func': cubic_func, 'bounds': (-np.inf, np.inf)},
    {'name': 'Power', 'func': power_func, 'bounds': ([0, -np.inf, -np.inf
    {'name': 'Exponential', 'func': exp_func, 'bounds': ([0, -np.inf, -np
    {'name': 'Logarithmic', 'func': log_func, 'bounds': ([0, 1e-10, -np.i
    {'name': 'Sinusoidal', 'func': sin_func, 'bounds': ([0, 0, -np.inf, -
]

def find_best_fit(x, y, dataset_name):
    best_r2 = -np.inf
    best_result = None

    print(f"\nAnalyzing {dataset_name} data (n={len(x)})")
    print("="*40)

    for func_info in functions:
        try:
            # Filter out NaN/inf values
```

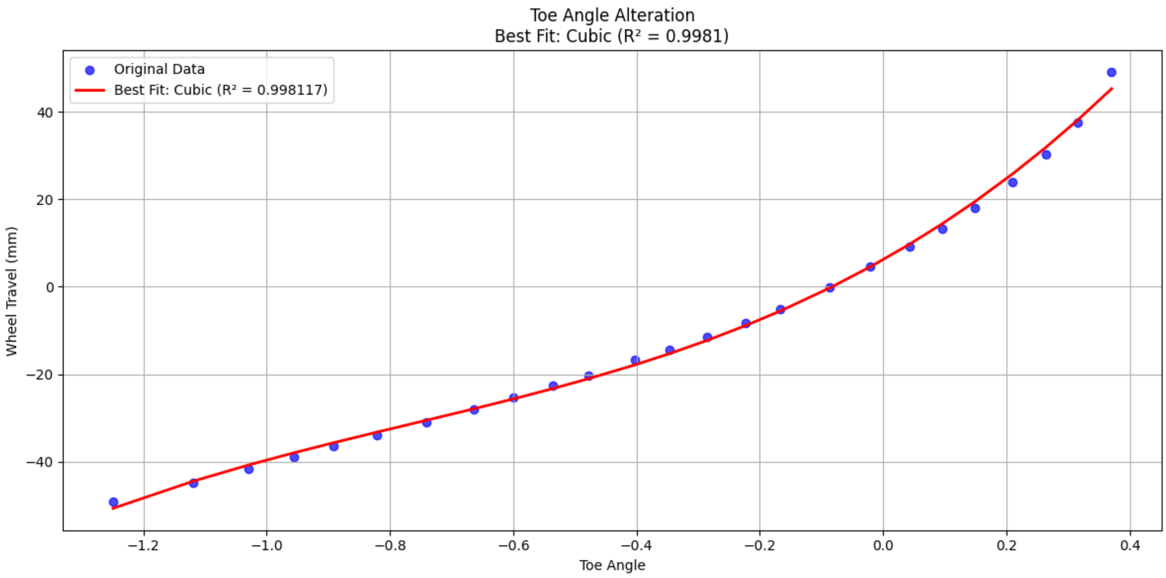
```
            mask = np.isfinite(x) & np.isfinite(y)
            x_clean = x[mask]
            y_clean = y[mask]

            # Initial parameter guesses
            p0 = [1.0] * len(func_info['func'].__code__.co_varnames[1:])

            # Perform the fit with bounds
            popt, pcov = curve_fit(func_info['func'], x_clean, y_clean,
                                   p0=p0, bounds=func_info['bounds'],
                                   maxfev=10000)

            # Calculate predictions and R²
            y_pred = func_info['func'](x_clean, *popt)
            r2 = r2_score(y_clean, y_pred)

            # Store all results
            result = {
                'name': func_info['name'],
                'func': func_info['func'],
                'params': popt,
                'r2': r2,
                'y_pred': func_info['func'](x, *popt)  # Prediction on or
            }

            print(f"{func_info['name']:12} - R²: {r2:.6f}")

            if r2 > best_r2 and np.isfinite(r2):
                best_r2 = r2
                best_result = result

        except Exception as e:
            print(f"{func_info['name']:12} - Failed: {str(e)[:50]}...")
            continue

    return best_result

def plot_results(x, y, result, dataset_name):
    plt.figure(figsize=(12, 6))
    plt.scatter(x, y, label='Original Data', color='blue', alpha=0.7)

    if result:
        # Sort for clean plotting
        sort_idx = np.argsort(x)
        x_sorted = x[sort_idx]
        y_pred_sorted = result['y_pred'][sort_idx]

        plt.plot(x_sorted, y_pred_sorted, 'r-', linewidth=2,
                label=f'Best Fit: {result["name"]} (R² = {result["r2"]:.6

        plt.title(f'{dataset_name} Alteration\nBest Fit: {result["name"]}
        print(f"\nBest fit for {dataset_name}: {result['name']}")
        print("Function parameters:", result['params'])
    else:
        plt.title(f'{dataset_name} Alteration - No suitable fit found')

    plt.xlabel(dataset_name)
    plt.ylabel("Wheel Travel (mm)")
    plt.legend()
    plt.grid(True)
```

```python
    plt.tight_layout()
    plt.show()

# Load all datasets
camber_df = pd.read_csv("camber_alteration.csv")
toe_df = pd.read_csv("toe_alteration.csv")
track_df = pd.read_csv("track_alteration.csv")

# Process Camber data
print("\n" + "="*60)
print("CAMBER DATA ANALYSIS".center(60))
print("="*60)
camber_result = find_best_fit(camber_df["x"].values, camber_df["y"].value
plot_results(camber_df["x"].values, camber_df["y"].values, camber_result,

# Process Toe data
print("\n" + "="*60)
print("TOE DATA ANALYSIS".center(60))
print("="*60)
toe_result = find_best_fit(toe_df["x"].values, toe_df["y"].values, "Toe A
plot_results(toe_df["x"].values, toe_df["y"].values, toe_result, "Toe Ang

# Process Track data
print("\n" + "="*60)
print("TRACK DATA ANALYSIS".center(60))
print("="*60)
track_result = find_best_fit(track_df["x"].values, track_df["y"].values,
plot_results(track_df["x"].values, track_df["y"].values, track_result, "T
```

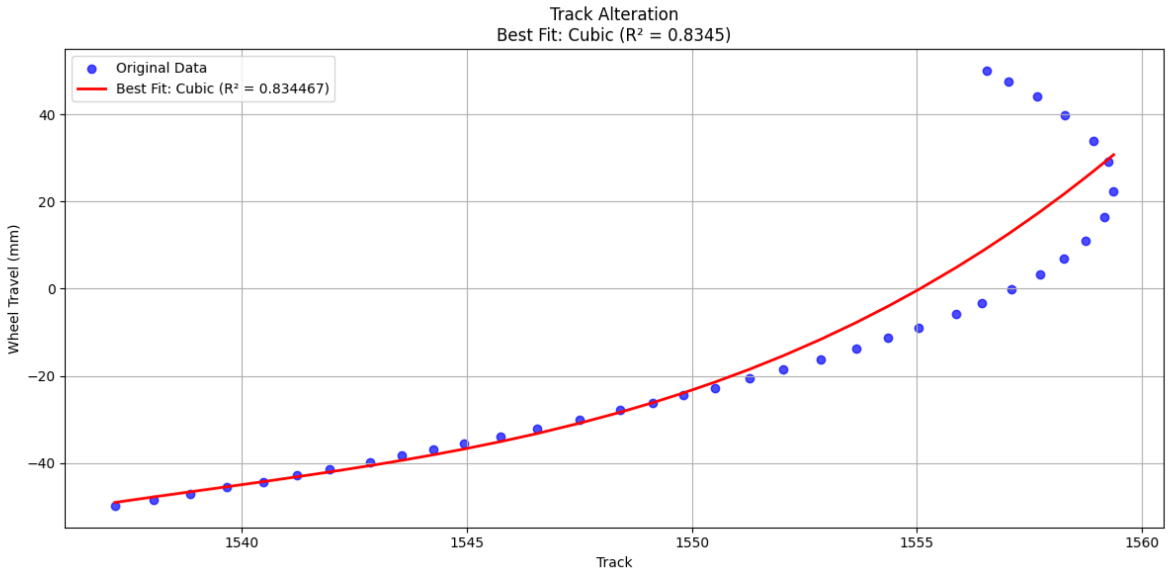
```
============================================================
                    CAMBER DATA ANALYSIS
============================================================

Analyzing Camber Angle data (n=20)
========================================
Linear       - R²: 0.937298
Quadratic    - R²: 0.982273
Cubic        - R²: 0.991234
Power        - R²: 0.807772
Exponential  - R²: 0.987590
Logarithmic  - R²: 0.424890
Sinusoidal   - R²: 0.174567

Best fit for Camber Angle: Cubic
Function parameters: [-48.76258113   3.03680145 -37.87645218 -28.72789553]
```




```
============================================================
                    TOE DATA ANALYSIS
============================================================

Analyzing Toe Angle data (n=25)
========================================
Linear       - R²: 0.947897
Quadratic    - R²: 0.990630
Cubic        - R²: 0.998117
Power        - R²: 0.394194
Exponential  - R²: 0.995422
Logarithmic  - R²: -0.000000
Sinusoidal   - R²: 0.990628

Best fit for Toe Angle: Cubic
Function parameters: [26.12141996 59.97397949 79.71489359  6.19447021]
```



```
============================================================
                    TRACK DATA ANALYSIS
============================================================

Analyzing Track data (n=38)
========================================
Linear        - R²: 0.782972
Quadratic     - R²: 0.832864
Cubic         - R²: 0.834467
Power         - Failed: Optimal parameters not found: The maximum number
o...
Exponential   - Failed: Residuals are not finite in the initial point....
Logarithmic   - R²: 0.782258
Sinusoidal    - R²: 0.012660

Best fit for Track: Cubic
Function parameters: [ 5.82353414e-03 -2.68903521e+01  4.13904475e+04 -2.1
2372593e+07]
```



Camber:

$$s(\epsilon) = -48.76258113x^3 + 3.03680145x^2 - 37.87645218x - 28.72789553$$

Toe:

$$s(\delta) = 26.12141996x^3 + 59.97397949x^2 + 79.71489359x + 6.19447021$$

Track:

$$s(b) = (5.82353414e-03)x^3 - (2.68903521e+01)x^2 + (4.13904475e+04)x - ($$

2. Dataset Extraction Based on the DOE Experiment (Sensitivity Analysis)

In [2]:
```
import csv

# Define the regression formulas
def calculate_camber(F1, F2, F3, F4, F5, F6, F7, F8, F9, F10, F11, F12):
    return (4.4686
            - 1.3297e-4 * F1
```

```python
            - 3.5092e-4 * F2
            + 3.2059e-3 * F3
            + 2.9229e-5 * F4
            - 7.2766e-5 * F5
            + 1.5516e-3 * F6
            - 5.2122e-4 * F7
            + 4.1222e-4 * F8
            - 4.5559e-3 * F9
            + 8.3043e-4 * F10
            + 5.7845e-3 * F11
            - 1.1116e-3 * F12)

def calculate_toe(F1, F2, F3, F4, F5, F6, F7, F8, F9, F10, F11, F12):
    return (1.5261
            + 2.3629e-5 * F1
            + 1.0878e-4 * F2
            - 5.5909e-3 * F3
            - 2.4836e-3 * F4
            - 3.0625e-4 * F5
            - 8.0725e-3 * F6
            + 3.1017e-3 * F7
            - 3.0418e-4 * F8
            + 1.5253e-2 * F9
            + 9.1023e-4 * F10
            + 1.3966e-3 * F11
            - 2.0041e-4 * F12)

def calculate_track(F1, F2, F3, F4, F5, F6, F7, F8, F9, F10, F11, F12):
    return (13.7968
            - 1.5747e-3 * F1
            - 5.5609e-3 * F2
            + 4.5056e-2 * F3
            - 1.7605e-3 * F4
            - 2.6833e-3 * F5
            + 3.2607e-2 * F6
            - 3.6524e-3 * F7
            + 6.4920e-3 * F8
            - 7.2687e-2 * F9
            + 1.4644e-3 * F10
            + 9.8420e-3 * F11
            - 1.9373e-3 * F12)

# Define nominal values for other factors (from the HTML file)
nominal_values = {
    'F1': 60,    # hpl_lca_front.x
    'F2': -400,  # hpl_lca_front.y
    'F4': 460,   # hpl_lca_rear.x
    'F5': -390,  # hpl_lca_rear.y
    'F6': 205,   # hpl_lca_rear.z
    'F7': 240,   # hpl_lca_outer.x
    'F8': -700,  # hpl_lca_outer.y
    'F10': 317.5, # hpl_top_mount.x
    'F12': 755    # hpl_top_mount.z
}

# Create ranges for the variables we want to vary
hpl_top_mount_y = np.linspace(-600, -560, 20)  # F11
hpl_lca_outer_z = np.linspace(155, 195, 20)    # F9
hpl_lca_front_z = np.linspace(170, 210, 20)    # F3
```

```python
# Generate all combinations
data = []
for F11 in hpl_top_mount_y:
    for F9 in hpl_lca_outer_z:
        for F3 in hpl_lca_front_z:
            # Set all factors (using nominal values for fixed factors)
            factors = {
                'F1': nominal_values['F1'],
                'F2': nominal_values['F2'],
                'F3': F3,
                'F4': nominal_values['F4'],
                'F5': nominal_values['F5'],
                'F6': nominal_values['F6'],
                'F7': nominal_values['F7'],
                'F8': nominal_values['F8'],
                'F9': F9,
                'F10': nominal_values['F10'],
                'F11': F11,
                'F12': nominal_values['F12']
            }

            # Calculate responses
            camber = calculate_camber(**factors)
            toe = calculate_toe(**factors)
            track = calculate_track(**factors)

            # Add to dataset
            data.append([F11, F9, F3, camber, toe, track])

# Write to CSV
filename = 'suspension_geometry.csv'
with open(filename, 'w', newline='') as csvfile:
    writer = csv.writer(csvfile)
    writer.writerow(['hpl_top_mount.y', 'hpl_lca_outer.z', 'hpl_lca_front
                     'camber_angle', 'toe_angle', 'track'])
    writer.writerows(data)

print(f"CSV file '{filename}' generated successfully with {len(data)} row
```

```
CSV file 'suspension_geometry.csv' generated successfully with 8000 rows.
```

In [3]:
```python
suspension_df = pd.read_csv("suspension_geometry.csv")
suspension_df
```

Out[3]:

	hpl_top_mount.y	hpl_lca_outer.z	hpl_lca_front.z	camber_angle	toe_angle
0	-600.0	155.0	170.000000	0.339787	0.476949
1	-600.0	155.0	172.105263	0.346536	0.465179
2	-600.0	155.0	174.210526	0.353286	0.453409
3	-600.0	155.0	176.315789	0.360035	0.441638
4	-600.0	155.0	178.421053	0.366784	0.429868
...	...	...	...	...	...
7995	-560.0	195.0	201.578947	0.490170	0.966378
7996	-560.0	195.0	203.684211	0.496919	0.954608
7997	-560.0	195.0	205.789474	0.503669	0.942838
7998	-560.0	195.0	207.894737	0.510418	0.931068
7999	-560.0	195.0	210.000000	0.517167	0.919297

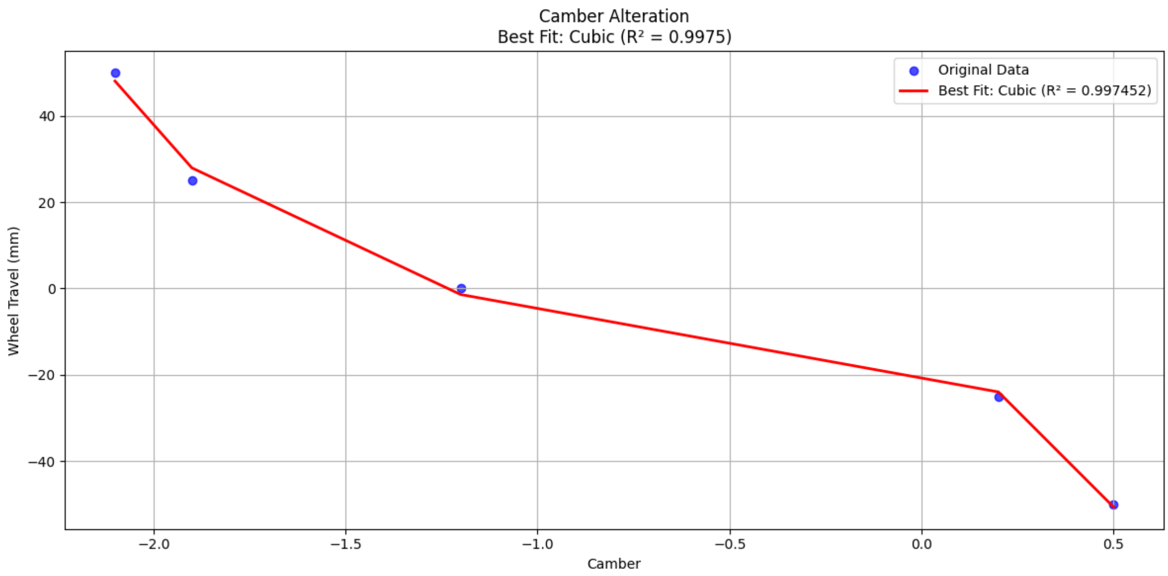
8000 rows × 6 columns

3. Target Optimized Curves (Objective Curves)

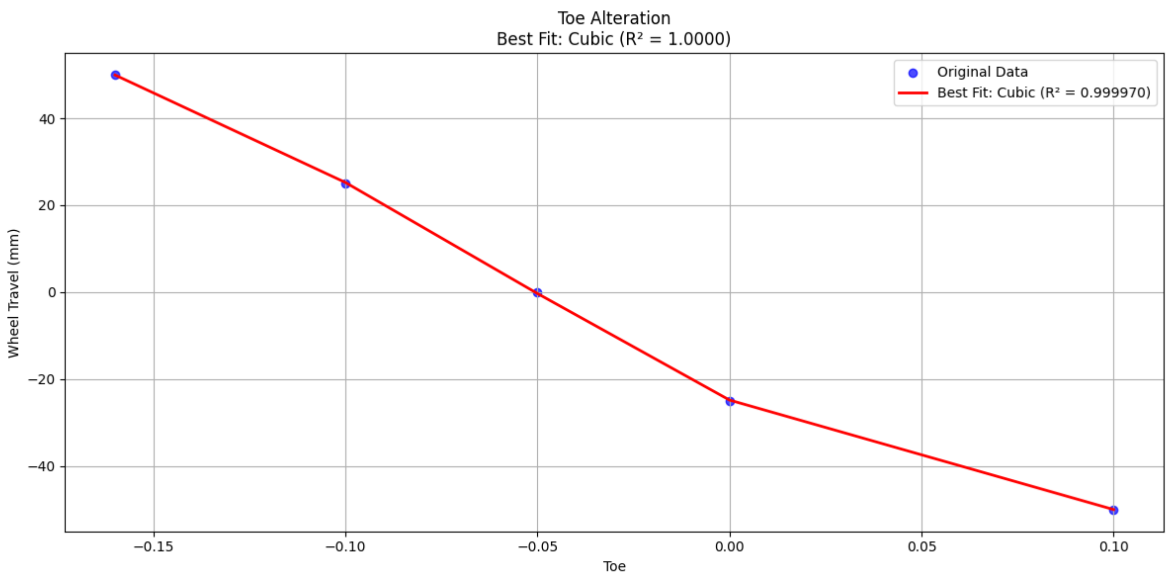
In [5]:

```
# Input data
camber_angles = np.array([0.5, 0.2, -1.2, -1.9, -2.1])  # degrees (indepe
wheel_travel = np.array([-50, -25, 0, 25, 50])          # mm (dependent)
toe_angles = np.array([0.1, 0, -0.05, -0.1, -0.16])   # degrees (independ

# Camber angle vs wheel travel analysis
print("\nCAMBER ANGLE → WHEEL TRAVEL")
camber_result = find_best_fit(camber_angles, wheel_travel, "Camber")
plot_results(camber_angles, wheel_travel, camber_result, "Camber")

# Toe angle vs wheel travel analysis
print("\nTOE ANGLE → WHEEL TRAVEL")
toe_result = find_best_fit(toe_angles, wheel_travel, "Toe")
plot_results(toe_angles, wheel_travel, toe_result, "Toe")
```

```
CAMBER ANGLE → WHEEL TRAVEL

Analyzing Camber data (n=5)
========================================
Linear       - R²: 0.934912
Quadratic    - R²: 0.947166
Cubic        - R²: 0.997452
Power        - R²: 0.905929
Exponential  - R²: 0.952600
Logarithmic  - R²: 0.662700
Sinusoidal   - R²: 0.947162

Best fit for Camber: Cubic
Function parameters: [-22.96634834 -54.31766864 -41.96792673 -13.22145237]
```




```
TOE ANGLE → WHEEL TRAVEL

Analyzing Toe data (n=5)
========================================
Linear       - R²: 0.978615
Quadratic    - R²: 0.993769
Cubic        - R²: 0.999970
Power        - R²: 0.009692
Exponential  - R²: 0.992751
Logarithmic  - R²: 0.232662
Sinusoidal   - R²: 0.999956

Best fit for Toe: Cubic
Function parameters: [6908.21532205 1243.58113283 -445.28034159  -24.83310
947]
```



Objective Camber Curve:

$$s(\epsilon) = -22.96634834x^3 - 54.31766864x^2 - 41.96792673x - 13.22145237$$

Objective Toe Curve:

$$s(\delta) = 6908.21532205x^3 + 1243.58113283x^2 - 445.28034159x - 24.83310947$$

4. Optimization

Implementation Steps: Regression-Based Approach

1. Fit a meta-model (e.g., polynomial or neural network) to predict camber/toe from hardpoints.
2. Optimize the meta-model to find hardpoints that minimize error.

4.1. Normalization and Spliting (Preprocessing)

In [6]:
```
from sklearn.preprocessing import StandardScaler
from sklearn.model_selection import train_test_split


X = suspension_df[["hpl_top_mount.y", "hpl_lca_outer.z", "hpl_lca_front.z
y = suspension_df[["camber_angle", "toe_angle"]]
# Split the data into training and temporary sets (80% train, 20% tempora
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,

# Standardization
sc = StandardScaler()
sc.fit(X_train)
X_train_scaled = sc.transform(X_train)
X_test_scaled = sc.transform(X_test)
```

4.2. Model Training

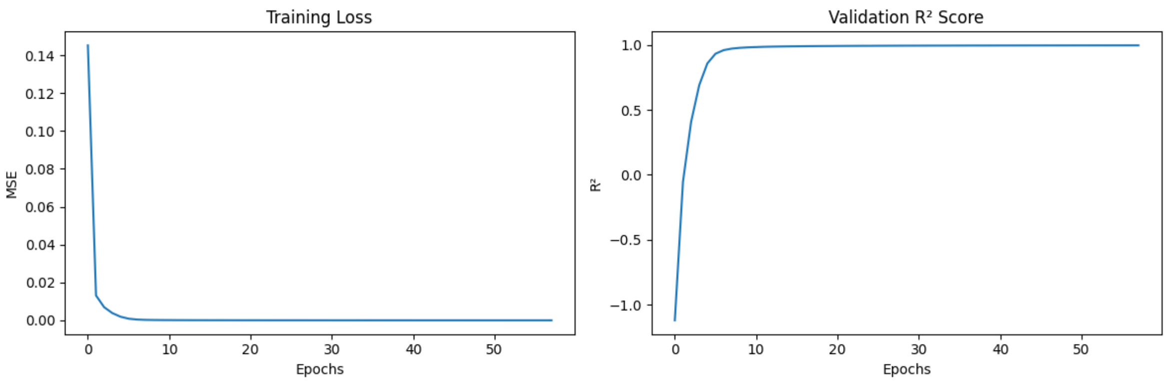
In [7]:
```
from sklearn.neural_network import MLPRegressor

# Initialize and train MLP
mlp = MLPRegressor(
    hidden_layer_sizes=(100, 50),  # 2 hidden layers
    activation='relu',
    solver='adam',
    max_iter=500,
    random_state=42,
    early_stopping=True,
    validation_fraction=0.2
)

mlp.fit(X_train_scaled, y_train)
```

Out[7]:
```
MLPRegressor
MLPRegressor(early_stopping=True, hidden_layer_sizes=(100, 50), m
ax_iter=500,
             random_state=42, validation_fraction=0.2)
```

4.3. Training Progress Visualization

In [8]:
```
plt.figure(figsize=(12, 4))

# Plot training loss
```

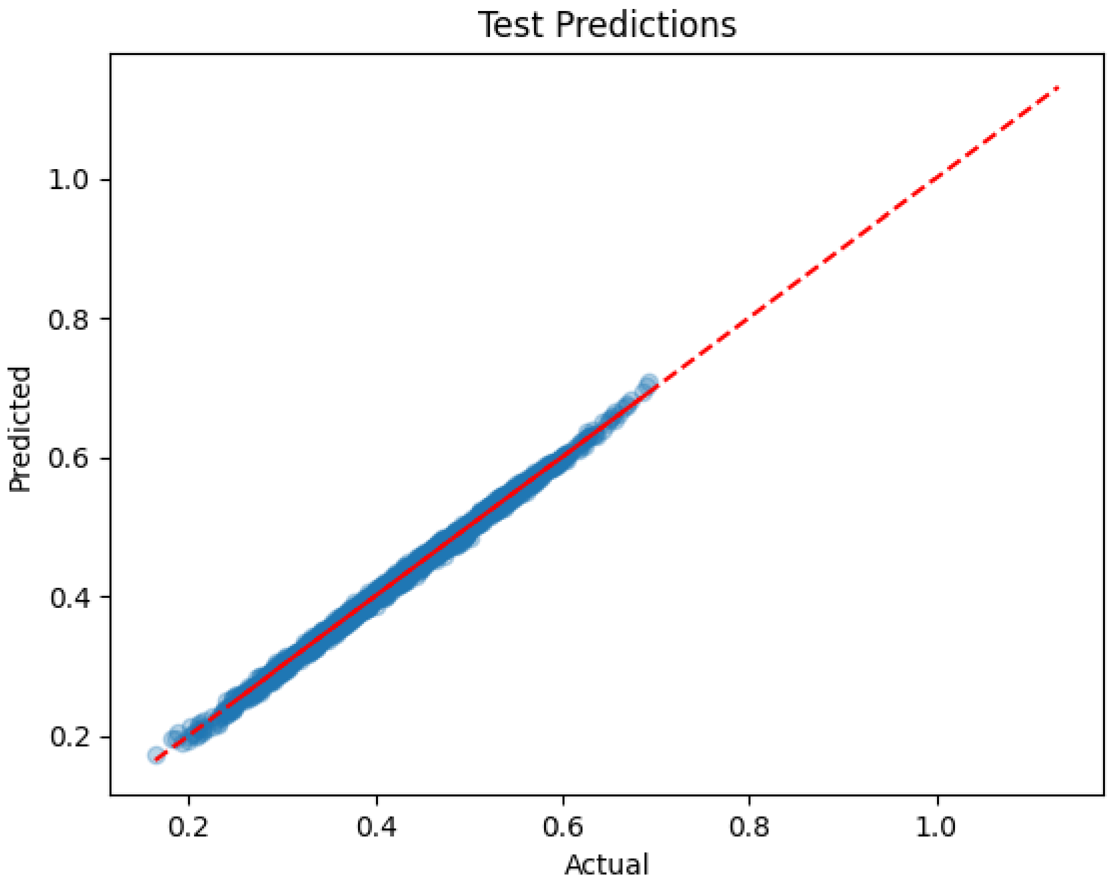
```
plt.subplot(1, 2, 1)
plt.plot(mlp.loss_curve_)
plt.title('Training Loss')
plt.xlabel('Epochs')
plt.ylabel('MSE')

# Plot validation score
plt.subplot(1, 2, 2)
plt.plot(mlp.validation_scores_)
plt.title('Validation R² Score')
plt.xlabel('Epochs')
plt.ylabel('R²')

plt.tight_layout()
plt.show()
```



4.3. Evaluation

In [9]:
```
from sklearn.metrics import mean_squared_error

# Make predictions
train_pred = mlp.predict(X_train_scaled)
test_pred = mlp.predict(X_test_scaled)

# Calculate metrics
train_r2 = r2_score(y_train, train_pred)
test_r2 = r2_score(y_test, test_pred)

print(f"Train R²: {train_r2:.3f}")
print(f"Test R²: {test_r2:.3f}")
print(f"Test MSE: {mean_squared_error(y_test, test_pred):.3f}")

# Quick prediction plot
plt.scatter(y_test.iloc[:, 0], test_pred[:, 0], alpha=0.3)
plt.plot([y_test.min(), y_test.max()], [y_test.min(), y_test.max()], 'r--
plt.xlabel('Actual')
plt.ylabel('Predicted')
plt.title('Test Predictions')
plt.show()
```

```
Train R²: 0.998
Test R²: 0.998
Test MSE: 0.000
```



4.4. Define the Optimization Problem and the Objective Functions

In [10]:
```
def camber_curve(x):
    return -48.76258113 * x**3 + 3.03680145 * x**2 - 37.87645218 * x - 28

def toe_curve(x):
    return 26.12141996 * x**3 + 59.97397949 * x**2 + 79.71489359 * x + 6.

def camber_objective(x):
    return -22.96634834 * x**3 - 54.31766864 * x**2 - 41.96792673 * x -13

def toe_objective(x):
    return 6908.21532205 * x**3 + 1243.58113283 * x**2 - 445.28034159 * x
```

In [11]:

```
def loss_function(hardpoints_scaled):
    # Predict camber and toe angles
    camber_pred, toe_pred = mlp.predict([hardpoints_scaled])[0]

    # Evaluate wheel travel from predicted angles
    s_camber = camber_curve(camber_pred)
    s_toe = toe_curve(toe_pred)

    # Evaluate objective wheel travel
    s_camber_obj = camber_objective(camber_pred)
    s_toe_obj = toe_objective(toe_pred)

    # Compute squared error
    camber_error = (s_camber - s_camber_obj)**2
    toe_error = (s_toe - s_toe_obj)**2

    return camber_error + toe_error
```

4.5. Constraints and Scaling

In [21]:

```
# Physical bounds
bounds_physical = {
    'hpl_top_mount.y': (-600, -560),
    'hpl_lca_outer.z': (155, 195),
    'hpl_lca_front.z': (170, 210)
}

# Convert bounds to scaled space
X_bounds = np.array([
    bounds_physical['hpl_top_mount.y'],
    bounds_physical['hpl_lca_outer.z'],
    bounds_physical['hpl_lca_front.z']
])

# Scale bounds
X_bounds_scaled = sc.transform(X_bounds.T).T  # shape (3, 2)
```

```
/Library/Frameworks/Python.framework/Versions/3.11/lib/python3.11/site-pac
kages/sklearn/base.py:465: UserWarning: X does not have valid feature name
s, but StandardScaler was fitted with feature names
  warnings.warn(
```

4.6. Optimize the Hardpoints

In [18]:

```
from scipy.optimize import minimize

# Initial guess: scaled mean of training data
initial_guess = np.mean(X_train_scaled, axis=0)

# Use L-BFGS-B with bounds
result = minimize(
    loss_function,
    initial_guess,
    method='L-BFGS-B',
    bounds=X_bounds_scaled,
    options={'maxiter': 1000}
)
```

```python
# Get optimized hardpoints in physical space
optimized_scaled = result.x
optimized_hardpoints = sc.inverse_transform([optimized_scaled])[0]

# Map to column names
optimized_dict = dict(zip(X.columns, optimized_hardpoints))
```

4.7. Optimization Results

In [19]:

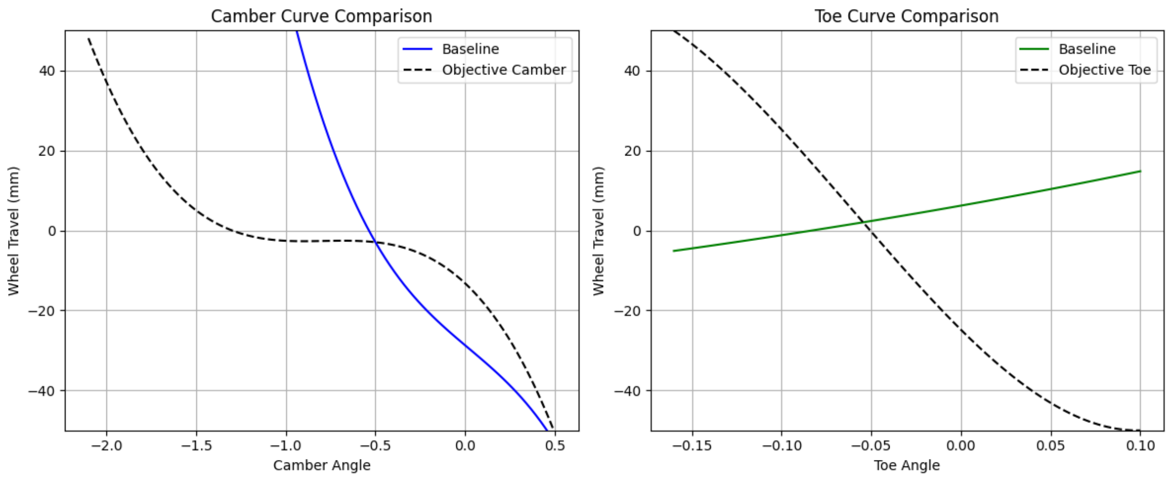
```python
optimized_dict
```

Out[19]:

```
{'hpl_top_mount.y': -599.9309968856223,
 'hpl_lca_outer.z': 155.0,
 'hpl_lca_front.z': 207.26123936523382}
```

In [17]:

```python
# Angle range
x_vals_camber = np.linspace(-2.1 , 0.5 , 100)
x_vals_toe = np.linspace(-0.16 , 0.1 , 100)

# Evaluate curves
s_camber_pred_vals = camber_curve(x_vals_camber)
s_camber_obj_vals = camber_objective(x_vals_camber)

s_toe_pred_vals = toe_curve(x_vals_toe)
s_toe_obj_vals = toe_objective(x_vals_toe)

# Plotting
plt.figure(figsize=(12, 5))

# Camber subplot
plt.subplot(1, 2, 1)
plt.plot(x_vals_camber, s_camber_pred_vals, label="Baseline", color='blue
plt.plot(x_vals_camber, s_camber_obj_vals, label="Objective Camber", line
# Set y-axis limits
plt.ylim(-50, 50)
plt.title("Camber Curve Comparison")
plt.xlabel("Camber Angle")
plt.ylabel("Wheel Travel (mm)")
plt.grid(True)
plt.legend()

# Toe subplot
plt.subplot(1, 2, 2)
plt.plot(x_vals_toe, s_toe_pred_vals, label="Baseline", color='green')
plt.plot(x_vals_toe, s_toe_obj_vals, label="Objective Toe", linestyle='--
# Set y-axis limits
plt.ylim(-50, 50)
plt.title("Toe Curve Comparison")
plt.xlabel("Toe Angle")
plt.ylabel("Wheel Travel (mm)")
plt.grid(True)
plt.legend()

plt.tight_layout()
plt.show()
```



Camber Curve Comparison
Baseline
Objective Camber
Wheel Travel (mm)
Camber Angle
40
20
0
−20
−40
−2.0
−1.5
−1.0
−0.5
0.0
0.5
Toe Curve Comparison
Baseline
Objective Toe
Wheel Travel (mm)
Toe Angle
−0.15
−0.10
−0.05
0.00
0.05
0.10

In [ ]: