

Faculty of Electrical Engineering

Final Project of Robust Control

**Design and Analysis of H∞ and μ-Synthesis-Based Attitude Controllers for an Underactuated Quadrotor**

By

**Kosar Aminjafari**

**Amirhossein Mohammadi**

Supervisor

**Hamid D. Taghirad**

July 2025

**Abstract**

**Table of Contents**

**List of Figures and Tables**

**1. Introduction**

**2. Modeling and System Identification**

**3. Robust H∞ Controller Design**

**4. μ-Synthesis-Based Controller Design**

**Definition of the Structured Singular Value (𝜇)**

The structured singular value, denoted by μ, quantifies the smallest amount of structured uncertainty that can destabilize a system or cause unacceptable performance degradation. Given a complex matrix M and a structured uncertainty set Δ, the structured singular value μΔ​(M) is defined as:

(I−MΔ)=0: indicates that the interconnection between M and Δ leads to an unstable or undefined system.

**Robust Stability μ vs. Robust Performance μ**

In robust control, structured singular value analysis is used to assess two key properties of a system under uncertainty: robust stability and robust performance. Although both use the same underlying mathematical tool — the structured singular value μ — they differ in their objectives, formulations, and the part of the system they evaluate.

**Robust Stability ( 𝜇 stab )**

Verify whether the closed-loop system remains stable for all admissible uncertainties Δ within a given structure.

M11​ is the submatrix of the full interconnection matrix M, mapping uncertainty inputs to uncertainty outputs.

If the above condition is satisfied, then no structured perturbation Δ with ∥Δ∥∞​≤1 can destabilize the system. A peak in μstab​ close to 1 suggests small robustness margins, meaning the system is near instability.

**Robust Performance μperf**​

Ensure the system maintains acceptable performance (e.g., disturbance rejection, tracking, noise attenuation) despite the presence of structured uncertainties.

M is the full augmented interconnection matrix that includes both the uncertainty paths and performance channels.

If the condition holds, the system satisfies performance requirements under all allowable structured uncertainties. If μperf​ >1 at any frequency, performance cannot be guaranteed at that frequency, indicating sensitivity to uncertainty.

**What to Do if 1μperf​>1?**

When the structured singular value for robust performance exceeds one it implies that the system cannot meet the specified performance objectives for all allowable structured uncertainties. In other words, the weighted performance norm is **violated**, and there exists at least one uncertainty configuration that causes the system to underperform.

**Possible Cause**

Overly Stringent Performance Requirements The performance weight may demand very small sensitivity (high disturbance rejection or tracking accuracy) at frequencies where it's physically or practically unattainable — particularly if:

**Remedies**

Adjust the Performance Weight 𝑊 𝑃 ​ To bring 𝜇 perf <, you can relax the performance specifications by modifying 𝑊 𝑃 ​ : Reduce the magnitude of W P ​ (s) at frequencies where μ perf ​ (jω) exceeds 1. This implies that you are tolerating more error (i.e., larger sensitivity) at those frequencies, making the performance requirement less strict and thus easier to satisfy in the presence of uncertainty.

**5. Results and Discussion**

**Conclusion**

**References**

[1] Ali Noormohammadi-Asl, Omid Esrafilian, Mojtaba Ahangar Arzati and Hamid D.Taghirad, *“System Identification and H∞-based Control of Quadrotor Attitude”*, Advanced Robotics and Automated Systems (ARAS), Faculty of Electrical Engineering, K. N. Toosi University of Technology, January 2020.

[2] Hamid D. Taghirad, *“Robust H∞ Control”*, K.N.Toosi University of Technology, Second Edition, 2024.