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K. N. Toosi University
of Technology

Department of Mechanical Engineering

Vehicle Dynamics First Project

Longitudinal Dynamics of a Vehicle

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October 2024

Abstract

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1. Introduction

When it comes to vehicle dynamics, there are many topics to consider. These topics can be divided into 4 main subsets that are listed in Figure 1.1. Each of these topics are analyzed in a separate article. The purpose of this paper is to study and simulate longitudinal dynamics of a real-world passenger car whose specifications are defined later.

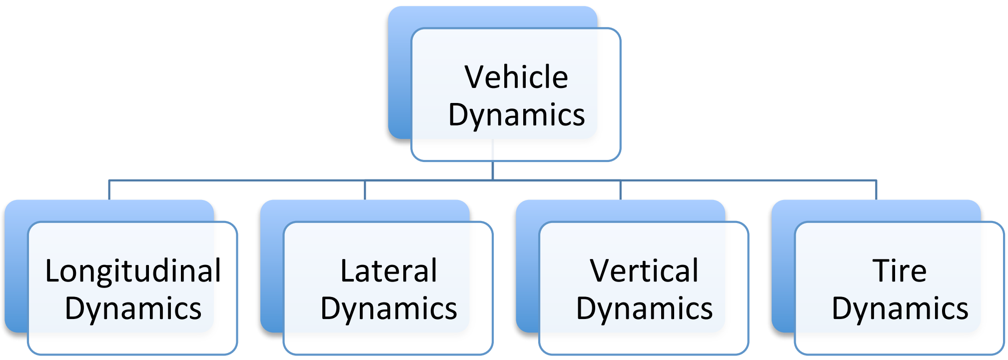

Figure 1.1. Vehicle dynamics main subsets

1.1. Importance of Longitudinal analysis

The longitudinal dynamics of a vehicle are essential to understanding its behavior under various driving conditions. This branch focuses on the forces and motions along the longitudinal axis of the vehicle. Concepts such as acceleration and braking performance, maximum speed, maximum slope, fuel consumption considerations and etc are important for engineers to optimize safety, efficiency, and comfort.

The importance of this topic cannot be overstated. By delving into longitudinal dynamics of a vehicle, the crucial insights mentioned earlier are unlocked that directly impact vehicle performance, safety and efficiency.

1.2. Objectives and Problem Definition

The vehicle reviewed in this article is one of SAIPA company's products called Tiba whose specifications are available in Table 1.1.

Table 1.1. Vehicle specifications

Characteristic	Symbol	Value	Unit
Total Weight	M	1050	kg
C.G to Front Axle	b	1.007	m
C.G to Rear Axle	c	1.408	m
C.G Weight	h_{CG}	0.498	m
Moment of Inertia (x direction)	I_x	350	$kg.m^2$
Moment of Inertia (y direction)	I_y	1520	$kg.m^2$
Moment of Inertia (z direction)	I_z	1340	$kg.m^2$
Front Spring Stifness	K_f	17650	N/m
Rear Spring Stifness	K_r	22600	N/m
Air Drag Coefficient	C_d	0.35	-
Cross Section of Air Resistance	A_0	2.34	m^2
Air Density	ρ	1.184	kg/m^3
Tire Size	175/60R14		
Tire Model	Fiala		
Normal Stifness	153		N/mm
Tire Rolling Radius (Unload)	267.5		mm
Tire Rolling Radius (Full load)	265.3		mm
1st Gear Ratio	3.454		
2nd Gear Ratio	1.944		
3rd Gear Ratio	1.275		
4th Gear Ratio	0.861		
5th Gear Ratio	0.692		
R	3.583		
Differential Ratio	3.777		

For longitudinal analysis, specially in steady conditions where instantaneous changes in time are ignored, there are 3 experimental curves defined as engine's performance characteristics which involve power-limited approach. These 3 characteristics are torque, power and specific fuel consumption (SFC) plots per the rotational speed of the engine. The related diagrams are shown in Figures 1.2, 1.3 and 1.4 that specify characteristics of the gasoline engine of the reviewed vehicle.

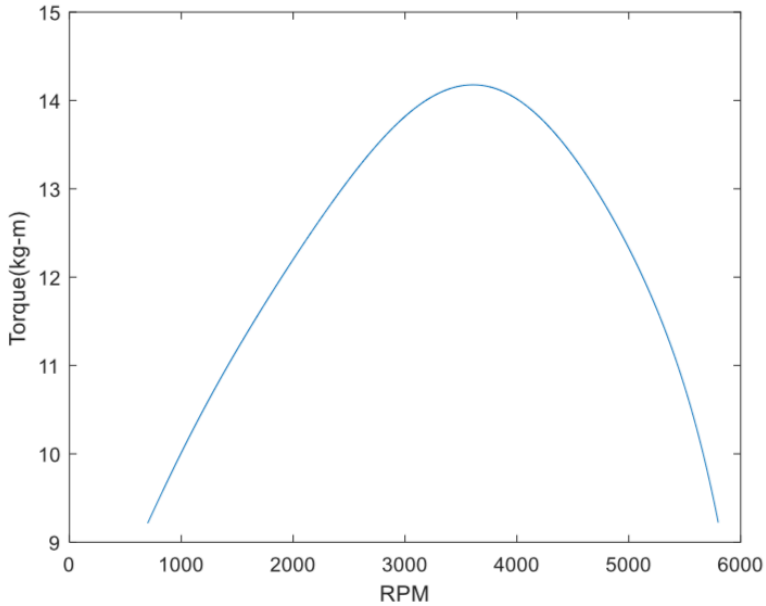

Figure 1.2. Torque per Rotational speed of the engine

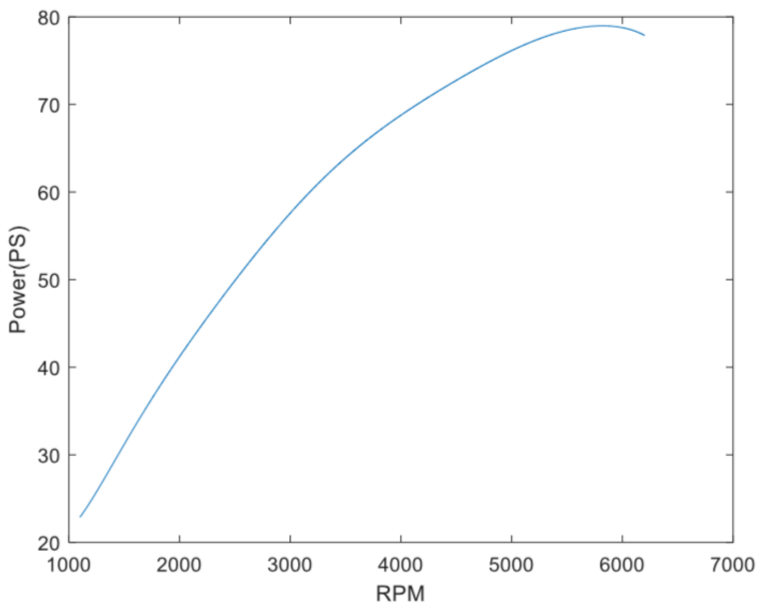

Figure 1.3. Power per Rotational speed of the engine

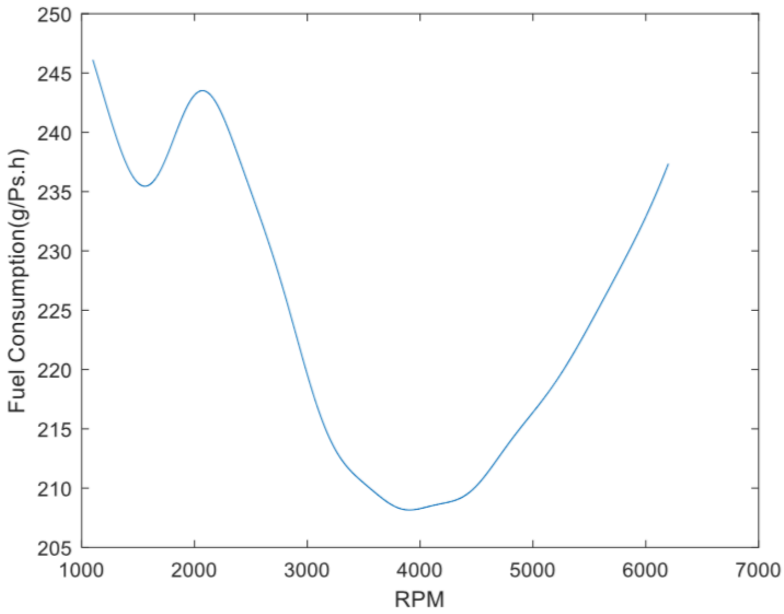

Figure 1.4. SFC per Rotational speed of the engine

According to the specification table (Table 1.1) as well as the characteristics diagrams, the demands of the problem are divided into 5 items below.

1. Simulation of the longitudinal dynamics of the vehicle in two modes: single-passenger and full-passenger with cargo.
2. The maximum speed of the vehicle in two modes: single-passenger and full-passenger with cargo.
3. Zero to 100 km/h acceleration of the vehicle in two modes: single-passenger and full-passenger with cargo.
4. The maximum incline the vehicle can handle at 5 km/h with a full load of passengers.
5. The minimum fuel consumption at constant speed.

It is assumed that the weight of each passenger is 68 kg and the amount of permissible load for each passenger is 7 kg. Also rolling resistance coefficient is 0.015 in this article.

2. Simulation of the longitudinal dynamics

In this part, the relationship between tractive force and longitudinal velocity of the vehicle and also engine's rotational speed must be determined. This relationship is different based on each of the gears of the manual transmission. By combining Newton's second law on the vehicle shown in Figure 2.1 and Euler's law of motion for the wheel, a simplified differential equation is obtained as shown in Equation 2.1.

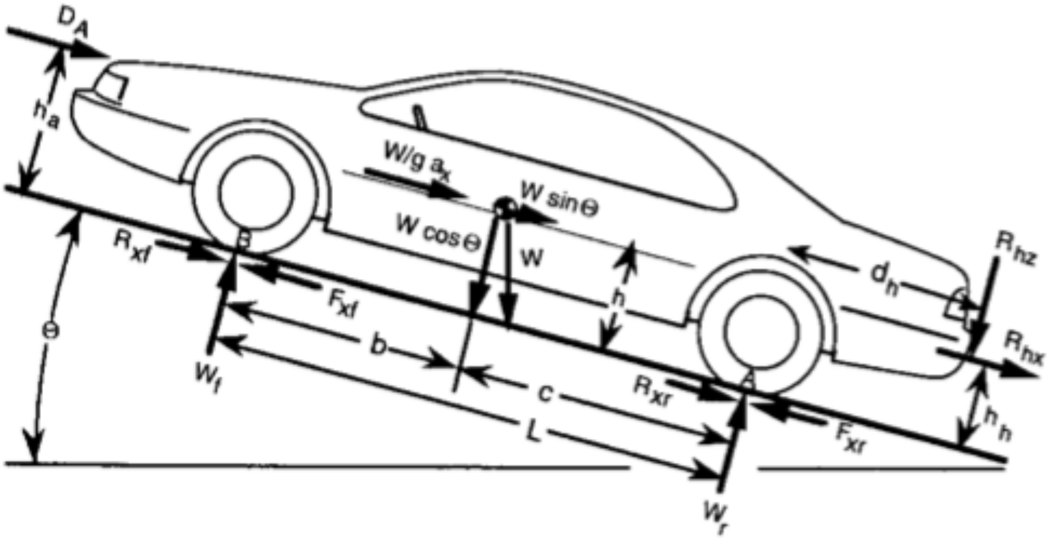

Figure 2.1. Arbitrary forces acting on e vehicle

$$\left(M+M_r\right) a_x = \frac{W+W_r}{g} a_x = \frac{T_e N_{tf} \eta_{tf}}{r} - R_x - D_A - R_{hx} - W \sin\theta \;(Equation\ 2.1)$$

where:

M = Mass of the vehicle = W/g

M_r = Equivalent mass of the rotating components

a_x = Longitudinal acceleration

T_e = Engine torque

$N_{tf} = N_t.N_f$ = Combined ratio of transmission and final drive

$\eta_{tf} = \eta_t.\eta_f$ = Combined efficiency of transmission and final drive

r = Radius of the wheels

R_x = Rolling resistance forces

D_A = Aerodynamic drag force

R_{hx} = Hitch (towing) forces

θ = Slope (incline) of the road

Note that the tractive force exists in Equation 2.1 and includes the engine torque and rotational inertia terms. Therefore, it is calculated from Equation 2.2.

$$Tractive\ Force = F_x = \frac{T_e N_{tf} \eta_{tf}}{r} - M_r a_x \ (Equation\ 2.2)$$

The combination of the two masses is the effective mass. There is also another parameter called mass factor that is used to calculate M_r. It is represented as Equation 2.3.

$$Mass\ Factor = \frac{M + M_r}{M} = 1 + 0.04\,N_{tf} + 0.0025\,N_{tf}{}^2 \ (Equation\ 2.3)$$

Since the longitudinal velocity is needed for simulation of longitudinal dynamics, the rotational speed of the engine is converted to longitudinal velocity of the vehicle using Equation 2.4 assuming no slip.

$$V_x = \frac{r\omega}{N_{tf}} \ (Equation\ 2.4)$$

where V_x is longitudinal velocity of the vehicle and ω is rotational speed of the engine.

To calculate tractive force at a certain velocity, 4 main steps are required and all of the 4 mentioned equations are used. From Figure 1.2 the values of engine's power at different rotational speeds are extracted. By having a single value of rotational speed the corresponding longitudinal velocity is calculated using Equation 2.4. The equivalent mass of rotating components is the only unknown parameter in Equation 2.3. By placing Longitudinal velocity (V_x) and M_r values in Equation 2.1 the longitudinal acceleration (a_x) is determined. Finally Equation 2.2 leads to tractive force at the certain velocity.

These 4 steps are only used to determine a single value of velocity and the corresponding tractive force. To plot a continuous diagram of tractive fore per velocity, the mentioned steps must be applied to all of the extracted points from Figure 1.2.

Torque per rotational speed of the engine values are extracted from Figure 1.2 using an online tool called PlotDigitizer. A CSV file is generated containing extracted discrete points from the digram. Figure 2.2 shows the related torque-speed diagram plotted by Matlab. It contains both extracted data (CSV file) and the fitted polynomial curve. The fitted curve is a 5th degree polynomial and its equation is displayed on the plot.

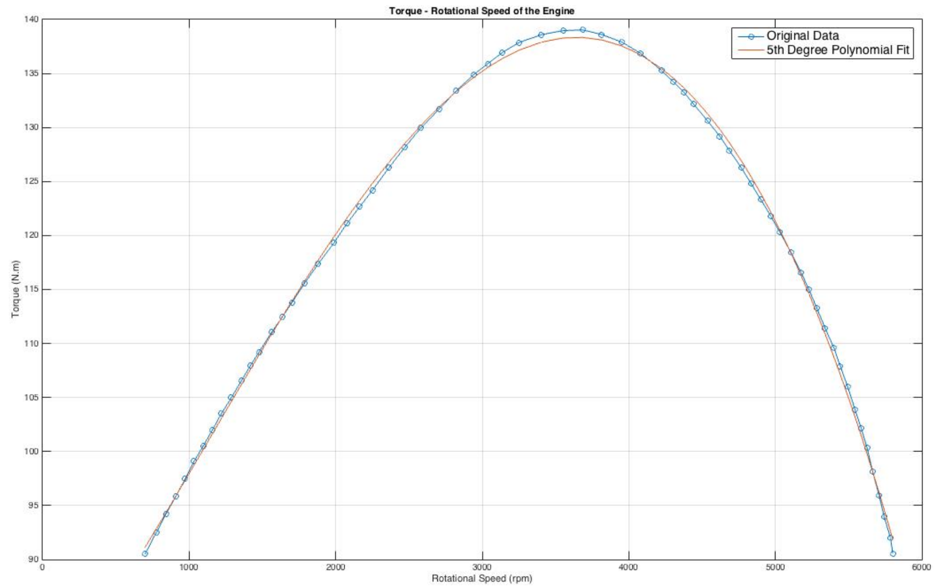

Figure 2.2. Torque per Rotational Speed in Matlab

Now the 4 mentioned steps must be applied to different points of the Figure 2.2. Here is an example scenario that explains theses steps altogether.

Example scenario:

Point (1000, 98) is located on the torque-speed curve. It means at rotational speed of 98 (rpm), the engine's torque is almost 1000 (N.m). By applying Equation 2.4 and assuming single-passenger, the longitudinal velocity is about 28 (m/s). Assuming 1st gear ratio, the mass factor is 1.947

from Equation 2.3. Therefore, M_r is 1058.75 (kg). By having longitudinal velocity and M_r the longitudinal acceleration is calculated from Equation 2.1 (assuming $\theta = 0°$, $\eta_{tf} = 0.85$ and there is to towing). Therefore the tractive force is obtained using Equation 2.2. Final results are visualized in Matlab for all of the 5 gear ratios in 2 modes of single-passenger and full load of passenger. These visualizations are shown in Figure 2.3 and Figure 2.4.

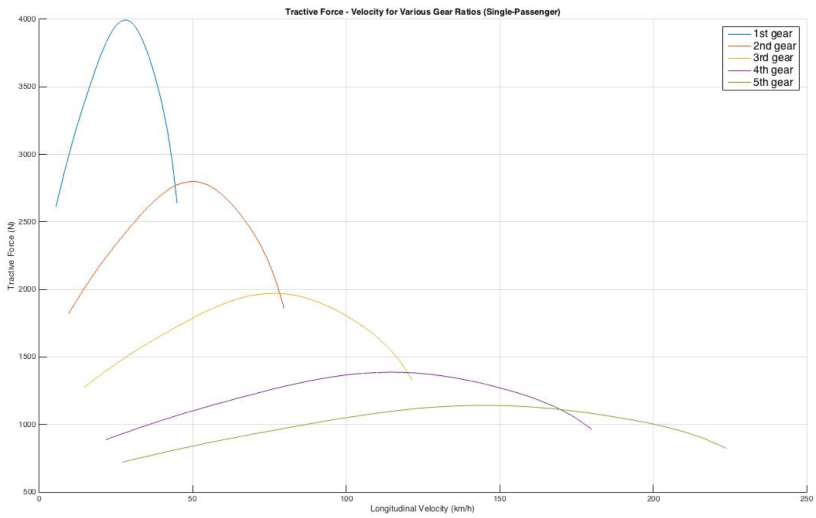

Figure 2.3. Tractive force (F_x) per Longitudinal velocity (V_x) for various gear ratios (Single-passenger)

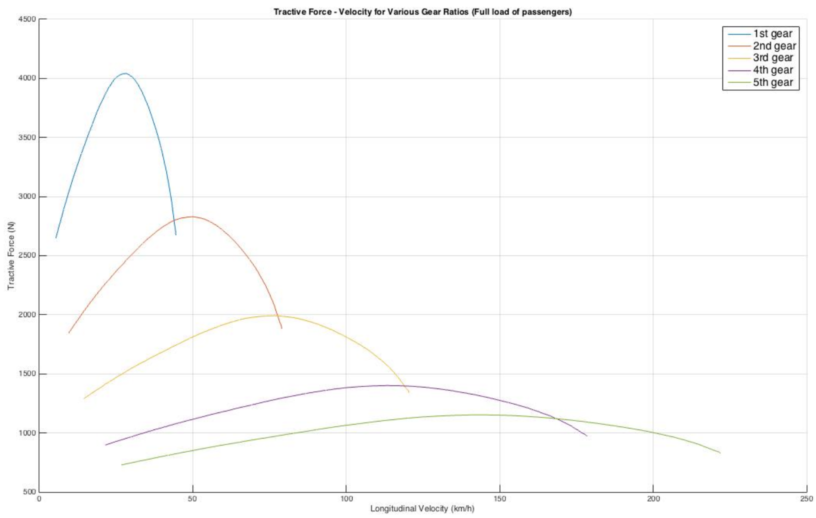

Figure 2.4. Tractive force (F_x) per Longitudinal velocity (V_x) for various gear ratios (Full load of passengers)

Note that the point of intersection of the curves is the best point to change between gears and thus optimized fuel consumption and performance.

All of the Matlab files and calculations are attached to this report.

3. Maximum velocity of the vehicle

Maximum velocity of the vehicle occurs in a horizontal road at the lowest gear ratio. Also the longitudinal acceleration is zero due to constant velocity at its peak. Therefore Equation 2.1 is modified as follows.

$$Ma_x = F_x - R_x - D_A = 0 \Rightarrow F_x = R_x + D_A \Rightarrow F_x = F_R \ (Equation\ 3.1)$$

where F_R is the total resistance force.

Equation 3.1 means the intersection point between the 2 curves F_x and F_R is where the maximum velocity happens. There are different ways to analytically solve this equation. Since tractive force per velocity is already plotted, the F_R - velocity curve can also be visualized the same way. Equation 3.2 shows that total resistance force is a function of longitudinal velocity.

$$F_R = \mu Mg + \frac{1}{2}\rho A_0 C_d V_x^2 \ (Equation\ 3.2)$$

Figure 3.1 shows both total resistance and tractive force plots for 5th gear which has the lowest ratio. This diagram is for single-passenger.

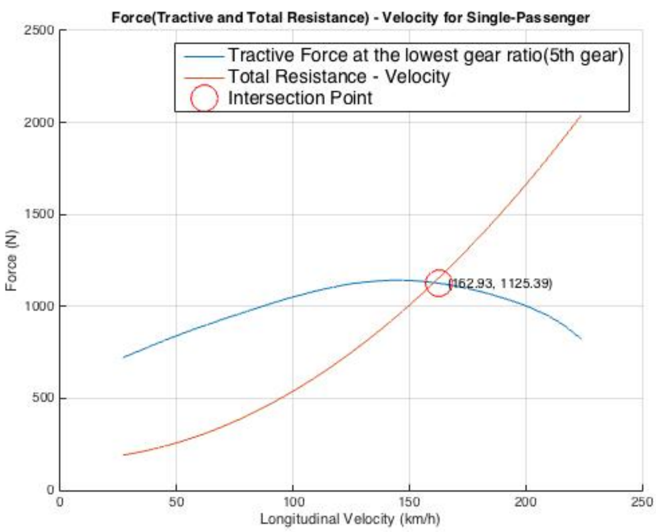

Figure 3.1. Tractive and total resistance forces per Velocity for single-passenger

Figure 3.2 shows the same diagram for full load of passengers.

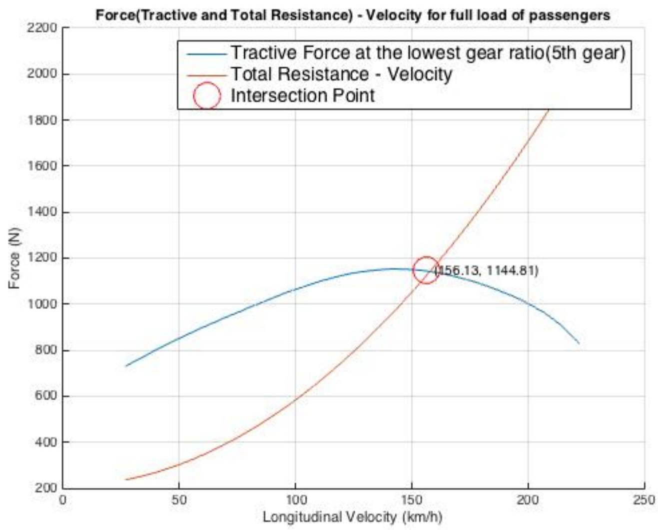

Figure 3.2. Tractive and total resistance forces per Velocity with full load of passengers

From the intersection points in Figures 3.1 and 3.2 the maximum longitudinal velocity of the vehicle is obtained. It is about 162 (km/h) in single-passenger mode and 156 (km/h) with a full load of passenger. Table 3.1 shows them both.

Table 3.1. Maximum longitudinal velocity

Passenger mode	**Maximum velocity (km/h)**
Single-passenger	162
Full load of passengers	156

Matlab code is available in the attached files.

4. Zero to 100 km/h acceleration of the vehicle

To calculate the time it takes to accelerate from 0 to 100(km/h), the integral of acceleration in terms of velocity must be calculated. Equation 4.1 shows the integration.

$$a = \frac{dV}{dt} \Rightarrow dt = \frac{dV}{a} \Rightarrow \int_{t_0}^{t} dt = \int_{V_0}^{V_1} \frac{dV}{a(V)} \Rightarrow \Delta t = \int_{0}^{100} \frac{dV}{a(V)} \quad (Equation\ 4.1)$$

The longitudinal acceleration is already calculated using previous equations and Matlab in previous chapters. Plotting 1/a(V) in terms of velocity is required to solve Equation 4.1. From Figures 2.3 and 2.4, it’s obvious that the velocity reaches 100(km/h) for the first time when the manual transmission is in the 3rd gear. Therefore, 1/a(V) must be visualized for the 1st, 2nd and 3rd gear ratios. The related digram for single-passenger and full-passenger is visualized in Matlab as shown in Figures 4.1 and 4.2. Note that the velocity must be in m/s not km/h so the time would be in seconds.

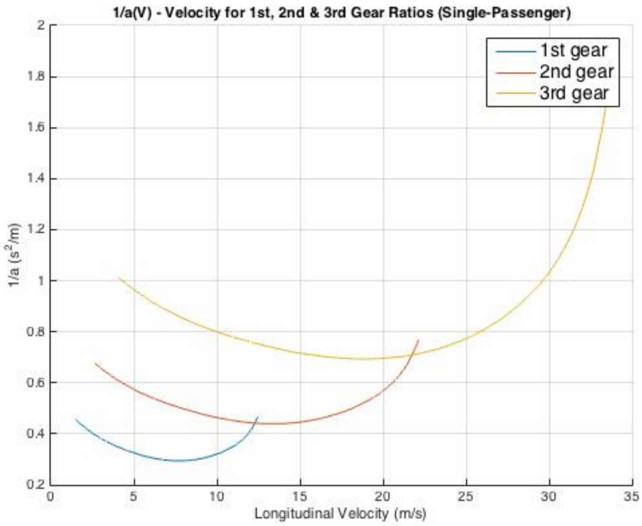

Figure 4.1. 1/a per velocity for single-passenger

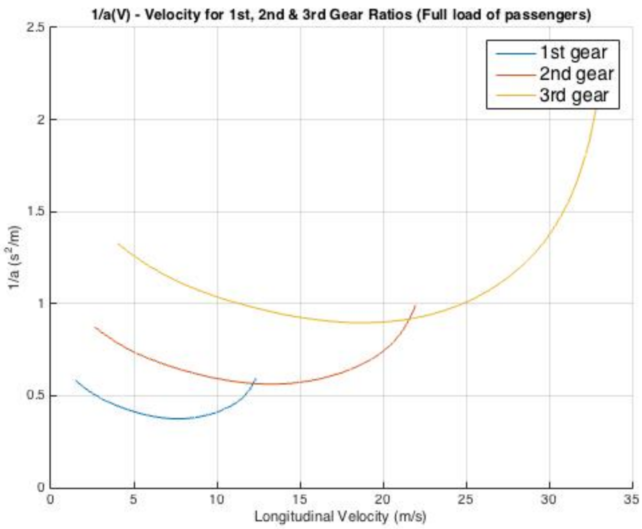

Figure 4.2. 1/a per velocity with full load of passenger

The integral's value is equal to the area under intersecting curves. It can be calculated in Matlab using trapz() function which is for numerical integration using trapezoidal rule. It estimates the integral(area under the curve) by dividing the data points into trapezoids.

The obtained values of Zero to 100(km/h) acceleration time are shown in Table 4.1. For more detail refer to the relevant Matlab file.

Table 4.1. Zero to 100(km/h) acceleration time

Passenger mode	Time(seconds)
Single-passenger	13.2
Full load of passengers	16.92

5. Maximum incline at velocity of 5(km/h)

The maximum incline the vehicle can handle at 5 km/h with a full load of passengers can be obtained from Equation 2.1. Since the maximum value of θ is desired, the longitudinal acceleration is 0. On the other hand rolling resistance is determined from Equation 5.1.

$R_x = \mu Mg \cos\theta \ (Equation\ 5.1)$

Therefore by applying it Equation 2.1 is modified as follows.

$$(\frac{T_e N_{tf} \eta_{tf}}{r} - D_A)/Mg = \mu \cos\theta + \sin\theta \ (Equation\ 5.2)$$

By having the velocity, the rotational speed or angular velocity of the engine is determined. Then, the corresponding engine torque is obtained from the fitted 5th degree polynomial in chapter 1 and Figure 2.2. Here are the calculations for the first gear ratio in-which the maximum incline is more likely to happen.

$$V_x = 5(km/h) \Rightarrow \omega_{engine} = \frac{V_x N_{tf}}{r} \Rightarrow \omega_{engine} = \frac{5(3.777)(3.454)}{(3.6)(0.2653)} \cdot \frac{60}{2\pi} = 652.19(rpm)$$

$$\Rightarrow T_e = 90.1(N.m) \Rightarrow \mu \cos\theta + \sin\theta = 0.2693 \ (Equation\ 5.3)$$

The answer to the above equation is where the trigonometric curve reaches the corresponding value. Figure 5.1 shows trigonometric curve of y = μ cos(θ) + sin(θ) in the range of 0 < θ < π/2 and for different corresponding values based on the gear ratio.

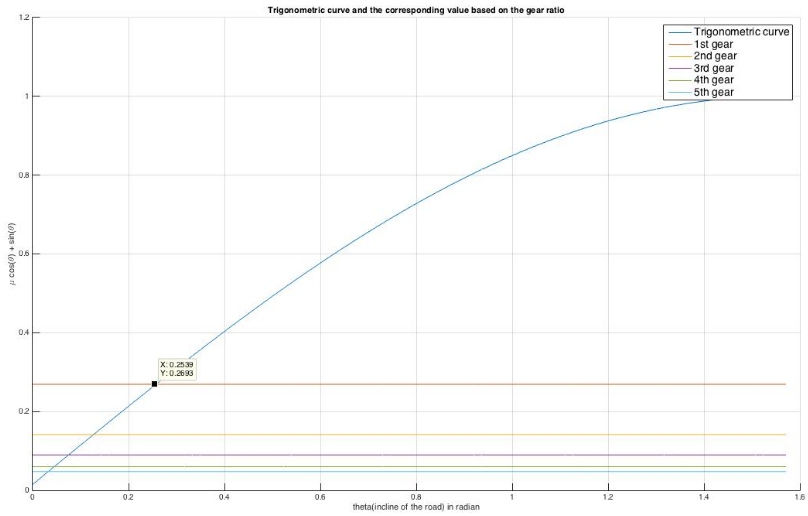

Figure 5.1. Trigonometric curve of y = μ cos(θ) + sin(θ) for different corresponding values based on the gear ratio

As predicted, the highest value of incline occurs in the first gear and is marked in figure 5.1. Therefore, here's the maximum value of incline.

$\theta = 0.27(rad) = 7.73^{\circ}$

6. The minimum fuel consumption at constant speed

For this chapter, both of the Figures 1.3 and 1.4 are required. Discrete points of these plots are extracted in the same way as described in the first chapter. Then, fuel consumption in Liter per hour can be defined as a function of rotational speed or longitudinal velocity. Equation 6.1 shows the fuel consumption function.

$$Fuel\ Consumption = \frac{SFC\,.\,P}{\rho}\ (Equation\ 6.1)$$

where:

SFC [g/Ps.h] = Specific fuel consumption that is a function of rotational speed based on Figure 1.4.

P [Ps] = Power that is a function of rotational speed based on Figure 1.3.

ρ [g/L] = Density of the fuel (for gasoline is 742.9 g/L)

To visualize fuel consumption per engine's speed plot, power and SFC curves must be estimated using curve fitting in Matlab. Figure 6.1 shows fitted curves for power and SFC.

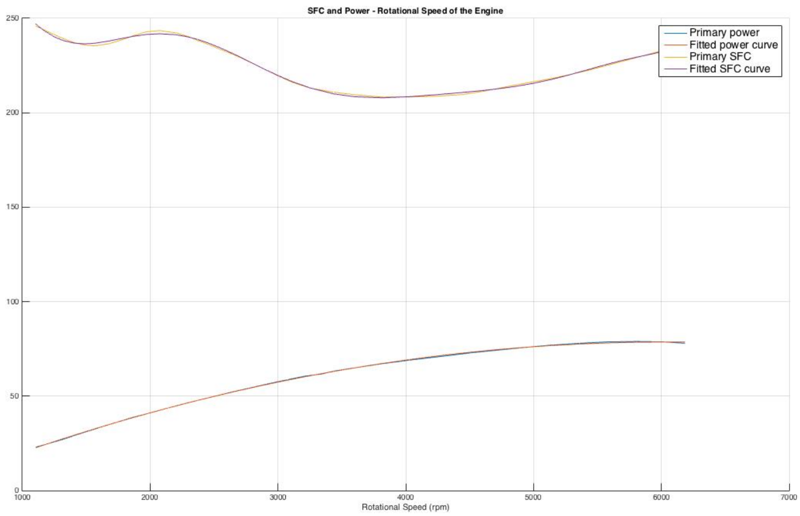

Figure 6.1. Power and SFC curves per rotational speed of the engine in Matlab

Having a functional rule governing the curves and also Equation 6.1, and Equation 2.4 lead to fuel consumption curve per longitudinal velocity. The points located on this curve are obtained in Matlab and finally it is plotted for 4th and 5th gear as shown in Figure 6.2 (assuming full load of passengers).

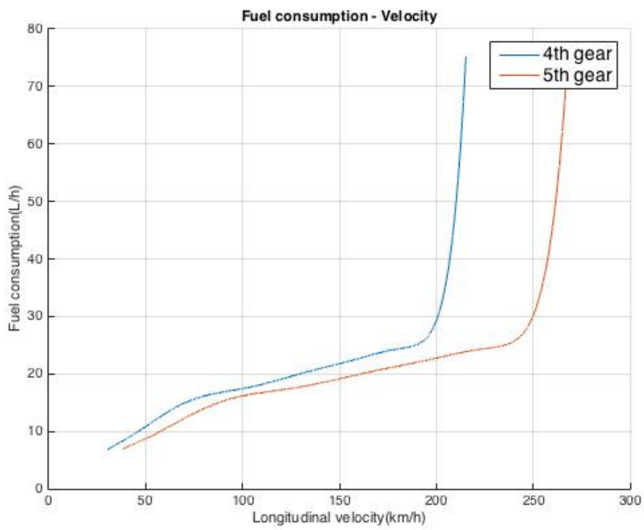

Figure 6.2. Fuel consumption per engine's rotational speed

The slope of the curve is fuel consumption in L/km. Therefore the minimum fuel consumption happens at minimum slope. Using numerical differential analysis in Matlab, Table 6.1 is completed as follows.

Table 6.1. Minimum fuel consumption and corresponding velocity

Gear	**Minimum fuel consumption(L/km)**	**Minimum fuel consumption(L per 100km)**	**Velocity(km/h)**
4th	0.057	5.7	94
5th	0.046	4.6	116

The table shows that the amount of minimum fuel consumption is 4.6 liters per 100 km at the velocity of 116 km/h in the 5th gear. Note that the engine rotates at about 3000 rpm in this condition.

Conclusion

References