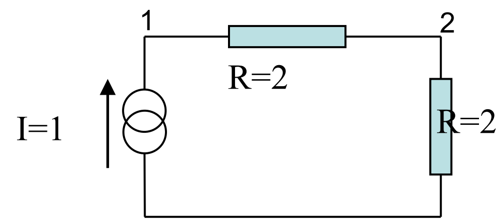
# IR Drop: Solving Resistive Networks

- Can you solve the circuit at right?



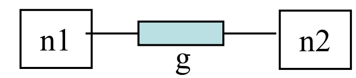

- A more systematic way:
  - Write circuit equation as $G\,\mathbf{V} = \mathbf{J}$
    - $G$ is an nxn conductance matrix
    - **V** is an nx1 vector of node voltages
    - **J** is an nx1 vector of excitations
  - Need to choose a ground node
  - Can build the equations by inspection

# Nodal Analysis

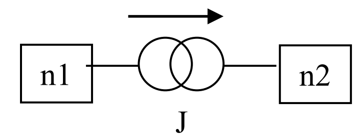
- "Stamps"
  - Conductance [1/resistor]: LHS only
  - Current source: RHS only

  $G\,\mathbf{V} = \mathbf{J}$

$$\begin{array}{c} n1 \\ \\ n2 \\ \\ \end{array} \begin{bmatrix} .. & +g & .. & -g & .. \\ .. & .. & .. & .. & .. \\ .. & -g & .. & +g & .. \\ .. & .. & .. & .. & .. \end{bmatrix}$$




$$\begin{array}{c} n1 \\ \\ n2 \\ \\ \end{array} \begin{bmatrix} .. & .. & .. & .. \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ .. & .. & .. & .. \end{bmatrix} \begin{bmatrix} : \\ : \\ : \\ : \end{bmatrix} = \begin{bmatrix} -\mathrm{J} \\ \\ +\mathrm{J} \\ \\ \end{bmatrix}$$



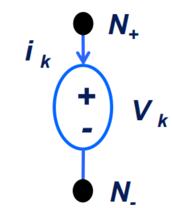

- For every such branch in the circuit, superpose the contributions
  - (Can also build stamps for other elements, extend to general circuits – basically, that's how SPICE works)

# Modified Nodal Analysis

- "Stamps"
  - Voltage source: Both sides additional rows and columns
  - $G\,\mathbf{V} = \mathbf{J}$

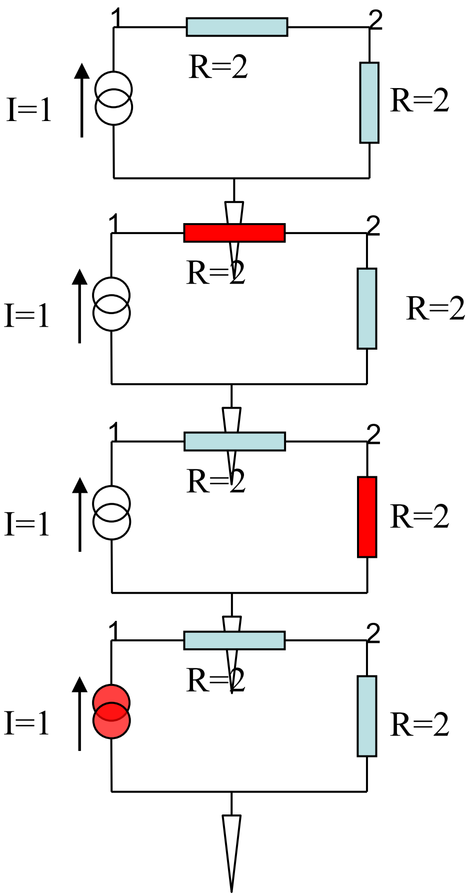
current introduced!

$$\begin{array}{c|c|c|c|} & N+ & N- & i_k \\ \hline N+ & 0 & 0 & 1 \\ \hline N- & 0 & 0 & -1 \\ \hline \text{branch } k & 1 & -1 & 0 \\ \hline \end{array} \qquad \text{RHS} \begin{bmatrix} 0 \\ 0 \\ V_k \end{bmatrix}$$




- For every such branch in the circuit, superpose the contributions
  - (Can also build stamps for other elements, extend to general circuits – basically, that's how SPICE works)

# IR Drop: Back to the circuit



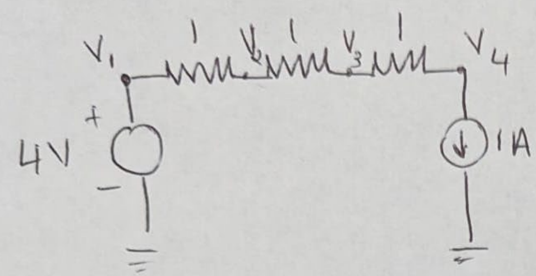

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1.0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & -0.5 \\ -0.5 & 1.0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\Rightarrow V_1 = 4 \;\; V_2 = 2$$



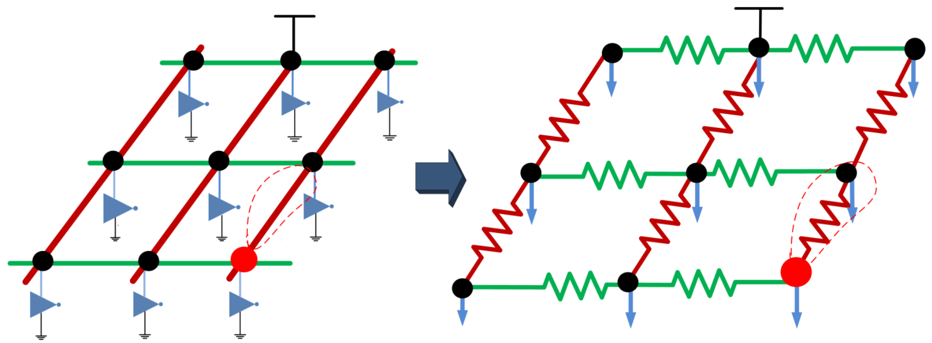

$GV = J$

Stamp in all element stamps into $GV = J$ system of equations.

$$
\begin{array}{c}
\\ n_1 \\ n_2 \\ n_3 \\ n_4 \\ b
\end{array}
\begin{array}{c}
\begin{array}{ccccc} n_1 & n_2 & n_3 & n_4 & b \end{array} \\
\left[\begin{array}{cccc|c}
+1 & -1 & 0 & 0 & +1 \\
-1 & +1+1 & -1 & 0 & 0 \\
0 & -1 & +1+1 & -1 & 0 \\
0 & 0 & -1 & +1 & 0 \\
\hline
+1 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ i_b \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ +4 \end{bmatrix}
$$

# So now you can solve any resistive network (or thermal network)

- Network of resistors and current sources, some nodes at a fixed voltage



- Power grid analysis and thermal analysis use the same core technology)