Final Exam

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1. The output of Minitab can be found in below tables.

a. $\hat{\beta} = (X'X)^{-1}X'y = \begin{bmatrix} 1.17991 & -7.30982E-3 & 7.3006E-4 \\ -7.30982E-3 & 7.9799E-5 & -1.23713E-4 \\ 7.3006E-4 & 1.23713E-4 & 4.6576E-4 \end{bmatrix} \begin{bmatrix} 220 \\ 36768 \\ 9965 \end{bmatrix} = \begin{bmatrix} -1.91221 \\ 0.09309 \\ 9.35059 \end{bmatrix} \Rightarrow$

$y = -1.91221 + 0.09309x_1 + 9.35059x_2$

b. $y = -1.91221 + 0.09309 \times 200 + 9.35059 \times 50 = 484.23529$

2. The output of Minitab can be found in below tables.

Regression Equation

a y = -7.63 + 0.003976 x2 + 0.2478 x7 - 0.00389 x8

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-7.63	c 7.85	-0.97	0.340	
x2	0.003976	0.000714	5.57	0.000	1.06
x7	0.2478	0.0890	2.79	0.010	1.97
x8	-0.00389	0.00130	-3.00	0.006	1.90

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.79711	g 76.29%	73.33%	69.30%

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	249.45	83.151	25.75	e 0.000
x2	1	100.28	100.282	31.05	f 0.000
x7	1	25.05	25.054	7.76	0.010
x8	1	29.16	29.158	9.03	0.006
Error	24	77.51	b 3.230		
Total	27	326.96			

Fits and Diagnostics for Unusual Observations

Obs	y	Fit	Resid	Std Resid	
1	10.000	6.466	3.534	2.02	R

Settings

Variable	Setting
x2	2000
x7	60
x8	1800

Prediction

Fit	SE Fit	95% CI	95% PI
d 8.17684	0.497392	(7.15027, 9.20341)	(4.32834, 12.0253)

a. $y = -7.63 + 0.003976x_2 + 0.2478x_7 - 0.00389x_8$

b. $\sigma^2 = \frac{SSE}{n-p} = \frac{77.51}{28-4} = 3.230$

c. $SE(\beta_0) = 7.85 \quad SE(\beta_2) = 0.000714 \quad SE(\beta_7) = 0.0890 \quad SE(\beta_8) = 0.00130$

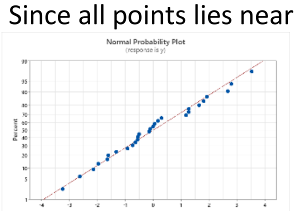
d. $y = -7.63 + 0.003976 \times 2000 + 0.2478 \times 60 - 0.00389 \times 1800 = 8.17684$

e. $\begin{cases} H_0: \beta_0 = \beta_2 = \beta_7 = \beta_8 = 0 \\ H_1: \exists i \ \beta_i \neq 0 \end{cases} \Rightarrow F_{Value} = 25.75 \Rightarrow P_{Value} = 0.000 < 0.05 \Rightarrow$ Model is significant.

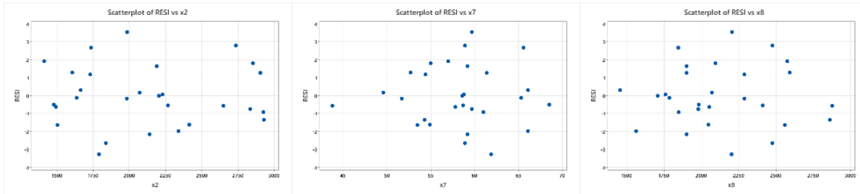
f. $P_{Value}(\beta_2) = 0.000 \quad P_{Value}(\beta_7) = 0.010 \quad P_{Value}(\beta_8) = 0.006 < 0.05 \Rightarrow$ All variables are significant.

g. $R^2 = 76.29\% \quad R^2_{adj} = 73.33\%$

h. Since all points lies near the line visually, we conclude that normality of residuals assumption is valid.



i. Since the points in a residual plot are randomly dispersed around the horizontal axis for all variables, a linear regression model is appropriate for the data and all variances are equal.



j. According to cook distance values, the points which are greater than $\frac{4}{n-k-1} = \frac{4}{28-3-1} = 0.166$ are influential point in this data. Thus, the 8th observation is influential point.

Observation	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
COOK	0.05	0.04	0.08	0.03	0.00	0.02	0.06	0.17	0.07	0.08	0.01	0.01	0.00	0.00	0.02	0.01	0.00	0.10	0.00	0.00	0.09	0.03	0.00	0.01	0.02	0.00	0.02	0.04

3. The output of Minitab can be found in below tables.

Regression Equation

a y = 7.46 - 0.030 x2 + 0.521 x3 - 0.1018 x4 - 2.16 x5

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	7.46	7.23	1.03	0.320	
x2	-0.030	0.263	-0.11	0.912	1.44
x3	0.521	0.136	3.83	0.002	1.07
x4	-0.1018	0.0534	-1.91	0.077	1.41
x5	-2.16	2.39	-0.90	0.382	1.36

c

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.882737	67.16%	57.78%	47.30%

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	4	22.3119	5.5780	7.16	0.002
x2	1	0.0099	0.0099	0.01	0.912
x3	1	11.4305	11.4305	14.67	0.002
x4	1	2.8328	2.8328	3.64	0.077
x5	1	0.6343	0.6343	0.81	0.382
Error	14	10.9091	b 0.7792		
Total	18	33.2211			

Fits and Diagnostics for Unusual Observations

Obs	y	Fit	Resid	Std Resid
2	8.300	10.065	-1.765	-2.07 R

Settings

Variable	Setting
x2	20
x3	30
x4	90
x5	2

Prediction

Fit	SE Fit	95% CI	95% PI
d 8.99568	0.472445	(7.98238, 10.0090)	(6.84829, 11.1431)

a. $y = 7.46 - 0.030x_2 + 0.521x_3 - 0.1018x_4 - 2.16x_5$

b. $\sigma^2 = \frac{SSE}{n-p} = \frac{10.9091}{19-5} = 0.7792$

c. $SE(\beta_0) = 7.23 \quad SE(\beta_2) = 0.263 \quad SE(\beta_3) = 0.136 \quad SE(\beta_4) = 0.0534 \quad SE(\beta_5) = 2.39$
Only the coefficient related to x_3 estimated precisely because its $P_{Value} < 0.05$.

d. $y = 7.46 - 0.030 \times 20 + 0.521 \times 30 - 0.1018 \times 90 - 2.16 \times 2 = 8.99568$

4. The output of Minitab can be found in below tables.

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values

Method	4	1, 2, 3, 4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Method	3	489740	163247	a 12.73	b 0.000
Error	12	153908	12826		
Total	15	643648			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
113.251	76.09%	70.11%	57.49%

Means

Method	N	Mean	StDev	95% CI
1	4	2971.0	120.6	(2847.6, 3094.4)
2	4	3156.3	136.0	(3032.9, 3279.6)
3	4	2933.8	108.3	(2810.4, 3057.1)
4	4	2666.3	81.0	(2542.9, 2789.6)

Pooled StDev = 113.251

Fisher Pairwise Comparisons

Grouping Information Using the Fisher LSD Method and 95% Confidence

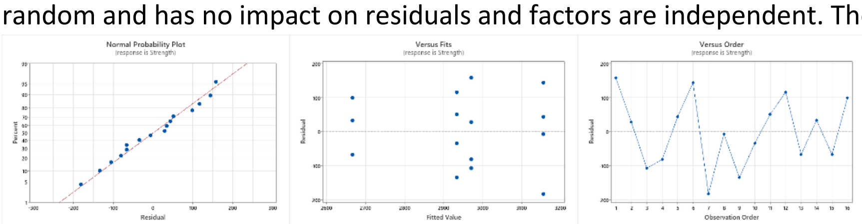
Method	N	Mean	Grouping		
2	4	3156.3	A		
d 1	4	2971.0		B	
3	4	2933.8		B	
4	4	2666.3			C

Means that do not share a letter are significantly different.

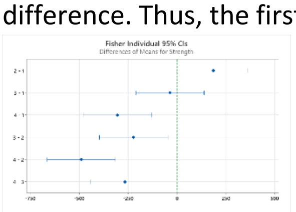
a. $\begin{cases} H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \\ H_1: \exists i,j \;\; \mu_i \neq \mu_j \end{cases} \Longrightarrow F_{Value} = 12.73 \Longrightarrow P_{Value} = 0.000 < 0.05 \Longrightarrow$ Mixing method has effect.

b. $F_{Value} = 12.73 \Longrightarrow P_{Value} = 0.000$

c. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.



d. It is also indicated that among the pairwise comparisons, the intervals that do not include zero have a significant difference. Thus, the first and third methods do not show a significant difference, but the others do.



5. The output of Minitab can be found in below tables.

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.05

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Type	4	EC1, EC10, EC10A, EC4

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Type	3	0.2402	0.08007	4.96	a $\boxed{0.020}$
Error	11	0.1775	0.01613		
Total	14	0.4177			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.127023	57.51%	45.92%	33.27%

Means

Type	N	Mean	StDev	95% CI
EC1	2	d 0.1300	0.0424	(-0.0677, 0.3277)
EC10	4	0.4900	0.0898	(0.3502, 0.6298)
EC10A	6	0.2617	0.1665	(0.1475, 0.3758)
EC4	3	0.2000	0.0800	(0.0386, 0.3614)

Pooled StDev = 0.127023

Fisher Pairwise Comparisons

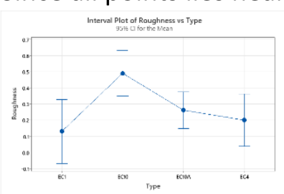
Grouping Information Using the Fisher LSD Method and 95% Confidence

Type	N	Mean	Grouping	
EC10	4	0.4900	A	
EC10A	6	0.2617		B
EC4	3	0.2000		B
EC1	2	0.1300		B

Means that do not share a letter are significantly different.

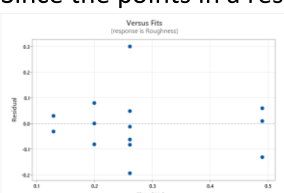
a. $\begin{cases} H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \\ H_1: \exists i,j \;\; \mu_i \neq \mu_j \end{cases} \Rightarrow F_{Value} = 4.96 \Rightarrow P_{Value} = 0.02 < 0.05 \Rightarrow$ Carbon material type has effect.

b. Since all points lies near the line visually, it can be concluded that normality of residuals assumption is valid.



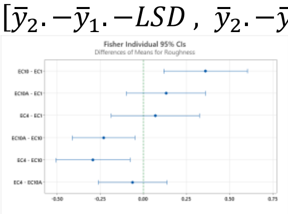
Type	EC10				EC10A						EC4			EC1	
Roughness	0.50	0.55	0.55	0.36	0.31	0.07	0.25	0.18	0.56	0.20	0.20	0.28	0.12	0.10	0.16
RESI	0.01	0.06	0.06	-0.13	0.05	-0.19	-0.01	-0.08	0.30	-0.06	0.00	0.08	-0.08	-0.03	0.03

c. Since the points in a residual plot are randomly dispersed around the horizontal axis, all variances are equal.



Type	EC10				EC10A						EC4			EC1	
Roughness	0.50	0.55	0.55	0.36	0.31	0.07	0.25	0.18	0.56	0.20	0.20	0.28	0.12	0.10	0.16
FITS	0.49	0.49	0.49	0.49	0.26	0.26	0.26	0.26	0.26	0.26	0.20	0.20	0.20	0.13	0.13

d. $[\bar{y}_2. - \bar{y}_1. - LSD\,,\; \bar{y}_2. - \bar{y}_1. + LSD] = [0.49 - 0.13 - 0.24\,, 0.49 - 0.13 + 0.24] = [0.12\,, 0.60]$



6. The output of Minitab can be found in below tables.

Method

Null hypothesis	All means are equal
Alternative hypothesis	Not all means are equal
Significance level	α = 0.01

Equal variances were assumed for the analysis.

Factor Information

Factor	Levels	Values
Temperature	3	70, 75, 80

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Temperature	2	16480	8240	a 7.84	b 0.007
Error	12	12610	1051		
Total	14	29090			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
32.4166	56.65%	49.43%	32.27%

Means

Temperature	N	Mean	StDev	99% CI
70	5	1249.00	20.74	(1204.72, 1293.28)
75	5	1221.00	16.73	(1176.72, 1265.28)
80	5	1169.0	49.4	(1124.7, 1213.3)

Pooled StDev = 32.4166

Fisher Pairwise Comparisons

Grouping Information Using the Fisher LSD Method and 95% Confidence

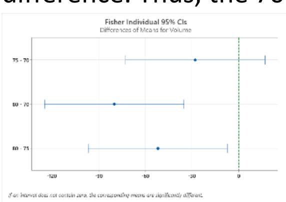
Temperature	N	Mean	Grouping	
c 70	5	1249.00	A	
75	5	1221.00	A	
80	5	1169.0		B

Means that do not share a letter are significantly different.

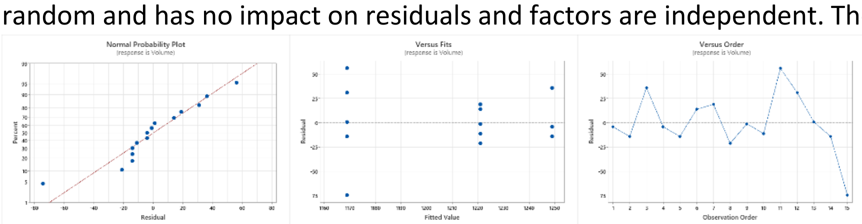
a. $\begin{cases} H_0: \mu_1 = \mu_2 = \mu_3 \\ H_1: \exists i,j \ \ \mu_i \neq \mu_j \end{cases} \Longrightarrow F_{Value} = 7.84 \Longrightarrow P_{Value} = 0.007 < 0.01 \Longrightarrow$ Temperature has effect.

b. $F_{Value} = 7.84 \Longrightarrow P_{Value} = 0.000$

c. It is also indicated that among the pairwise comparisons, the intervals that do not include zero have a significant difference. Thus, the 70°C and 75°C do not show a significant difference, but the others do.



d. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.



7. The output of Minitab can be found in below tables.

Factor Information

Factor	Levels	Values
Glass Type	2	1, 2
Phosphor Type	3	1, 2, 3

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	15516.7	3103.3	a 58.80	0.000
Linear	3	15383.3	5127.8	97.16	0.000
Glass Type	1	14450.0	14450.0	273.79	0.000
Phosphor Type	2	933.3	466.7	8.84	0.004
2-Way Interactions	2	133.3	66.7	1.26	0.318
Glass Type*Phosphor Type	2	133.3	66.7	1.26	0.318
Error	12	633.3	52.8		
Total	17	16150.0			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
7.26483	96.08%	94.44%	91.18%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	263.33	1.71	153.79	0.000	
Glass Type					
1	28.33	1.71	16.55	0.000	1.00
Phosphor Type					
1	-3.33	2.42	-1.38	0.194	1.33
2	10.00	2.42	4.13	0.001	1.33
Glass Type*Phosphor Type					
1 1	-3.33	2.42	-1.38	0.194	1.33
1 2	-0.00	2.42	-0.00	1.000	1.33

Regression Equation

C7 = 263.33 + 28.33 Glass Type_1 - 28.33 Glass Type_2 - 3.33 Phosphor Type_1 + 10.00 Phosphor Type_2 - 6.67 Phosphor Type_3
- 3.33 Glass Type*Phosphor Type_1 1 - 0.00 Glass Type*Phosphor Type_1 2 + 3.33 Glass Type*Phosphor Type_1 3
+ 3.33 Glass Type*Phosphor Type_2 1 + 0.00 Glass Type*Phosphor Type_2 2 - 3.33 Glass Type*Phosphor Type_2 3

Fits and Diagnostics for Unusual Observations

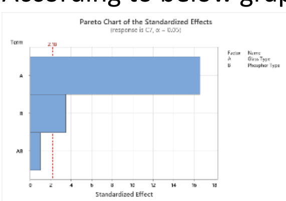
Obs	C7	Fit	Resid	Std Resid	
5	260.00	245.00	15.00	2.53	R

a. $\begin{cases} H_0: \forall i \ \tau_i = 0 \\ H_1: \exists i \ \tau_i \neq 0 \end{cases} \Longrightarrow F_{Value} = 1.90 \Longrightarrow P_{Value} = 0.000 < 0.05 \Longrightarrow$ The effect of glass type is significant.

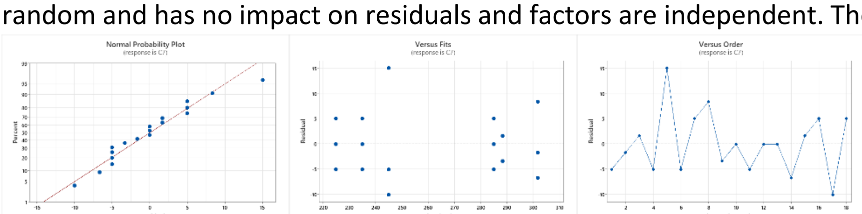
$\begin{cases} H_0: \forall j \ \beta_j = 0 \\ H_1: \exists j \ \beta_j \neq 0 \end{cases} \Longrightarrow F_{Value} = 0.07 \Longrightarrow P_{Value} = 0.004 < 0.05 \Longrightarrow$ The effect of phosphor type is significant.

$\begin{cases} H_0: \forall i,j \ (\tau\beta)_{ij} = 0 \\ H_1: \exists i,j \ (\tau\beta)_{ij} \neq 0 \end{cases} \Longrightarrow F_{Value} = 5.03 \Longrightarrow P_{Value} = 0.318 > 0.05 \Longrightarrow$ The effect of interaction is insignificant.

b. According to below graph, glass type and phosphor type are significant separately, but their interaction is not.



c. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.



8. The output of Minitab can be found in below tables.

Factor Information

Factor	Levels	Values
Paint	2	1, 2
Drying Time	3	20, 25, 30

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	5	2261.78	452.36	a 2.42	0.097
Linear	3	383.00	127.67	0.68	0.579
Paint	1	355.56	355.56	1.90	0.193
Drying Time	2	27.44	13.72	0.07	0.930
2-Way Interactions	2	1878.78	939.39	5.03	0.026
Paint*Drying Time	2	1878.78	939.39	5.03	0.026
Error	12	2242.67	186.89		
Total	17	4504.44			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
13.6707	50.21%	29.47%	0.00%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	73.44	3.22	22.79	0.000	
Paint					
1	-4.44	3.22	-1.38	0.193	1.00
Drying Time					
20	-1.11	4.56	-0.24	0.811	1.33
25	-0.61	4.56	-0.13	0.896	1.33
Paint*Drying Time					
1 20	-5.22	4.56	-1.15	0.274	1.33
1 25	-9.06	4.56	-1.99	0.070	1.33

Regression Equation

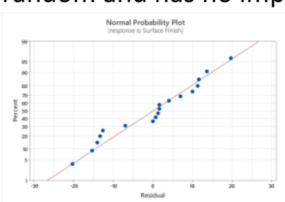
Surface Finish = 73.44 - 4.44 Paint_1 + 4.44 Paint_2 - 1.11 Drying Time_20
- 0.61 Drying Time_25 + 1.72 Drying Time_30 - 5.22 Paint*Drying Time_1 20
- 9.06 Paint*Drying Time_1 25 + 14.28 Paint*Drying Time_1 30
+ 5.22 Paint*Drying Time_2 20 + 9.06 Paint*Drying Time_2 25
- 14.28 Paint*Drying Time_2 30

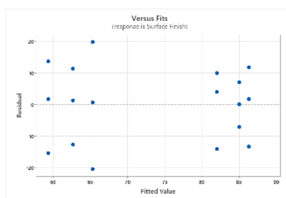
a. $\begin{cases} H_0: \forall i \ \tau_i = 0 \\ H_1: \exists i \ \tau_i \neq 0 \end{cases} \Longrightarrow F_{Value} = 1.90 \Longrightarrow P_{Value} = 0.193 > 0.05 \Longrightarrow$ The effect of paint is insignificant.

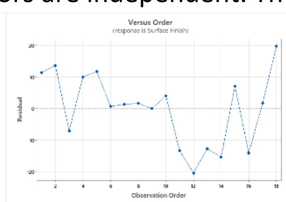
$\begin{cases} H_0: \forall j \ \beta_j = 0 \\ H_1: \exists j \ \beta_j \neq 0 \end{cases} \Longrightarrow F_{Value} = 0.07 \Longrightarrow P_{Value} = 0.930 > 0.05 \Longrightarrow$ The effect of drying time is insignificant.

$\begin{cases} H_0: \forall i,j \ (\tau\beta)_{ij} = 0 \\ H_1: \exists i,j \ (\tau\beta)_{ij} \neq 0 \end{cases} \Longrightarrow F_{Value} = 5.03 \Longrightarrow P_{Value} = 0.026 < 0.05 \Longrightarrow$ The effect of interaction is significant.

b. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.







9. The output of Minitab can be found in below tables.

Factor Information

Factor	Levels	Values
Concentration	3	10, 15, 20
Cooking Time	2	1.5, 2.0
Freeness	3	350, 500, 650

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	17	60.440	3.5553	a 6.41	b 0.000
Linear	5	47.588	9.5176	17.17	0.000
Concentration	2	8.375	4.1875	7.55	0.004
Cooking Time	1	17.361	17.3611	31.31	0.000
Freeness	2	21.852	10.9258	19.71	0.000
2-Way Interactions	8	10.768	1.3460	2.43	0.056
Concentration*Cooking Time	2	3.204	1.6019	2.89	0.082
Concentration*Freeness	4	6.513	1.6283	2.94	0.050
Cooking Time*Freeness	2	1.051	0.5253	0.95	0.406
3-Way Interactions	4	2.084	0.5211	0.94	0.463
Concentration*Cooking Time*Freeness	4	2.084	0.5211	0.94	0.463
Error	18	9.980	0.5544		
Total	35	70.420			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.744610	85.83%	72.44%	43.31%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	98.000	0.124	789.67	0.000	
Concentration					
10	0.667	0.176	3.80	0.001	1.33
15	-0.208	0.176	-1.19	0.251	1.33
Cooking Time					
1.5	-0.694	0.124	-5.60	0.000	1.00

Freeness					
350	-0.417	0.176	-2.37	0.029	1.33
500	-0.675	0.176	-3.85	0.001	1.33
Concentration*Cooking Time					
10 1.5	-0.389	0.176	-2.22	0.040	1.33
15 1.5	0.336	0.176	1.92	0.072	1.33
Concentration*Freeness					
10 350	-0.850	0.248	-3.42	0.003	1.78
10 500	0.433	0.248	1.75	0.098	1.78
15 350	0.450	0.248	1.81	0.087	1.78
15 500	-0.217	0.248	-0.87	0.394	1.78
Cooking Time*Freeness					
1.5 350	0.178	0.176	1.01	0.325	1.33
1.5 500	-0.231	0.176	-1.31	0.205	1.33
Concentration*Cooking Time*Freeness					
10 1.5 350	-0.194	0.248	-0.78	0.444	1.78
10 1.5 500	-0.261	0.248	-1.05	0.307	1.78
15 1.5 350	0.206	0.248	0.83	0.418	1.78
15 1.5 500	0.139	0.248	0.56	0.583	1.78

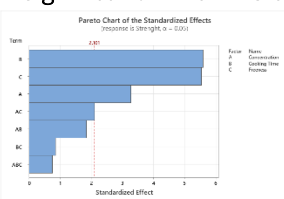
Regression Equation

It has been deleted because of large size of equation. You can find it in attached Minitab file.

Fits and Diagnostics for Unusual Observations

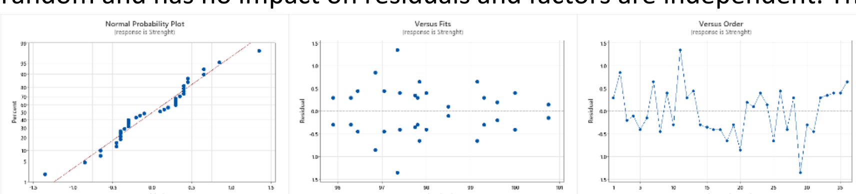
Obs	Strenght	Fit	Resid	Std Resid	
11	98.700	97.350	1.350	2.56	R
29	96.000	97.350	-1.350	-2.56	R

a. According to below graph, the Concentration*Cooking Time, Cooking Time*Freeness, Concentration*Cooking Time*Freeness factors are insignificant which we can exclude them from model, but the other factors are significant.



b. The values of P_{Value} are determined in last page in analysis of variance section. The values which are less than 5% show that the related factor is significant.

c. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.



10. The output of Minitab can be found in below tables.

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		413.1	12.4	33.30	0.000	
A	18.2	9.1	12.4	0.74	0.483	1.00
B	84.3	42.1	12.4	3.40	0.009	1.00
C	71.8	35.9	12.4	2.89	0.020	1.00
A*B	-11.3	-5.6	12.4	-0.45	0.662	1.00
A*C	-119.3	-59.6	12.4	-4.81	0.001	1.00
B*C	-24.3	-12.1	12.4	-0.98	0.357	1.00
A*B*C	-34.7	-17.4	12.4	-1.40	0.199	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
49.6236	85.36%	72.56%	41.45%

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	114888	16412.5	a 6.66	0.008
Linear	3	50317	16772.3	6.81	0.014
A	1	1332	1332.3	0.54	0.483

B	1	28392	28392.3	11.53	0.009
C	1	20592	20592.3	8.36	0.020
2-Way Interactions	3	59741	19913.6	8.09	0.008
A*B	1	506	506.3	0.21	0.662
A*C	1	56882	56882.2	23.10	0.001
B*C	1	2352	2352.3	0.96	0.357
3-Way Interactions	1	4830	4830.2	1.96	0.199
A*B*C	1	4830	4830.2	1.96	0.199
Error	8	19700	2462.5		
Total	15	134588			

Regression Equation in Uncoded Units

Life = 413.1 + 9.1 A + 42.1 B + 35.9 C - 5.6 A*B - 59.6 A*C - 12.1 B*C - 17.4 A*B*C

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		413.1	12.2	33.78	0.000	
B	84.3	42.1	12.2	3.44	0.005	1.00
C	71.7	35.9	12.2	2.93	0.013	1.00
A*C	-119.2	-59.6	12.2	-4.88	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
48.9226	78.66%	73.33%	62.06%

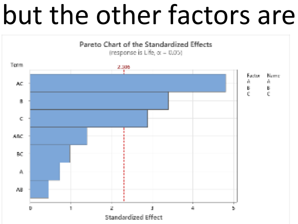
Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	3	105867	35289	14.74	0.000
Linear	2	48985	24492	10.23	0.003
B	1	28392	28392	11.86	0.005
C	1	20592	20592	8.60	0.013
2-Way Interactions	1	56882	56882	23.77	0.000
A*C	1	56882	56882	23.77	0.000
Error	12	28721	2393		
Lack-of-Fit	4	9021	2255	0.92	0.499
Pure Error	8	19700	2463		
Total	15	134588			

Regression Equation in Coded Units

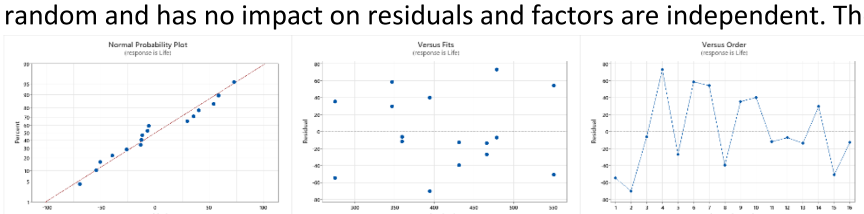
b Life = 413.1 + 42.1 B + 35.9 C - 59.6 A*C

a. According to below graph, the A, A*B, B*C, A*B*C factors are insignificant which we can exclude them from model, but the other factors are significant because their P_{Values} are less than 5%.



b. $y = 413.1 + 42.1B + 35.9C - 59.6AC$

c. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.



11. The output of Minitab can be found in below tables.

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		2888	*	*	*	
A	1462.1	731.1	*	*	*	1.00
B	3538	1769	*	*	*	1.00
C	-137.13	-68.56	*	*	*	1.00
D	474.6	237.3	*	*	*	1.00

a

E	425.4	212.7	*	*	*	1.00
A*B	1199.6	599.8	*	*	*	1.00
A*C	124.63	62.31	*	*	*	1.00
A*D	62.87	31.44	*	*	*	1.00
A*E	62.12	31.06	*	*	*	1.00
B*C	-99.63	-49.81	*	*	*	1.00
B*D	-12.875	-6.437	*	*	*	1.00
B*E	-12.125	-6.062	*	*	*	1.00
C*D	112.12	56.06	*	*	*	1.00
C*E	-62.12	-31.06	*	*	*	1.00
D*E	224.6	112.3	*	*	*	1.00
A*B*C	-62.88	-31.44	*	*	*	1.00
A*B*D	200.4	100.2	*	*	*	1.00
A*B*E	49.63	24.81	*	*	*	1.00
A*C*D	75.38	37.69	*	*	*	1.00
A*C*E	99.63	49.81	*	*	*	1.00
A*D*E	-87.12	-43.56	*	*	*	1.00
B*C*D	99.62	49.81	*	*	*	1.00
B*C*E	-74.62	-37.31	*	*	*	1.00
B*D*E	-62.88	-31.44	*	*	*	1.00
C*D*E	37.12	18.56	*	*	*	1.00
A*B*C*D	-12.125	-6.063	*	*	*	1.00
A*B*C*E	12.125	6.062	*	*	*	1.00
A*B*D*E	0.3750	0.1875	*	*	*	1.00
A*C*D*E	150.38	75.19	*	*	*	1.00
B*C*D*E	-25.38	-12.69	*	*	*	1.00
A*B*C*D*E	62.87	31.44	*	*	*	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
*	100.00%	*	*

Analysis of Variance

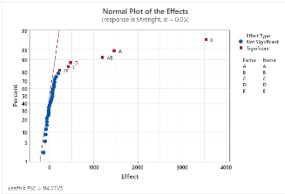
Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	31	133896385	4319238	*	*
Linear	5	120635081	24127016	*	*
A	1	17102476	17102476	*	*
B	1	100132476	100132476	*	*
C	1	150426	150426	*	*
D	1	1802151	1802151	*	*
E	1	1447551	1447551	*	*
2-Way Interactions	10	12316561	1231656	*	*
A*B	1	11512801	11512801	*	*
A*C	1	124251	124251	*	*
A*D	1	31626	31626	*	*
A*E	1	30876	30876	*	*
B*C	1	79401	79401	*	*
B*D	1	1326	1326	*	*
B*E	1	1176	1176	*	*
C*D	1	100576	100576	*	*
C*E	1	30876	30876	*	*
D*E	1	403651	403651	*	*
3-Way Interactions	10	724711	72471	*	*
A*B*C	1	31626	31626	*	*
A*B*D	1	321201	321201	*	*
A*B*E	1	19701	19701	*	*
A*C*D	1	45451	45451	*	*
A*C*E	1	79401	79401	*	*
A*D*E	1	60726	60726	*	*
B*C*D	1	79401	79401	*	*
B*C*E	1	44551	44551	*	*
B*D*E	1	31626	31626	*	*
C*D*E	1	11026	11026	*	*
4-Way Interactions	5	188406	37681	*	*
A*B*C*D	1	1176	1176	*	*
A*B*C*E	1	1176	1176	*	*
A*B*D*E	1	1	1	*	*
A*C*D*E	1	180901	180901	*	*
B*C*D*E	1	5151	5151	*	*
5-Way Interactions	1	31626	31626	*	*
A*B*C*D*E	1	31626	31626	*	*
Error	0	*	*		
Total	31	133896385			

Regression Equation in Uncoded Units

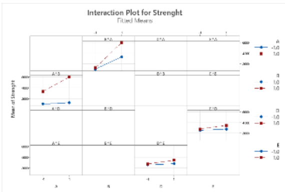
Strenght = 2888 + 731.1 A + 1769 B - 68.56 C + 237.3 D + 212.7 E + 599.8 A*B + 62.31 A*C
+ 31.44 A*D + 31.06 A*E - 49.81 B*C - 6.437 B*D - 6.062 B*E + 56.06 C*D
- 31.06 C*E + 112.3 D*E - 31.44 A*B*C + 100.2 A*B*D + 24.81 A*B*E + 37.69 A*C*D
+ 49.81 A*C*E - 43.56 A*D*E + 49.81 B*C*D - 37.31 B*C*E - 31.44 B*D*E
+ 18.56 C*D*E - 6.063 A*B*C*D + 6.062 A*B*C*E + 0.1875 A*B*D*E + 75.19 A*C*D*E
- 12.69 B*C*D*E + 31.44 A*B*C*D*E

a. The estimation of factor effects are determined in last page in coded coefficient section.

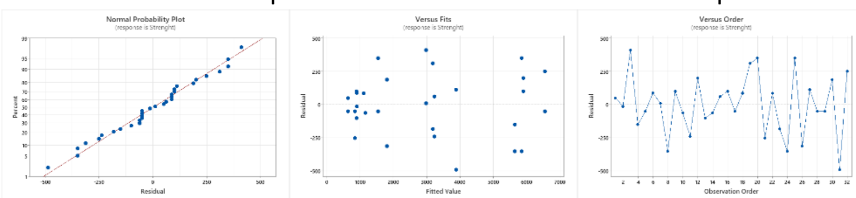
b. As you can see in below chart the red colored points which are far from the line are important factors with high effect. The A, B, A*B, D, E, D*E factor effects appear important and other factors are excluded from model.



c. Based on below chart, to maximize strength of concrete, the directions of all factors must be in upper direction.

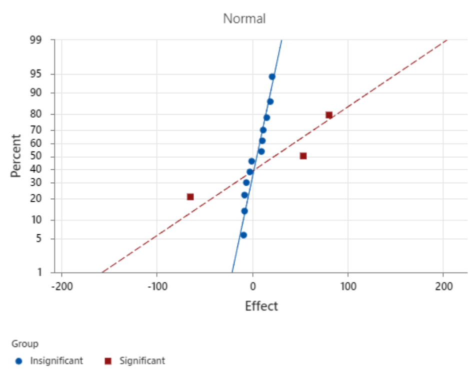


d. Since all points lies near the line visually in left graph, it can be concluded that normality of residuals assumption is valid for fitting linear regression and while the points in a residual plot are randomly dispersed around the horizontal axis in the middle graph, all variances are equal. The right graph shows that experiments order was random and has no impact on residuals and factors are independent. Therefore, all three assumptions are valid.

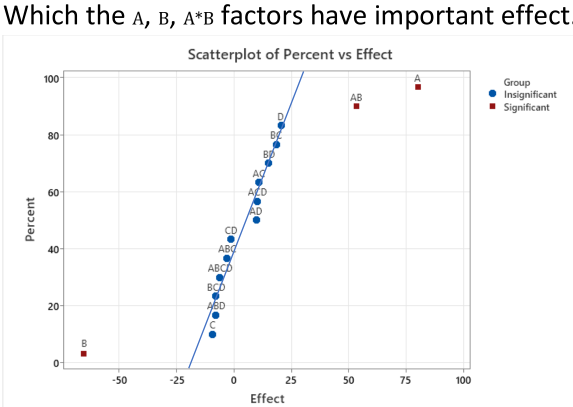


12. The output of Minitab can be found in below tables.

a. The Probability plot of data is as follow.



b. Which the A, B, A*B factors have important effect.



c. he coefficients of factors are half of their effect in regression model and intercept is mean of all data. So the regression equation will be $y = 40.125x_1 - 32.75x_2 + 26.625x_1x_2 + 400$