

Engineering Probability and Statistics

Computer Homework 1

Amirabbas Rezasoltani

November 2023

1. Let X be a random variable with probability density function.

$$f(x) = (\theta + 1)(1 - x)^\theta, \quad 0 \leq x \leq 1, \theta > 0$$

In python:

- (a) Find the cumulative density function and draw it (You can either first find equation and then draw it or just draw it using discrete point with small gap).
 - (b) What is $P(X \leq 0.25 | \theta = 2)$? (Use the data in part (a))
 - (c) Fix the seed at 80, and generate 100,000 realizations of X with $\theta = 2$. What are the mean and variance of the random sample?
 - (d) Calculate the **theoretical** mean and variance of X where $\theta = 2$.
 - (e) Compare **theoretical** and sampling results.
2. The time, in minutes, that a car is parked in a mall has the following density function:

$$f(x) = \begin{cases} \frac{1}{50} e^{-\frac{x}{50}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

- (a) Find the probability that a car stays more than 1 hour (using python is optional).
 - (b) Let $Y = 0.5 + 0.03X$ be the cost in dollars that the mall has to pay a security service per parked car. Find the mean parking cost for 1000 cars using python.
3. Using python, Construct a plot for the probability mass function and the cumulative probability distribution of a binomial random variable $Bin(n = 8, p = 0.3)$. Also using python, Find the smallest value of k such that $P(X \leq k) \geq 0.44$ when $X \sim Bin(n = 8, p = 0.7)$. Calculate $P(Y \geq 3)$ if $Y \sim Bin(20, 0.2)$.