Modeling of the Stability of Highly Inclined Boreholes in Anisotropic Rock Formations

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Summary. Simulators have been developed to study the fracture and collapse behavior of boreholes. To take into account the directional properties of real rocks, an anisotropic stress model is used. The model takes into account anisotropic elastic properties, directional shear, and directional tensile strengths. The orientation of the borehole, the in-situ stresses, and the bedding plane can all be arbitrarily related to each other to model actual field situations. This paper presents some of the results of the model. It is shown that neglecting the anisotropic effects introduces an error. Also, studying the anisotropic model yields further insight into the behavior of the borehole.

Introduction

The integrity of the borehole plays an important role in many well operations. During drilling, lost circulation and borehole collapse cause economic losses, and in production operations, controlled fracturing and sand control are of utmost importance for the economy of an oil field. Therefore, a better understanding of the rock mechanics is necessary to improve the economy of the well operations. In the past, accurate methods to predict critical fracturing or collapse pressures have been unavailable. Simple isotropic stress equations have been used to some extent, but these fail to take into account real rock properties that are clearly anisotropic. Sedimentary rocks have a laminated structure, with directional elastic properties as well as directional shear and tensile strengths. To understand field situations better, a complete mathematical model was developed that takes into account all directional properties.

Fig. 1 gives an overview of typical borehole problems. Shown are the only two cases covered in this paper—i.e., borehole collapse at low borehole pressures and fractures initiating along the borehole axis at high pressures. Our analysis is limited to wells >2,000 ft [>610 m] deep.[1] The model is based on linear elasticity, and neglects plastic or time-dependent effects. Fig. 1 shows the versatility of the simulators. The borehole, the in-situ stresses, and the rock-property reference frame may assume any orientation. This gives us a tool to model any field situation.

Mathematical Model

The core of the simulators is the mathematical model for stresses around boreholes in anisotropic materials. The complicated model is derived in detail by Aadnoy[2] and is based on a generalized plane-strain concept and linear elasticity. It results in two coupled partial-differential equations to determine the parameters of the stress functions. Aadnoy[2] applies the model with a transversely isotropic type of anisotropy. This is a good description for sedimentary rocks; all parameters of elasticity are equal in a horizontal plane, but differ vertically. The appendix briefly outlines the model. Table 1 is taken from Chenevert's[3] work and gives a summary of the elastic parameters as applied to different rock types. It is observed that the degree of anisotropy, K_{ani}, is lithology-dependent. Therefore, the sandstone can be classified as very anisotropic, the shales as moderately anisotropic, and the limestone as isotropic for this particular data set.

The two simulators, ANISFRAC for borehole fracture analysis and ANISCOLL for collapse analysis, also perform all transformations necessary to implement randomly oriented boreholes, in-situ stresses, and bedding planes, and thus are able to model any field situation.

The introduction of anisotropy vastly complicates the mathematics involved, but it is believed that it describes real field behavior better. From Chenevert's[3] work, a set of directional tensile-strength data for common sedimentary rocks is also obtained. Table 2 summarizes these data. It is observed that the tensile strength is 20 to 35% lower parallel to the bedding plane than perpendicular to it. This, of course, will affect the behavior of inclined boreholes. Finally, the simulators use shear data also obtained by Chenevert[3] for the same rock types. An extended Mohr-Coulomb criterion with a plane of weakness is introduced, allowing both the cohesive strength and the angle of internal friction to vary. The results are summarized in Table 3. Note that the limestone is isotropic in both its elastic properties (Table 1) and its shear properties (Table 3). Some of the resultant shear envelopes are plotted in Fig. 2. For $\beta=15°$ [0.26 rad], the material is weakest because it fails along the bedding plane. At 0 or 90° [0 or 1.6 rad], it fails across the bedding plane and is therefore strongest for these directions.

Applications in Borehole Fracturing

The simulator ANISFRAC will be used to study a few field cases. The failure criteria applied are that when the least effective principal stress exceeds the strength of the rock in tension, a tensile failure occurs. Two different tensile strengths will be used. One is the measured values given in Table 2 and the other is zero tensile strength, assuming that the rock contains cracks that are merely reopened.[1,6] One shale and one sandstone are studied. The shales are generally almost nonpermeable but porous and contain pore fluid at a given pressure. Therefore, these will be modeled with a pore pressure inside the porous rock with an instantaneous rise (a step function) to the borehole pressure outside the borehole wall.

The shales can also be abnormally pressurized.[4] Thus, the pore pressure is also a very important parameter in shales because the fracture gradient is strongly sensitive to the magnitude of the pore pressure. The sandstone is somewhat different from the shales because it is permeable. Two different situations will be studied. In the first one, a perfect mudcake is assumed, and we have an analogous situation for the shales. Inside the borehole is the borehole pressure, and immediately inside the rock wall is the pore pressure, the transition being a step function. The second situation assumes no mudcake; therefore, fluid communications between the formation and the borehole are allowed. This means that during fracturing operations, the pore pressure immediately inside the porous rock wall is equal to the borehole pressure. Stress contributions caused by the flowing fluid are neglected.

The results of the calculations for the Permian shale are shown in Fig. 3. The parameters used are typical for an intermediate deep well and are given in the figure. The bedding plane and the in-situ stresses are assumed to lie in the horizontal plane. The numerical values used in the following simulations are not chosen to simulate actual oil fields, but to study different parameters.

The hatched band defines the two limits of tensile strength. The upper curve uses data from Table 2, while the bottom curve assumes

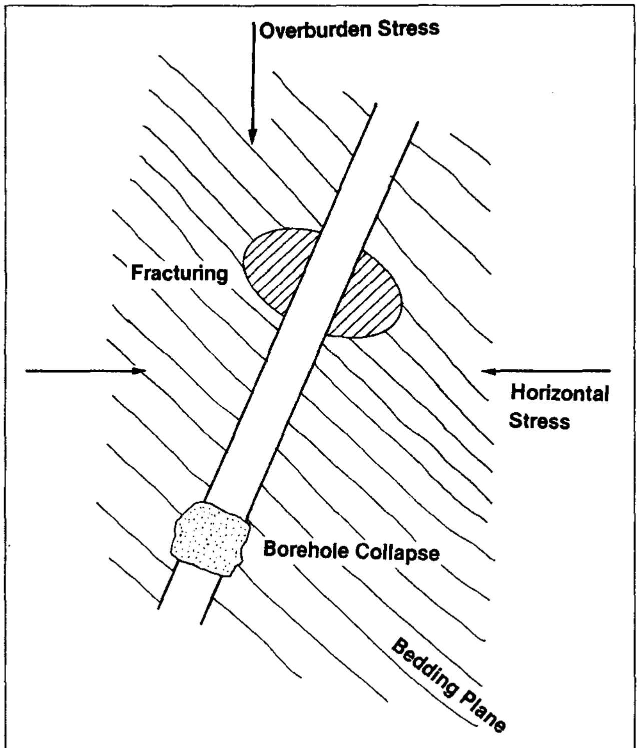

Fig. 1—Definition of the model.

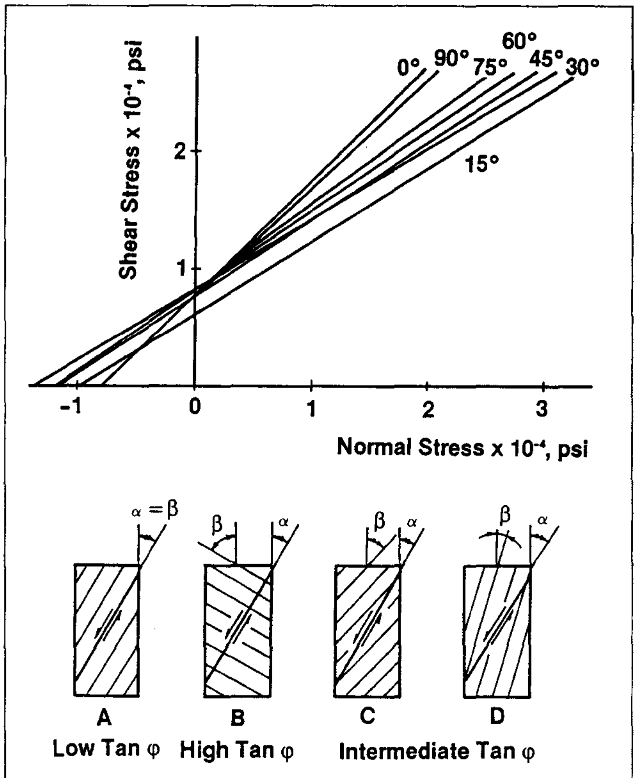

Fig. 2—Mohr's envelope for Green River shale and failure plane vs. bedding plane.

TABLE 1—SUMMARY OF ELASTIC PARAMETERS

Rock	$E \cdot 10^{-6}$	μ	K_{ani}
Leuders limestone	3.5	0.22	0.97
Arkansas sandstone	2.8	0.20	0.61
Green River shale	4.3	0.20	0.84
Permian shale	3.5	0.24	0.73

TABLE 2—TENSILE STRENGTHS DETERMINED BY THE DIAMETRAL-COMPRESSION METHOD

Rock	Tensile Strength Perpendicular to Bedding (psi)	Tensile Strength Parallel to Bedding (psi)
Arkansas sandstone	1,698	1,387
Green River shale	3,136	1,971
Permian shale	2,500	1,661

TABLE 3—EXPERIMENTALLY DETERMINED SHEAR DATA

Rock	τ_0 (psi)	ϕ (degrees)	β (degrees)
Leuders limestone	2,500	35	All β
Arkansas sandstone	5,000	57.5	$0<\beta<15$
	5,000	57.5	$35<\beta<90$
	4,200	50	$15<\beta<35$
Green River shale	7,250	41	0
	6,000	32	15
	8,250	30	30
	7,500	33.4	45
	7,500	35	60
	7,800	36.5	75
	7,250	43	90

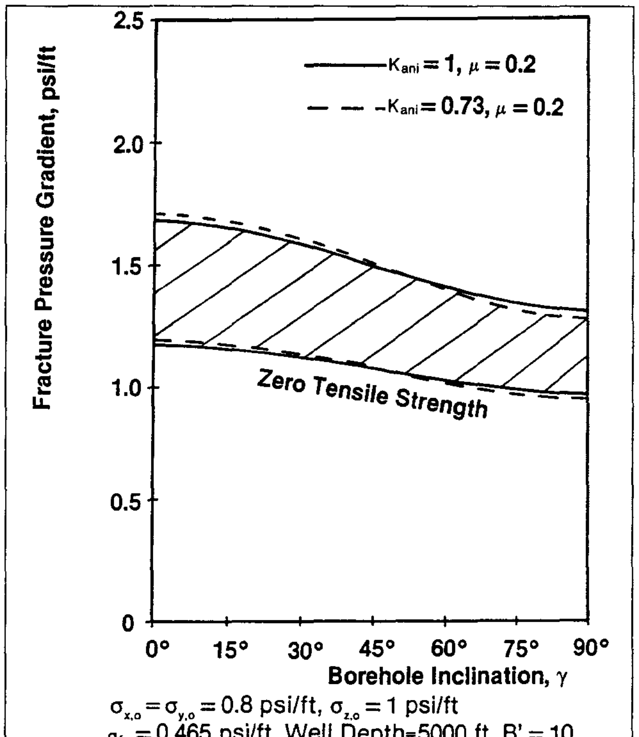

Fig. 3—Fracture gradients vs. borehole angle for Permian shale.

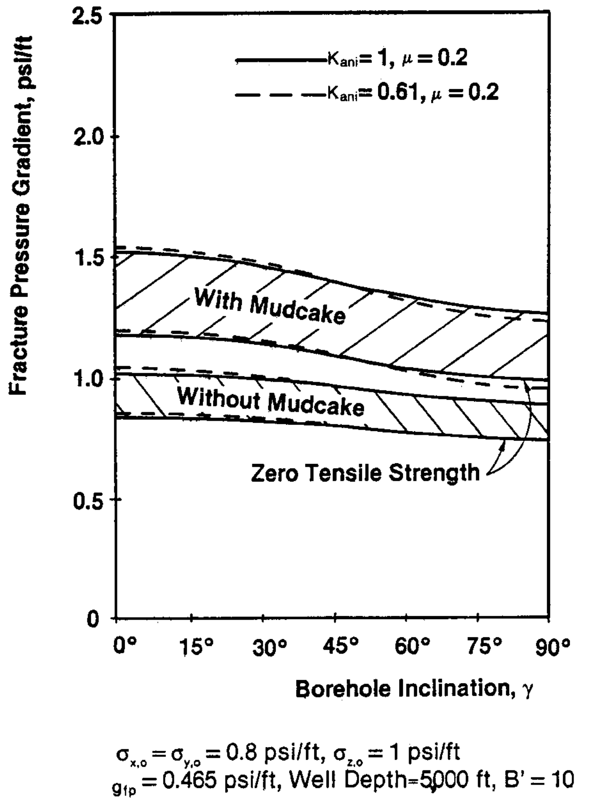

Fig. 4—Fracture gradients vs. borehole angle for Arkansas sandstone.

zero tensile strength. The difference in fracturing gradient between these two limits is large, about 30% for a vertical borehole. This spread is less for a horizontal borehole because the tensile strength decreases with increased borehole angle. The solid lines give the isotropic solution, and we see that the error is only 2%. It is evident that for this rock, the tensile strength is by far the governing parameter, and the effect of elastic anisotropy is negligible.

Fig. 4 gives the results of the computations for a similar well in a sandstone formation. First, let us study the upper band, which corresponds to the previous case. Here a perfect mudcake is assumed, which means that no pressure from the borehole is transmitted into the porous rock. The bandwidth is now about 20% at most, a result of low tensile strength. The broken lines show the isotropic solution, the error now being 3% if elastic anisotropy is neglected.

The bottom band of Fig. 4 shows a very interesting effect. Here it is assumed that the borehole pressure can transmit into the porous rock—i.e., no mudcake exists. The fracture gradient is now lowered by more than 30%. This is explained by the increased pressure of the fluids inside the pores, aiding in fracturing the borehole wall. However, the bandwidth is now smaller than for the case with mudcake. The effects of the elastic anisotropy are again overshadowed by the possible variation in tensile strength and account for only a few percent deviation from the isotropic behavior. Also, for such permeable rocks as Arkansas sandstone, the total fracturing pressure band may cover all cases shown in Fig. 4, depending on the quality of the mudcake. For this case, the variations in fracture gradients may therefore be more than 43%. This figure also explains why shales typically have higher fracture gradients than sandstones. The shale is essentially impermeable, while the sandstone may transmit pressure from the borehole into the porous rock, thereby effectively lowering the fracture pressure.

Conclusions of the Fracturing Study. The detailed conclusions from this analysis[2] show that the effects of anisotropic elasticity parameters on the fracturing pressure are small. Therefore, with all possible sources of error involved in the interpretation of the fracturing pressure, the anisotropy may be neglected. The anisotropy has a definite effect, however, on the position of the fracture around the hole wall.

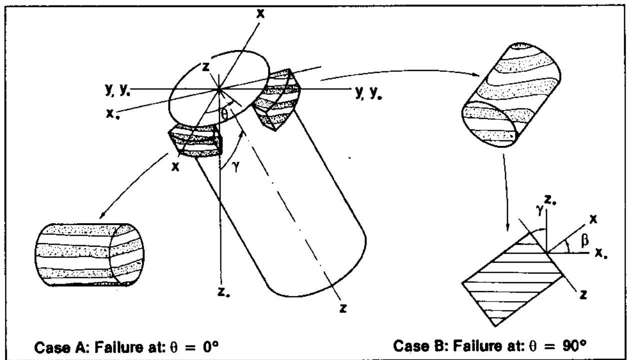

Fig. 5—Adaptation of core-plug data to a real borehole.

During back-calculation of directions and magnitudes of the in-situ stresses from borehole measurements, the anisotropic solution is required. The principal strains and stresses generally will have different directions in the anisotropic solution. Therefore, use of a simpler isotropic model may give erroneous results for the in-situ stresses and their directions.

Permeability is an important parameter. Such impermeable rocks as shales have high fracture pressures. For permeable rocks, the wellbore pressure may be transmitted into the porous rock, thereby lowering the fracture pressure. This explanation matches observed behavior in the field and also shows the importance of a good mudcake.

The tensile strength of the rock is an important parameter in the fracturing situation. Because real rocks often contain cracks or fractures, the tensile strength may show considerable variation from one location to another. Therefore, it is convenient to use two values for the fracture pressure, one with the tensile strength of a good rock sample (which has directional variations) and the other with zero tensile strength. The band between these two points represents the uncertainty associated with the fracture pressure. Note that the tensile strength is an absolute value, regardless of the general stress state. Because the general stress level increases as we drill deeper, the fracture band will decrease. Therefore, there is less uncertainty in estimating the fracture pressure for deeper wells than for shallower wells. The most important single factor in the fracturing analysis is probably the magnitude of the in-situ stress field.

Applications to Borehole-Collapse Problems

It is difficult to estimate the collapse pressure of an inclined borehole with accuracy. If the borehole has collapsed, however, and we measure the position of the failure, then we have a measure for the direction of the least normal stress on the borehole wall.[2]

Chenevert[3] provides a number of shear measurements on core plugs as a function of the bedding-plane orientation. These are summarized in Table 3. For these data to be used constructively, they must be put into proper context. Fig. 5 illustrates a borehole in a laminated rock formation, with a borehole inclination, γ, in the x-z plane. Two infinitesimally small pieces of rocks are shown on the borehole wall. The Mohr-Coulomb failure criterion takes into account only the major and minor principal stresses and neglects the intermediate principal stress.

The borehole typically will fail at 0° [0 rad] (Case A) or 90° [1.6 rad] (Case B), as in Fig. 5. If the applied stress in the x-direction is the smallest, the borehole will fail as in Case A, and if the y-direction has the least applied stress, the borehole will fail as in Case B. For a typical collapse, the radial stress is smallest, followed by the axial stress, and the tangential stress is largest. Therefore, in adapting our laboratory data, the axial stress is neglected, because the Mohr-Coulomb failure criterion neglects the intermediate principal stress. The radial stress is the minor principal stress, and the tangential (or hoop) stress is the major principal stress. The radial stress is always in a principal direction. The tangential stress is not exactly in a principal direction because some

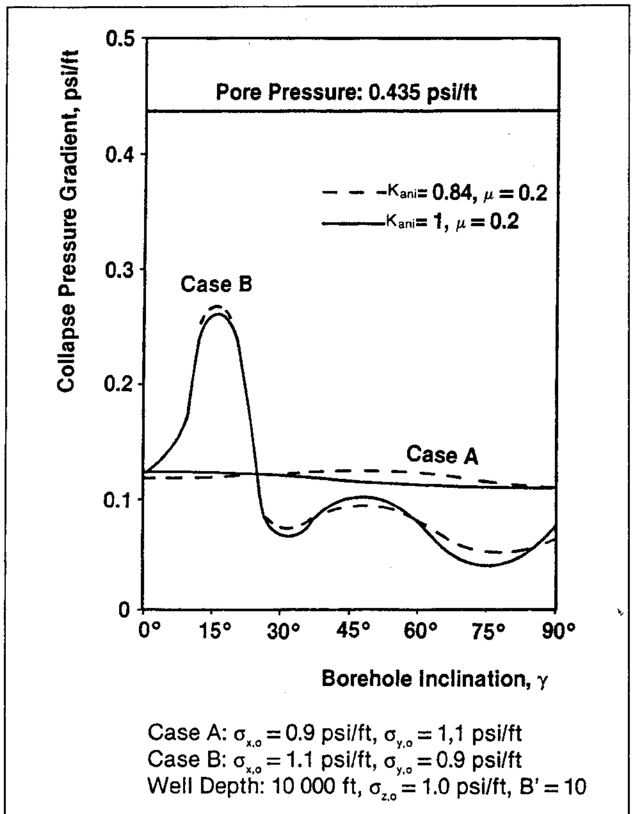

Fig. 6—Collapse gradients vs. borehole angle for Green River shale.

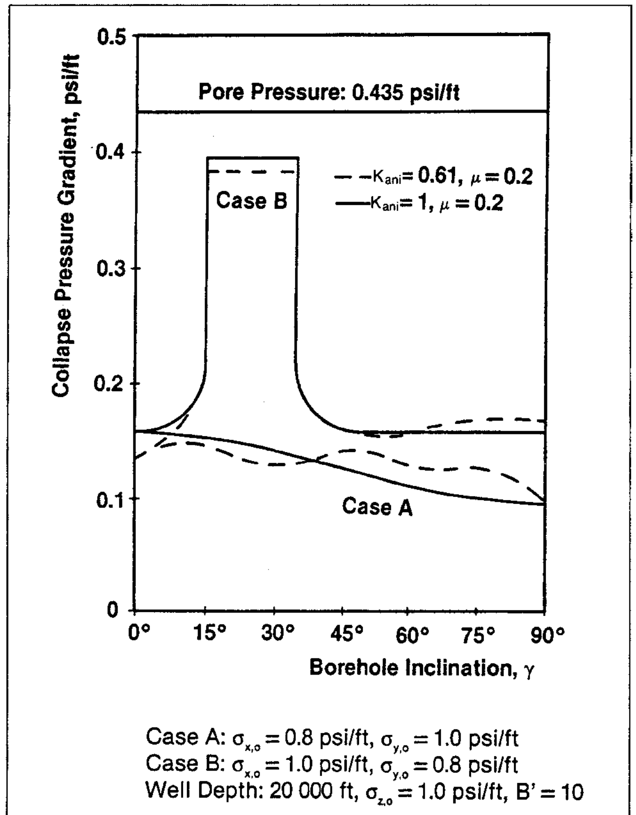

Fig. 7—Collapse gradients vs. borehole angle for Arkansas sandstone.

shear-stress components change the direction slightly. For simplicity, we will neglect this small deviation from the principal direction and assume that the major principal-stress direction coincides with the tangential stress direction.

The equivalent core plugs are shown in Fig. 5. For Case A, the tangential stress acts parallel to the bedding plane; therefore, $\beta=0°$ [0 rad] for this case, regardless of the inclination between the borehole and the bedding plane. It is seen that for Case A, one set of shear data applies for all borehole angles. In Case B, the tangential stress applies at an angle with respect to the bedding plane, and the value is now $\beta=\gamma$. For this case, the directional characteristics as a function of borehole angle with respect to the bedding planes apply. The borehole has at any point a stress state consisting of three different principal stresses. The core plug assumes two equal principal stresses (the confining pressure) and one larger stress (the axial load). These two stress states are clearly different, but the accuracy of applying core-plug data to real boreholes is not known. The following field cases will illustrate the concept further.

Fig. 6 gives the results of a borehole simulation located in a Green River shale. Case A corresponds to Case A of Fig. 5—i.e., the smallest in-situ stress is along the x axis at 0.9 psi/ft [20.4 kPa/m], while the stress is 1.1 psi/ft [24.9 kPa/m] along the y axis. For this case, only the shear measurement at $\beta=0°$ [0 rad] applies, regardless of the borehole inclination, γ. In Case A of Fig. 6, the resultant curve is shown. There is a large difference between the pore-pressure line and the borehole-collapse line. This makes the borehole very stable because the minimum pressure seen in a real borehole is usually not much lower than the pore pressure. Also, this difference is nearly constant with borehole angle, which implies that for this loading situation the borehole is very stable and is not sensitive to the borehole angle.

Next, Case B of Fig. 5 is evaluated. The in-situ stresses along the x and y axes are now interchanged, with 1.1 psi/ft [24.9 kPa/m] along the x axis and 0.9 psi/ft [20.4 kPa/m] along the y axis. Now the directional shear properties come into account. This is shown in Case B of Fig. 6. Note that the borehole for this case is more stable than for Case A, except for the region $10°<\gamma<25°$ [0.17 rad$<\gamma<$0.44 rad]. In this region, the stability decreases significantly, with a peak at $\gamma=15°$ [0.26 rad]. The isotropic solution is shown in solid lines, while the broken lines show the anisotropic solution. We observe that for this weakly anisotropic rock, the error introduced with the isotropic solution is $<10\%$. This error is overshadowed, however, by the effect of the directional shear strength.

A similar analysis is conducted with the Arkansas sandstone data. The results are shown in Fig. 7. For this very anisotropic rock, the effect of the plane of weakness is even more pronounced. Although the error introduced with the isotropic elastic solution is at most 25%, this is overshadowed by the effect of the directional shear strength. When the borehole pressure is lower than the pore pressure, there will be a flow of fluids into the borehole. A mudcake cannot be formed under this condition.

Conclusions of the Collapse Analysis. If the least normal stress at the borehole wall is in the same plane as the borehole axis and the normal axis to the bedding plane, one Mohr-Coulomb envelope applies for all borehole angles. This is the least serious and the preferred case. On the other hand, if the least in-situ stress is normal to the plane of the borehole axis and the axis is normal to the bedding plane, the directional-shear-strength properties come into account. Now the borehole has a potential collapse problem in the inclination range $15°<\gamma<35°$ [0.26 rad$<\gamma<$0.61 rad]. This applies only to sedimentary rocks with a plane of weakness. For a tectonically relaxed basin with equal horizontal in-situ stresses, this always applies.

It is found that use of isotropic instead of anisotropic elastic properties introduced an error of only a few percent for such weakly anisotropic rocks as chalks. For such strongly anisotropic rocks as sandstones, however, the error was larger. Also, it was found that pore pressure is an important parameter. Generally, the higher the pore pressure, the more sensitive the borehole is toward collapse. For such impermeable rocks as shales, the pore pressure may be considered constant, regardless of the loading.

Also, simulations on weak rocks—e.g., Leuders limestone—showed a high sensitivity toward borehole collapse. Simulations

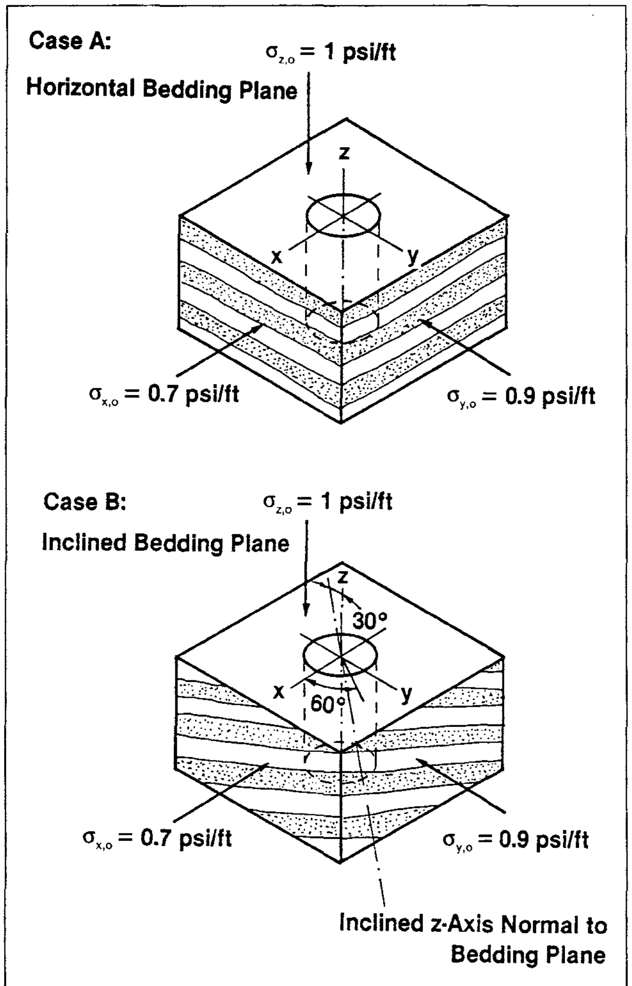

Fig. 8—Cases for studies of borehole deformation.

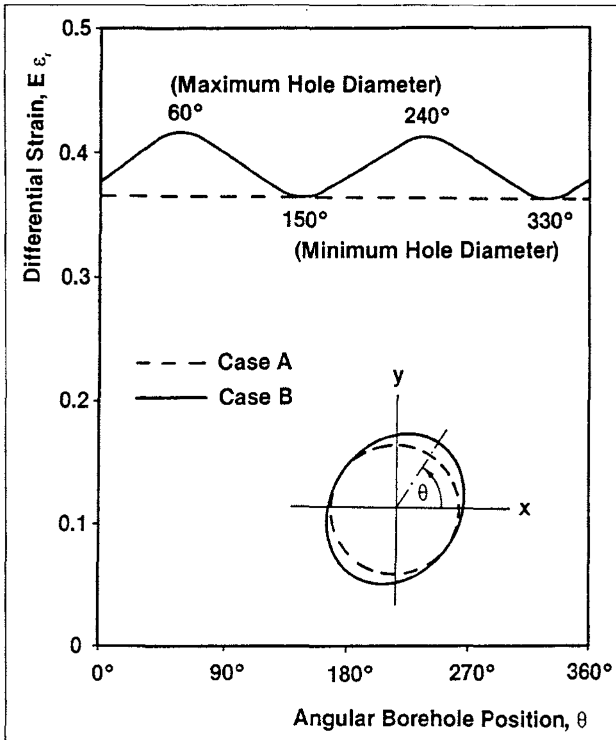

Fig. 9—Differential strain around the borehole.

on a sandstone showed that it had a pronounced plane of weakness and became very sensitive toward collapse for given inclinations. Finally, simulations showed that collapse would occur at a slightly different position around the borehole wall for anisotropic vs. isotropic rocks. This is important when the principal in-situ stress directions from collapsed holes are interpreted.

Deformation of the Borehole

In this section, we present the way borehole strain acts on changing borehole pressure to see whether it behaves differently under anisotropic conditions. Two different field cases are considered. Both assume a vertical hole. The two horizontal in-situ stresses are different, 70 and 90% of the overburden stress, respectively. For simplicity, the rock is assumed to be nonporous and elasticity data for Arkansas sandstone from Table 1 are used as input parameters. The two cases discussed are shown in Fig. 8. Case A assumes a transversely isotropic rock with horizontal bedding planes, while Case B assumes the same rock type, but now the axis normal to the bedding plane is oriented 30° [0.52 rad] from vertical and the azimuth angle of this axis is set at 60° [1.05 rad]. The only difference between these two cases is the orientation of the bedding plane.

The borehole is assumed to be drilled with a borehole pressure of 50% of the overburden stress. During this process, the unequal in-situ stresses may distort the shape of the borehole, but we assume that the drill bit acts as a reamer and leaves a circular borehole behind, where the stress field around the hole is in equilibrium with the in-situ stresses. At this stage, we have eliminated the effects of the in-situ stresses because of the reaming process. After the hole is drilled, it is assumed to be pressurized toward the overburden stress state, and the deformation around the borehole wall caused by this pressure increase is computed. The results are shown in Fig. 9.

Case A with a horizontal bedding plane acts like an isotropic material because of complete symmetry. We see that the deformation is constant around the borehole wall. This is interesting because even if the in-situ stresses are different, the strain is constant in any direction around the wall, and the hole remains circular. The strain is seen to depend on the inside boundary condition, which for our case is a hydrostatic pressure. This is expected because the effects of the in-situ stresses were eliminated during the reaming process.

Case B with inclined bedding plane shows strain, which varies around the borehole. Note that the deformation in this case is larger, even with the same loading conditions. The difference between the maximum and minimum differential strains is about 12%, a significant value. Fig. 9 shows that the borehole has a maximum diameter across the $\theta=60$ and 240° [1.05- and 4.19-rad] positions and a minimum diameter across the $\theta=150$ and 330° [2.62- and 5.76-rad] positions around the borehole wall. Numerical simulations showed that the maximum deformation occurred at a position that corresponds to the azimuth of the axis normal to the bedding plane—i.e., the deformation was completely independent of the relative values of the in-situ stresses, and the positions of its maximum and minimum values were only a function of the orientation of the bedding plane relative to the borehole. This was also expected because the only change has been to increase the hydrostatic borehole pressure. The preceding analysis gives some very interesting observations. A new technique of determining the maximum and minimum in-situ stresses has been developed. An oriented core is recovered from a deep borehole; as soon as it reaches surface, strain gauges are installed to determine changes as a function of time. The relative deformation is then used to calculate the in-situ stresses by assuming that the principal strain directions coincide with the directions of the principal stresses.

Although this method reportedly measures anelastic deformation, the insight provided in Fig. 9 may be used to argue that possibly the anisotropic elasticity effects also are measured. At this time, the core-recovery process is not fully understood; therefore, rather than drawing any final conclusions, the point here is to bring the element of anisotropy into the picture of this complex issue. Another factor that is not understood is how the reaming or grinding of the core bit affects the release of the in-situ stresses during the coring process.

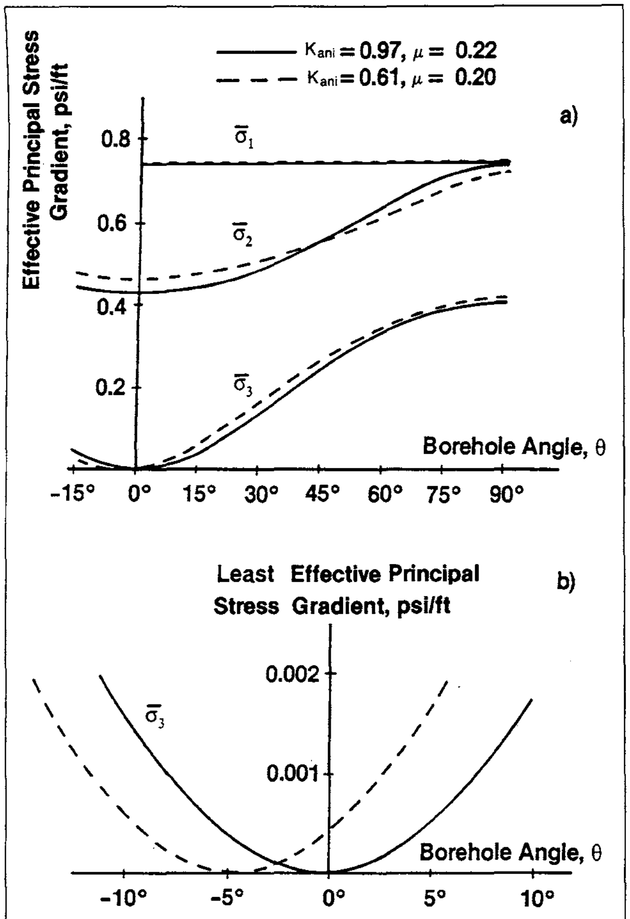

Fig. 10—Effective principal stresses around the borehole during fracturing.

Direction of the Fracture on the Hole Wall

In petroleum engineering, borehole fracturing operations are frequently carried out. Offshore, where most wells are inclined at high angles, fracturing is not performed as often as onshore. One argument is that the fracture direction is not known offshore. If the fracture cuts across the borehole, only a small additional fluid flow path is created. On the other hand, a fracture extending parallel to the hole axis will give a significant flow path. The myth of fractures cutting across the borehole generally is not true; it will be shown why fractures in inclined boreholes >2,000 ft [>610 m] deep[1] are initiated parallel to the borehole axis, regardless of the inclination of the borehole. If a fracture cutting across the borehole should be observed, it is probably the result of faulting or some other geologic phenomenon. A numerical example will be used to show the fracture direction. A borehole inclination of $\gamma=60°$ [1.05 rad] is arbitrarily chosen. The fracture gradient is 1.09 psi/ft [24.7 kPa/m] for both the isotropic Leuders limestone and the anisotropic Arkansas sandstone used in the analysis. The pore-pressure gradient is 0.4 psi/ft [9.0 kPa/m]. Overburden stress is set at 1.0 psi/ft [22.6 kPa/m], and the horizontal in-situ stresses are both equal to 80% of this. The tensile strength is set to zero. For this case, the three principal stresses are plotted as a function of the angle around the borehole, as seen in Fig. 10a. The maximum principal stress is the radial stress, the intermediate is the axial stress, and the minimum principal stress is the tangential stress at $\theta=0°$ [0 rad]. The largest effective principal stress is constant and equal to the borehole pressure, while the smallest principal stress shows quite a variation. Because there is symmetry around the borehole, only 90° [1.6 rad] is shown. For the isotropic case, the smallest effective principal stress is zero only at $\theta=0°$ [0 rad] (and $\theta=180°$ [3.14 rad]). Outside this region, it goes into a compressive state. For the anisotropic Arkansas sandstone, this minimum is at $\theta=-5°$ [-0.09 rad]. It is observed in Fig. 10a that the principal stresses are only slightly different for the anisotropic and isotropic cases, but not significantly. In Fig. 10b, an expanded scale is used near the minimum points. It is observed that the only significant difference between the anisotropic and isotropic cases is that the minimum point is shifted by 5° [0.09 rad] around the borehole wall. During the computations, it was observed that the direction of the least principal stress does not always correspond to the normal to the borehole axis. In fact, for horizontal or vertical boreholes, the fracture would always extend along the borehole axis, while for inclined boreholes, a small deviation from the borehole axis sometimes was seen. Inspection of Fig. 10a, however, reveals that even if the fracture starts out at an angle, it may not extend a long distance because the least principal stress goes compressive by moving only a small angle θ around the borehole wall. Thus, it may be concluded that the fracture is confined to a narrow region and possibly extends in a zigzag form along the borehole axis, as indicated in Fig. 11b. A similar conclusion is given

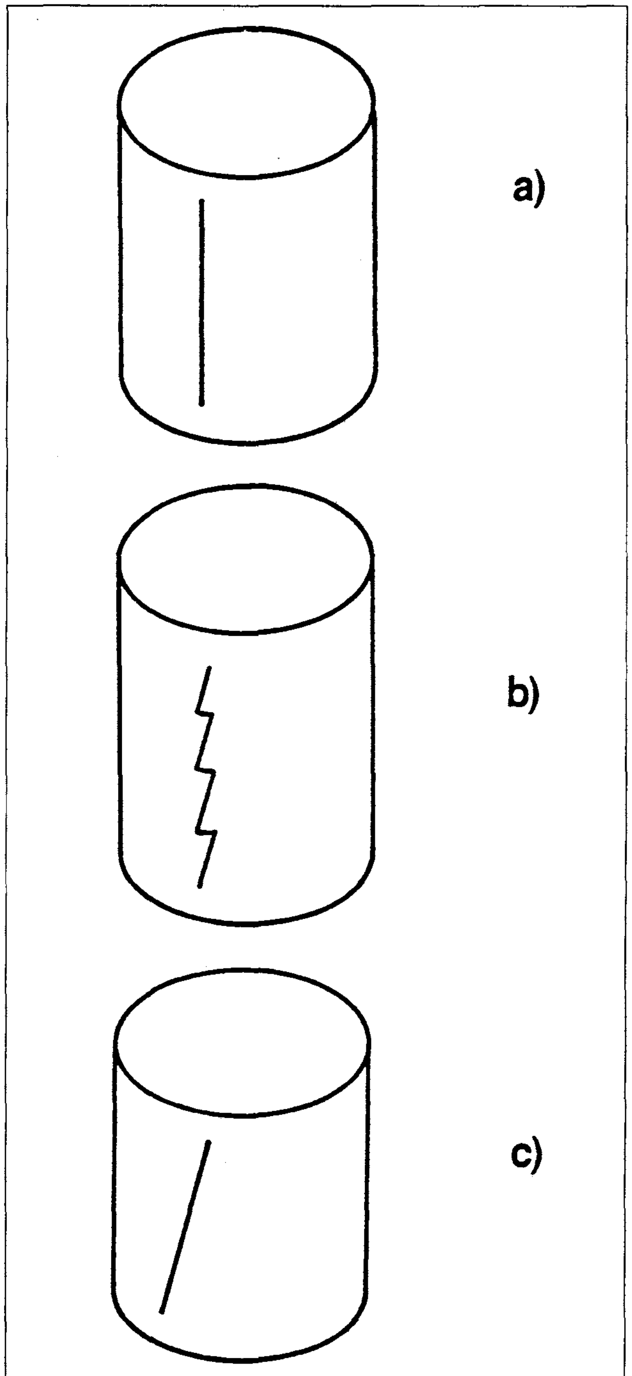

Fig. 11—Fracture pattern.

by Fairhurst.[5] Possible fracture patterns are seen in Fig. 11. In Fig. 11a, a fracture extending along the borehole axis is shown, as expected, in vertical or horizontal boreholes. Fig. 11b shows the expected fracture for an inclined borehole and Fig. 11c shows still another possible fracture type. Referring again to Fig. 11b, we see that a potential shear action apparently connects each locus. These loci can be very short, thus giving the appearance of a continuous, but rough, surface. Therefore, contrary to earlier belief, the anisotropic rock model does not predict fractures cutting the hole at an angle. Fig. 11 shows the fracture direction at the borehole wall only. The fracture will orient itself normal to the least in-situ stress as it propagates away from the borehole.

Heterogeneous and Discontinuous Rock

The mathematical solution used in this paper is *strictly not valid* for this case, but it will be used intuitively. Consider the same case as before, with the following modifications. The rock has a finite tensile strength and contains an existing crack along the borehole axis located at $\theta = -10°$ $[-0.17$ rad$]$. The least effective stress at failure is shown in Fig. 12. When the borehole pressure is increased, the stress curves will shift downward until the tensile strength is exceeded or a weakness (a crack) reaches a critical stress state. In Fig. 12, we see that the existing crack will open up at Point A when the least effective stress approaches the tensile strength. Observe that the point of fracture no longer corresponds to the position of the least effective stress and note also that for this case, the two rocks will fracture at different stress levels, which means that they will fracture at different borehole pressures. From this discussion, the following can be deduced. Rock heterogeneity and discontinuity may determine the position of fracture initiation as well as the stress level at which fracture occurs. The stress level might be influenced by the degree of anisotropy of the rock. Heterogeneities and discontinuities generally reduce the accuracy of fracture-pressure and fracture-position predictions.

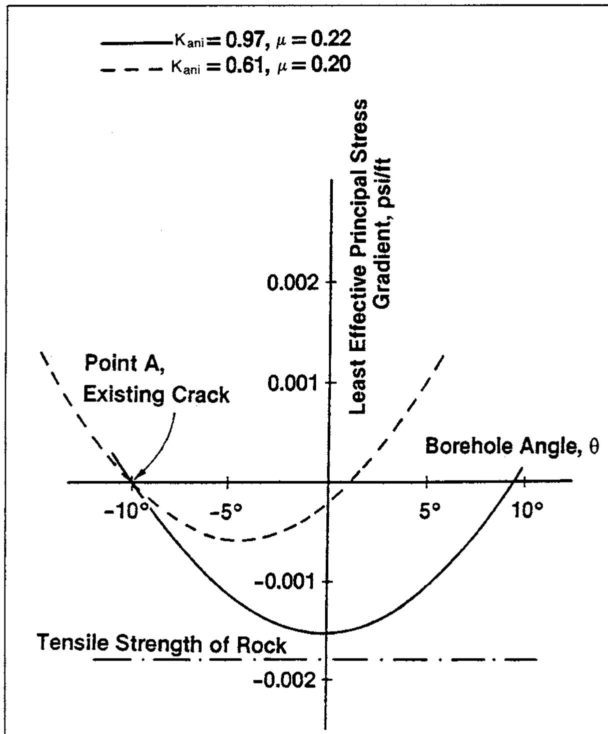

Fig. 12—Fracturing of rock containing a crack.

Application for the Petroleum Industry

In the foregoing, some results of simulations of a complex mathematical model are shown. It is believed that this model more accurately describes real sedimentary rocks, which typically have a laminated structure. The model shows very interesting results, but how can these be applied? A few ideas regarding application are presented next.

Drilling Operations. There is a clear connection between degree of anisotropy and lithology, which implies that borehole stability is lithology-dependent. Typically, this factor has previously been neglected in the petroleum industry. During drilling, lost circulation usually occurs unexpectedly, and one explanation relates to lithology. Leak-off pressures are typically taken in competent shale sections, while lost circulation often occurs in sandstones.[8]

Also during drilling, a criterion for avoiding collapse (plane of weakness) is given. If a hole has serious collapse problems, sidetracking 90° [1.6 rad] to this hole direction can reduce the problems. Experiences from the North Sea support this theory.

Production Operations. The model shows that during fracturing operations, the fracture will extend along the hole axis for deeper wells, regardless of the inclination of the borehole. This argument may be used to stimulate the application of hydraulic fracturing in offshore wells. In the past, this type of well stimulation has not been used because the direction of the fracture, across or along the hole, is not known. This analysis may help eliminate such arguments against choosing this stimulation method.

Sand production has a connection not only to shear strength of the rock, but also to the total stress field around the hole. A possible extension of the "plane-of-weakness" concept is to apply it to sand production. At this point, this is only speculation, but it seems logical to assume that sand production and borehole collapse are related phenomena.

In Norway, borehole stability has become an important research area. The task is to be able not only to predict fracturing or collapse pressures, but to model the whole field with its stress state. Therefore, this analysis points the way to a more rigorous approach in solving borehole problems than has been used in the past. The reader is also referred to Refs. 1, 6, 7, and 8, which give a good overview of borehole stability problems.

Conclusions

A mathematical model has been developed to take into account real rock properties, which are typically anisotropic.

First, fracturing of the borehole at high pressures was investigated. The result was that assuming isotropic rock properties instead of the real anisotropic elastic constants introduced only a small error in the fracture-pressure prediction. More important is the tensile strength of the rock. Using measured tensile strengths and zero tensile strength (as assumed in the field with cracks present) as extreme limits, the fracture pressure was given in a range with maybe more than 50% variation between the upper and lower limits. The real tensile strength is an important parameter in fracturing-pressure predictions. Because the tensile strength is an absolute value, it is found that the above-measured range decreases with increased well depth. Therefore, there is less uncertainty associated with estimating fracture pressure for deeper wells if all in-situ parameters are known. The position of fracture initiation is erroneous if the simplified isotropic equations are used for anisotropic rocks. Also, impermeable rocks (e.g., shales) were distinguished from permeable rocks (e.g., sandstones). If the borehole pressure could not be transmitted through the mudcake or into an impermeable rock, a higher fracture pressure was observed.

Borehole collapse was investigated with an extended Mohr-Coulomb criterion. It was found that the higher the pore pressure, the more sensitive the borehole became to collapse. For relaxed depositional basins with equal horizontal in-situ stresses, the borehole is sensitive to collapse for inclination between 10 and 35° [0.17 and 0.61 rad]. This applies only to laminated rocks with a plane of weakness. If the horizontal in-situ stresses are different, a borehole very stable against collapse can be drilled by inclining the hole in the direction of the least in-situ stress. The collapse analysis showed that with the simplified isotropic stress equations, a significant error is introduced for such highly anisotropic rocks as sandstones. Also, it was found that the least in-situ stress was not perfectly aligned with the direction of collapse for general anisotropic cases.

The deformation pattern of the borehole was investigated when the borehole pressure is increased. If initially circular, the hole will deform in a circular manner if the hole is perfectly normal to the bedding plane (perfect symmetry). If the angle between the bedding plane and the borehole is different from 90° [1.6 rad], however, the hole will deform elliptically. The maximum diameter of the hole thus obtained corresponds to the orientation of the borehole relative to the bedding plane. In other words, the geometry and the material properties, rather than the in-situ stresses, may determine the deformation pattern. These observations are valid for a time-independent material only and are introduced to show that anisotropy may have a significant effect. For time-dependent materials, the conclusions may no longer be valid.

Simulations also show that the plane-strain anisotropic model predicts only fractures extending parallel to the borehole axis for boreholes >2,000 ft [>610 m] deep.[1] Fractures cutting across or at an angle were never predicted. For an anisotropic rock, however, the fracture would initiate at a different location around the borehole wall compared with an isotropic rock. Therefore, an isotropic model to calculate the direction of the in-situ stresses generally gives erroneous results. Finally, it was deduced that heterogeneities and discontinuities in the rock might be the most important factors in a field situation, reducing the accuracy of the predictions.

Knowledge of the actual in-situ stress state is probably the most important element in all rock-mechanics-related work.

Nomenclature

a = constant
a_{ij} = elements of matrix of compliance
$[A_s]$ = matrix of compliance for interpore material
B' = E_i/E
E = elastic modulus of porous material
E_i = elastic modulus of interpore material
F = stress function
g_{fp} = formation pressure gradient
$[I]$ = identity matrix
K_{ani} = anisotropy coefficient; horizontal/vertical elasticity ratio
$[L]$ = matrix of elastic constants for porous material in rock
x,y,z = reference frame for borehole
x_0,y_0,z_0 = reference frame for virgin in-situ stresses
α = angle between applied force and failure plane during triaxial core testing, degrees [rad]
β = angle between applied force and bedding plane during triaxial core testing, degrees [rad]
γ = borehole inclination from vertical
$\gamma_{xy},\gamma_{xz},\gamma_{yz}$ = shear strains
ϵ_r = borehole strain in the radial direction
$\epsilon_x,\epsilon_y,\epsilon_z$ = normal strains
θ = angle around borehole from x axis, degrees [rad]
μ = Poisson's ratio
σ = normal stress
$\sigma_x,\sigma_y,\sigma_z$ = normal stresses in x, y, z directions
$\sigma_{x,o},\sigma_{y,o},\sigma_{z,o}$ = virgin in-situ stress field
$\bar{\sigma}_1,\bar{\sigma}_2,\bar{\sigma}_3$ = principal effective stresses
τ = shear stress
$\tau_{xy},\tau_{xz},\tau_{yz}$ = shear stresses in x, y, z coordinate system
τ_0 = cohesive strength of rock
ϕ = angle of internal friction in Mohr-Coulomb envelope, degrees [rad]
Ψ = stress function

Subscripts

i,j = 1,2,3

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Appendix—Mathematical Model

Ref. 1 defines the stresses around the borehole, the transformation equations, the principal stresses, and the effective stresses for isotropic materials. In the following, we will look at a few parameters that are different for anisotropic materials.

The strain components are related to the stress components through the constitutive relations of the anisotropic body as follows:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \quad \text{(A-1)}$$

Introducing the generalized plane-strain concept, $\epsilon_z=0$, the axial stress component can be expressed as

$$\sigma_z = -\frac{1}{a_{33}}(a_{31}\sigma_x + a_{32}\sigma_y + a_{34}\tau_{yz} + a_{35}\tau_{xz} + a_{36}\tau_{xy}). \quad \text{(A-2)}$$

Please observe that the three material constants—a_{34}, a_{35}, and a_{36}—may obtain nonzero values for anisotropic materials, thereby coupling the normal and shear stresses. For isotropic materials, this coupling does not exist.

By introducing the following stress functions and strain compatibility conditions, a system of differential equations results:

$$\sigma_x = \frac{\partial^2 F}{\partial y^2}, \quad \text{(A-3a)}$$

$$\sigma_y = \frac{\partial^2 F}{\partial x^2}, \quad \text{(A-3b)}$$

$$\tau_{xy} = \frac{\partial^2 F}{\partial x \partial y}, \quad \text{(A-3c)}$$

$$\tau_{xz}=\frac{\partial\psi}{\partial y}, \quad \text{(A-3d)}$$

and

$$\tau_{yz}=-\frac{\partial\psi}{\partial x}. \quad \text{(A-3e)}$$

This yields

$$L_4F+L_3\psi=0 \quad \text{(A-4a)}$$

and

$$L_3F+L_2\psi=0, \quad \text{(A-4b)}$$

where

$$L_2=\beta_{44}\frac{\partial^2}{\partial x^2}-2\beta_{45}\frac{\partial^2}{\partial x\partial y}+\beta_{55}\frac{\partial^2}{\partial y^2},$$

$$L_3=-\beta_{24}\frac{\partial^3}{\partial x^3}+(\beta_{25}+\beta_{46})\frac{\partial^3}{\partial x^2\partial y}-(\beta_{14}+\beta_{56})\frac{\partial^3}{\partial x\partial y^2}+\beta_{15}\frac{\partial^3}{\partial y^3},$$

$$L_4=\beta_{22}\frac{\partial^4}{\partial x^4}-2\beta_{26}\frac{\partial^4}{\partial x^3\partial y}-2\beta_{16}\frac{\partial^4}{\partial x\partial y^3}+(2\beta_{12}+\beta_{66})\frac{\partial^4}{\partial x^2\partial y^2}+\beta_{11}\frac{\partial^4}{\partial y^4},$$

and

$$\beta_{ij}=a_{ij}-\frac{a_{i3}a_{j3}}{a_{33}}(i,j=1,2,4,5,6).$$

It is readily observed that the solution depends on the elements of the constitutive relation matrix of Eq. A-1. Solved in terms of F, we are left with a sixth-order differential equation:

$$(L_4L_2-L_3^2)F=0. \quad \text{(A-5)}$$

Inspection of Eq. A-4 can give us some insight into the properties of the solution. If $L_3=0$, the two stress functions, F and Ψ, are no longer coupled, and we have a reduced case that is similar to an isotropic solution. Amadei[9] defines the following conditions of anisotropy that correspond to the condition $L_3=0$: (1) orthotropic material with one plane of elastic symmetry perpendicular to the hole axis and two other planes parallel to the hole axis, (2) transversal isotropy in a plane striking parallel to the hole axis, (3) transversal isotropy in a plane perpendicular to the hole axis, and (4) isotropy.

From these equations, it is deduced that $L_3=0$ only for a horizontal or vertical hole in a horizontally oriented transversely isotropic rock. For all cases in between, L_3 will not vanish.

Inspection of Eq. A-4 reveals that the anisotropic stress equations are very complicated. Indeed, the solution to Eq. A-5 always gives complex or imaginary roots. The resultant stress equations are rather lengthy; Aadnoy[2] provides complete expressions.

The model assumes that a general stress field at infinite radius is the outer boundary condition and the borehole pressure is the inner boundary condition. Five independent material constants generally are necessary to use such a model. A simplified three-parameter concept of van Cauvelaert[10] is adopted, however, which yields for a laminated rock with the laminae oriented in the x-y plane:

$$\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \epsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{bmatrix}=\frac{K_{ani}}{E}\times\begin{bmatrix}1 & -\mu & -\mu & 0 & 0 & 0\\ -\mu & 1 & -\mu & 0 & 0 & 0\\ -\mu & -\mu & 1/K_{ani} & 0 & 0 & 0\\ 0 & 0 & 0 & [1/K_{ani}+(1+2\mu)] & 0 & 0\\ 0 & 0 & 0 & 0 & [1/K_{ani}+(1+2\mu)] & 0\\ 0 & 0 & 0 & 0 & 0 & 2(1+\mu)\end{bmatrix}\times\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{bmatrix}. \quad \text{(A-6)}$$

The constitutive relations are defined in Eq. A-6. The three constants are defined in Table 1. For fracturing analysis, a strength criterion was defined assuming either that failure occurred when the least principal stress exceeded the tensile strength of the rock or that the tensile strength was zero because of existing cracks. The data from Table 2 were used in the fracturing analysis and were interpolated for intermediate orientations. For borehole-collapse analysis, a failure envelope was needed because collapse is defined as a shear failure. The variable cohesive strength/variable angle of the internal friction Mohr-Coulomb criterion was used. It is based on the plane-of-weakness theory that says that the rock would be weaker when it fails along the bedding plane.

The Mohr-Coulomb failure model is presented as

$$\tau=\tau_0+\bar{\sigma}\tan\phi. \quad \text{(A-7)}$$

The data from Table 3 were used in Eq. A-7 for the collapse analysis. Now some of the most important parameters have been defined briefly. Because we are dealing with porous rocks containing fluids under pressure, an effective-stress concept must be defined. For anisotropic rocks, the following effective-stress concept is applied.

$$[\bar{\sigma}]_{xyz}=[\sigma]_{xyz}-g_{fp}\left\{[I]-\frac{1}{B'}[L][A_s]\right\}\begin{bmatrix}1\\1\\1\\0\\0\\0\end{bmatrix}, \quad \text{(A-8)}$$

where

g_{fp} = pore pressure,
$[A_s]$ = matrix of compliance for interpore material,
$[L]$ = matrix of elastic constants for porous material in rock, and
$[I]$ = identity matrix.

The anisotropy may be either *structural* (anisotropic pore geometry) or *intrinsic* (anisotropic solid material) or both. For an isotropic material, the product $[L][A_s]=[I]$.

Numerical Simulators

The mathematical models described in this paper were implemented in two numerical simulators, one for borehole fracturing, and one for borehole collapse. In addition to the anisotropic stress equations, transformation equations were included to take into account various boreholes, the bedding plane, and in-situ stress orientations. The complete simulators have been evaluated with respect to possible simplifications of the equations involved. It was found that for the general case, the full solution must be used. Such simplifications as decoupling of the two differential equations in Eq. A-4 introduce an error.

SI Metric Conversion Factors

degrees	×	1.745 329	E−02	=	rad
ft	×	3.048*	E−01	=	m
psi	×	6.894 757	E+00	=	kPa
psi/ft	×	2.262 059	E+01	=	kPa/m

*Conversion factor is exact.

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Discussion of Modeling of the Stability of Highly Inclined Boreholes in Anisotropic Rock Formations

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Introduction

Aadnoy (Sept. 1988 *SPEDE,* Pages 259–68) should be congratulated for his continuous effort toward the better understanding of deep-wellbore stability. We also would like to contribute to this subject by commenting on some of Aadnoy's conclusions. This discussion will provide independent data and information that may indicate that (1) the reported results from borehole fracturing are not sufficient to justify the conclusion that "...in the interpretation of the fracturing pressure, the anisotropy may be neglected"; (2) the reasoning on the direction of the fracture on the hole wall is not sufficient to justify the conclusion that "...fractures...are initiated parallel to the borehole axis, regardless of the inclination of the borehole"; and (3) the use of the Mohr-Coulomb failure criterion to evaluate the collapse gradient in the case of shale formation is not appropriate. These conclusions should be tested further before they are accepted by the drilling engineering community.

Fracturing Studies

To evaluate the influence of rock formation anisotropy upon wellbore fracturing pressure, Aadnoy selected two case studies that cannot be considered representative of the possible spectrum of practical situations because the chosen K_{ani} values are too high and because the in-situ effective horizontal stresses are hydrostatic.

The K_{ani} values of 0.73 and 0.61, chosen to model the degree of rock anisotropy, may not be representative of highly anisotropic rocks. Barla[1] reported results of K_{ani} as low as 0.2 for some schists. Because few data are available for sedimentary rocks, more effort must be made to obtain the appropriate range of K_{ani} for oil-bearing sedimentary rock formations. Before this range can be accurately determined, however, K_{ani} values smaller than those used by Aadnoy should be considered to study the effects of rock anisotropy. Also, in light of previous experience, it is not surprising that K_{ani} values of 0.61 indicated only a negligible influence of rock anisotropy upon stress distribution around a wellbore in an elastic medium. Hoek[2] suggested that for K_{ani} values greater than 0.33, the stress distribution is not significantly different from the one obtained for the isotropic elastic medium ($K_{ani}=1.0$).

The chosen in-situ state of stress is not representative of possible situations in the field because the horizontal stresses are hydrostatic. In this choice, the direction of the wellbore was made irrelevant. Previous experience has shown, however, that for isotropic rock formations, both in-situ stress ratio and wellbore direction with respect to in-situ stresses may greatly influence the stress distribution around the wellbore and, consequently, the fracturing pressure. Recent papers[3,4] clearly show this influence. The directional tensile strength of an anisotropic material may add to this influence.

Direction of the Fracture on the Hole Wall

Aadnoy concluded that "...fractures...are initiated parallel to the borehole axis, regardless of the inclination of the borehole," but we think the example on which the author based this conclusion is too particular. This example actually represents a situation that can be analyzed by an isotropic model because K_{ani} is too high and because the tensile strength is zero. Aadnoy also considered a hydrostatic condition for the in-situ effective horizontal principal stresses, which added to the limitations of the example.

Aadnoy's conclusion is not in agreement with the results of Daneshy[5] and of Roegiers and Detournay.[4] Both investigations showed that the fracture can be inclined with respect to the borehole axis, depending upon the geometric combination (i.e., borehole inclination and direction). Again, we would expect to observe an influence of the rock anisotropy on the direction of the fracture on the borehole wall if the proper variation of the tensile strength with direction is considered.

Borehole Collapse Problems

To evaluate failure condition around a wellbore properly, a three-dimensional failure criterion for the rock mass should be considered. For the case of anisotropic rocks, Amadei[6] presented a comprehensive criterion to consider the effect of all principal stresses upon the slip condition along the plane of anisotropy. It is necessary, however, to calculate the direction of principal stresses around the wellbore. Unfortunately, Aadnoy neglected the possible rotation of principal stresses in planes tangent to the wellbore, which can be quite considerable.[4,5]

To apply the Mohr-Coulomb failure criterion, the stresses must be compressive. It is not valid to extend the straight line for the region of tension because this may overestimate the strength of the material. In Aadnoy's example for impermeable shale, the drilling was done under highly *underbalanced* conditions—i.e., the internal mud pressure was much smaller than the pore fluid pressure, generating a tensile radial stress. Therefore, for this case, the stresses leading to failure (tangential and radial), as analyzed by Aadnoy, would lead to an underestimation of the collapse gradient.

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SPEDE

Author's Reply to Discussion of Modeling of the Stability of Highly Inclined Boreholes in Anisotropic Rock Formations

B.S. Aadnoy, SPE, Rogaland U.

I thank da Fontoura and dos Santos for their interest in my paper and for their discussion. I certainly agree that more effort must be spent in obtaining data for oil-bearing sedimentary rocks.

Their discussion correctly points out that the degree of anisotropy shown in my examples is not extreme. I showed that the error is small when anisotropy is neglected compared to uncertainties associated with other factors, such as mudcake quality and tensile strength of the rock. With use of very low K_{ani} values, my conclusion is still the same. My data were chosen for commonly encountered oilfield rocks. I also agree that the chosen in-situ stress states may not represent the "possible spectrum of practical situations." I never intended my cases to cover all possible field situations specifically. The cases were chosen to represent a common oil field, mainly to study the effect of rock anisotropy. Other sets of data would, of course, result in different fracture or collapse pressures, but I still believe that the same general trends would be present. The hydrostatic in-situ stress state is chosen mainly because the principal in-situ stresses are not known. A recently published paper,[1] however, gives a method to estimate the nonhydrostatic in-situ stress field from leakoff data, providing a major improvement in the modeling of inclined boreholes.

In my discussion on fracture initiation, I pointed out that the fracture may initiate at an angle to the borehole axis, but that it is confined to extend axially within a narrow band. The model therefore predicts a fracture that may be irregular but that will initiate along the axis of the hole as illustrated in Fig. 11b of my paper. Daneshy's[2] experimental work seems to be in agreement with my model. He, however, observed some deviations at the endpoints of the fractures. This observation could, in my opinion, result from end effects in the experimental setup. Roegiers and Detournay's[3] paper is a well-written discussion about fracture initiation. Again, I see no direct disagreement because my model predicts fractures that initiate at an angle to the borehole axis but that extend along the hole axis within a confined band. The tangential stress typically goes tensile only at narrow regions of the borehole during fracturing, explaining the confinement within the band. As da Fontoura and dos Santos correctly point out, the particular fracture shape (see Fig. 11b of my paper) depends on the geometric combination and loading situation.

It is further suggested that a three-dimensional failure criterion must be considered for borehole-collapse analysis. I agree that this would provide an interesting extension of my work. da Fontoura and dos Santos suggest a criterion suitable for jointed rocks that seems promising. I point out, however, that there is apparently no consistent view on the effect of the intermediate principal stress. I therefore find the Mohr-Coulomb failure criterion acceptable along with other criteria. I agree that the maximum-principal-stress direction deviates somewhat from the direction of the tangential stress component. For the examples shown in the paper, this deviation was small. One of the main points was to show the effect of the plane of weakness along the bedding plane.

I also agree that extrapolating the Mohr-Coulomb envelope to negative stresses may introduce errors. The core plugs used to obtain rock strength data are typically tested without a pore pressure or at specific pore-pressure levels. When these data are applied in a simulator, the effective stress concept is used to model the borehole response at any pore pressures. There is, of course, a possible source of error here. The validity of the effective stress principle may be questioned, particularly for impermeable shales. In addition, cores are usually not tested under in-situ temperatures. Also, it is often difficult to find agreement between laboratory-obtained data and borehole response data from the field. Here, I believe, it is not only a matter of accurately determining the rock properties. The determination of the in-situ stress field[1] is more important. Borehole collapse is much more difficult to model than fracturing, and my paper is one step toward full understanding of this problem.

In conclusion, I think da Fontoura and dos Santos correctly emphasized the complexity of borehole mechanics with its variety of rock types and loading conditions. Also, they point out that there are failure models other than the ones I have chosen. Future work will show how these compare. Within the conditions and limitations given, I still think my conclusions hold. As da Fontoura and dos Santos imply, however, borehole mechanics is an area where a lot of analytical and experimental work remains to be done.

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