In The Name of God.
The Merciful, The Compassionate.

Properties of Determinants

notes on Gilbert Strang videos, Lecture 18

1 Properties of determinants

1. $det(I) = 1$

2. Exchange rows: reverse the sign of det. So, det of permutation matrices are 1 or -1.

3. a) det(multiply a row of A by t) $= tdet(A)$.

   b) det(A with its $row_i + \vec{r}$) $= det(A) + det$(A with row_i replaced by $\vec{r}$)
   $\Longrightarrow$ det is linear for each row.

4. Two equal rows $\rightarrow det = 0$, proof by exchanging rows

5. Subtract $l * row_i$ from row_k: det doesn't change

6. Row of zeros: $det = 0$, proof by multiplying zero row by a scalar t

7. $det(U)$ (upper triangular) $= \prod_i U_{(i,i)} := (\pm)$ product of pivots.

   - proof: kill off diagonal with row operations (suppose $d_i \neq 0:$ if it is zero we will get a zero row and $det = 0$)
     The result is a diagonal matrix: the product of the elements is the det. by property 3.a.

8. $det(A)$ is zero exactly when A is singular.

9. $det(AB) = det(A)det(B)$
   $\Rightarrow det(A^{-1}) = 1/det(A)$
   $\Rightarrow det(A^2) = det(A)^2$
   $\Rightarrow det(2A) = 2^n det(A)$

10. $det(A^T) = det(A)$
    proof: $|A| = |LU| \Rightarrow |A^T| = |U^T L^T| = |U^T||L^T|$