

In The Name of God.
The Merciful, The Compassionate.

Linear Transformations, Change of basis

notes on Gilbert Strang videos, Lecture 30-32

1 Linear Transformations

- $T(cV + dW) = cT(V) + dT(w)$
- $T(0) = 0$
- $T(x) = Ax$ is a linear transformation
- Coordinates come from a basis: coordinates of $v = c_1v_1 + c_2v_2 + \dots + c_nv_n$ are c_1, c_2, \dots
- Construct matrix A that represents linear transformation T :
 - $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$
 - Choose bases v_1, \dots, v_n for inputs.
 - Choose bases w_1, \dots, w_m for outputs.
- Rule to find A given bases v_1, \dots, v_n and w_1, \dots, w_m :
 - first column of A : write $T(v_1) = a_{11}w_1 + a_{21}w_2 + \dots + a_{m1}w_m$
 - second column of A : write $T(v_2) = a_{12}w_1 + \dots + a_{m2}w_m$
 - $A[\text{input coordinates}] = [\text{output coordinates}]$
- Example: $T = \frac{d}{dx}$
 - Input: $c_1 + c_2x + c_3x^2$, basis: $1, x, x^2$
 - Output: $c_2 + 2c_3x$, basis: $1, x$
 - $A \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} c_2 \\ 2c_3 \end{bmatrix}$
 - $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$