In The Name of God.
The Merciful, The Compassionate.

Markov Matrices and Fourier Series

notes on Gilbert Strang videos, Lecture 24

1 Markov matrices

1. all entries ≥ 0
2. all columns add to 1

- Steady state: $\rightarrow \lambda = 1$
- Two facts:
  1. $\lambda = 1$ is an eigenvalue, $x_1 \geq 0$
     - $A - 1I$ = matrix with all columns add to 0. So, it is singular and 1 is an eigenvalue for A
     - Also note that $\mathbf{1}^T$ is in $n((A-I)^T)$.
     - x_1 is in $n(A-I)$
  2. All other $|\lambda_i| < 1$
- $\Rightarrow u_k = A^k u_0 = c_1 {\lambda_1}^k x_1 + c_2 {\lambda_2}^k x_2 + \ldots \qquad \rightarrow \qquad c_1 x_1$ as $k \rightarrow \infty$
- x_1 part of u_0 is the steady state.

2 Projection with orthonormal basis $\{q_1, \ldots, q_n\}$

- Any $v = x_1 q_1 + x_2 q_2 + \ldots + x_n q_n$
- ${q_1}^T v = x_1, \ldots$
- $\begin{bmatrix} | & & | \\ q_1 & \ldots & q_n \\ | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = v$
- $Qx = v \Rightarrow x = Q^{-1} v = Q^T v$

3 Fourier series

- $f(x) = a_0 + a_1 cos(x) + b_1 sin(x) + a_2 cos(2x) + b_2 sin(2x) + \ldots$
- infinite, orthogonal bases (working in function space).
- bases are $1, cos(x), sin(x), cos(2x), sin(2x), \ldots$
- inner product of functions: $f^T g = \int f(x)g(x)\,\mathrm{d}x = \int_0^{2\pi} f(x)g(x)\,\mathrm{d}x \rightarrow$ orthonormal infinite bases
- $a_1 = ?$
- $\int_0^{2\pi} f(x)cos(x)dx = a_1 \int_0^{2\pi} (cos(x))^2\,\mathrm{d}x$
- $\Rightarrow a_1 = \frac{1}{\pi} \int_0^{2\pi} f(x)cos(x)\,\mathrm{d}x$