In The Name of God.
The Merciful, The Compassionate.

Complex Matrices, Fast Fourier Transform (FFT)

notes on Gilbert Strang videos, Lecture 26

1 Complex Matrices

- $z \in \mathbb{C}^n$, squared length of z is $\bar{z}^T z = z^H z$ (Hermitian).
- Inner product: $\bar{y}^T x = y^H x$
- Symmetric changes to Hermitian matrices: $\bar{A}^T = A$
- Orthogonal changes to Unitary Matrices: $I = Q^H Q$

2 Fourier Matrices

$$F_n = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & W & W^2 & \dots & w^{n-1} \\ 1 & W^2 & W^4 & \dots & W^{2(n-1)} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & W^{n-1} & W^{2(n-1)} & \dots & w^{(n-1)^2} \end{bmatrix}$$

$$(F_n)_{i,j} = W^{ij}, i, j = 0, \dots, n-1$$

$$W^n = 1 \Rightarrow W = e^{i2\pi/n} = \cos(\frac{2\pi}{n}) + i \sin(\frac{2\pi}{n})$$

- Columns are orthogonal
- n point Fourier transform. The inverse matrix is the inverse Fourier transform for n-D vectors.
- $F_n \to \frac{1}{\sqrt{n}} F_n$ has orthonormal columns.
- ${F_n}^H F_n = I$

3 Fast Fourier Transform

- ${W_{64}}^2 = W_{32}$

$$\left[F_{64}\right] = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & & & & \\ & W & & & \\ & & W^2 & & \\ & & & \ddots & \\ & & & & W^{31} \end{bmatrix}$$

- Complexity: $64^2 = 2(32)^2 + 32 \rightarrow \frac{n}{2}\log_2 n$