

In The Name of God.
The Merciful, The Compassionate.

Multivariate Gaussian

$p(\mathbf{x}) \sim \mathcal{N}(\mu, \Sigma)$:

$$p(\mathbf{x}) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp \left[-\frac{1}{2} (\mathbf{x} - \mu)^t \Sigma^{-1} (\mathbf{x} - \mu) \right], \quad (1)$$

where $\mathbf{x} \in \mathbb{R}^d$.

Now suppose \mathbf{y} is a linear transformation of \mathbf{x} :

$$\mathbf{y} = \mathbf{A}^t \mathbf{x} \quad (2)$$

where $\mathbf{A} \in \mathbb{R}^{d \times k}$ and $\mathbf{y} \in \mathbb{R}^k$. Then, $p(\mathbf{y}) \sim \mathcal{N}(\mathbf{A}^t \mu, \mathbf{A}^t \Sigma \mathbf{A})$

Now suppose $k = 1$ and \mathbf{A} is a unit vector \mathbf{a} . $y = \mathbf{a}^t \mathbf{y}$ represents the projection of \mathbf{x} onto a line in the direction of \mathbf{a} .

Example

If $\mathbf{x} = \{x_1, x_2\}$, then the projection onto x_1 is represented by $\mathbf{a}^t = [1, 0]$. If $y = \mathbf{a}^t \mathbf{x}$, then $y \sim \mathcal{N}(\mu' = \mathbf{a}^t \mu, \sigma' = \mathbf{a}^t \Sigma \mathbf{a})$ and

$$\begin{aligned} \mu' &= [1, 0] [\mu_1, \mu_2]^t = \mu_1 \\ \sigma' &= [1, 0] \begin{bmatrix} \sigma_{1,1} & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2} \end{bmatrix} [1, 0]^t = \sigma_{1,1} \end{aligned}$$