In The Name of God.
The Merciful, The Compassionate.

Eigenvalues, Eigenvectors, Diagonalization, and Powers of A

notes on Gilbert Strang videos, Lecture 21,22

1 Eigenvalues and eigenvectors

- $Ax = \lambda x$, eigenvectors parallel to x
  - If A is singular, then $\lambda = 0$ is an eigenvalue.
  - What are the x's, and λ's of projection matrix P? $\longrightarrow \lambda = 0, 1$
    - Any x in the projection plane : $Px = x$, $\lambda = 1$
    - Any $x \perp$ to the plane: $Px = 0$, $\lambda = 0$
  - How about permutation matrix?
    - $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
    - swap rows!
    - $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\lambda = 1$, $Ax = x$
    - $x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\lambda = -1$, $Ax = -x$
  - FACT: sum of λ's $= a_{11} + a_{22} + \ldots + a_{nn}$
  - Solve $Ax = \lambda x$: $(A - \lambda I)x = 0$
    - $\Rightarrow det(A - \lambda I) = 0$. Find λ's first.
    - For 2×2 matrices *trace* is the linear coefficient, and *det* is constant of degree 2 equation for eigenvalues.
    - For degenerate equations (e.g., when A is upper triangular), the eigenvalues, and eigenvectors are repeated. A double or higher order degeneracy will cause a lost in one(or more) equations. We have infinite number of ways to choose eigenvectors. The most traditional way (almost in 2^{nd} order degeneracy) is to choose the first one to be the trivial normalized one. and the other vector to be orthogonal to the first one.