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Introduction

This exercise explores polynomial regression, a form of regression analysis where the relationship between the independent variable (X) and the dependent variable (y) is modeled as an (n)th degree polynomial. We will create a synthetic dataset, train models with varying degrees of polynomials, and evaluate their performance on different test sets.

Task

We want to use polynomial regression to model the relationship between the independent variable (X) and the dependent variable (y).

Dataset

We will create a synthetic dataset using the given equation. We will use this dataset to train polynomial regression models with varying degrees of polynomials. We will also create different test sets to evaluate the models' performance.

Objective

We want to find the degree of the polynomial that best fits the relationship between the independent variable (X) and the dependent variable (y). We will use the mean squared error (MSE) to evaluate the models' performance on different test sets.

Plan

1. Create a synthetic dataset using the given equation.
2. Split the dataset into training and testing sets.

3. Train polynomial regression models with varying degrees of polynomials.
4. Evaluate the models' performance on different test sets using the mean squared error (MSE).
5. Find the degree of the polynomial that best fits the relationship between the independent variable (X) and the dependent variable (y).

Implementation (100 Points)

We will start by creating a synthetic dataset using the given equation.

1. Create a synthetic dataset (20 Points)

First, define a function to generate a dataset with 1000 samples in the range of **-4 to -1** from a polynomial with added noise. Assume that the noise is from a normal distribution and has a mean of 0 and std of 5.

The relationship between the independent variable (X) and the dependent variable (y) is given by the following equation:

[y = 0.1X^5 + X^4 - 2X^3 + 7X^2 - 9X + 3]

Generate the independent variable (X) using the `numpy` library and then calculate the dependent variable (y) using the given equation.

In [1]:
```
import numpy as np

def generate_dataset(range_start, range_end, num_samples, noise_std=5.0):
    X = np.random.uniform(range_start, range_end, num_samples)
    y = 0.1 * X**5 + X**4 - 2 * X**3 + 7 * X**2 - 9 * X + 3
    y += np.random.normal(0, noise_std, num_samples)
    return X, y
```

In [2]:
```
X, y = generate_dataset(-4, -1, 1000)
print(len(X))
print(len(y))
```

```
1000
1000
```

2. Splitting the Dataset (5 Points)

Split the dataset into training and two test sets with the same length, ensuring one test set is in the range of the training data and the other is in the **range of 5 to 7.**

In [3]:
```
from sklearn.model_selection import train_test_split

degree = 5
X_train, X_test_in_range, y_train, y_test_in_range = train_test_split(X, y, test_size=0.2, ranc
X_test_out_of_range, y_test_out_of_range = generate_dataset(5, 7, 200)
print(len(X_test_in_range), len(X_test_out_of_range))
print(len(y_test_in_range), len(y_test_out_of_range))
```

```
200 200
200 200
```

3. Polynomial Regression Training (30 Points)

Train polynomial regression models of varying degrees from degree = 2 to degree = 7. You can use the `preprocessing` , `linear_model` , and `pipeline` classes of the `sklearn` library.

In [4]:
```
from sklearn.preprocessing import PolynomialFeatures
from sklearn.linear_model import LinearRegression
from sklearn.pipeline import make_pipeline

def train_polynomial_regression(X, y, degree):
    polynomial_features = PolynomialFeatures(degree=degree)
    linear_regression = LinearRegression()
    model = make_pipeline(polynomial_features, linear_regression)
    model.fit(X.reshape(-1, 1), y)

    return model
```

In [5]:
```
models = {}
# TODO: Put the models with different degrees in the models dict
for d in range(2, degree + 3):
    models[d] = train_polynomial_regression(X_train, y_train, d)
```

4. Model Evaluation (15 Points)

Evaluate the models on both test sets using MSE. You can use the `mean_squared_error` from `sklearn.metrics` .

In [6]:
```python
from sklearn.metrics import mean_squared_error

def evaluate_models(models, X_test, y_test):
    losses = {}
    for d, model in models.items():
        y_pred = model.predict(X_test.reshape(-1, 1))
        losses[d] = mean_squared_error(y_test, y_pred)
    return losses
```

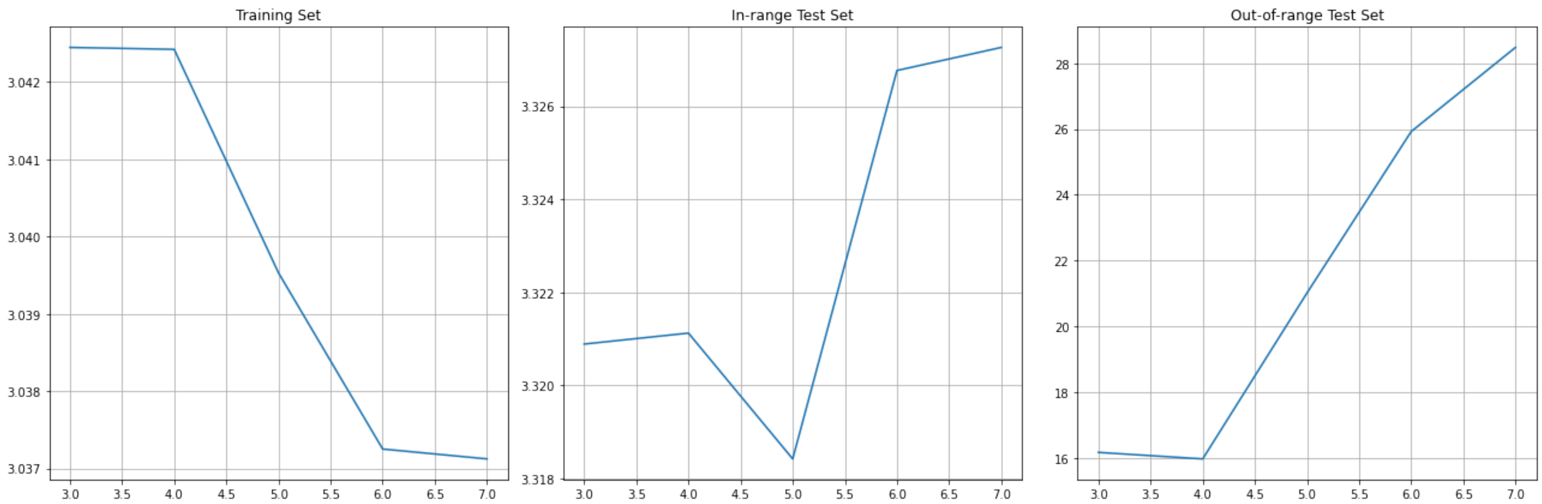
In [7]:
```python
losses_train = evaluate_models(models, X_train, y_train)
losses_in_range = evaluate_models(models, X_test_in_range, y_test_in_range)
losses_out_of_range = evaluate_models(models, X_test_out_of_range, y_test_out_of_range)
```

5. Plotting Model Scores (20 Points)

Now to evaluate the performance of the polynomial regression models, plot the **logarithm** of losses of the training and two test sets based on the model degree varying from 3 to 7. Train the model a few times to get a sense of how the results can change.

In [8]:

```
from matplotlib import pyplot as plt

log_losses_train = {degree: np.log(loss) for degree, loss in losses_train.items()}
log_losses_in_range = {degree: np.log(loss) for degree, loss in losses_in_range.items()}
log_losses_out_of_range = {degree: np.log(loss) for degree, loss in losses_out_of_range.items()

fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(18, 6))

degrees = range(3, 8)
ax1.plot(degrees, [log_losses_train[d] for d in degrees])
ax1.set_title('Training Set')
ax1.grid(True)

ax2.plot(degrees, [log_losses_in_range[d] for d in degrees])
ax2.set_title('In-range Test Set')
ax2.grid(True)

ax3.plot(degrees, [log_losses_out_of_range[d] for d in degrees])
ax3.set_title('Out-of-range Test Set')
ax3.grid(True)

plt.tight_layout()
plt.show()
```



Conclusion (10 Points)

Now print out the losses on each set here.

In [9]:
```
print("Training:")
for degree, loss in losses_train.items():
    print(f"{degree}: {loss}")

print("\nIn-range:")
for degree, loss in losses_in_range.items():
    print(f"{degree}: {loss}")

print("\nOut-of-range:")
for degree, loss in losses_out_of_range.items():
    print(f"{degree}: {loss}")
```

```
Training:
2: 28.87281006974992
3: 20.956463281315077
4: 20.95594849022611
5: 20.895476043570234
6: 20.847875258090724
7: 20.84521608598634

In-range:
2: 34.55560106764296
3: 27.68496506397417
4: 27.69148249493489
5: 27.616747583263106
6: 27.84815685681635
7: 27.861953172673797

Out-of-range:
2: 247482.04042456188
3: 10483498.52382846
4: 8603541.094693016
5: 1338022432.5033252
6: 181935180903.09692
7: 2349037160869.303
```

With comparing the loss of **training set and the in-range test set** and also **in-range test set and the out-of-range test set**, specially in higher degrees, what we can conclude about the bias - variance trade off?

Answer:

```
The loss function exhibits an upward trend as the degree increases in case
s of out-of-range data. As anticipated, the disparity in loss between the
training set and the in-range test set remains minimal. This observation h
ighlights that lower degrees exhibit high bias and low variance for out-of
-range test sets, while higher degrees manifest both high bias and high va
riance. Overfitting on the training set results in reduced generalization
```

Expected to explain overfitiing on the training set and also the face that model has less generalization to unseen data.