95503統資軟體課程講義

maple在統計上的應用

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統計中的各項機率分配用MAPLE來做探討

一.連續機率分配中包含了

1.連續均勻分配(Uniform)

2.指數分配(Exponetial)

3.Gamma分配

4.韋伯分配(Weibull)

5. 常態分配

6.對數常態分配(Lognormal)

7.Beta分配

8.對數邏輯分配(Log-logistic)

統計中的各項機率分配用MAPLE來做探討

二.不連續機率分配中包含了

1.白努利分配(Bernoulli)

2.不連續均勻分配(Discrete Uniform)、

3.二項分配(Binomial)、

4.幾何分配(Geometric)、

5.負二項分配(Negative binomial)、

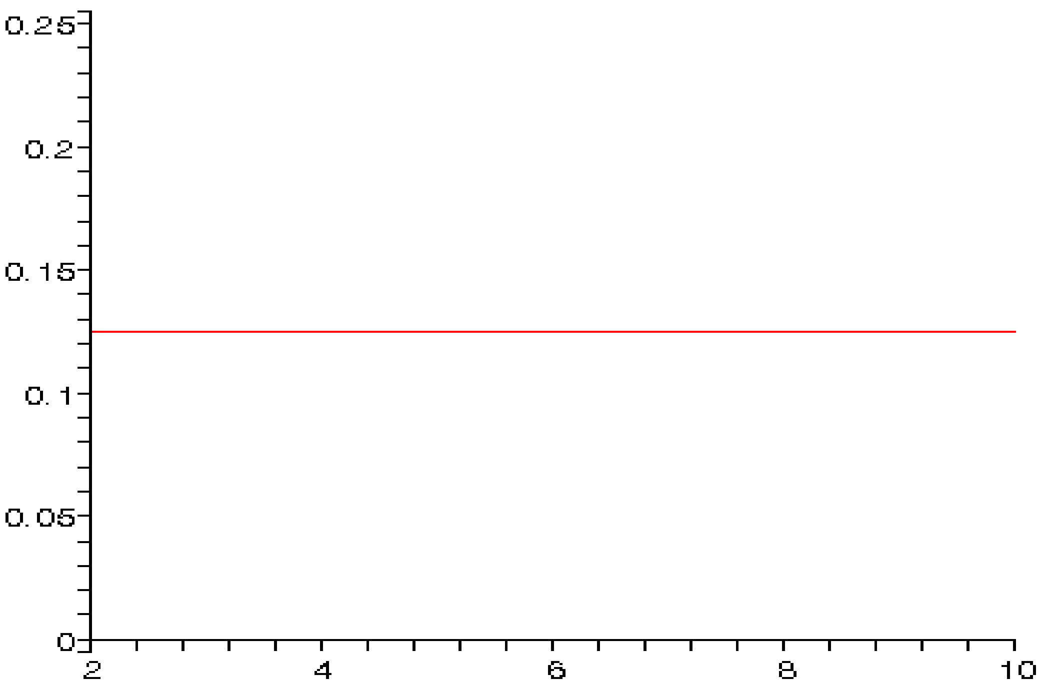
6.波氏分配(Poisson)。

1.均匀分配

```
> f:=x->1/(b-a);        #定義均勻分配的機率密度函數
  f:=x->1/(10-2);       #令a=2，b=10
  plot(f,2..10);        #畫出函數圖形，且令x的範圍從2到10
```

$$f := x \rightarrow \frac{1}{b - a}$$

$$f := x \rightarrow \frac{1}{8}$$



1.均勻分配

:

```
> Ex:=int(x*f(x),x=a..b);                    #期望值
```

$$Ex := \frac{b^2 - a^2}{2\,(b - a)}$$

:

```
> Vx:=simplify(int(x^2*f(x),x=a..b)-Ex^2);   #變異數
>
```

$$Vx := \frac{1}{12}\,b^2 - \frac{1}{6}\,a\,b + \frac{1}{12}\,a^2$$

:

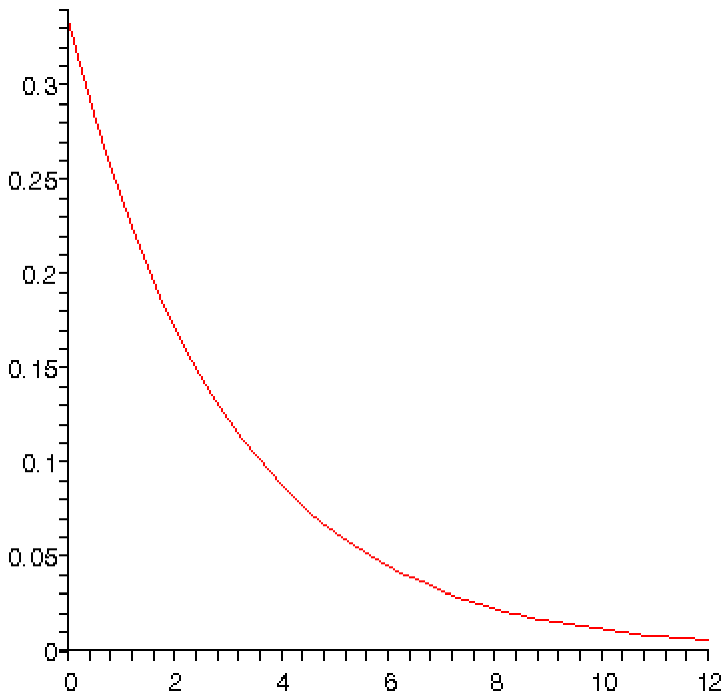
```
> Mxt:=int(f(x)*exp(x*t),x=a..b);            #  動差生成函數
```

$$Mxt := -\frac{e^{(a\,t)} - e^{(b\,t)}}{(b - a)\,t}$$

: .

2.指數分配(Exponential Distribution) example

```
> expf:=x->(1/beta)*exp(-x/beta);      #定義指數分配的機率密度函數
  beta:=3;                             #令β等於3
  plot(expf,0..4*beta);                #畫出函數圖形，且令x的範圍從0到4倍平均數
```



$$expf := x \to \frac{e^{\left(-\frac{x}{\beta}\right)}}{\beta}$$

$$\beta := 3$$

2.指數分配(Exponential Distribution)

```
> beta:=3;
  Ex:=int(x*expf(x),x=0..infinity);                    #期望值
```

$$\beta := 3$$

$$Ex := 3$$

```
=
> Vx:=simplify(int(x^2*expf(x),x=0..infinity)-Ex^2);     #變異數
>
```

$$Vx := 9$$

```
=
> beta:='beta':
  Mxt:=int(expf(x)*exp(x*t),x=0..infinity);               #  動差生成函數
```

$$Mxt := \lim_{x \to \infty} \left(\frac{e^{\left(\frac{x\,(-1+t\,\beta)}{\beta}\right)} - 1}{-1 + t\,\beta} \right)$$

```
=
>
```

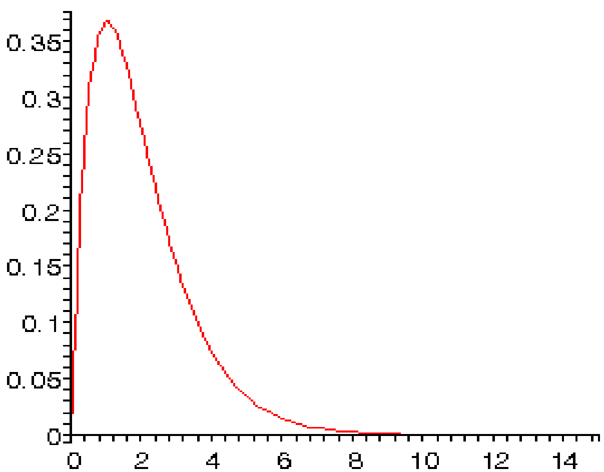
3.Gamma分配 example

```
> f:=x->1/(GAMMA(alpha)*beta^alpha)*x^(alpha-1)*exp(-x/beta);     #定義GAMMA分配的機率密度函數
  alpha:=2;beta:=1;     #令α等於2、β等於1
  plot(f,0..15);     #畫出函數圖形，且令x的範圍從0到15
```

$$f := x \rightarrow \frac{x^{(\alpha-1)}\, e^{\left(-\frac{x}{\beta}\right)}}{\Gamma(\alpha)\,\beta^{\alpha}}$$

$$\alpha := 2$$

$$\beta := 1$$



3.Gamma分配

```
>
  Ex:=int(x*f(x),x=0..infinity);                 #期望值
```

$$Ex := \left(\frac{1}{\beta}\right)^{(-\alpha)} \beta^{(-\alpha+1)} \alpha$$

```
> Vx:=simplify(int(x^2*f(x),x=0..infinity)-Ex^2);     #變異數
>
```

$$Vx := \left(\frac{1}{\beta}\right)^{(-\alpha)} \beta^{(2-\alpha)} \alpha^2 + \left(\frac{1}{\beta}\right)^{(-\alpha)} \beta^{(2-\alpha)} \alpha - \left(\frac{1}{\beta}\right)^{(-2\alpha)} \beta^{(-2\alpha+2)} \alpha^2$$

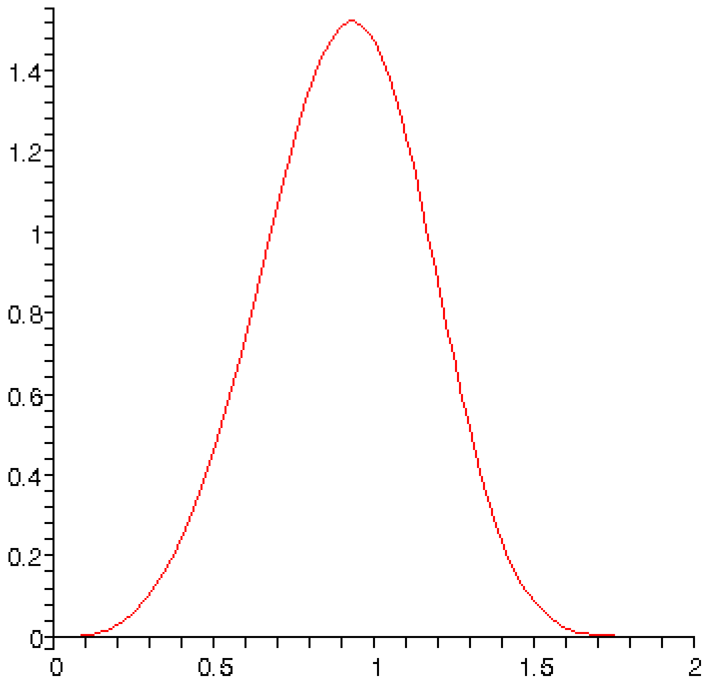
```
>
  Mxt:=int(f(x)*exp(x*t),x=0..infinity);          #  動差生成函數
```

$$Mxt := \beta^{(-\alpha)} (-t)^{(-\alpha)} \left(1 - \frac{1}{\beta t}\right)^{(-\alpha)}$$

4.韋伯分配(Weibull) example

```
> f:=x->alpha*beta^(-alpha)*x^(alpha-1)*exp(-(x/beta)^alpha);        #定義韋伯 分配的機率密度函數
  alpha:=4;beta:=1;                    #令α等於4、β等於1

  plot(f,0..2);                        #畫出函數圖形，且令x的範圍從0到2
```



$$f := x \to \alpha\, \beta^{(-\alpha)}\, x^{(\alpha-1)}\, e^{\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right)}$$

$$\alpha := 4$$

$$\beta := 1$$

4.韋伯分配(Weibull)

```
>
  Ex:=int(x*f(x),x=0..infinity);                    #期望值
```

$$Ex := \left(\frac{1}{\beta}\right)^{\left(-\frac{1}{\alpha}\right)} \Gamma\left(1+\frac{1}{\alpha}\right)$$

```
> Vx:=simplify(int(x^2*f(x),x=0..infinity)-Ex^2);    #變異數
>
```

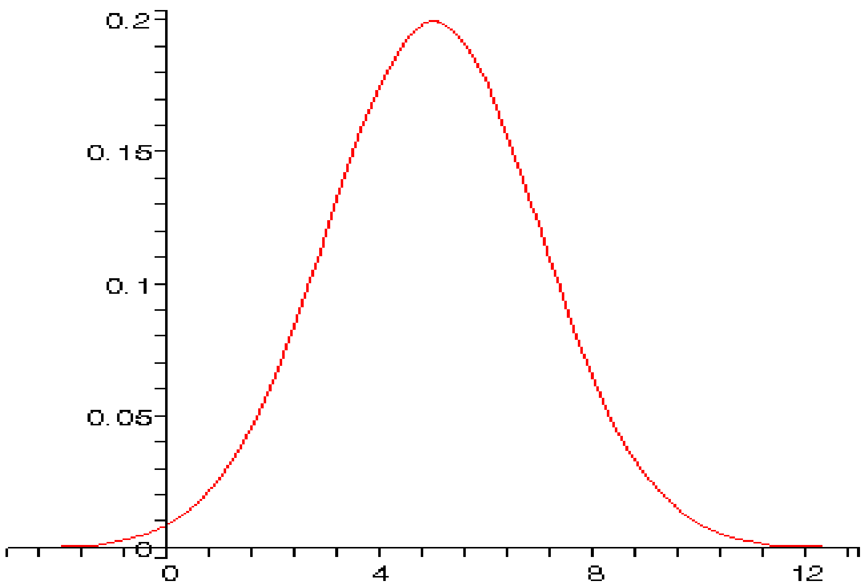
$$Vx := \left(\frac{1}{\beta}\right)^{\left(-\frac{2}{\alpha}\right)} \left(\Gamma\left(\frac{2+\alpha}{\alpha}\right) - \Gamma\left(\frac{1+\alpha}{\alpha}\right)^{2}\right)$$

```
>
  Exn:=int(f(x)*x^n,x=0..infinity);                 #  動差生成函數
```

$$Exn := \left(\frac{1}{\beta}\right)^{\left(-\frac{n}{\alpha}\right)} \Gamma\left(\frac{\alpha-1+n}{\alpha}+\frac{1}{\alpha}\right)$$

5. 常態分配
example

```
> f:= x -> exp( -((x-mu)^2)/ (2*sigma^2) )/(sigma*sqrt(2*Pi)); # 定義常 態分配的機率密度函數
  mu:=5;sigma:=2; # 令 μ 等於5、σ 等於2
  plot(f,-3..13); # 畫出函數圖形，且令x的範圍從-3到13
```



$$f := x \to \frac{e^{\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)}}{\sigma\sqrt{2\pi}}$$

$$\mu := 5$$

$$\sigma := 2$$

5. 常態分配

```
> mu:=0;sigma:=1;
  Ex:=int(x*f(x),x=-infinity..infinity);                    #期望值
```

$$\mu := 0$$

$$\sigma := 1$$

$$Ex := 0$$

=

```
> Vx:=simplify(int(x^2*f(x),x=-infinity..infinity)-Ex^2);      #變異數
>
```

$$Vx := 1$$

=

```
> mu:='mu';sigma:='sigma';
  Mxt:=int(f(x)*exp(x*t),x=-infinity..infinity);             #  動差生成函數
```

$$\mu := \mu$$

$$\sigma := \sigma$$

$$Mxt := \begin{cases} \mathbf{e}^{\left(\frac{1}{2} t \left(2 \mu + t \sigma^2\right)\right)} \mathrm{csgn}(\overline{\sigma}) & \mathrm{csgn}\left(\overline{\sigma}^{2}\right) = 1 \\ \infty & otherwise \end{cases}$$

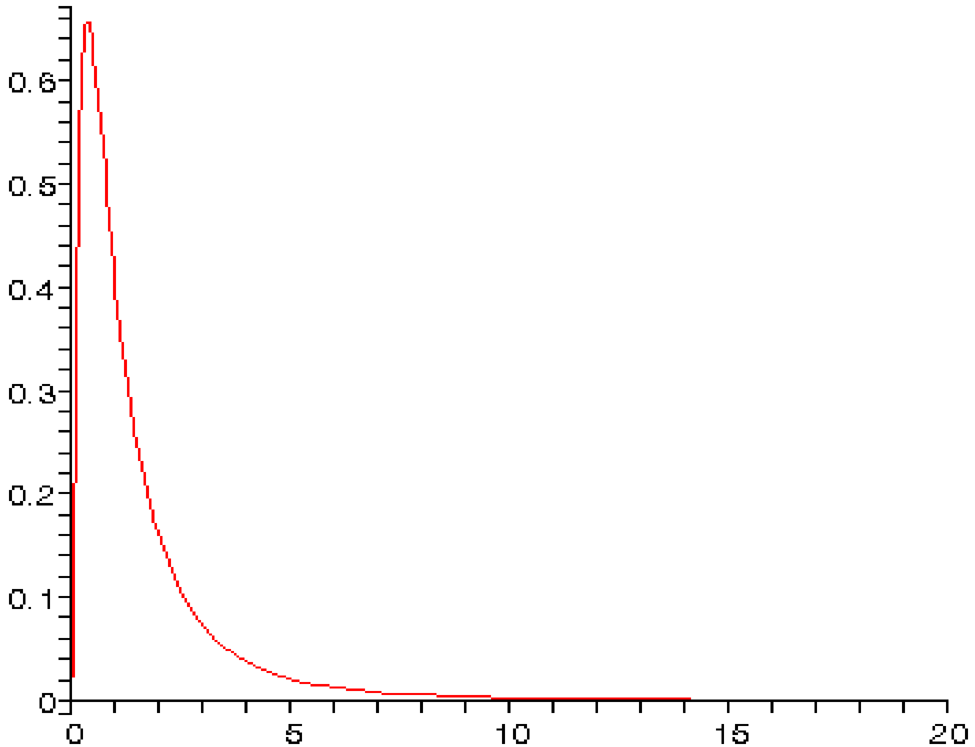
6.對數常態分配(Lognormal) example

```
> f:=x->exp( (-(ln(x)-mu)^2)/(2*sigma^2) )/(x*sqrt(2*Pi*sigma^2));          # 定義 對數常態分配的機率密度函數
  mu:=0;sigma:=1;                                                           # 令μ等於0、σ等於1
  plot(f,0..20);                                                            # 畫出函數圖形，且令x的範圍從0到20
```

$$f := x \to \frac{e^{\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right)}}{x\sqrt{2\pi\sigma^2}}$$

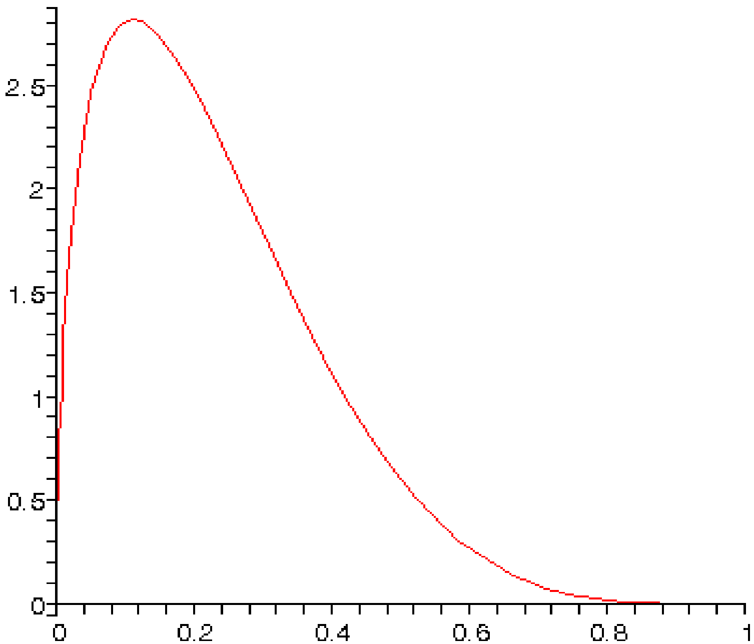
$$\mu := 0$$

$$\sigma := 1$$



7. Beta 分配
example

```
> f:=x->GAMMA(alpha+beta)/(GAMMA(alpha)*GAMMA(beta))*x^(alpha-1)*(1-x)^(beta-1); # 定義Beta分配的機率密度函數
  alpha:=1.5;beta:=5; # 令α等於1.5、β等於5
  plot(f,0..1); # 畫出函數圖形，且令x的範圍從0到1
```



$$f := x \to \frac{\Gamma(\alpha+\beta)\, x^{(\alpha-1)}\, (1-x)^{(\beta-1)}}{\Gamma(\alpha)\, \Gamma(\beta)}$$

$$\alpha := 1.5$$

$$\beta := 5$$

7. Beta 分配

```
>
  Ex:=int(x*f(x),x=0..1);                #期望值
```

$$Ex := \frac{\alpha}{\alpha+\beta}$$

```
> Vx:=simplify(int(x^2*f(x),x=0..1)-Ex^2);      #變異數
>
```

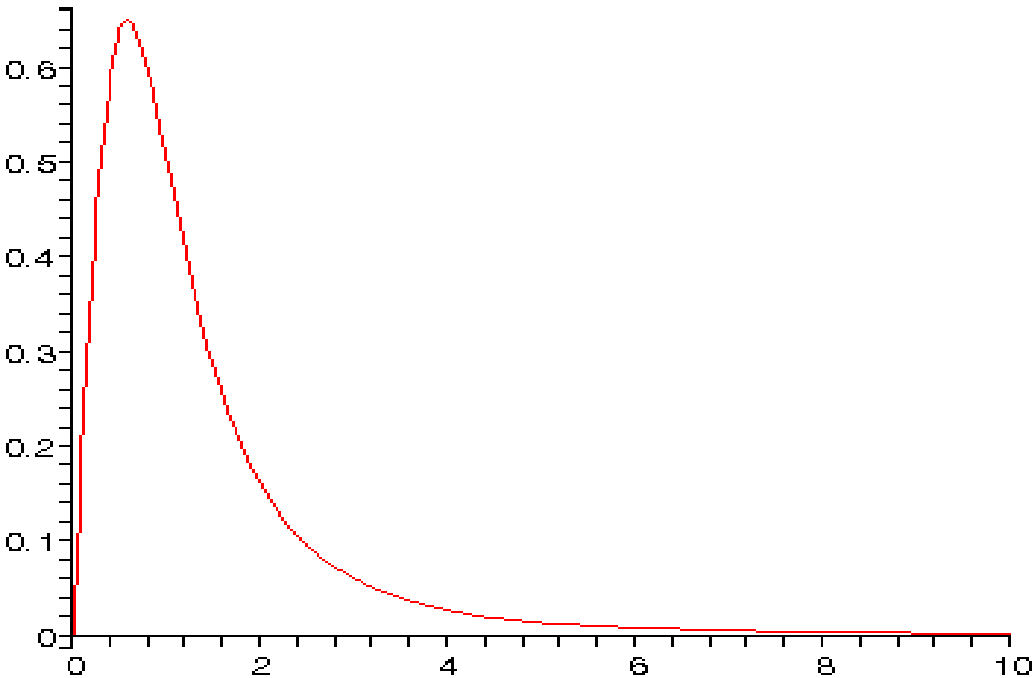
$$Vx := \frac{\alpha\,\beta}{(\alpha+\beta)^2\,(\beta+1+\alpha)}$$

```
> Mxt:=int(f(x)*exp(x*t),x=0..1);            #  動差生成函數
```

$$Mxt := -\frac{(-t)^{(-\alpha)}\,t\,\Gamma(\alpha+\beta)\left(-\frac{(-t)^{\alpha}\,(t+\alpha+\beta)\,LaguerreL(-\alpha,\alpha+\beta,t)\,\pi}{t\,\sin(\pi\,\alpha)\,\Gamma(\beta+1)}+\frac{(-t)^{\alpha}\,LaguerreL(-\alpha,\beta+1+\alpha,t)\,\pi}{\sin(\pi\,\alpha)\,\Gamma(\beta+1)}\right)}{\Gamma(\alpha)}$$

8.對數邏輯分配(Log-logistic) example

```
> f:=x->alpha/(beta*(1+(x/beta)^alpha)^2)*(x/beta)^(alpha-1);    #定義對數邏輯分配的機率密度函數
  alpha:=2;beta:=1;                                              #令α等於2、β等於1
  plot(f,0..10);                                                 # 畫出函數圖形，且令x的範圍從0到10
```



$$f := x \to \frac{\alpha \left(\frac{x}{\beta}\right)^{(\alpha - 1)}}{\beta \left(1 + \left(\frac{x}{\beta}\right)^{\alpha}\right)^2}$$

$$\alpha := 2$$

$$\beta := 1$$

1.白努利分配(Bernoulli) example

```
> binom:=proc(n::integer,p::float,x::integer)local q;q:=1-p;binomial(1,x)*p^x*q^(1-x);
end proc; #定義百努力分配的機率密度函數
binom(1,0.7,0); # 令n等於1、p等於0.7
seq(binom(1,0.7*i),i=0..1);
p:=i->[[i-0.5,0],[i-0.5,binom(1,0.7,i)],[i+0.5,binom(1,0.7,i)],[i+0.5,0]]:
p(0);
with(plottools):
l := polygon(p(0), color=pink, linestyle=1,
thickness=1),polygon(p(1), color=green, linestyle=1,
thickness=1);
plots[display](l); #出函數圖形，且令x的範圍從0到1
```

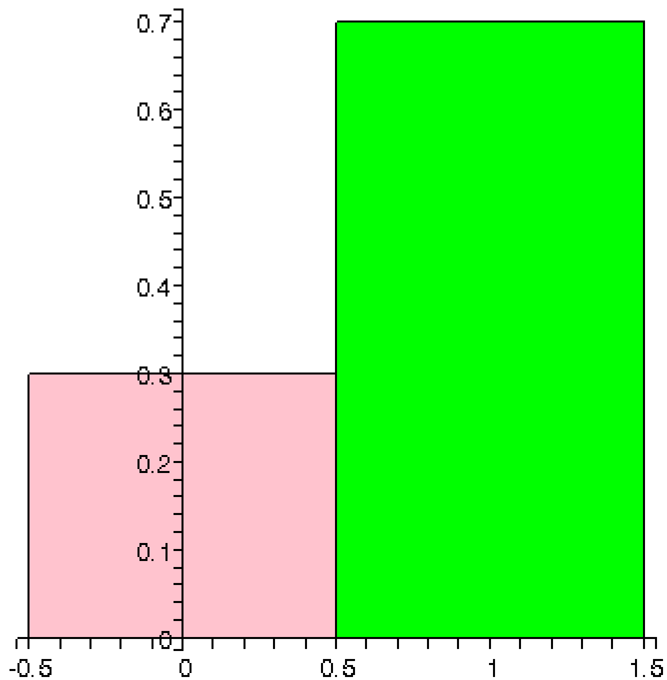
$$binom := \mathbf{proc}(n{::}integer, p{::}float, x{::}integer)\ \mathbf{local}\ q;\ q := 1 - p;\ \mathrm{binomial}(1, x)*p\text{^}x*q\text{^}(1 - x)\ \mathbf{end\ proc}$$

$$0.3$$

1.白努利分配(Bernoulli) example

[[-.5, 0], [-.5, 0.3], [0.5, 0.3], [0.5, 0]]

l := *POLYGONS*([[-.5, 0.], [-.5, 0.3], [0.5, 0.3], [0.5, 0.]], *COLOUR*(*RGB*, 1, 0.752941176, 0.7960784314), *THICKNESS*(1), *LINESTYLE*(1)),
POLYGONS([[0.5, 0.], [0.5, 0.7], [1.5, 0.7], [1.5, 0.]], *COLOUR*(*RGB*, 0., 1.00000000, 0.), *THICKNESS*(1), *LINESTYLE*(1))



1.白努利分配(Bernoulli)

```
> restart;
  f:=x->p^x*(1-p)^(1-x);                    #PDF
```

$$f := x \to p^x (1-p)^{(1-x)}$$

:

```
> Ex:=sum(k*f(k),k=0..1);                   #期望值
```

$$Ex := p$$

:

```
> Vx:=sum(k^2*f(k),k=0..1)-Ex^2;            #變異數
```

$$Vx := p - p^2$$

:

```
> Mxt:=sum(exp(k*t)*f(k),k=0..1);           #動差生成函數
```

$$Mxt := 1 - p + e^t p$$

:

2. 不連續均等分配（Discrite uniform）example

:

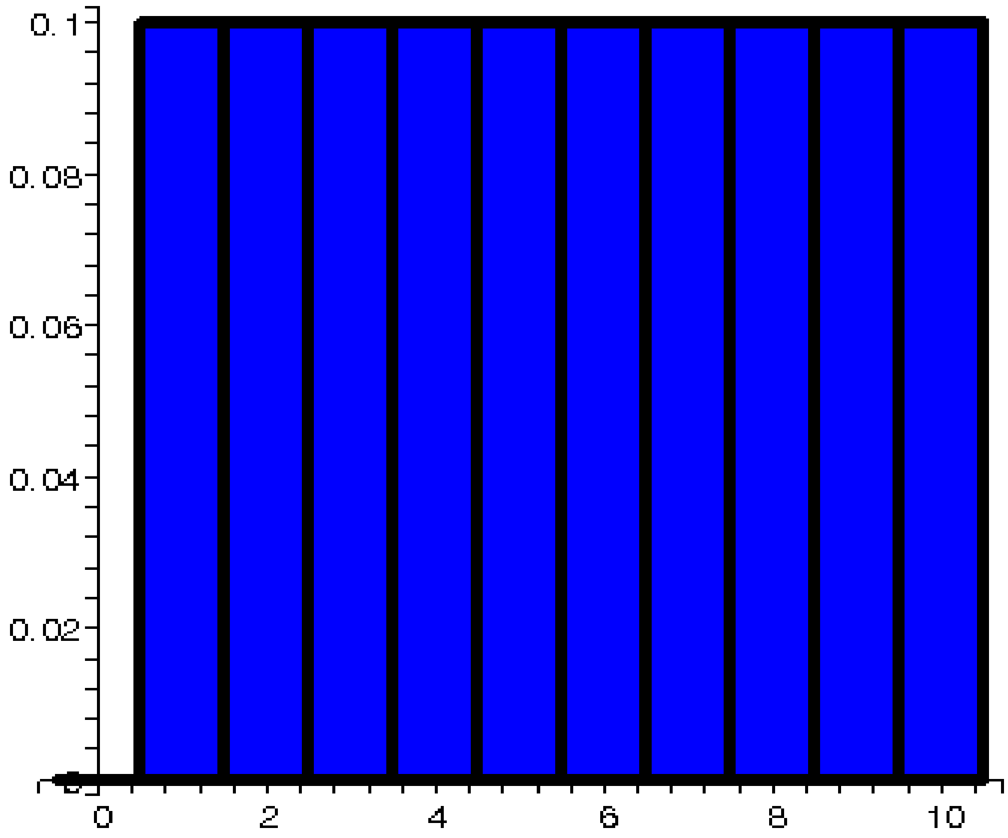
```
> uniform:=proc(i::integer,j::integer,x ::integer);if  i<=x and x<=j then 1/j-i+1;end if;end;
  #定義二項分配的機率密度函數
  seq(uniform(1,10,x),x=0..10);
  # 令i等於1、j等於10
  pl:=x->[[x-0.5,0],[x-0.5, uniform(1,10,x)],[x+0.5,uniform(1,10,x)],[x+0.5,0]]:
  pl(0);
  with(plottools):
  l := seq(polygon(pl(x), color=blue, linestyle=1, thickness=3),x=0..10):
  plots[display](l); #畫出函數圖形，且令x的範圍從0到1
```

$$uniform := \mathbf{proc}(i{::}integer, j{::}integer, x{::}integer)\ \mathbf{if}\ i <= x\ \mathbf{and}\ x <= j\ \mathbf{then}\ (1)/j - i + 1\ \mathbf{end\ if\ end\ proc}$$

$$\frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}$$

$$[[-.5, 0], [-.5], [0.5], [0.5, 0]]$$

2. 不連續均等分（Discrete uniform）example



2. 不連續均等分（Discrite uniform）

```
>
  Ex:=simplify(sum(k*f(k),k=1..N));                #期望值
```

$$Ex := \frac{1}{2}N + \frac{1}{2}$$

```
> Vx:=simplify(sum(k^2*f(k),k=0..N)-Ex^2);          #變異數
```

$$Vx := \frac{1}{12}N^2 - \frac{1}{12}$$

```
> Mxt:=sum(exp(k*t)*f(k),k=0..N);                  #動差生成函數
```

$$Mxt := \frac{e^{((N+1)t)}}{N\left(e^t - 1\right)} - \frac{1}{N\left(e^t - 1\right)}$$

3.二項分配(Binomial distribution) example

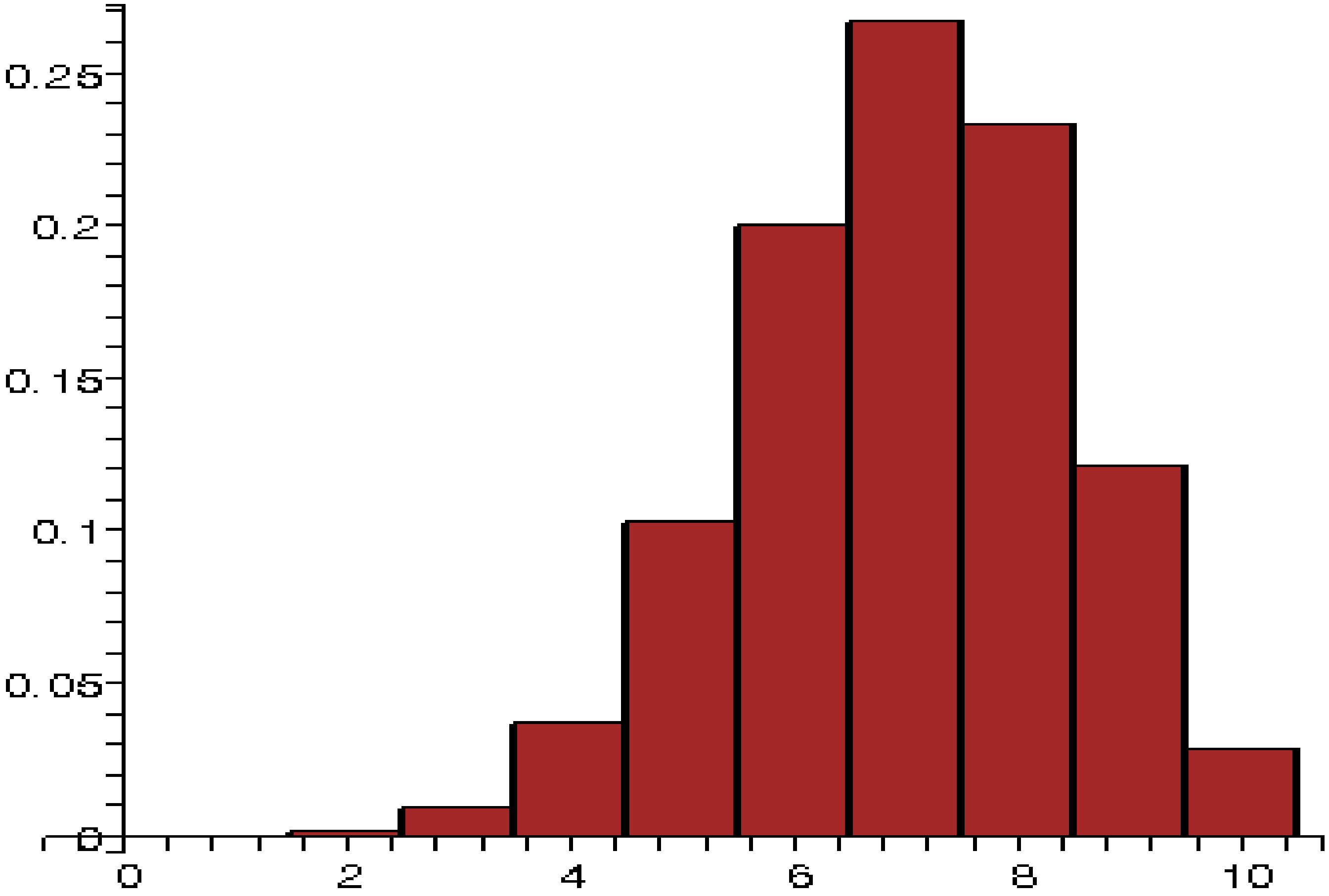
```
> binom:=proc(n::integer,p::float,x::integer)local q;q:=1-p;
   binomial(10,x)*p^x*q^(10-x);
  end proc: #定義對二項分配的機率密度函數
  binom(10,0.7,0);
  seq(binom(10,0.7,i),i=0..10):
  p:=i->[[i-0.5,0],[i-0.5,binom(1,0.7,i)],[i+0.5,binom(1,0.7,i)],[i+0.5,0]]:
  p(0);
  with(plottools):
  l := seq(polygon(p(i), color=brown, linestyle=1, thickness=2),i=0..10): #畫出函數圖形，且令x的範圍從0到10

  plots[display](l); #第一種畫法
```

0.0000059049

[[-.5, 0], [-.5, 0.0000059049], [0.5, 0.0000059049], [0.5, 0]]

3.二項分配(Binomial distribution) example



3.二項分配(Binomial distribution)

```
>
  Ex:=simplify(sum(k*f(k),k=0..n));            #期望值
```

$$Ex := n\,p\,(1-p)^{n}\left(-\frac{1}{-1+p}\right)^{n}$$

```
:
> Vx:=sum(k^2*f(k),k=0..n)-Ex^2;            #變異數
```

$$Vx := -\frac{(1-p)^{(n-1)}\,n!\left(-\frac{1}{-1+p}\right)^{n} p\,(-1+p)\,(n\,p+1-p)}{(n-1)!} - n^{2}\,p^{2}\,\left((1-p)^{n}\right)^{2}\left(\left(-\frac{1}{-1+p}\right)^{n}\right)^{2}$$

```
:
> Mxt:=sum(exp(k*t)*f(k),k=0..n);            #動差生成函數
```

$$Mxt := (1-p)^{n}\left(-\frac{\mathrm{e}^{t}\,p-p+1}{-1+p}\right)^{n}$$

4.幾何分配(Geometric) example

```
> geom:=proc(p :: float, x :: integer)
  local q;
  q:=1-p;
  p*q^(x-1);
  end proc; #定義對幾何分配的機率密度函數
  geom(0.25,5);#令p=0.25,x=5代入
  seq(geom(0.25,i),i=1..15);#產生15個幾何分配
  p:=i -> [[i-0.5,0],[i-0.5,geom(0.5,i)],[i+0.5,geom(0.5,i)],[i+0.5,0]]:
  p(0);
  with(plottools):
  l:=seq(polygon(p(i), color=gray, linestyle=1,thickness=2),i=1..15):
  plots[display](l);#第一種畫法
```

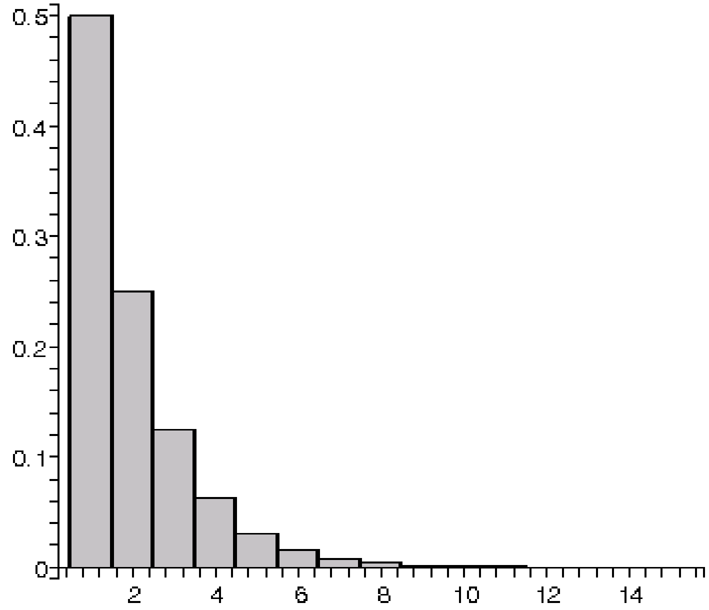
4.幾何分配(Geometric) example

geom := **proc**(*p*::*float*, *x*::*integer*) **local** *q*; *q* := 1 - *p*; *p***q*^(*x* - 1) **end proc**

0.0791015625

0.25, 0.1875, 0.140625, 0.10546875, 0.0791015625, 0.05932617188, 0.04449462890, 0.03337097168, 0.02502822875, 0.01877117157, 0.01407837868, 0.01055878401, 0.007919088005, 0.005939316005, 0.004454487002

[[-.5, 0], [-.5, 1.000000000], [0.5, 1.000000000], [0.5, 0]]



4.幾何分配(Geometric)

\>

Ex:=simplify(sum(k*f(k),k=0..infinity)); #期望值

$$Ex := \frac{1}{p}$$

\> Vx:=sum(k^2*f(k),k=0..infinity)-Ex^2; #變異數

$$Vx := \frac{2-p}{p^2} - \frac{1}{p^2}$$

\> Mxt:=sum(exp(k*t)*f(k),k=0..infinity); #動差生成函數

$$Mxt := -\frac{p}{(-1+p)\left(1+e^t(-1+p)\right)}$$

5.負二項分配(Negative binomial) example

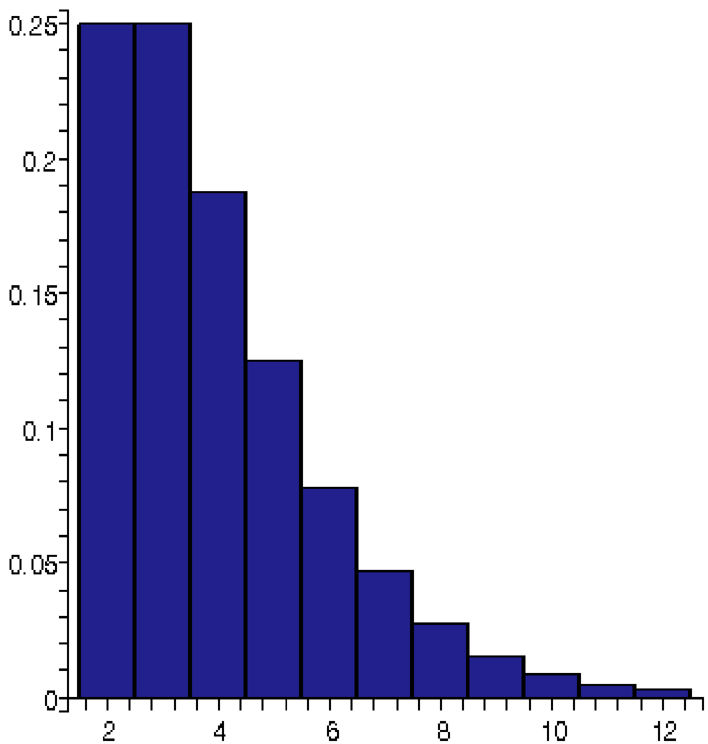
```
:
> negbin:=proc(r :: integer, p :: float, x :: integer)
  local q;
  q:=1-p;
  binomial(x-1,x-r)*p^r*q^(x-r);
  end proc: #定義負二項分配的機率密度函數
  negbin(2,0.5,2);#令r=2，p=0.5，x=2代入
  seq(negbin(2,0.5,i), i=2..12);#產生11個函數
  p:=i -> [[i-0.5,0],[i-0.5,negbin(2,0.5,i)],[i+0.5,negbin(2,0.5,i)],[i+0.5,0]]:
  p(0);
  with(plottools):
  l:=seq(polygon(p(i), color=navy, linestyle=1,thickness=2),i=2..12):
  plots[display](l);#第一種畫法
```

5.負二項分配(Negative binomial) example

0.25

0.25, 0.250, 0.1875, 0.12500, 0.078125, 0.0468750, 0.02734375, 0.015625000, 0.0087890625, 0.00488281250, 0.002685546875

[[-.5, 0], [-.5, -1.000000000], [0.5, -1.000000000], [0.5, 0]]



5.負二項分配(Negative binomial)

```
> restart;
  f:=x->(r+x-1)!/(x!*(r-1)!)*p^r*(1-p)^x;                    #PDF
>
```

$$f := x \to \frac{(r+x-1)!\, p^r (1-p)^x}{x!\,(r-1)!}$$

```
>
  Ex:=simplify(sum(k*f(k),k=0..infinity));                   #期望值
```

$$Ex := -\frac{r(-1+p)}{p}$$

```
> Vx:=simplify(sum(k^2*f(k),k=0..infinity)-Ex^2);            #變異數
```

$$Vx := -\frac{r(-1+p)}{p^2}$$

```
> Mxt:=sum(exp(k*t)*f(k),k=0..infinity);                     #動差生成函數
```

$$Mxt := \frac{p^r}{\left(1+e^t p - e^t\right)^r}$$

6.卜瓦松分佈(Poisson) example

```
> poi:=proc(lambda :: positive, x :: integer)
  exp(1)^(-lambda)*lambda^x/x!;
  end proc; #定義poisson分配的機率密度函數
  poi(2,0);#令λ=2,x=0代入
  seq(poi(2,i), i=0..10);#產生10個函數
  p:=i -> [[i-0.5,0],[i-0.5,poi(2,i)],[i+0.5,poi(2,i)],[i+0.5,0]]:
  p(0);
  with(plottools):
  l:=seq(polygon(p(i), color=white, linestyle=1,thickness=2),i=0..10):
  plots[display](l);#第一種畫法
```

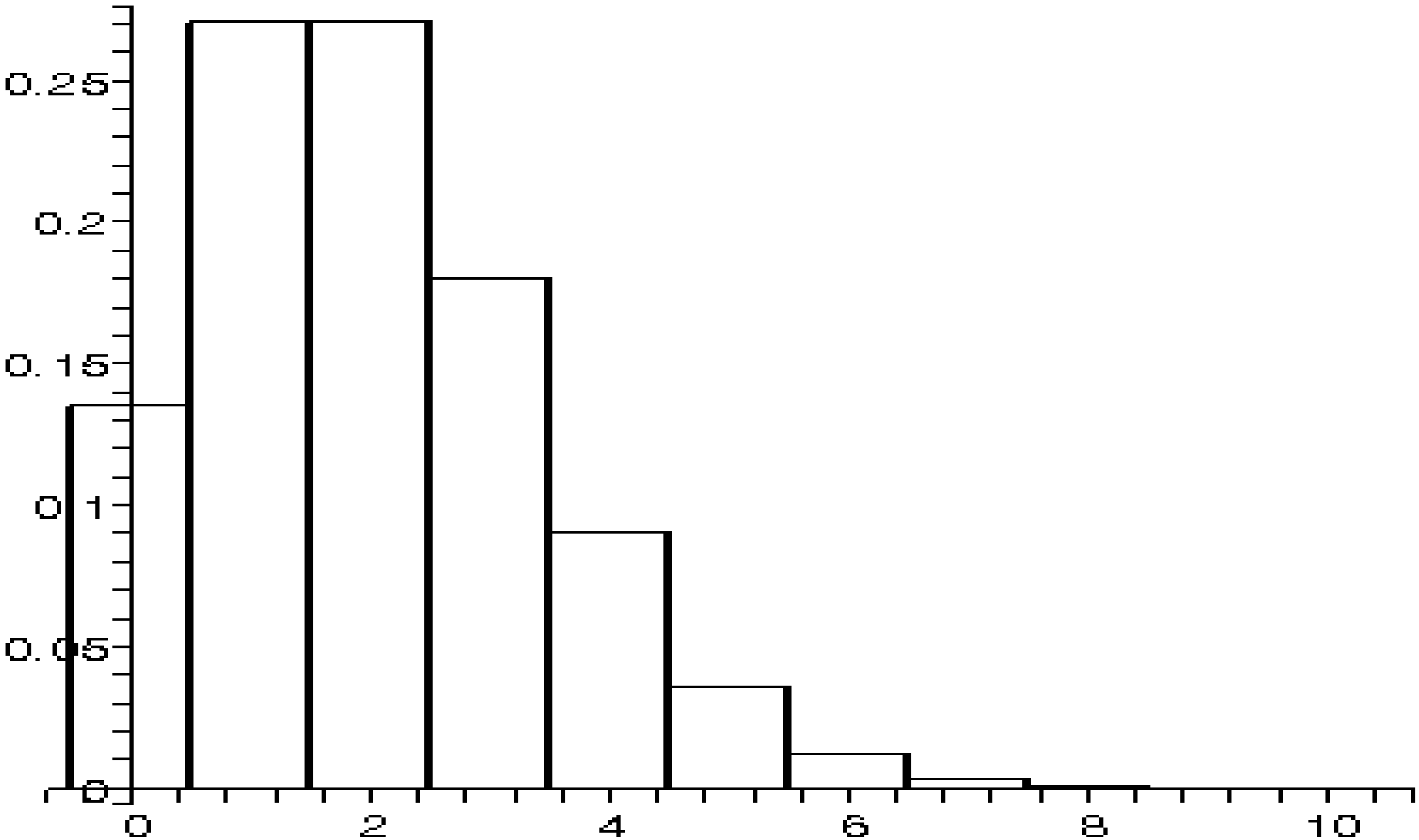
poi := **proc**(*lambda*::*positive*, *x*::*integer*) (\`*exp*\`(1)^(-*lambda*)**lambda*^*x*)/\`*factorial*\`(*x*) **end proc**

$$\frac{1}{(\mathbf{e})^2}$$

$$\frac{1}{(\mathbf{e})^2}, \frac{2}{(\mathbf{e})^2}, \frac{2}{(\mathbf{e})^2}, \frac{4}{3\,(\mathbf{e})^2}, \frac{2}{3\,(\mathbf{e})^2}, \frac{4}{15\,(\mathbf{e})^2}, \frac{4}{45\,(\mathbf{e})^2}, \frac{8}{315\,(\mathbf{e})^2}, \frac{2}{315\,(\mathbf{e})^2}, \frac{4}{2835\,(\mathbf{e})^2}, \frac{4}{14175\,(\mathbf{e})^2}$$

$$\left[[-.5, 0], \left[-.5, \frac{1}{(\mathbf{e})^2}\right], \left[0.5, \frac{1}{(\mathbf{e})^2}\right], [0.5, 0]\right]$$

6. 卜瓦松分佈(Poisson) example



6.卜瓦松分佈(Poisson)

```
> restart;
  f:=x->exp(-lambda)*lambda^x/x!;                    #PDF
>
```

$$f := x \to \frac{e^{(-\lambda)} \lambda^x}{x!}$$

```
>
  Ex:=simplify(sum(k*f(k),k=0..infinity));              #期望值
```

$$Ex := \lambda$$

```
> Vx:=simplify(sum(k^2*f(k),k=0..infinity)-Ex^2);       #變異數
```

$$Vx := \lambda$$

```
> Mxt:=sum(exp(k*t)*f(k),k=0..infinity);              #動差生成函數
```

$$Mxt := e^{(-\lambda + e^t \lambda)}$$

```
>
```

Tests of Hypotheses and Significance

a) P-value

(i) For the Right-tailed test: $H_1: \mu > \mu_0$, *P value* = $P(Z>z_0)$

(ii) For the Left-tailed test: $H_1: \mu < \mu_0$, *P value* = $P(Z<z_0)$

(iii) For the Two-tailed test: $H_1: \mu \neq \mu_0$, *P value* = $P(|Z|>|z_0|) = P(Z>|z_0|) + P(Z<-|z_0|) = 2P(Z>|z_0|)$

Example for calculating P-values

Determine the *P value* when testing the null hypothesis $H_0: \mu = 12$, against the alternative hypotheses:

(i) Right-tailed test: $H_1: \mu > 12$

(ii) Left-tailed test: $H_1: \mu < 12$

(iii) Two-tailed test: $H_1: \mu \neq 12$

Tests of Hypotheses and Significance

based on a sample of size $n = 36$, if = 12.95, and σ = 3. In this case, $\mu_0 = 12$.

```
> restart:with(stats):with(statevalf):
mu[0]:=12;n:=36;xbar:=12.95;sigma:=3;
z[0]:=(xbar-mu[0])/(sigma/sqrt(n));
1-cdf[normald[0,1]](1.9);          #For the right-tailed test: H[1]: mu > 12, P value = P(Z>1.9)
cdf[normald[0,1]](1.9);            #For the left-tailed test: H[1]: mu < 12, P value = P(Z<1.9)
2*(1-cdf[normald[0,1]](abs(1.9))); #For the two-tailed test: H[1]: mu <> 12, P value = P(|Z|>1.9) = P(Z>1.9) + P(Z<-1.9) = 2P(Z>1.9)
```

$$\mu_0 := 12$$

$$n := 36$$

$$xbar := 12.95$$

$$\sigma := 3$$

$$z_0 := 1.900000000$$

0.0287165598

0.9712834402

0.057433120

Special tests of significance for large samples

b) means

Typically, a hypothesis test in such case involves a null hypothesis of the form $H_0: \mu = \mu_0$, against one of the following alternative hypothesis:

(i) Right-tailed test: $H_1: \mu > \mu_0$;

(ii) Left-tailed test: $H_1: \mu < \mu_0$;

(iii) Two-tailed test: $H_1: \mu \neq \mu_0$

Example of hypothesis testing on one mean

Test the null hypothesis $H_0: \mu = 10.5$ against each of the alternative hypotheses:

(i) Right-tailed test: $H_1: \mu > 10.5$

(ii) Left-tailed test: $H_1: \mu < 10.5$

(iii) Two-tailed test: $H_1: \mu \neq 10.5$

at significance level $\alpha = 0.05$, based on a sample of size 50 with a mean of 11 and a standard deviation (based on *n*-1) of 2.5.

```
> restart:with(stats):with(statevalf):
  alpha:=0.05;mu[0]:=10.5;n:=50;xbar:=11;s:=2.5;
  z[0]:=evalf((xbar-mu[0])/(s/sqrt(n)));        #The test statistic to use is
  z_alpha:=icdf[normald[0,1]](1-alpha);
  z_alpha_2:=icdf[normald[0,1]](1-alpha/2);    #The critical values, z[alpha] and z[alpha/2] are
```

$$\alpha := 0.05$$

$$\mu_0 := 10.5$$

$$n := 50$$

$$xbar := 11$$

$$s := 2.5$$

$$z_0 := 1.414213562$$

$$z\_alpha := 1.644853627$$

$$z\_alpha\_2 := 1.959963985$$

Check the criteria for rejecting the null hypothesis:

If the alternative hypothesis is:	Reject H_0 at level α if:	For this case
(i) Right-tailed test: $H_1: \mu > \mu_0$;	$z_0 > z_\alpha$	$1.41 < 1.64$, reject H_0
(ii) Left-tailed test: $H_1: \mu < \mu_0$;	$z_0 < -z_\alpha$	$1.41 > -1.64$, do not reject H_0
(iii) Two-tailed test: $H_1: \mu \neq \mu_0$	$z_0 > z_{\frac{\alpha}{2}}$, or $z_0 < -z_{\frac{\alpha}{2}}$	$1.41 < 1.95$, $1.41 > -1.95$, do not reject H_0

Alternatively, we could use the P value, as follows:

```
P1:=1-cdf[normald[0,1]](z[0]); #For the right-tailed H1 [see (i) above]
P2:=cdf[normald[0,1]](z[0]); #For the left-tailed H1 [see (ii) above]
P3:=2*(1-cdf[normald[0,1]](abs(z[0]))); #For the two-tailed H1 [see (iii) above]
```

$$P1 := 0.0786496036$$

$$P2 := 0.9213503964$$

$$P3 := 0.157299207$$

Since P1 < α (0.0786 < 0.05), the decision is to <u>reject</u> H_0 against the right-tailed alternative hypothesis in (i).

Since P2 > α (0.921 > 0.05), the decision is to <u>not reject</u> H_0 against the left-tailed alternative hypothesis in (ii)

P3 > α (0.157299 > 0.05), the decision is to <u>not reject</u> H_0 against the two-tailed alternative hypothesis in (ii)

c) Proportions

(i) Right-tailed test: $H_1: p > p_0$;

(ii) Left-tailed test: $H_1: p < p_0$;

(iii) Two-tailed test: $H_1: p \neq p_0$

Example of hypothesis testing on a proportion

To determine the proportion of defective temperature sensors in a large building (assume it is an infinite population), 50 sensors are selected at random and 12 are found to be defective. It is proposed that we test the null hypothesis that the proportion of defective temperature sensors in the building is 0.15 (i.e., $H_0: p = 0.15$) , against the three possible alternative hypotheses:

(i) Right-tailed test: $H_1: p > 0.15$;

(ii) Left-tailed test: $H_1: p < 0.15$;

(iii) Two-tailed test: $H_1: p \neq 0.15$

```
> restart:with(stats):with(statevalf):
  p[0]:=0.15;alpha:=0.01;n:=50;X:=12;
  P:=evalf(X/n);  #The sample proportion is calculated as
  z[0]:=(P-p[0])/sqrt(p[0]*(1-p[0]));  #One possibility for the test statistic is
  z_alpha:=icdf[normald[0,1]](1-0.01);
  z_alpha_2:=icdf[normald[0,1]](1-0.01/2);
```

$$p_0 := 0.15$$

$$\alpha := 0.01$$

$$n := 50$$

$$X := 12$$

$$P := 0.2400000000$$

$$z_0 := 0.2520504151$$

$$z\_alpha := 2.326347874$$

$$z\_alpha\_2 := 2.575829304$$

Next, we check the rejection criteria:

If the alternative hypothesis is:	Reject H_0 at level α if:	For this case:
(i) Right-tailed test: $H_1: p > p_0$;	$z_0 > z_\alpha$	$0.2520 < 0.4960$, do not reject H_0
(ii) Left-tailed test: $H_1: p < p_0$;	$z_0 < -z_\alpha$	$0.2520 > -0.4960$, do not reject H_0
(iii) Two-tailed test: $H_1: p \neq p_0$	$z_0 > z_{\frac{\alpha}{2}}$, or $z_0 < -z_{\frac{\alpha}{2}}$	$0.2520 < 2.5758$, $0.2520 > -2.5758$, do not reject H_0

Alternatively, we could use the *P value*, as follows:

```
> P1:=1-cdf[normald[0,1]](z[0]); #For the right-tailed H1 [see (i) above]
```

$$P1 := 0.4005010479$$

Since P1 > α (*0.400* > *0.01*), the decision is to not reject H_0 against the right-tailed alternative hypothesis in (i).

```
> P2:=cdf[normald[0,1]](z[0]); #For the left-tailed H1 [see (ii) above]
```

$$P2 := 0.5994989521$$

Since P2 > α (*0.921* > *0.01*), the decision is to not reject H_0 against the left-tailed alternative hypothesis in (ii).

```
> P3:=2*(1-cdf[normald[0,1]](abs(z[0]))); #For the two-tailed H1 [see (iii) above]
```

$$P3 := 0.801002096$$

Since P3 > α (*0.8010* > *0.01*), the decision is to not reject H_0 against the two-tailed alternative hypothesis in (ii).

d) difference of means

(i) Right-tailed test: H_1: $\mu_1 - \mu_2 > \delta$;

(ii) Left-tailed test: H_1: $\mu_1 - \mu_2 < \delta$;

(iii) Two-tailed test: H_1: $\mu_1 - \mu_2 \neq \delta$

Example of hypothesis testing on the difference of two means

A traffic study is performed at a given intersection to determine the difference on the waiting time required for a left-turning vehicle during the morning and the afternoon rush hours. A sample of 40 left-turning vehicles during the morning rush hour indicates a mean value of 4.5 minutes with a standard deviation of 1.0 minute, while a sample of 50 left-turning vehicles during the afternoon rush hour indicates a mean value of 5.2 minutes with a standard deviation of 1.2 minutes. At a significance level $\alpha = 0.10$, test the null hypothesis that the mean waiting time for left-turning vehicles is the same in the morning rush hour as in the afternoon rush hour, i.e., H_0: $\mu_1 - \mu_2 = 0$, against each of the alternative hypotheses:

(i) Right-tailed test: H_1: $\mu_1 - \mu_2 > 0$;

(ii) Left-tailed test: H_1: $\mu_1 - \mu_2 < 0$;

(iii) Two-tailed test: H_1: $\mu_1 - \mu_2 \neq 0$

```
> restart:with(stats):with(statevalf):
n[1]:=40;x1bar:=4.5;s1hat:=1.0;n[2]:=50;x2bar:=5.2;s2hat:=1.2;delta:=0;alpha:=0.10;
z[0]:=((x1bar-x2bar)-delta)/sqrt(s1hat^2/n[1]+s2hat^2/n[2]);#The test statistic to be used
z_alpha:=icdf[normald[0,1]](1-alpha);
z_alpha_2:=icdf[normald[0,1]](1-alpha/2);#which needs to be compared to either z[alpha]
or z[alpha/2], depending on the alternative hypothesis selected.  Let's find those values:
```

$$n_1 := 40$$

$$x1bar := 4.5$$

$$s1hat := 1.0$$

$$n_2 := 50$$

$$x2bar := 5.2$$

$$s2hat := 1.2$$

$$\delta := 0$$

$$\alpha := 0.10$$

$$z_0 := -3.017914295$$

$$z\_alpha := 1.281551566$$

$$z\_alpha\_2 := 1.644853627$$

Next, we check the rejection criteria:

If the alternative hypothesis is:	Reject H_0 at level α if:	For this case:
(i) Right-tailed test: H_1: $\mu_1 - \mu_2 > 0$;	$z_0 > z_\alpha$	*-3.02 < 1.28*, do not reject H_0
(ii) Left-tailed test: H_1: $\mu_1 - \mu_2 < 0$;	$z_0 < -z_\alpha$	*-3.02 < - 1.28*, reject H_0
(iii) Two-tailed test: H_1: $\mu_1 - \mu_2 \neq 0$	$z_0 > z_{\frac{\alpha}{2}}$, or $z_0 < -z_{\frac{\alpha}{2}}$	*-3.02 < 1.64*, *-3.02 < -1.64*, reject H_0

Alternatively, we could use the *P value*, as follows:

```
> P1:=1-cdf[normald[0,1]](z[0]); #For the right-tailed H1 [see (i) above]
```

$$P1 := 0.9987273956$$

Since P1 > α (*0.998 > 0.1*), the decision is to not reject H_0 against the right-tailed alternative hypothesis in (i).

```
> P2:=cdf[normald[0,1]](z[0]); #For the left-tailed H1 [see (ii) above]
```

$$P2 := 0.001272604376$$

Since P2 < α (*0.0012 < 0.01*), the decision is to reject H_0 against the left-tailed alternative hypothesis in (ii).

```
> P3:=2*(1-cdf[normald[0,1]](abs(z[0]))); #For the two-tailed H1 [see (iii) above]
```

$$P3 := 0.002545209$$

Since P3 > α (*0.002545 > 0.01*), the decision is to not reject H_0 against the two-tailed alternative hypothesis in (ii).

資料來源

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2. http://www.engineering.usu.edu/cee/faculty/gurro/Software_Calculators/Maple_Docs/Maple_Statistics.htm

Thanks for your listening!