Q1. $T(n) = 3T(n-1) + 12n$

$T(2) = 3T(1) + 12(2)$

$\Rightarrow T(2) = 3T(1) + 24 \Rightarrow T(2) = 3(27) + 24$

$= 81 + 24$

$\Downarrow$

$\boxed{T(2) = 105}$

$T(1) = 3\cancel{T(0)} + 12(1)$

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$\Rightarrow T(1) = 15 + 12 = 27$

Q2.

a). $T(n) = T(n-1) + c \Rightarrow T(n) = T(n-2) + 2c$

$T(n-1) = T(n-2) + c$

& so on, $T(n) = T(n-k) + kc$

Considering, $T(n) = 1$ when $n = 1$

$\Rightarrow n - k = 1 \Rightarrow k = n - 1$

$\therefore T(n) = 1 + (n-1)c \rightarrow \boxed{O(n)}$

b). $T(n) = 2T(n/2) + n$

$T(n/2) = 2T(n/2^2) + \frac{n}{2}$

$\Rightarrow T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$

$T(n) = 2^2\, T\left(\frac{n}{2^2}\right) + 2n$

$T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$

$$\Rightarrow T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + 2n$$

$$T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$$

If $T(1) = 1 \Rightarrow \frac{n}{2^k} = 1 \Rightarrow n = 2^k$

$\Downarrow$

$\log_2 n = k$

$$\Rightarrow T(n) = 2^{\log_2 n} T(1) + n\log_2 n$$

$$T(n) = n \cancel{(1+k)} n + n\log_2 n$$

$$\Rightarrow \boxed{T.C = O(n\log_2 n)}$$

c). $T(n) = 2T(n/2) + c$

$$T(n/2) = 2T(n/2^2) + c$$

$$\Rightarrow T(n) = 2(2T(n/2^2) + c) + c$$

$$T(n) = 2^2 T(n/2^2) + \cancel{2c}\, 2c + c$$

$$T(n/2^2) = 2T(n/2^3) + c$$

$$\Rightarrow T(n) = 2^2\left(2T\left(\frac{n}{2^3}\right) + c\right) + \overset{2c+c}{\cancel{3c}}$$

$$= 2^3 T(n/2^3) + 2^2c + 2c + c$$

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$\therefore T(n) = 2^k T\left(\frac{n}{2^k}\right) + 2^k c + 2^{k-1} c \ldots + 2^2 c + 2c + c$

$= 2^k T\left(\frac{n}{2^k}\right) + c \cdot 1(2^k - 1)$

$\frac{n}{2^k} = 1 \Rightarrow \log_2 n = k$

$= n + c \cdot (n-1) \Rightarrow \boxed{O(n)}$

d). $T(n) = T(n/2) + c$

$T(n/2) = T(n/2^2) + c$

$\Rightarrow T(n) = T(n/2^2) + 2c$

$T(n/2^2) = T(n/2^3) + c$

$\Rightarrow T(n) = T(n/2^3) + 3c$

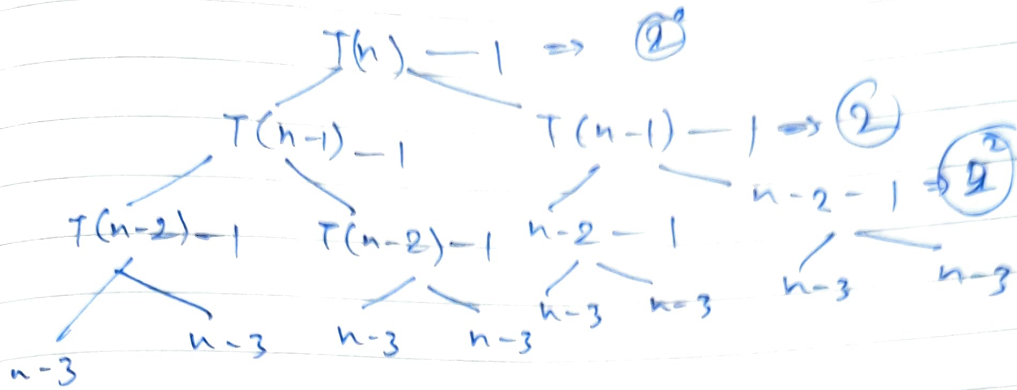
$\Rightarrow T(n) = T\left(\frac{n}{2^k}\right) + kc$

$k = \log_2 n$ if $n = 1$ & $T(1) = 1$

$\Rightarrow T(n) = 1 + \log_2 n \, c.$

$\Rightarrow \boxed{O(\log_2 n)}$

Q3.
a). $T(n) = 2T(n-1) + 1 = T(n-1) + T(n-1) + 1$



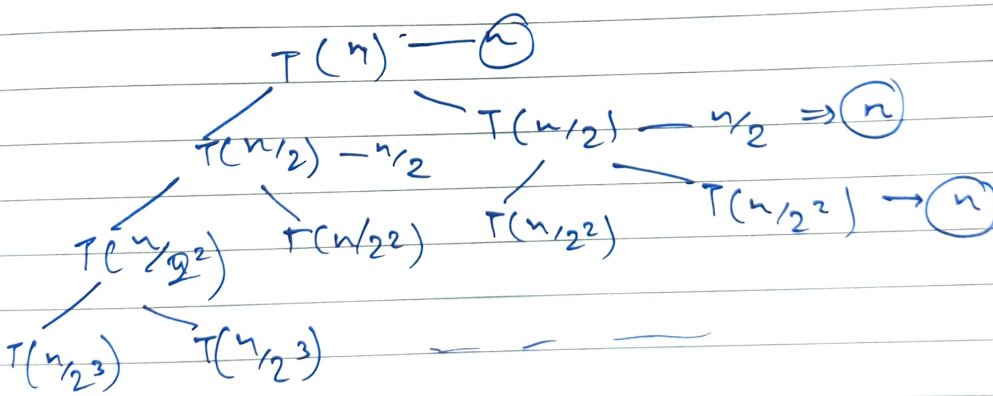
∴ keep repeating for k levels

$\Rightarrow n-k=0 \Rightarrow \boxed{n=k}$

$T.C \Rightarrow 2^0 + 2^1 + 2^2 \ldots + 2^n$

$\Rightarrow \boxed{O(2^n)}$

b). $T(n) = T(n/2) + T(n/2) + n$



$\therefore \frac{n}{2^k} = 1 \Rightarrow$ base case reached

$\Rightarrow \cancel{n=k}\ k = \log_2 n$

$\therefore \boxed{T.C = O(\log_2 n \cdot n)}$