Bayesian networks

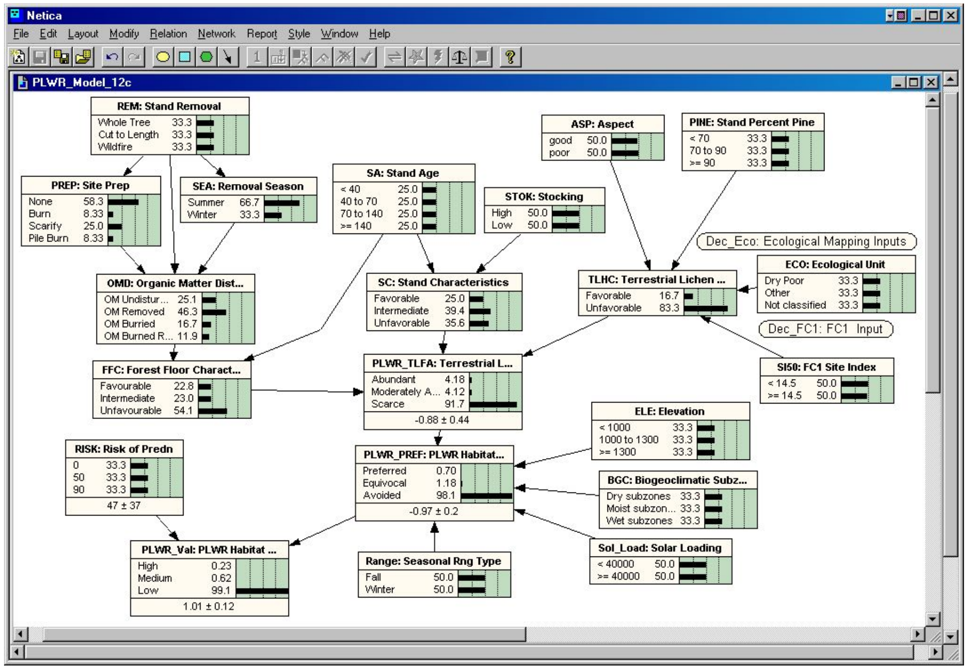
CS4881 Artificial Intelligence

Jay Urbain, PhD

Credits:
Judea Pearl, “Causality: Models, Reasoning, and Inference”
Russell and Norvig, AIMA

Outline

- Introduction to Bayesian Networks
- Syntax
- Semantics



Bayesian networks

- Graphical notation for conditional independence assertions of full joint distributions.

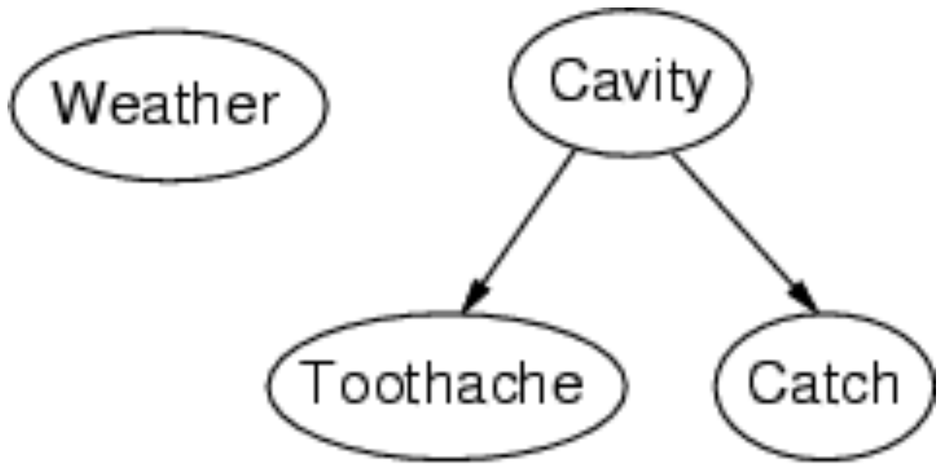
- Syntax:
  - a set of nodes, one per random variable
  - a directed, acyclic graph (Semantics: link ≈ "directly influences“ or “causes”)
  - a conditional distribution for each node given its parents:

$$\mathbf{P}\,(X_i \mid \text{Parents}\,(X_i))$$

- In the simplest case, conditional distributions are represented as a conditional probability table (CPT) giving the distribution over each random variable X_i for each combination of parent values.

Example

- Topology of network encodes conditional independence assertions:



- *Weather* is independent of the other variables.
- *Toothache* and *Catch* are conditionally independent given *Cavity.*

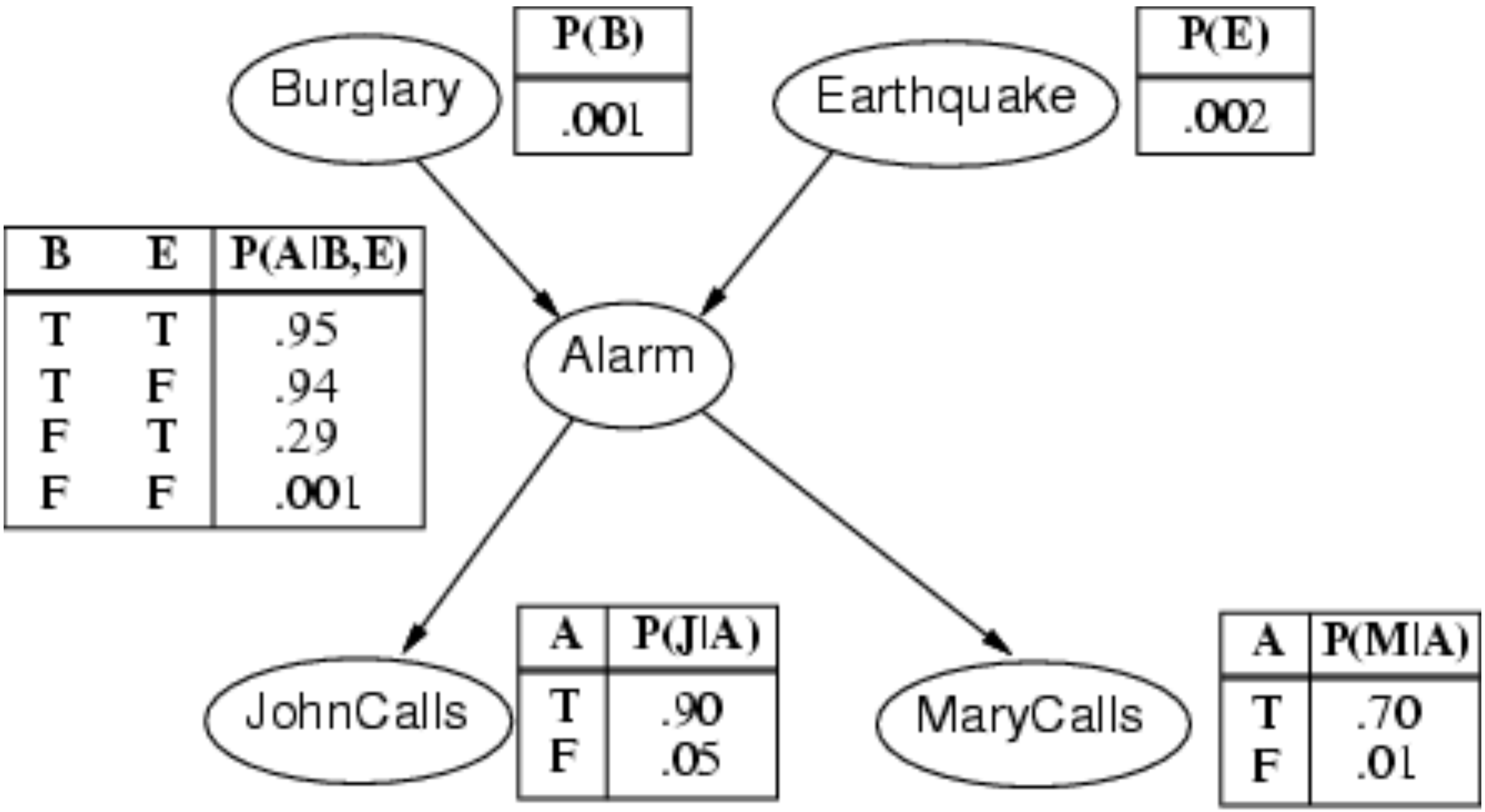
Example

- I'm at work, neighbor John calls to say my alarm is ringing, but neighbor Mary doesn't call. Sometimes it's set off by minor earthquakes. Is there a burglar?

- Variables: *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, *MaryCalls*

- Network topology reflects "causal" knowledge:
  - A burglar can set the alarm off
  - An earthquake can set the alarm off
  - The alarm can cause Mary to call
  - The alarm can cause John to call

Example cont’d.



P(B)
.001

P(E)
.002

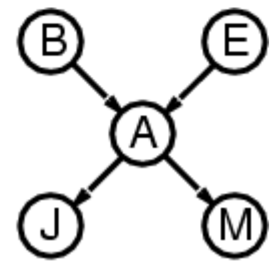
B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

Compactness

- A CPT for Boolean random variable X_i with k Boolean parents has 2^k rows for the combinations of parent values, i.e., 2 parents => 4 rows

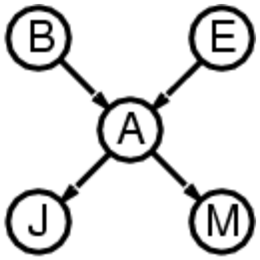


- Each row requires one number p for X_i = *true* (the number for X_i = *false* is just $1-p$)
- If each variable has no more than k parents, the complete network of n variables requires $O(n \cdot 2^k)$ numbers
- I.e., the network grows linearly in n, vs. $O(2^n)$ for the full joint distribution
- For burglary net, 1 + 1 + 4 + 2 + 2 = 10 numbers (vs. 2^5-1 = 31)
- *Note: Show factorization!*

Semantics

The full joint distribution is defined as the product of the local conditional distributions:

$$\mathbf{P}\,(X_1, \ldots, X_n) = \pi_{i=1}^{n}\, \mathbf{P}\,(X_i \mid Parents(X_i))$$



e.g., $\mathbf{P}(j \land m \land a \land \neg b \land \neg e)$

$= \mathbf{P}\,(j \mid a)\, \mathbf{P}\,(m \mid a)\, \mathbf{P}\,(a \mid \neg b, \neg e)\, \mathbf{P}\,(\neg b)\, \mathbf{P}\,(\neg e)$

$= 0.90 * 0.70 * 0.001 * 0.999 * 0.998$

Constructing Bayesian networks

- 1. Choose an ordering of variables $X_1, \ldots, X_n$
- 2. For i = 1 to n
  - add X_i to the network
  - select parents from $X_1, \ldots, X_{i-1}$

$$\mathbf{P}(X_i \mid Parents(X_i)) = \mathbf{P}(X_i \mid X_1, \ldots X_{i-1})$$

This choice of parents guarantees (conditional indepedance):

$$\mathbf{P}(X_1, \ldots, X_n) = \pi_{i=1}^{n} \mathbf{P}(X_i \mid X_1, \ldots, X_{i-1}) \text{ (chain rule)}$$

$$= \pi_{i=1} \mathbf{P}(X_i \mid^{n} Parents(X_i)) \text{ (by construction)}$$

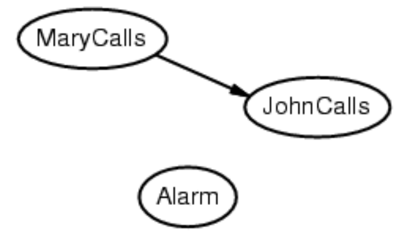
Example

- Suppose we choose the ordering *M, J, A, B, E*

$\mathbf{P}(J \mid M) = \mathbf{P}(J)$?

Example

- Suppose we choose the ordering *M, J, A, B, E*

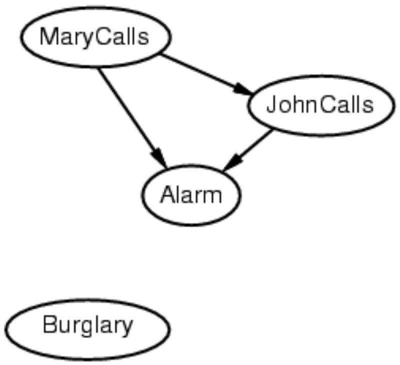


$\boldsymbol{P}(J \mid M) = \boldsymbol{P}(J)$? **No**

$\boldsymbol{P}(A \mid J, M) = \boldsymbol{P}(A \mid J)$?, $\boldsymbol{P}(A \mid J, M) = \boldsymbol{P}(A)$?

Example

- Suppose we choose the ordering *M, J, A, B, E*



$\boldsymbol{P}(J \mid M) = \boldsymbol{P}(J)$? **No**

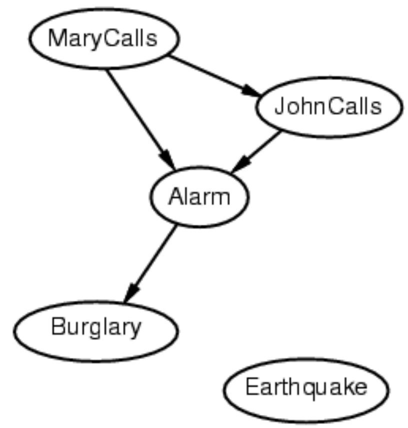
$\boldsymbol{P}(A \mid J, M) = \boldsymbol{P}(A \mid J)$? $\boldsymbol{P}(A \mid J, M) = \boldsymbol{P}(A)$? **No**

$\boldsymbol{P}(B \mid A, J, M) = \boldsymbol{P}(B \mid A)$?

$\boldsymbol{P}(B \mid A, J, M) = \boldsymbol{P}(B)$?

Example

- Suppose we choose the ordering M, J, A, B, E



P$(J \mid M)$ = ***P***(J)? **No**

P$(A \mid J, M)$ = ***P***$(A \mid J)$? ***P***$(A \mid J, M)$ = ***P***(A)? **No**

P$(B \mid A, J, M)$ = ***P***$(B \mid A)$? **Yes**

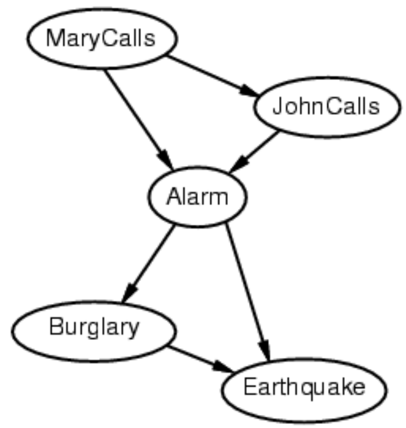
P$(B \mid A, J, M)$ = ***P***(B)? **No**

P$(E \mid B, A ,J, M)$ = ***P***$(E \mid A)$?

P$(E \mid B, A, J, M)$ = ***P***$(E \mid A, B)$?

Example

- Suppose we choose the ordering M, J, A, B, E



$\mathbf{P}(J \mid M) = \mathbf{P}(J)$? **No**

$\mathbf{P}(A \mid J, M) = \mathbf{P}(A \mid J)$? $\mathbf{P}(A \mid J, M) = \mathbf{P}(A)$? **No**

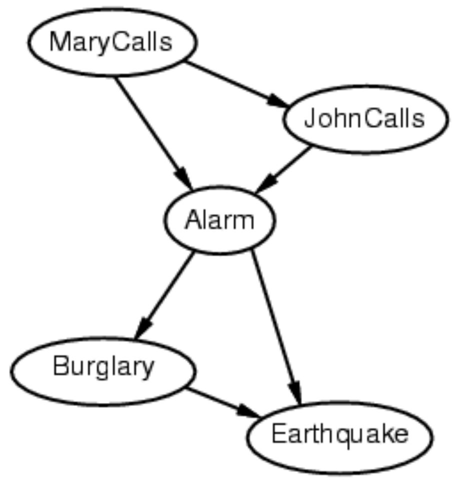
$\mathbf{P}(B \mid A, J, M) = \mathbf{P}(B \mid A)$? **Yes**

$\mathbf{P}(B \mid A, J, M) = \mathbf{P}(B)$? **No**

$\mathbf{P}(E \mid B, A ,J, M) = \mathbf{P}(E \mid A)$? **No/Yes?**

$\mathbf{P}(E \mid B, A, J, M) = \mathbf{P}(E \mid A, B)$? **Yes**

Example contd.



- **Deciding conditional independence is hard in *non-causal* directions**
- Causal models and conditional independence seem hardwired for humans!
- Network is less compact: 1 + 2 + 4 + 2 + 4 = 13 numbers needed

Summary

- Bayesian networks provide a natural representation for (causally induced) conditional independence
- Topology + CPTs = compact representation of joint distribution
- Generally easy for domain experts to construct
- Very powerful real world systems
- *Note: Example…*