In-Class Assignment 3

In [8]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import statsmodels.api as sm
from statsmodels.formula.api import ols
from sklearn.model_selection import train_test_split
import scipy.stats as ss

advertising_df = pd.read_csv("Advertising.csv", index_col=0)
advertising_df['intercept'] = 1
y_var = 'sales'
advertising_df.head()
```

Out[8]:

	TV	radio	newspaper	sales	intercept
1	230.1	37.8	69.2	22.1	1
2	44.5	39.3	45.1	10.4	1
3	17.2	45.9	69.3	9.3	1
4	151.5	41.3	58.5	18.5	1
5	180.8	10.8	58.4	12.9	1

Estimating the Coefficients

In [10]:
```
# x_vars = ['intercept', 'TV']
# model = sm.OLS(advertising_df[y_var], advertising_df[x_vars])

model = ols(f"{y_var} ~ TV", data=advertising_df)
results = model.fit()
results.summary()
```

Out[10]:

OLS Regression Results

Dep. Variable:	sales	**R-squared:**	0.612
Model:	OLS	**Adj. R-squared:**	0.610
Method:	Least Squares	**F-statistic:**	312.1
Date:	Wed, 04 Sep 2024	**Prob (F-statistic):**	1.47e-42
Time:	19:50:55	**Log-Likelihood:**	-519.05
No. Observations:	200	**AIC:**	1042.
Df Residuals:	198	**BIC:**	1049.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	7.0326	0.458	15.360	0.000	6.130	7.935
TV	0.0475	0.003	17.668	0.000	0.042	0.053

Omnibus:	0.531	**Durbin-Watson:**	1.935
Prob(Omnibus):	0.767	**Jarque-Bera (JB):**	0.669
Skew:	-0.089	**Prob(JB):**	0.716
Kurtosis:	2.779	**Cond. No.**	338.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

We've optimized the parameters β_0 and β_1, what is the objective?

Let $e_i = y_i - \hat{y}_i$ represent the residual of a prediction $\hat{y}_i$. Then the sum of squared residuals is $RSS = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

We can think of this as the amount of variablility that is left in the response after performing the regression.

In [12]:
```
RSS = ((results.predict(advertising_df[['intercept', 'TV']]) - advertising_df['sale
RSS
```

Out[12]:
```
2102.530583131351
```

In [14]:
```
sm.stats.anova_lm(results)
```

Out[14]:

	df	sum_sq	mean_sq	F	PR(>F)
TV	1.0	3314.618167	3314.618167	312.144994	1.467390e-42
Residual	198.0	2102.530583	10.618841	NaN	NaN

Where does the standard error come from? Do we know the true standard deviation of the errors in the model?

$$\hat{\sigma} = RSE = \sqrt{RSS/(n-p-1)}$$

In [16]:
```
# degrees of freedom = n - p - 1, where p is the number of predictor variables
degrees_of_freedom = (len(advertising_df) - 1 - 1)

RSE = np.sqrt(RSS / degrees_of_freedom)
RSE
```

Out[16]:
```
3.258656368650462
```

In [6]:
```
SE_beta_1 = np.sqrt((RSE**2) / ((advertising_df['TV'] - advertising_df['TV'].mean()
SE_beta_1.round(3)
```

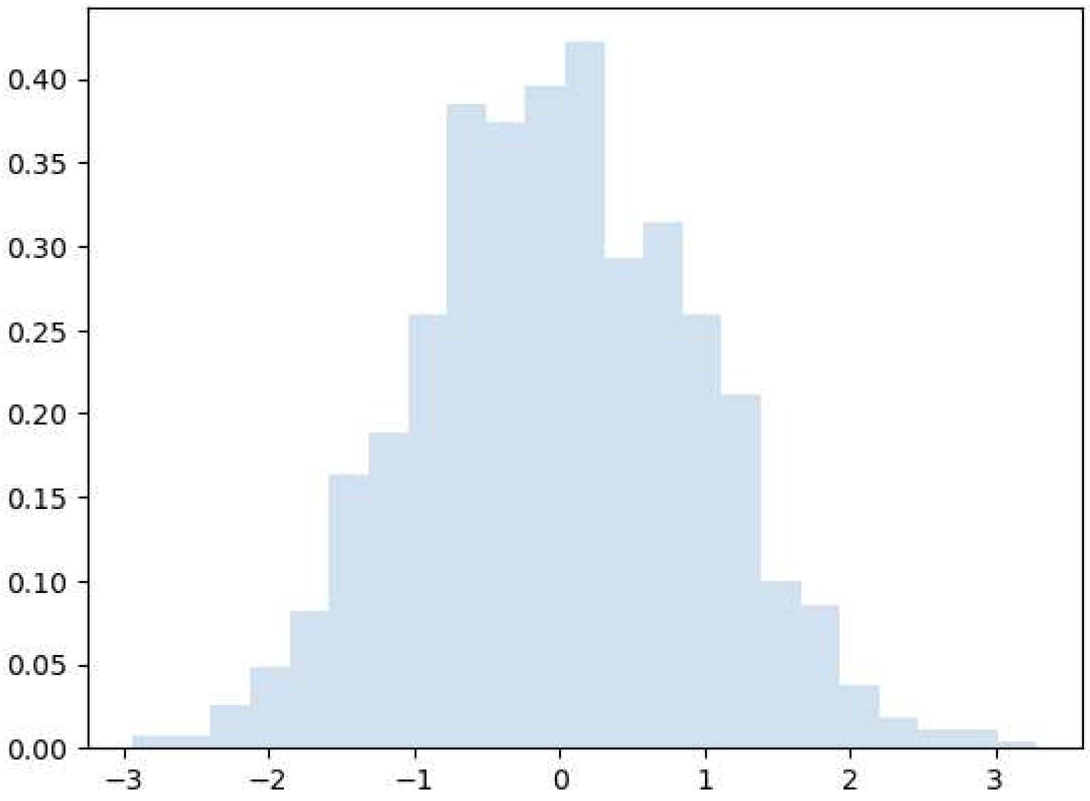
Out[6]:
```
0.003
```

What is this t?

In [7]:
```
t_var = ss.t(degrees_of_freedom)
sample = t_var.rvs(size=1000)

fig, ax = plt.subplots()
ax.hist(sample, density=True, bins='auto', histtype='stepfilled', alpha=0.2)
ax.legend(loc='best', frameon=False)
plt.show()
```

```
No artists with labels found to put in legend.  Note that artists whose label start
with an underscore are ignored when legend() is called with no argument.
```



Let's calculate a t-statistic, p-value, and confidence interval

- Note: the t-statistic for a parameter is measured with the hypothesis that the parameter equals 0, i.e. it has no relationship with the outcome

In [8]:
```
results.params
```

Out[8]:
```
Intercept    7.032594
TV           0.047537
dtype: float64
```

In [9]:
```
beta_1 = results.params['TV']
t_stat_beta_1 = (beta_1 - 0) / SE_beta_1

p_val_t_stat_beta_1 = 1 - t_var.cdf(t_stat_beta_1)

lb_beta_1 = beta_1 + t_var.ppf(.025) * SE_beta_1
ub_beta_1 = beta_1 + t_var.ppf(.975) * SE_beta_1

print("t-statistic:", round(t_stat_beta_1, 3))
print("p-val of t-statistic:", p_val_t_stat_beta_1)
print("95% Confidence Interval:", round(lb_beta_1, 3), "to", round(ub_beta_1, 3))
```

```
t-statistic: 17.668
p-val of t-statistic: 0.0
95% Confidence Interval: 0.042 to 0.053
```

What proportion of the variance in sales is explained by this model?

Let the sum of squared difference between the outcomes (y_i) and the mean of all outcomes ($\bar{y}$) be TSS, i.e.

$$TSS = \sum_{i=1}^{n}(y_i - \bar{y})^2 \tag{1}$$

$$R^2 = \frac{TSS - RSS}{TSS} \tag{2}$$

```
In [10]: TSS = ((advertising_df['sales'] - advertising_df['sales'].mean())**2).sum()
         R_squared = (TSS - RSS) / TSS
         R_squared
```

```
Out[10]: 0.6118750508500711
```

Multiple regression

```
In [37]: model_multiple = ols(f"{y_var} ~ TV + radio + newspaper", data=advertising_df)
         results_multiple = model_multiple.fit()
         results_multiple.summary()
```

Out[37]:

OLS Regression Results

Dep. Variable:	sales	**R-squared:**	0.897
Model:	OLS	**Adj. R-squared:**	0.896
Method:	Least Squares	**F-statistic:**	570.3
Date:	Wed, 04 Sep 2024	**Prob (F-statistic):**	1.58e-96
Time:	12:10:53	**Log-Likelihood:**	-386.18
No. Observations:	200	**AIC:**	780.4
Df Residuals:	196	**BIC:**	793.6
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
Intercept	2.9389	0.312	9.422	0.000	2.324	3.554
TV	0.0458	0.001	32.809	0.000	0.043	0.049
radio	0.1885	0.009	21.893	0.000	0.172	0.206
newspaper	-0.0010	0.006	-0.177	0.860	-0.013	0.011

Omnibus:	60.414	**Durbin-Watson:**	2.084
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	151.241
Skew:	-1.327	**Prob(JB):**	1.44e-33
Kurtosis:	6.332	**Cond. No.**	454.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

We can see all of the previous statistics, now let's perform a hypothesis test on whether the model variables have a relationship with the response

- Another way of stating this is that our hypothesis is that all of the estimated model parameters on the predictor variables are zero, i.e. $\beta_1 = \beta_2 = \beta_3 = 0$
- We might want to think that we can just look at the p-values of the variables, but this is not valid. If we had 100 variables, we expect about 5 of those to have a p-value of less than .05!

The F statistic can help us with this: $F = \frac{(TSS - RSS)/p}{RSS/(n-p-1)}$

- If our model assumptions are correct we expect that the numerator and denominator both equal the variance of the error distribution, i.e. $F = \frac{\sigma^2}{\sigma^2} = \frac{Var(\epsilon)}{Var(\epsilon)}$ where the model is $Y \sim f(X) + \epsilon$.

```
In [38]: TSS_multiple = results_multiple.centered_tss
         RSS_multiple = results_multiple.ssr

         p = 3
         n = len(advertising_df)

         F_stat_multiple = ((TSS_multiple - RSS_multiple) / p) / (RSS_multiple / (n - p - 1)

         print("TSS:", TSS_multiple)
         print("RSS:", RSS_multiple)
         print("F-stat:", F_stat_multiple)
```

```
TSS: 5417.14875
RSS: 556.8252629021874
F-stat: 570.270703659094
```

Questions

- Base all of your answers on the additive model $Y \sim \beta_0 + \beta_1 * TV + \beta_2 * radio + \beta_3 * newspaper$.

1. Is there a relationship between sales and advertising budget? What test would you use to determine this?

- Answer: Yes, F test

2. How strong is the relationship? What statistic would you use to measure this?

- Answer: 89.7% of the variance is explained by our model based on the R^2 statistic

3. Which of the media are associated with sales? What statistic would you use to measure this?

- Answer: TV and Radio, by using t-statistic

4. For each unit increase in TV advertising budget, what change in sales should we expect? What is a 95% confidence interval of that effect?

- Answer: We expect an average change of 0.046 in sales for a unit increase in TV ad expenditure. The confidence interval at 95% is [0.043, 0.049]

```
In [ ]:
```