Case Assignment 1
BAN 630 – Optimization for Analytics

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Case Study - Rougir Cosmetics

a. What are the costs of producing the three products in-house?

To calculate the costs of producing Face Cream, Body Cream, and Hand Cream in-house, we evaluated the following scenarios:

- Stage 1 and Stage 2 in Shift 1.
- Stage 1 and Stage 2 in Shift 2
- Stage 1 in Shift 1 and Stage 2 in Shift 2.

We used the below variables to represent them:

- **FS1**: Face Cream (Stage 1 & Stage 2 in Shift 1).
- **FS12**: Face Cream (Stage 1 in Shift 1, Stage 2 in Shift 2).
- **FS2**: Face Cream (Stage 1 & Stage 2 in Shift 2).
- **BS1**: Body Cream (Stage 1 & Stage 2 in Shift 1).
- **BS12**: Body Cream (Stage 1 in Shift 1, Stage 2 in Shift 2).
- **BS2**: Body Cream (Stage 1 & Stage 2 in Shift 2).
- **HS1**: Hand Cream (Stage 1 & Stage 2 in Shift 1).
- **HS12**: Hand Cream (Stage 1 in Shift 1, Stage 2 in Shift 2).
- **HS2**: Hand Cream (Stage 1 & Stage 2 in Shift 2).

We then have calculated final costs based on material costs and labor costs.

Variable	Cost per Unit ($)
FS1	$32.15
FS12	$32.89
FS2	$34.17
BS1	$37.35
BS12	$38.28
BS2	$39.81
HS1	$25.53
HS12	$25.99
HS2	$26.84

- **Shift 1 Only** (e.g., FS1, BS1, HS1): Producing entirely in Shift 1 is the cheapest option but may strain capacity.
- **Split Shifts** (e.g., FS12, BS12, HS12): Splitting stages across shifts increases costs slightly but utilizes resources efficiently.
- **Shift 2 Only** (e.g., FS2, BS2, HS2): Producing entirely in Shift 2 is the most expensive option due to higher labor costs.

b. Develop a linear programming model for this problem and find the optimal schedule.

The goal is to minimize the total cost of producing Face Cream, Body Cream, and Hand Cream while meeting demand and resource constraints.

Decision Variables:
The variables represent the number of cartons produced in specific stages and shifts, or outsourced:

- **FS1**: Face Cream (Stage 1 & Stage 2 in Shift 1)
- **FS2**: Face Cream (Stage 1 & Stage 2 in Shift 2)
- **FS12**: Face Cream (Stage 1 in Shift 1, Stage 2 in Shift 2)
- **BS1**: Body Cream (Stage 1 & Stage 2 in Shift 1)
- **BS2**: Body Cream (Stage 1 & Stage 2 in Shift 2)
- **BS12**: Body Cream (Stage 1 in Shift 1, Stage 2 in Shift 2)
- **HS1**: Hand Cream (Stage 1 & Stage 2 in Shift 1)
- **HS2**: Hand Cream (Stage 1 & Stage 2 in Shift 2)
- **HS12**: Hand Cream (Stage 1 in Shift 1, Stage 2 in Shift 2)
- **FO**: Face Cream outsourced
- **BO**: Body Cream outsourced

Objective Function:

Our objective function is to minimize the total cost.

Minimize $Z =$
$FS1 \cdot 32.15 + FS2 \cdot 34.17 + FS12 \cdot 32.89 + BS1 \cdot 37.35 + BS2 \cdot 38.91 + BS12 \cdot 38.28 + HS1 \cdot 25.53 + HS2 \cdot 26.84 + HS12 \cdot 25.99 + FQ \cdot 40 + BD \cdot 55$

Constraints

1. Demand Constraints

Ensure total production (in-house and outsourced) meets demand:

- **Face Cream**: $FS1+FS2+FS12+FQ \geq 12{,}000$
- **Body Cream**: $BS1+BS2+BS12+BD \geq 8{,}000$
- **Hand Cream**: $HS1+HS2+HS12 \geq 18{,}000$

2. Labor Time Constraints

Each shift's available labor hours for Stage 1 and Stage 2 must not exceed capacity:

- **Stage 1 (Shift 1)**: $1.5 \cdot (FS1+FS12) + 1.8 \cdot (BS1+BS12) + 1.0 \cdot (HS1+HS12) \leq 15{,}000$
- **Stage 1 (Shift 2)**: $1.5 \cdot FS2 + 1.8 \cdot BS2 + 1.0 \cdot HS2 \leq 13{,}500$
- **Stage 2 (Shift 1)**: $0.8 \cdot FS1 + 1.0 \cdot BS1 + 0.5 \cdot HS1 \leq 10{,}000$

- **Stage 2 (Shift 2)**: $0.8 \cdot (FS2 + FS12) + 1.0 \cdot (BS2 + BS12) + 0.5 \cdot (HS2 + HS12) \leq 9{,}000$

3. Material Constraints

Available raw materials must not be exceeded:

- **Water**: $8 \cdot (FS1 + FS2 + FS12) + 6 \cdot (BS1 + BS2 + BS12) + 7 \cdot (HS1 + HS2 + HS12) \leq 200{,}000$
- **Oil**: $1 \cdot (FS1 + FS2 + FS12) + 3 \cdot (BS1 + BS2 + BS12) + 2 \cdot (HS1 + HS2 + HS12) \leq 50{,}000$
- **Scents and Colors**: $0.5 \cdot (FS1 + FS2 + FS12) + 0.3 \cdot (BS1 + BS2 + BS12) + 0.4 \cdot (HS1 + HS2 + HS12) \leq 7{,}500$
- **Emulsifiers**: $0.5 \cdot (FS1 + FS2 + FS12) + 0.7 \cdot (BS1 + BS2 + BS12) + 0.6 \cdot (HS1 + HS2 + HS12) \leq 15{,}000$

4. Non-Negativity

All decision variables must be non-negative:

$FS1, FS2, FS12, BS1, BS2, BS12, HS1, HS2, HS12, FO, BO \geq 0$

Final Costs

- Total cost = **$1,368,100**.
- The cost breakdown:
    - Face Cream outsourced at $40/unit.
    - Body Cream primarily outsourced at $55/unit, with some in-house production in Shift 1.
    - Hand Cream produced entirely in-house, utilizing both shifts.

Insights

1. **Outsourcing**:
    - Face Cream is fully outsourced as it minimizes costs and alleviates labor and material constraints.
    - Body Cream is partially outsourced due to limited capacity and higher in-house production costs.
2. **Shift Utilization**:
    - Hand Cream leverages both shifts effectively, utilizing available slack in Shift 2.
3. **Material Constraints**:
    - Constraints on Scents and Colors and Emulsifiers are tight but met.

Linear Programming Model and Optimal Schedule

			cost, $ per unit	total raw material cost	labour cost per product	
face cream stage 1 & 2 in shift 1	FS1		32.15	12.00	20.15	
face cream stage 1 & 2 in shift 2	FS2		34.17	12.00	22.17	
face cream stage 1 in shift 1 & stage 2 in shift 2	FS12		32.89	12.00	20.89	
body cream stage 1 & 2 in shift 1	BS1		37.35	12.80	24.55	
body cream stage 1 & 2 in shift 1	*BS2*		39.81	12.80	27.01	
body cream stage 1 in shift 1 & stage 2 in shift 2	BS12		38.28	12.80	25.48	
hand cream stage 1 & 2 in shift 1	*HS1*		25.53	12.40	13.13	
hand cream stage 1 & 2 in shift 1	HS2		26.84	12.40	14.44	
hand cream stage 1 in shift 1 & stage 2 in shift 2	HS12		25.99	12.40	13.59	
face cream out sourced	FO		40.00			
body cream out sourced	BO		55.00			

Criteria	Face Cream	Body Cream	Hand Cream
Labor (hours/carton)			
Stage 1	1.5	1.8	1
Stage 2	0.8	1	0.5
Materials (pounds/carton)			
Water	8	6	7
Oil	1	3	2
Scents/Colors	0.5	0.3	0.4
Emulsifiers	0.5	0.7	0.6

	cost of raw material, $
Water	1
Oil	1.5
Scents/Colors	3
Emulsifiers	2

	FIRST SHIFT	SECOND SHIFT
LABOUR HOURS		
STAGE 1	15000	13500
STAGE 2	10000	9000
COST OF LABOUR		
STAGE 1	8.5	9.35
STAGE 2	9.25	10.175

Decision variables			
no of FS1 to produce	0		
no of FS2 to produce	0		
no of FS12 to produce	0		
no of BS1 to produce	1000		
no of BS2 to produce	0		
no of BS12 to produce	0		
no of HS1 to produce	13200		
no of HS2 to produce	4800		
no of HS12 to produce	0		
no of FO	12000		
no of BO	7000		
Objective Function			
min z	1368100		
constraints			
Demand constrain :			
face cream	12000	>=	12000
body cream	8000	>=	8000
hand cream	18000	>=	18000
Time constrain:			
stage 1 shift 1	15000	<=	15000
stage 2 shift 1	7600	<=	10000
stage 1 shift 2	4800	<=	13500
stage 2 shift 2	2400	<=	9000
raw mat constrain			
Water	132000	<=	200000
Oil	39000	<=	50000
Scents/Colors	7500	<=	7500
Emulsifiers	11500	<=	15000

c. Explore how sensitive your final solution is (both the objective and decision variables) with respect to some of the parameters in the problem.

Final values of decision variables:

- FS1: Face cream produced in shift 1 (both stages) = 0
- FS2: Face cream produced in shift 2 (both stages) = 0
- FS12: Face cream produced in shift 1&2 (Stage 1 in shift 1 and Stage 2 in shift 2) = 0
- FO: Face cream outsourced = 12,000
- BS1: Body cream produced in shift 1 (both stages) = 1,000
- BS2: Body cream produced in shift 2 (both stages) = 0
- BS12: Body cream produced in shift 1&2 (Stage 1 in shift 1 and Stage 2 in shift 2) = 0
- BD: Body cream outsourced = 7,000
- HS1: Hand cream produced in shift 1 (both stages) = 13,200
- HS2: Hand cream produced in shift 2 (both stages) = 4,800
- HS12: Hand cream produced in shift 1&2 (Stage 1 in shift 1 and Stage 2 in shift 2) = 0

Reduced cost interpretation:

- FS1(Face cream produced in shift 1 (both stages)), the reduced cost is 19.597917. This means the cost per unit for FS1 would need to decrease by at least 19. 597917 before it is worth producing
- A reduced cost of 0 (e.g. HS1, HS2, BS1, FO, BO) means the variable is already in the optimal solution
- HS12(Hand cream produced in shift 1&2 (Stage 1 in shift 1 and Stage 2 in shift 2)), the reduced cost is 0.4625. This means the cost per unit for HS12 would need to decrease by at least 0.4625 before it is worth producing

Allowable Decrease:

- FS1 has a allowable decrease of 19.597917. This means the final value of FS1 produced “0” here doesn't change unless the FS1 cost reduced by 19.597917

Allowable Increase:

- FO (Face cream outsourced) has an allowable increase of 19.597917. This means the final value of FO produced “12000” here doesn't change unless the FO cost is increased by 19.597917 or higher

Shadow price:

- The shadow price of scents/colours is -50.95833333, meaning if one additional pound of scent/colour is being added to existing stock of raw materials the objective function will decrease by $50.95833333.
- The shadow price of hand cream is **$47.22**, meaning that if one additional carton of hand cream is produced, the total production cost will increase by **$47.22**, provided the production remains within the allowable range of increase or decrease. The shadow price will remain constant as long as production increases by no more than **750 cartons which is allowable increase** (up to 18,750) or decreases by no more than **5,250 cartons which is allowable decrease** (down to 12,750). Beyond these ranges, the shadow price will change, reflecting a shift in resource utilization or binding constraints. The current production level of **18,000 cartons** will not change unless the demand constraint for hand cream is adjusted to exceed these allowable limits.

General Recommendations:

- The shadow price for scents/colors is -50.95833333, indicating that for each additional pound of scents/colors added to the existing stock, the total production cost decreases by $50.96. Rougir Cosmetics International (RCI) should consider increasing the availability of this raw material.
- Review the final values of resource constraints, especially for materials like water, oil, scents/colors, and emulsifiers. If a material shows significant slack (eg water, oil), consider reducing its stock to better align with actual production needs.
- The solution shows that all 12,000 cartons of face cream are outsourced at $40 per carton, while no in-house production occurs, indicating outsourcing is currently the most cost-effective option. Although the in-house cost of producing face cream ranges from $32.15 to $34.17 per carton, which is slightly lower than outsourcing, labor and material constraints make it more practical to outsource and allocate resources to other products. In contrast, body cream's outsourcing cost is $55 per carton, significantly higher than the in-house cost of $37.35 to $38.91 per carton, making in-house production a better choice for body cream. RCI should continue outsourcing face cream unless in-house production constraints improve, while prioritizing body cream production to reduce reliance on costly outsourcing.