Sequences & Series - Class Notes

1. Introduction

- Sequence: Ordered list of numbers (a_1, a_2, a_3...).

- Series: Sum of terms of a sequence.

- nth term (general term) = position formula.

2. Arithmetic Progression (AP)

Definition: Difference between consecutive terms is constant.

Formulas:

$a_n = a + (n-1)d$

$S_n = n/2 [2a + (n-1)d]$ or $S_n = n/2 (a_1+a_n)$

Example 1: Find sum of first 10 terms of 2,5,8...

a=2, d=3, n=10

$S_n = 10/2 [2*2+(9*3)] = 155$

Example 2: Insert 5 AMs between 3 and 13

d=(13-3)/6=5/3

Sequence = 3, 14/3, 19/3, 8, 29/3, 34/3, 13

Trick: Pair first+last terms.

3. Geometric Progression (GP)

Definition: Ratio between consecutive terms is constant.

Formulas:

$a_n = ar^{(n-1)}$

$S_n = a(r^n-1)/(r-1)$

$S_{inf} = a/(1-r)$, $|r|<1$

Example 3: 2,6,18 -> GP with r=3

Example 4: Insert 2 GMs between 4,108

r=(108/4)^(1/3)=3

  Sequence = 4,12,36,108

Example 5: Sum 5+2.5+1.25+... to infinity

  a=5, r=1/2

  S=5/(1-1/2)=10

4. Harmonic Progression (HP)

Definition: Reciprocals form AP.

Formula: HM = 2ab/(a+b)

Example 6: Insert 3 HMs between 6,12

  Sequence = 6,7.2,8.57,10.28,12

5. Means (AM-GM-HM)

AM = (a+b)/2, GM=sqrt(ab), HM=2ab/(a+b)

Inequality: AM >= GM >= HM

Example 7: For a=3,b=12

  AM=7.5, GM=6, HM=4.8

6. Arithmetic-Geometric Progression (AGP)

Definition: Terms formed by AP * GP.

Formula:

  Sn = [a-(a+nd)r^n]/(1-r) + [dr(1-r^n)]/(1-r)^2

Example 8: 1+3(0.5)+5(0.5^2)+...

  Use AGP formula with a=1,d=2,r=0.5

7. Method of Differences

Trick: Write terms as differences to cancel.

Example 9: Sum 1/[k(k+1)], k=1->n

= Sum(1/k - 1/(k+1)) = 1-1/(n+1) = n/(n+1)

8. Miscellaneous Series

Example 10: Sum k^2 = n(n+1)(2n+1)/6

Example 11: Sum k^3 = [n(n+1)/2]^2

Example 12: 1+2+4+...+2^n = 2^(n+1)-1

9. PYQ JEE Main 2020

Q: Insert 3 AMs & 3 GMs between 3 and 243 such that 4th AM = 2nd GM.

Solution:

- Insert 3 AMs -> AP terms: 3,63,123,183,243

- Insert 3 GMs -> GP terms: 3,9,27,81,243

- 4th AM=183, 2nd GM=9 -> mismatch -> solve general formulas.

Key Tricks & Results:

- AP: an=a+(n-1)d

- GP: an=ar^(n-1)

- HP: reciprocals->AP

- Inserted AMs: d=(b-a)/(n+1)

- Inserted GMs: r=(b/a)^(1/(n+1))

- Product of GMs=(ab)^(n/2)

- Telescoping sums by cancellation

- AGP formula

- AM >= GM >= HM