5
Fundamentals of Grasping

Robotic manipulation, where a robot *physically* interacts and changes the environment, is one of the most challenging tasks in robot autonomy from the perspectives of perception, planning, and control. Consider a simple pick-and-place problem: the robot needs to identify the object, find a good place to grasp, stably pick up the object, and move it to a new location, all while ensuring no part of the robot collides with the environment. In practice even this simple task can become much harder, for example if other objects are in the way and must be moved first, if the object does not have particularly good grasping features, if the weight, size, and surface texture of the object is unknown, or if the lighting is poor[1]. Manipulation tasks are also commonly composed of sequences of interactions, such as making a sandwich or opening a locked door. This chapter focuses on *grasping*[2], which is a fundamental component to all manipulation tasks.

[1] Generally speaking the infinite variability of the real world makes *robust* manipulation extremely difficult.

[2] D. Prattichizzo and J. C. Trinkle. "Grasping". In: *Springer Handbook of Robotics*. Springer, 2016, pp. 955–988

Grasping

Grasping is a fundamental component of robotic manipulation that focuses on obtaining complete control of an object's motion (in contrast to other interactions such as pushing).

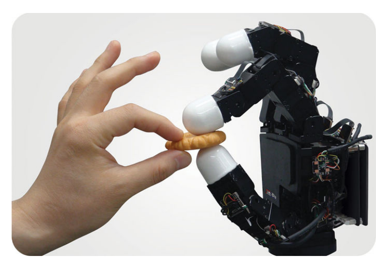

Figure 5.1: The Allegro Hand. Image retrieved from `wiki.wonikrobotics.com`.

Definition 5.0.1 (Grasp)**.** *A grasp is an act of restraining an object's motion through application of forces and torques at a set of contact points.*

Grasping is challenging for several reasons:

1. The configuration of the gripper may be high-dimensional. For example the Allegro Hand (Figure 5.1) has 4 fingers with 3 joints each for a total of 12 dimensions. Plus there are an additional 6 degrees of freedom in the wrist posture (position and orientation), and all of these degrees of freedom vary continuously.

2. Choosing contact points can be difficult. An ideal choice of contact points would lead to a robust grasp, but the space of feasible contacts is restricted

by the gripper's geometry. A rigid body object also has 6 degrees freedom, which affects where the contact points are located in the robot's workspace.

3. While the robot is attempting the grasp it must be sure that its entire body does not come into collision with the environment.

4. Once a grasp has been performed it is important to evaluate how robust the grasp is. While the grasp quality would ideally be optimized during the planning step, it may be important to also check retroactively in case uncertainty led to a different grasp than planned.

To address each of these challenges, the grasp can be subdivided into parts: planning, acquisition/maintenance, and evaluation. This chapter will focus on the fundamentals of how a grasp can be *modeled* and *evaluated* from a mathematical perspective.

5.1 Grasp Modeling

A grasp plan may be parameterized in several ways, including by the approach vector or wrist orientation of the gripper, by the initial finger configuration, or directly by points of contact with the object. However, regardless of the planning parameterization the resulting contacts between the gripper and the object will define the quality of the grasp. Therefore it is useful and convenient for grasp modeling to consider the contact points as the interaction interface between the gripper and object.

5.1.1 Contact Types

There are generally three types of contact that can occur in grasping scenarios:

1. *Point*: a point contact occurs when a single point comes in contact with either another point, a line, or a plane. A point contact is only stable if it is a point-on-plane contact[3], point-on-point or point-on-line contacts are unstable.

2. *Line*: line contacts occur when a line comes in contact with another line or a plane. Line-on-plane and line-on-nonparallel line contacts are stable, but line-on-parallel line contacts are unstable. Line contacts can also be represented as two point contacts.

3. *Plane*: plane-on-plane contacts are always stable. Plane contacts can also be represented as point contacts by converting a distribution of normal forces across a region into a weighted sum of point forces at the vertices of the region's convex hull.

[3] Point-on-plane contacts are by far the most commonly modeled contact types and will almost always be used in grasp analysis.

5.1.2 Point-on-Plane Contact Models

Point-on-plane contact models are by far the most commonly used for grasping since the possible contact points for most objects are almost always surface points (and not sharp edges or points). The purpose of the contact model is to specify the admissible forces and torques that can be transmitted through a particular contact. Considering a local reference frame defined at the contact point with the z direction pointing along the object's surface normal (with the positive direction defined as into the object), the force f can be written as:

$$f = f_{\text{normal}} + f_{\text{tangent}},$$

where $f_{\text{normal}} = [0,\ 0,\ f_z]^T$ is the vector component along the normal direction (with magnitude f_z) and $f_{\text{tangent}} = [f_x,\ f_y,\ 0]^T$ is the vector component tangent to the surface. For all types of contact only an inward force can be applied, therefore $f_z \geq 0$. Three types of contact models are commonly used:

1. Frictionless Point Contact: forces can only be applied along the surface normal, no torques or forces tangential with the surface are possible ($f_{\text{tangent}} = 0$). These types of contact models are more common in form closure grasps.

2. Point Contact with Friction[4]: it is possible to apply forces in directions other than just the surface normal. The admissible forces (i.e. forces that don't lead to slipping) are typically defined by a *friction cone*, which can be described mathematically by the condition:

$$\|f_{\text{tangent}}\| \leq \mu_s \|f_{\text{normal}}\|,$$

   where μ_s is the static friction coefficient associated with the surface (see Figure 5.2).

   A pyramidal inner-approximation of the friction cone is often more useful from a computational standpoint, since its definition only requires a *finite* set of vectors (see Figure 5.3). The point contact with friction model is more common in force closure grasps.

3. Soft-finger Contact Model: allows for torque around the surface normal axis and also includes a friction cone for the forces as in the point contact with friction model. The admissible torques are also constrained by friction:

$$|\tau_{\text{normal}}| \leq \gamma f_z,$$

   where τ_{normal} is the torque about the surface normal axis and $\gamma > 0$ is the torsional friction coefficient.

[4] Also referred to as the *hard finger* model.

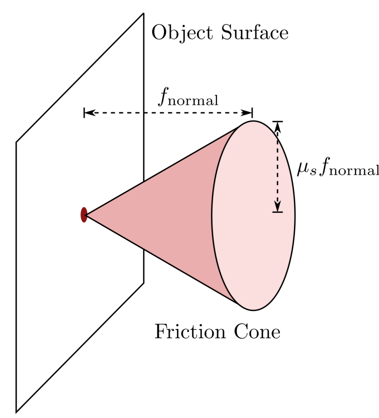

Figure 5.2: Friction cone defined by a static coefficient of friction μ_s.

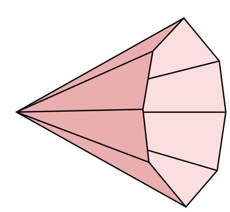

Figure 5.3: Linearized friction cone to *inner* approximate the true cone.

5.1.3 *Wrenches and Grasp Wrench Space*

Under the assumption of a specific contact model, a grasp (defined by a set of contact points) can be quantified and evaluated by determining the *grasp wrench space*, which defines how the grasp can influence the object through an applied wrench.

Definition 5.1.1 (Wrench)**.** *A wrench is a vector valued quantity that describes the forces and torques being applied to an object. For a force $\boldsymbol{f} \in \mathbb{R}^3$ and torque $\boldsymbol{\tau} \in \mathbb{R}^3$ applied at the object's center of mass, the wrench is the stacked vector:*

$$\boldsymbol{w} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{\tau} \end{bmatrix} \in \mathbb{R}^6,$$

and is typically written with respect to a frame fixed in the body.

Each contact point i in a grasp applies a wrench to the object. Additionally, since the torque $\boldsymbol{\tau}_i$ can be computed by $\boldsymbol{\tau}_i = \lambda(\boldsymbol{d}_i \times \boldsymbol{f}_i)$ where $\boldsymbol{d}_i$ is the vector defining the position of the i-th contact point with respect to the object's center of mass and λ is a constant that relates forces to torques, the wrench can be written as:

$$\boldsymbol{w}_i = \begin{bmatrix} \boldsymbol{f}_i \\ \lambda(\boldsymbol{d}_i \times \boldsymbol{f}_i) \end{bmatrix}.$$

Using this definition of a wrench, a grasp can be defined as the set of all possible wrenches that can be achieved by the grasp's contact points. Mathematically, an admissible force $\boldsymbol{f}_i$ applied at the i-th contact point can be linearly mapped into the corresponding wrench on the object (expressed in a reference frame common to all contact points) as $G_i \boldsymbol{f}_i$, where G_i is a wrench basis matrix. Therefore the total wrench on the object from all contacts is:

$$\boldsymbol{w} = \sum_{i=1}^{k} G_i \boldsymbol{f}_i = G \begin{bmatrix} \boldsymbol{f}_1 \\ \vdots \\ \boldsymbol{f}_k \end{bmatrix}, \quad G = \begin{bmatrix} G_1 & \dots & G_k \end{bmatrix}, \tag{5.1}$$

where the combined matrix G is referred to as the *grasp map* (which varies depending on the type of contact model used).

The *grasp wrench space*[5] can then be defined as the set of all possible wrenches that can be applied to the object with a given set of contact points and admissible forces[6]. In other words, the grasp wrench space is defined by the output of (5.1) over all possible applied force combinations $\{\boldsymbol{f}_i\}_{i=1}^{k}$. If the grasp wrench space is large the grasp can compensate for a bigger set of external wrenches that might be applied to the object, leading to a more robust grasp.

[5] In 3D grasping problems the wrench space is a set in $\mathbb{R}^6$, but in planar problems the wrench space is only in $\mathbb{R}^3$ since a wrench would be defined by a 2D force and a 1D torque.

[6] A force is admissible if it lies within the friction cone.

Example 5.1.1 (Computing a Grasp Wrench Space)**.** For a grasping problem with k contact points, let contact point i be associated with a linearized friction cone that is defined by the set of m force vectors:

$$\{\boldsymbol{f}_{i,1}, \boldsymbol{f}_{i,2}, \dots \boldsymbol{f}_{i,m}\}.$$

Then, the set of *all* possible wrenches on the object resulting from admissible forces at contact i is defined by the convex hull of the m wrench vectors:

$$\boldsymbol{w}_{i,j} = \begin{bmatrix} \boldsymbol{f}_{i,j} \\ \lambda(\boldsymbol{d}_i \times \boldsymbol{f}_{i,j}) \end{bmatrix}, \quad \forall j = 1, \dots, m.$$

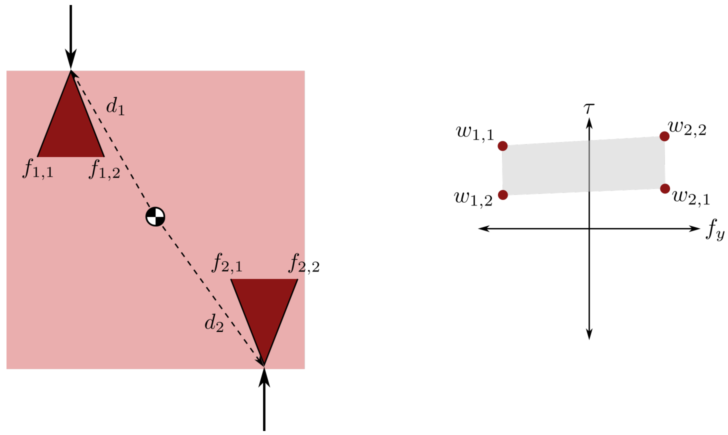

Figure 5.4: An example 2D grasp consisting of two point contacts with friction. The friction cones shown in the figure on the left yield the grasp wrench space in the figure on the right (showing only the vertical force and torque dimensions). This grasp is not stable.

Now consider a 2D problem shown in Figure 5.4, where there are $k = 2$ contact points with friction. The friction cones are defined by the convex hull of the vectors $\{\boldsymbol{f}_{1,1}, \boldsymbol{f}_{1,2}\}$ and $\{\boldsymbol{f}_{2,1}, \boldsymbol{f}_{2,2}\}$ and the distance vectors from the center of mass to the contact points are $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$. The force vectors $\boldsymbol{f}_{i,j}$ are then mapped into the wrenches $\boldsymbol{w}_{i,j}$ (shown on a 2D plot of vertical force f_y and torque τ in Figure 5.4, ignoring the horizontal force components f_x). The grasp wrench space is then given by the convex hull of the wrenches $\boldsymbol{w}_{i,j}$.

However, it turns out that this particular grasp is not stable since the grasp wrench space in Figure 5.4 only contains positive torque values[7]. This means that this grasp cannot apply a negative torque to counteract potential disturbances. This could potentially be fixed in two ways: moving one of the contact points or adding a new contact point. For example, Figure 5.5 demonstrates how adding a third contact point ensures the grasp achieves stability.

[7] The grasp wrench space should contain the origin to ensure stability.

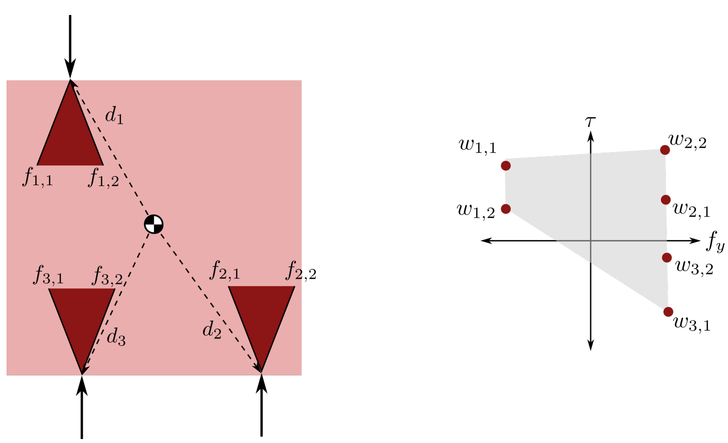

Figure 5.5: An example 2D grasp consisting of three point contacts with friction. The friction cones shown in the figure on the left yield the grasp wrench space in the figure on the right (showing only the vertical force and torque dimensions). This grasp is stable.

5.2 Grasp Evaluation

Now that the basics of grasp modeling have been introduced[8] it is possible to explore techniques for evaluating whether a grasp is "good". In particular, an ideal grasp is one that has *closure*.

[8] contact types, contact models, grasp wrench spaces

Definition 5.2.1 (Grasp Closure)**.** *Grasp closure occurs when the grasp can be maintained for every possible disturbance load.*

For example having grasp closure on a book would enable the gripper to maintain its grasp even if the book was hit by another object or if another book was suddenly stacked on top of it. In practice it may not be reasonable to assume that every *magnitude* disturbance load could be accounted for, but the concept of closure is useful nonetheless.

It can also be helpful to distinguish between two types of grasp closure. A *form closure*[9] grasp typically has the gripper joint angles locked and there is no "wiggle" room for the object (i.e. the object is kinematically constrained). Alternatively, a *force closure*[10] grasp uses forces applied at contact points to be able to *resist* any external wrench. Force closure grasps typically rely on *friction* and generally require fewer contact points than are required for form closure, but may not be able to actually cancel all disturbance wrenches if the friction forces are too weak. This chapter will primarily focus on evaluating force closure grasps since these are most often seen with common robotic gripper hardware.

[9] Also called power grasps or enveloping grasps. A grasp must have at least seven contacts to provide form closure for a 3D object.

[10] Also called a precision grasp. Under a point contact with friction model, a grasp must have at least three contacts to provide force closure for a 3D object.

Figure 5.6: Examples of grasps with form closure (left) and force closure under the soft-finger contact model (right).

5.2.1 Grasp Quality

Grasp quality can be quantified by leveraging the definition of the grasp wrench space from Section 5.1. In particular, a useful metric for quantifying the grasp quality is the radius of the largest ball centered at the origin that is completely contained in the grasp wrench space. This is shown for one of the cases from Example 5.1.1 in Figure 5.7. This metric quantifies the grasp quality in a worst-case sense because it measures the smallest disturbance that could not be compensated for by the grasp. This metric also reinforces the analysis that the grasp in Figure 5.4 is not stable since the origin is not contained within the grasp wrench space.

Another method for quantifying the grasp quality is to compute the volume of the grasp wrench space. This approach provides more of an average-case metric rather than a worst-case metric, and can help differentiate between different grasp spaces that have the same worst-case metric.

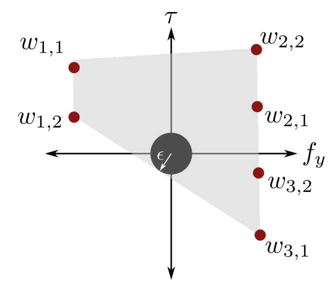

Figure 5.7: Grasp quality can be measured as the radius ϵ of the largest ball contained in the grasp wrench space centered at the origin.