Tutorial-04

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Soln1 $T(n) = 3T(n/2) + n^2$

$\Rightarrow a = 3$

$b = 2$

$f(n) = n^2$

$\because$ a & b are constant and $f(n)$ is a (+ve) function

$\because$ Master's theorem is applicable

$$c = \log_b a$$
$$= \log_2 3 = 1.58$$

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \theta(n^2)}$

Solu2 $T(n) = 4T(n/2) + n^2$

$\Rightarrow a = 4$

$b = 2$

$f(n) = n^2$

$\because$ 'a' & 'b' are constant and $f(n)$ is (+ve) function

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\therefore n^c = n^2$

$\because n^c = f(n)$

$\therefore$ case 2 is applied here

$\Rightarrow T(n) = \theta(n^2 \log n)$

Solu3 $T(n) = T(n/2) + 2^n$

$\Rightarrow a = 1$

$b = 2$

$f(n) = 2^n$

$\because$ 'a' & 'b' are constant and $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$

$\because f(n) > n^c$

$\therefore$ case 3 is applied here

$\Rightarrow T(n) = \theta(2^n)$

Solu 4 $T(n) = 2^n T(n/2) + n^n$

$\Rightarrow a = 2^n$

$b = 2$

$f(n) = n^n$

$\because$ 'a' is not constant, it's value depends on 'n'

$\therefore$ Master's theorem is not applicable here

Solu 5 $T(n) = 16T(n/4) + n$

$\Rightarrow a = 16$

$b = 4$

$f(n) = n$

$\because$ 'a' & 'b' are constant and $f(n)$ is a (+ve) function

$c = \log_b a$

$= \log_4 16 \Rightarrow \log_4 (4)^2 = 2\log_4 4 = 2$

$\Rightarrow n^c = n^2$

$\because f(n) < n^c$

$\because$ case 1 is applied here

$T(n) = \theta(n^2)$

Solu6 $T(n) = 2T(n/2) + n\log n$

$\Rightarrow a = 2$

$b = 2$

$f(n) = n\log n$

$\because$ 'a' and 'b' are constant and $f(n)$ is a (+ve) function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

$\because f(n) > n^c$

$\therefore$ case 3 is applied

$\Rightarrow T(n) = \Theta(n\log n)$

Solu7 $T(n) = 2T(n/2) + n/\log n$

$\Rightarrow a = 2$

$b = 2$

$f(n) = \frac{n}{\log n}$

$\because$ 'a' and 'b' are constant & $f(n)$ is a (+ve) function

$c = \log_b a = \log_2 2 = 1$

$n^c = n^1 = n$

$\because$ non-polynomial difference between $f(n)$ & n^c

$\therefore$ Master's theorem is not applicable

Solu8 $T(n) = 2T(n/4) + n^{0.51}$

$\Rightarrow a = 2$

$b = 4$

$f(n) = n^{0.51}$

$\because$ 'a' & 'b' are constant & $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$\because f(n) > n^c$

$\therefore$ case 3 is applicable

$T(n) = \Theta(n^{0.50})$

Solu9 $T(n) = 0.5T(n/2) + 1/n$

$\Rightarrow a = 0.5$

$b = 2$

$f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable

Solu10 $T(n) = 16T(n/4) + n!$

$\Rightarrow a = 16$

$b = 4$

$f(n) = n!$

∵ 'a' & 'b' are constant & f(n) is a (+ve) function

∴ Master's theorem is applicable

$c = \log_b a$

$= \log_4 16 = 2$

$n^c = n^2$

∵ $f(n) > n^c$

∴ case 3 is applied here

⇒ $\boxed{T(n) = \theta(n!)}$

Soln $T(n) = 4T(n/2) + \log n$

⇒ $a = 4$

$b = 2$

$f(n) = \log n$

∵ 'a' & 'b' are constant & f(n) is a (+ve) function

∴ Master's theorem is applicable

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

∵ $f(n) < n^2$

∴ case 1 is applied here

⇒ $\boxed{T(n) = \theta(n^2)}$

Soln 12 $T(n) = \sqrt{n}\,T(n/2) + \log n$

$\Rightarrow a = \sqrt{n}$

$b = 2$

$f(n) = \log n$

$\because$ 'a' is not constant

$\therefore$ Master's theorem is not applic

Soln 13 $T(n) = 3T(n/2) + n$

$\Rightarrow a = 3$

$b = 2$

$f(n) = n$

$\because$ 'a' & 'b' are constant & $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 01.58$

$\because n^c = n^{1.58}$

$\therefore f(n) < n^c$

$\therefore$ case 1 is applied here

$\Rightarrow \boxed{T(n) = \theta(n^{1.58})}$

Soln 14 $T(n) = 3T(n/3) + \sqrt{n}$

$\Rightarrow a = 3$

$b = 3$

$f(n) = \sqrt{n}$

$\because$ 'a' and 'b' are constant & $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable

$c = \log_b a = 1$

$n^c = n^2$

$\Rightarrow f(n) < n^c$

$\therefore$ case 1 is applied here

$\Rightarrow \boxed{T(n) = \theta(n)}$

Soln 5 $T(n) = 3T(n/4) + n\log n$

$\Rightarrow a = 3$

$b = 4$

$f(n) = n\log n$

$\therefore$ 'a' & 'b' are constant & $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$\therefore f(n) > n^c$

$\therefore$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \theta(n\log n)}$

Soln 7 $T(n) = 3T(n/3) + n/2$

$\Rightarrow a = 3$

$b = 3$

$f(n) = n/2$

$\therefore$ 'a' and 'b' are constant & $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_3 3 = 1$

$\therefore n^c = n^1$

$\because f(n) = n^c$

$\therefore$ case 2 is applied here

$\Rightarrow \boxed{T(n) = \theta(n \log n)}$

Soln 8 $T(n) = 6T(n/3) + n^2 \log n$

$\Rightarrow a = 6$

$b = 3$

$f(n) = n^2 \log n$

$\because$ 'a' & 'b' are constant & $f(n)$ is a (+ve) function

$\therefore$ Master's theorem is applicable here

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$\therefore f(n) > n^c$

$\Rightarrow$ case 3 is applied here

$\Rightarrow \boxed{T(n) = \theta(n^2 \log n)}$

Soln 9 $T(n) = 4T(n/2) + n/\log n$

$\Rightarrow a = 4$

$b = 2$

$f(n) = \frac{n}{\log n}$

∵ 'a' and 'b' are constant and f(n) is a (+ve) function

∴ Master's theorem is applicable here

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

∵ $f(n) < n^c$

∴ case 1 is applied here

⇒ $\boxed{T(n) = \Theta(n^2)}$

Soln 2) $T(n) = 64\,T(n/8) + n^2 \log n$

⇒ $a = 64$

$b = 8$

∵ 'a' and 'b' are constant but f(n) is a (+ve) function

∴ Master's theorem is not applicable here

Soln 2) $T(n) = 7\,T(n/3) + n^2$

⇒ $a = 7$

$b = 3$

$f(n) = n^2$

∵ 'a', 'b' are constant and f(n) is a (+ve) function

∴ Master's theorem is applied here

⇒ $c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

∵ $f(n) > n^c$

∴ case 3 is applied here

⇒ $\boxed{T(n) = \Theta(n^2)}$

Soln22 $T(n) = T(n/2) + n(2-\cos n)$

$\Rightarrow a = 1$

$b = 2$

$\because$ 'a' and 'b' are constant but $f(n)$ is not a regular function

$\therefore$ Master's theorem is not applicable here