Taking derivative by convolution

Partial derivatives with convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

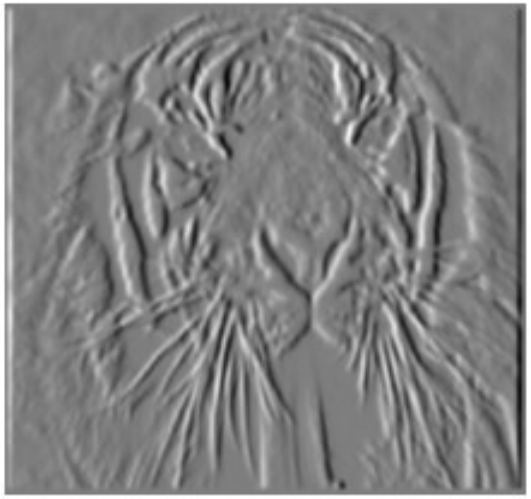
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement above as convolution, what would be the associated filter?

Source: K. Grauman

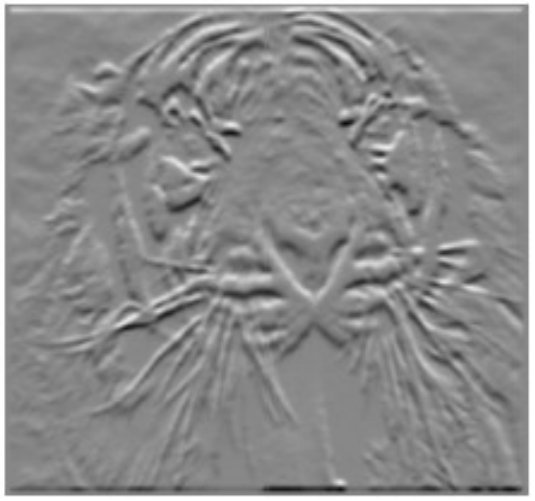
Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$



-1	1

$$\frac{\partial f(x,y)}{\partial y}$$



-1
1

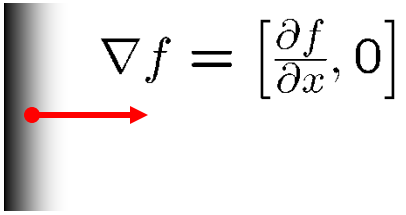
or

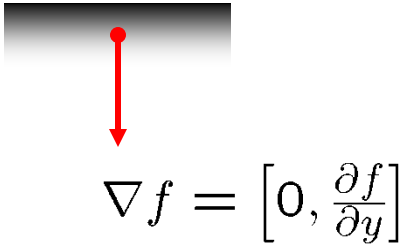
1
-1

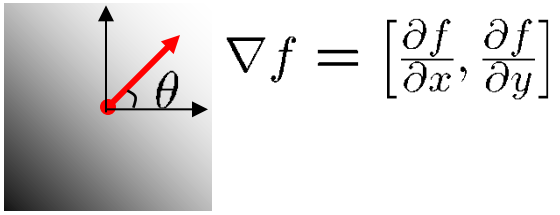
Which shows changes with respect to x?

Image gradient

The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$







The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

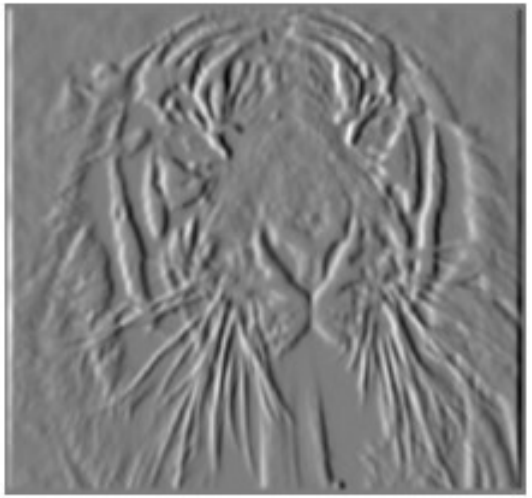
The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The *edge strength* is given by the gradient magnitude

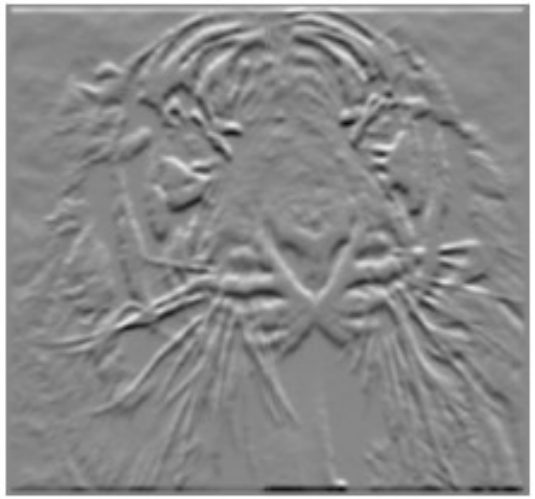
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Image Gradient



$$\frac{\partial f(x,y)}{\partial x}$$



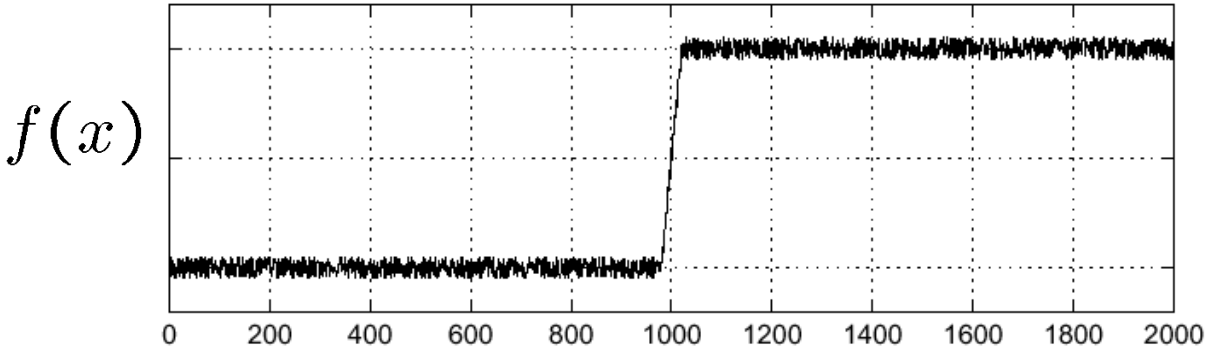
$$\frac{\partial f(x,y)}{\partial y}$$

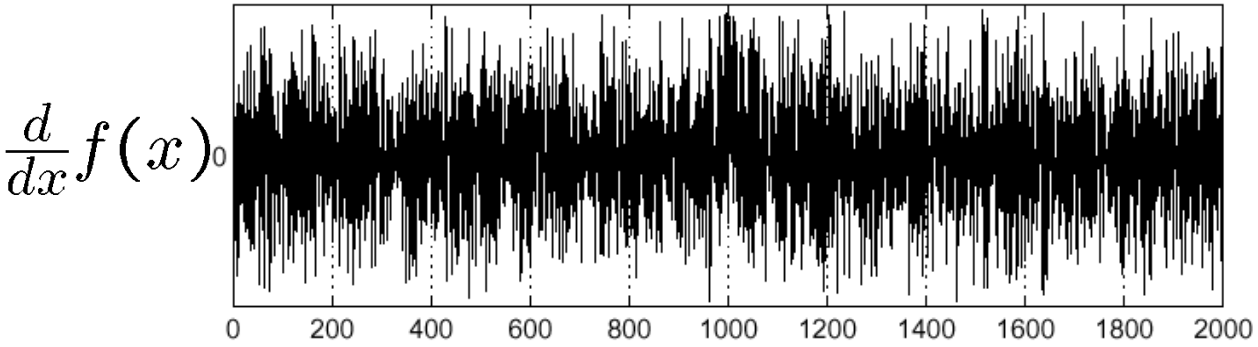
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Effects of noise

Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

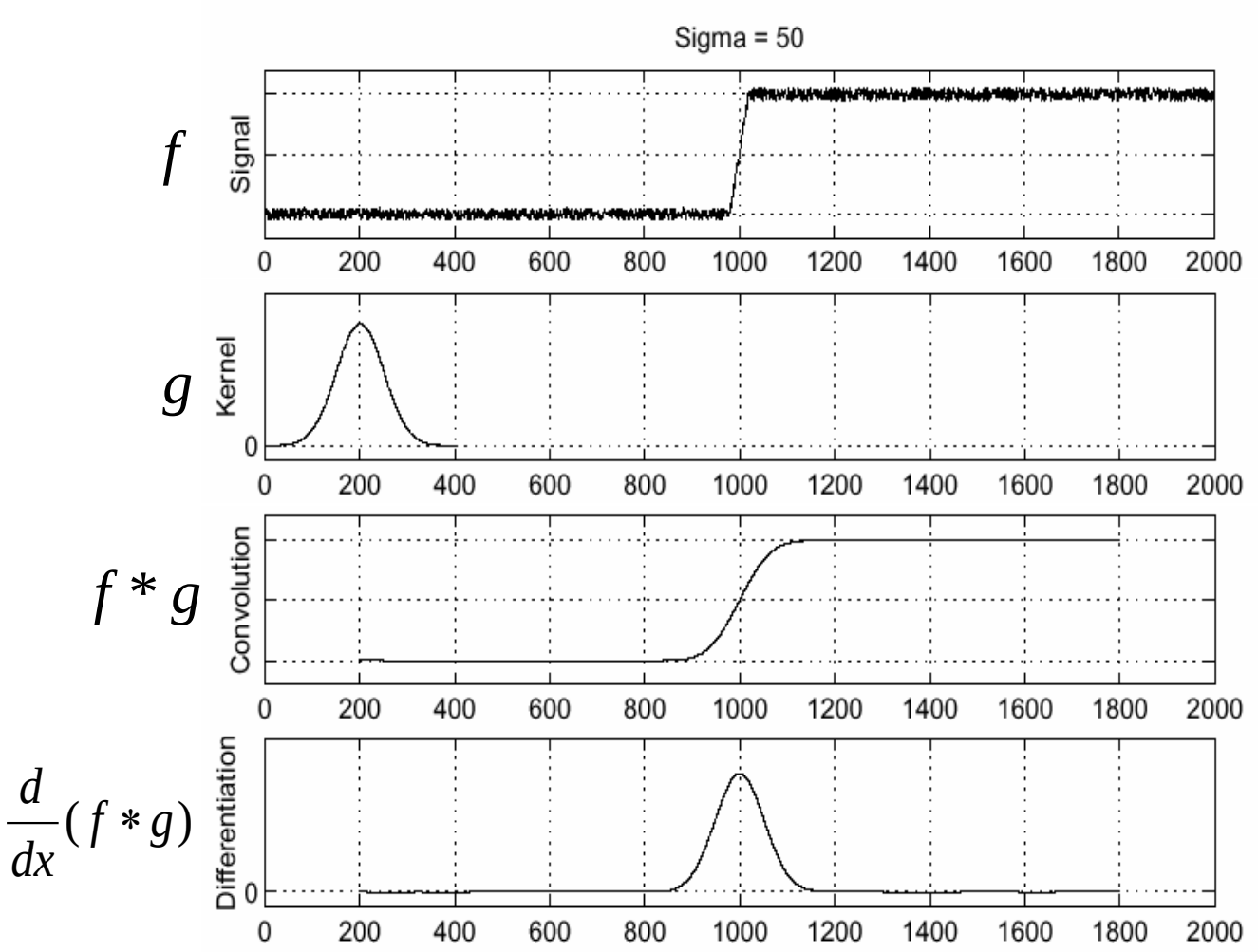




Where is the edge?

Source: S. Seitz

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

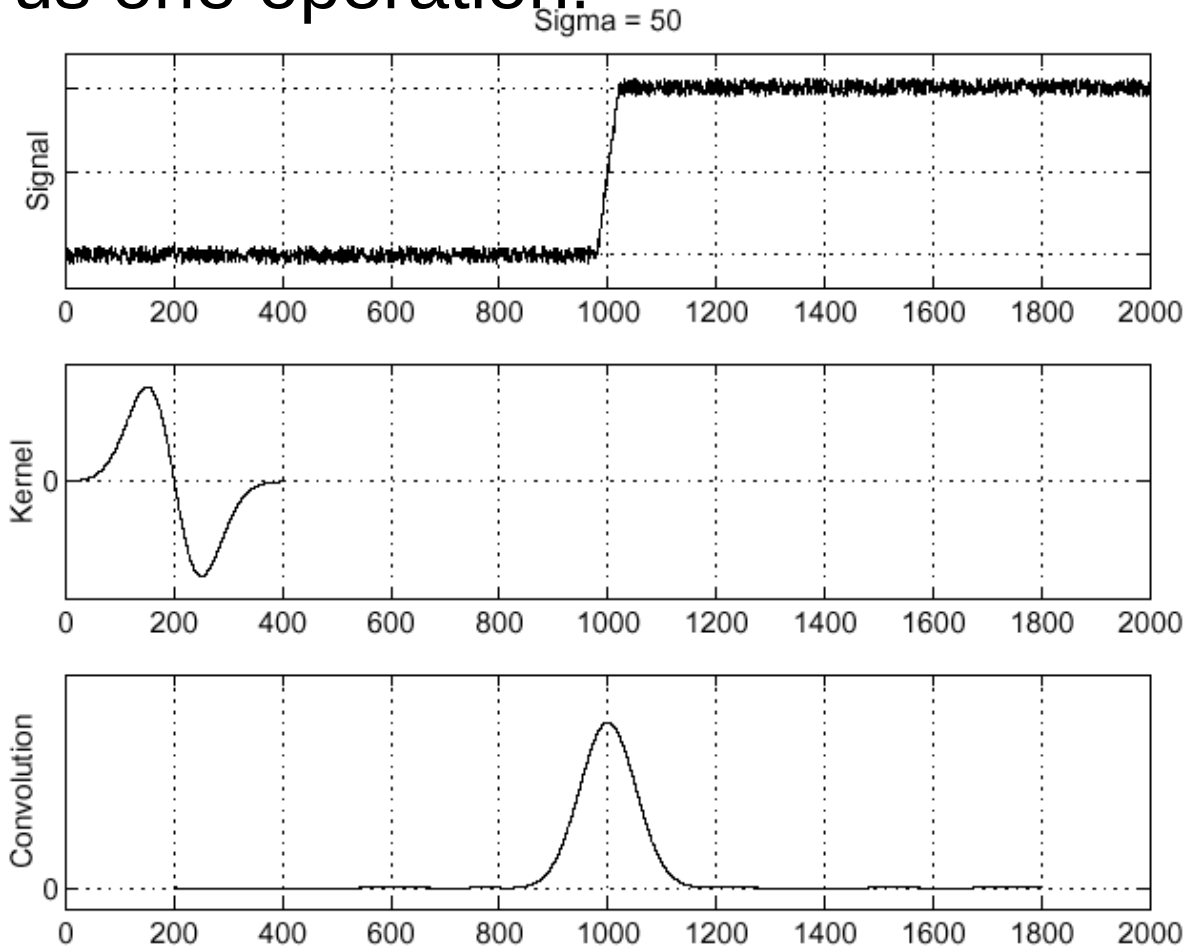
Derivative theorem of convolution

$$\frac{\partial}{\partial x}(h \star f) = (\frac{\partial}{\partial x}h) \star f$$

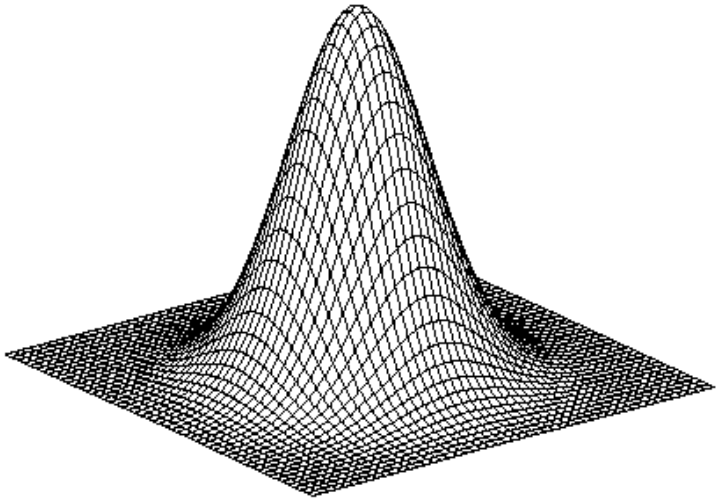
This saves us one operation:

$\frac{\partial}{\partial x}h$

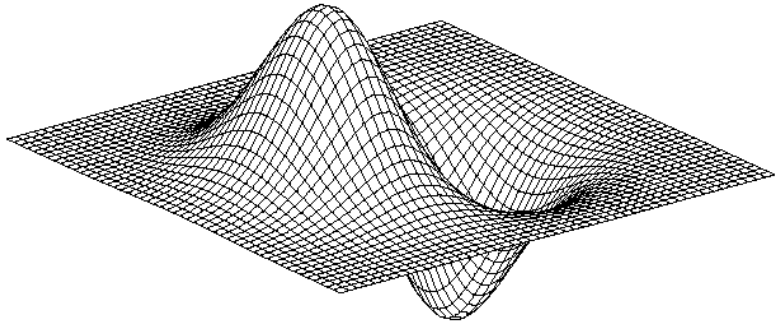
$(\frac{\partial}{\partial x}h) \star f$



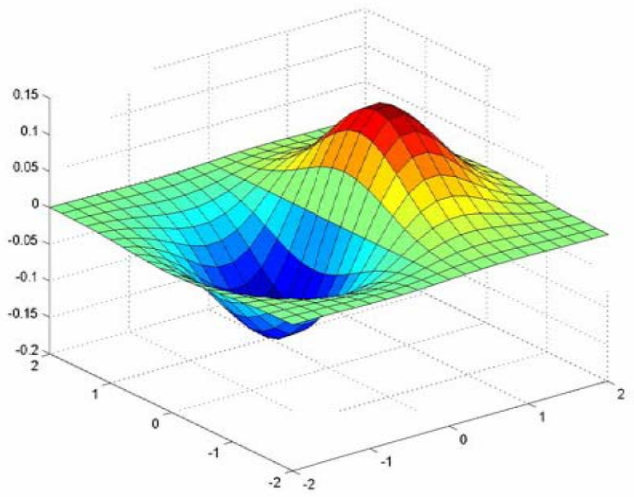
Derivative of Gaussian filter



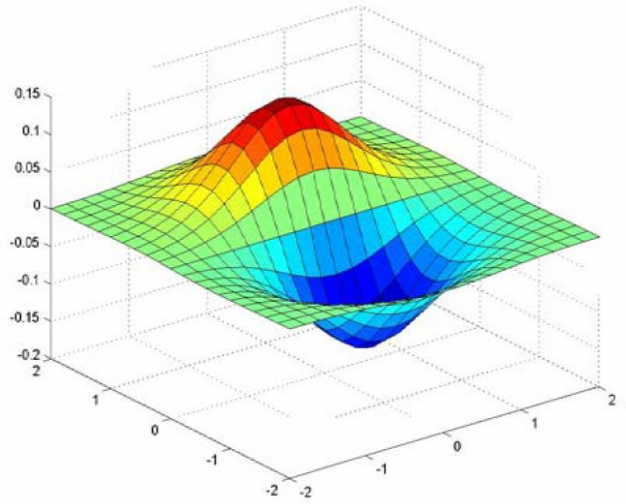
* [1 -1] =



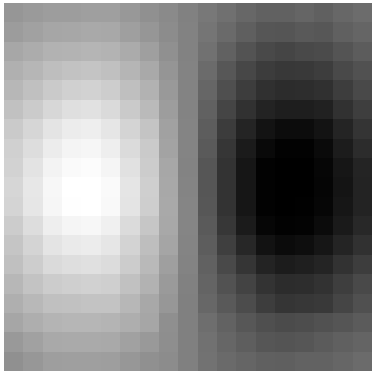
Derivative of Gaussian filter

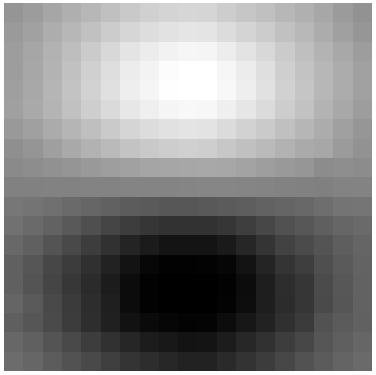


x-direction



y-direction





Which one finds horizontal/vertical edges?

Example

input image (“Lena”)

Compute Gradients (DoG)

X-Derivative of Gaussian

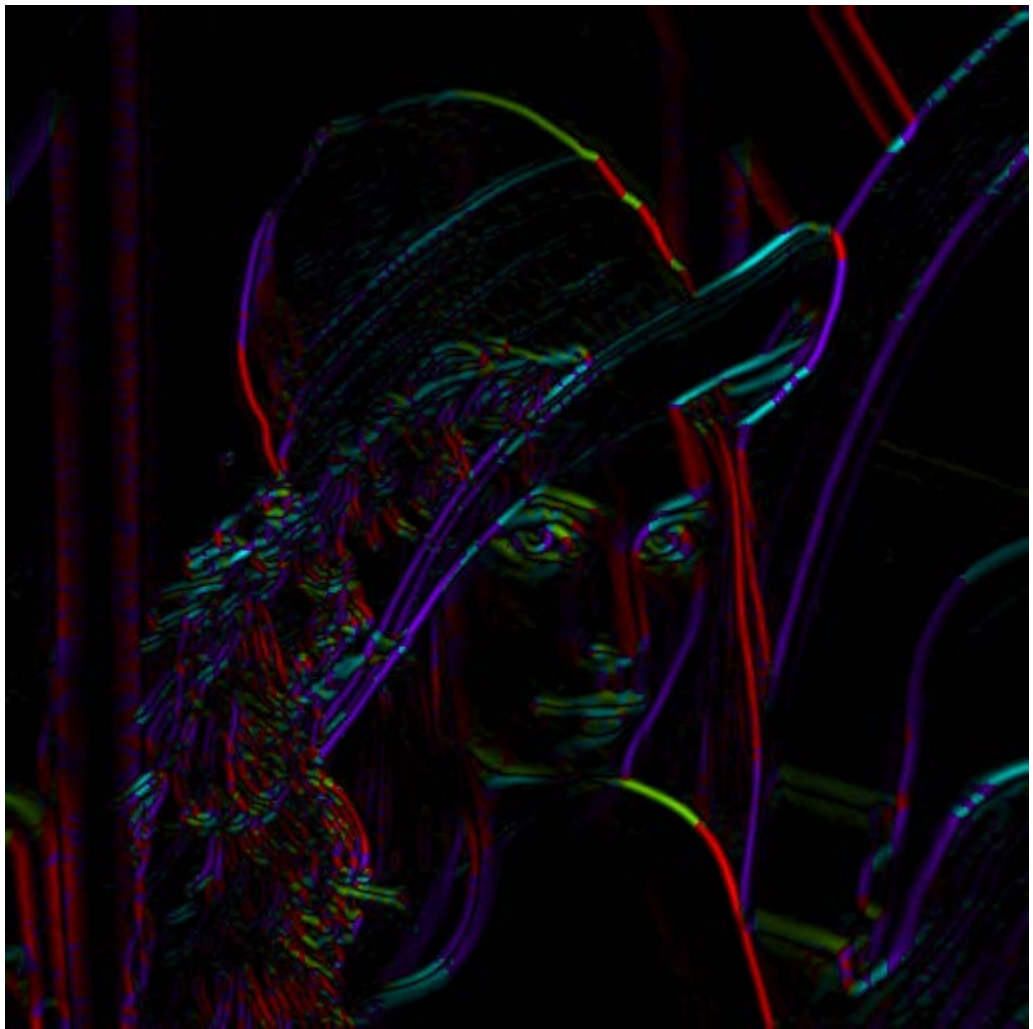
Y-Derivative of Gaussian

Gradient Magnitude

Get Orientation at Each Pixel

Threshold at minimum level

Get orientation



theta = atan2(-gy, gx)

MATLAB demo

```
im = im2double(imread(filemane));
g = fspecial('gaussian',15,2);
imagesc(g);
surfl(g);
gim = conv2(im,g,'same');
imagesc(conv2(im,[-1 1],'same'));
imagesc(conv2(gim,[-1 1],'same'));
dx = conv2(g,[-1 1],'same');
Surfl(dx);
imagesc(conv2(im,dx,'same'));
```

Finite difference filters

Other approximations of derivative filters exist:

Prewitt: $M_x =$

-1	0	1
-1	0	1
-1	0	1

; $M_y =$

1	1	1
0	0	0
-1	-1	-1

Sobel: $M_x =$

-1	0	1
-2	0	2
-1	0	1

; $M_y =$

1	2	1
0	0	0
-1	-2	-1

Roberts: $M_x =$

0	1
-1	0

; $M_y =$

1	0
0	-1

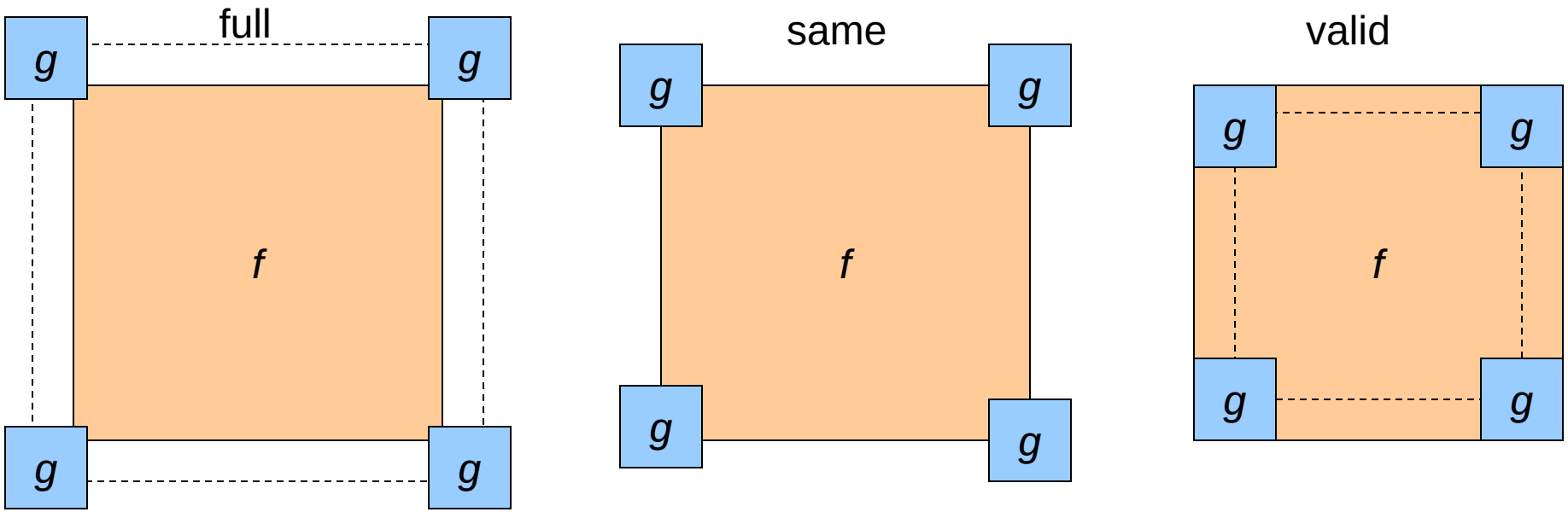
Source: K. Grauman

Practical matters

What is the size of the output?

MATLAB: filter2(g, f, *shape*) or conv2(g,f,shape)

- *shape* = ‘full’: output size is sum of sizes of f and g
- *shape* = ‘same’: output size is same as f
- *shape* = ‘valid’: output size is difference of sizes of f and g



Source: S. Lazebnik

Practical matters

What about near the edge?

- the filter window falls off the edge of the image
- need to extrapolate
- methods:
  - clip filter (black)
  - wrap around
  - copy edge
  - reflect across edge

Source: S. Marschner

Q?

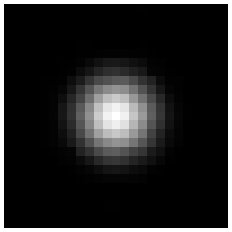
Practical matters

- methods (MATLAB):
  - clip filter (black): imfilter(f, g, 0)
  - wrap around: imfilter(f, g, ‘circular’)
  - copy edge: imfilter(f, g, ‘replicate’)
  - reflect across edge: imfilter(f, g, ‘symmetric’)

Source: S. Marschner

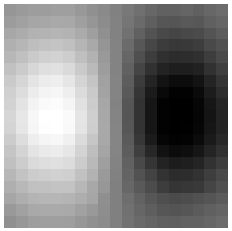
Review: Smoothing vs. derivative filters

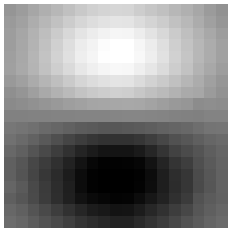
Smoothing filters



- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter

Derivative filters





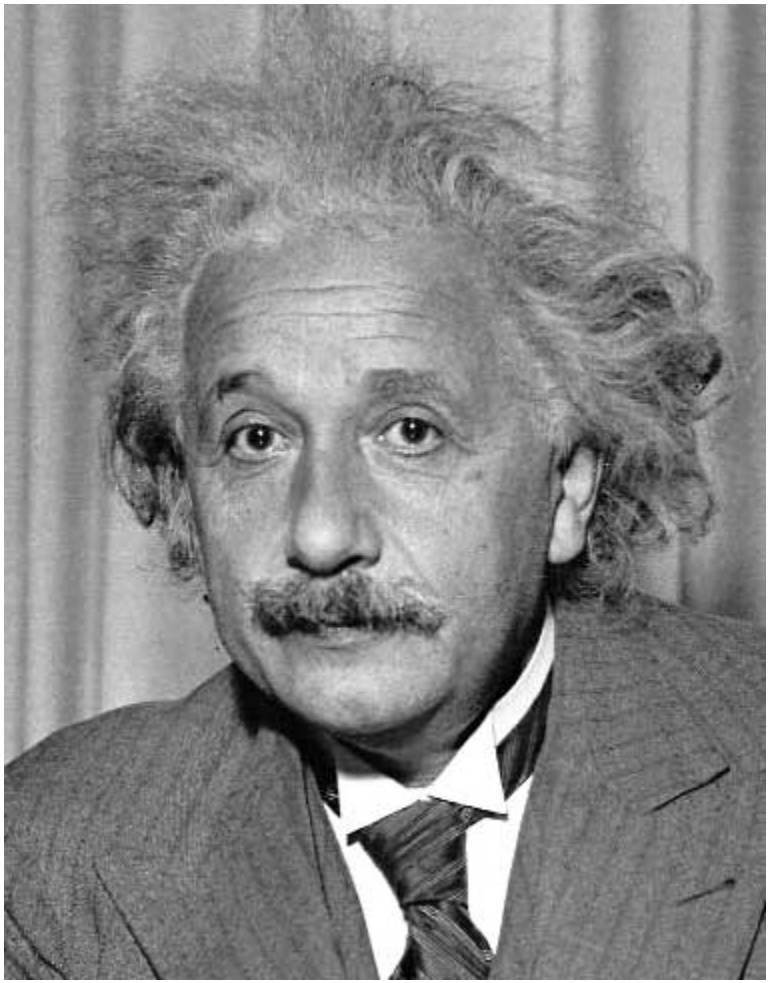
- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions
- High absolute value at points of high contrast

Template matching

Goal: find  in image

Main challenge: What is a good similarity or distance measure between two patches?

- Correlation
- Zero-mean correlation
- Sum Square Difference
- Normalized Cross Correlation



Side by Derek Hoiem

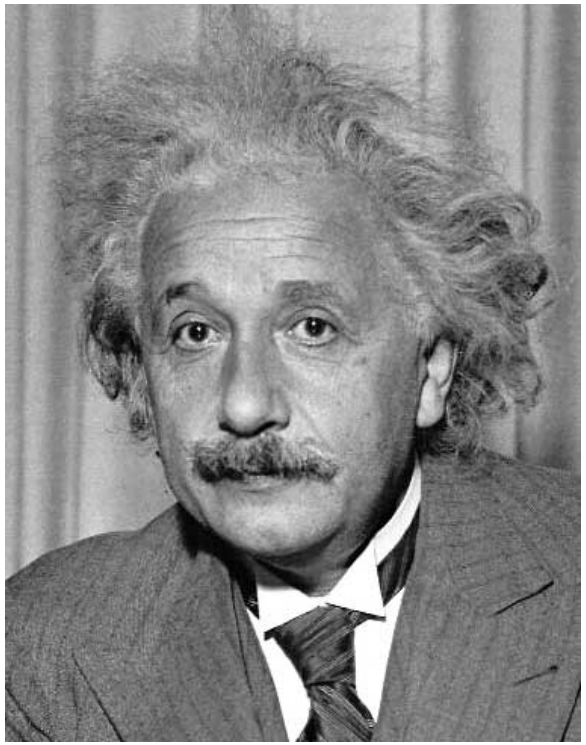
Matching with filters

Goal: find  in image

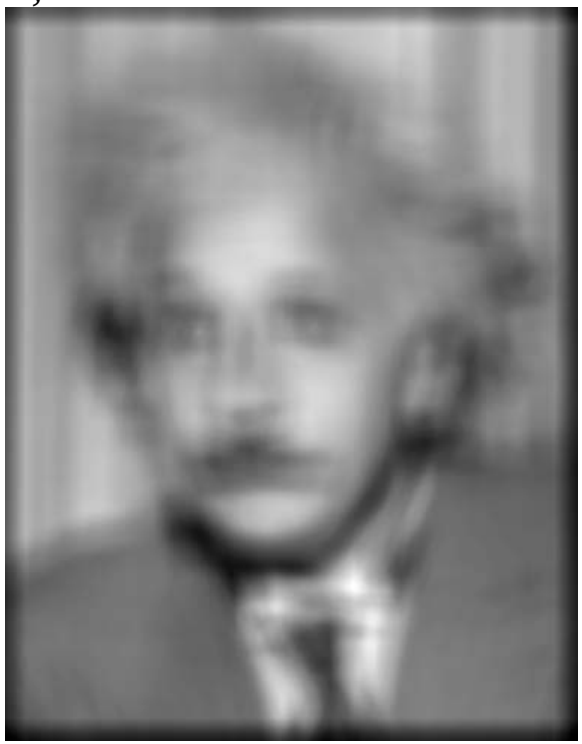
Method 0: filter the image with eye patch

$$h[m,n] = \sum_{k,l} g[k,l]\ f[m+k, n+l]$$

f = image
g = filter



Input



Filtered Image

What went wrong?

Side by Derek Hoiem

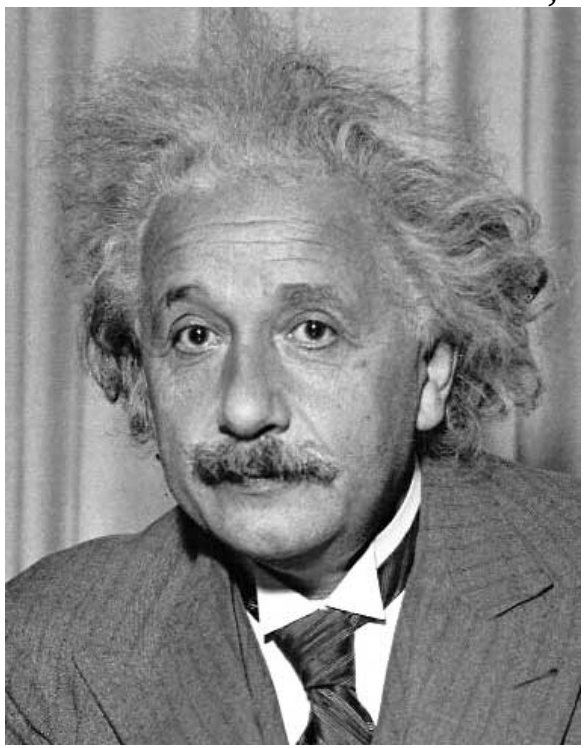
Matching with filters

Goal: find  in image

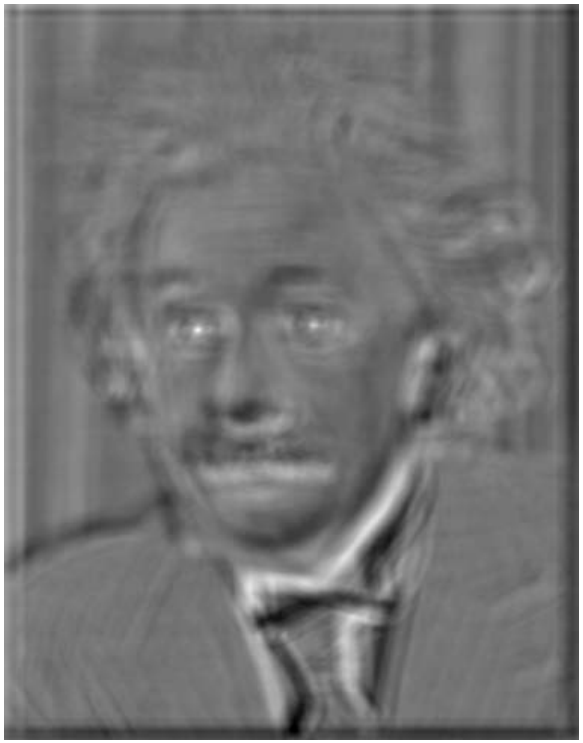
Method 1: filter the image with zero-mean eye

$$h[m,n] = \sum_{k,l} (f[k,l] - \bar{f})(g[m+k, n+l])$$

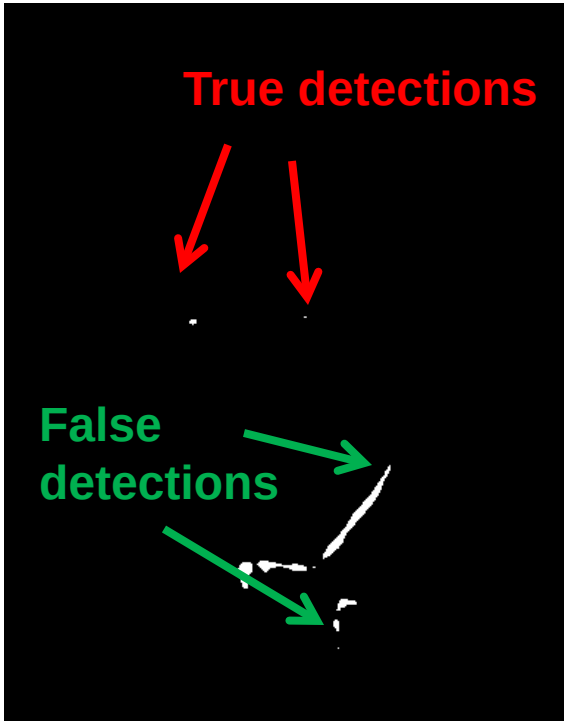
mean of f



Input



Filtered Image (scaled)



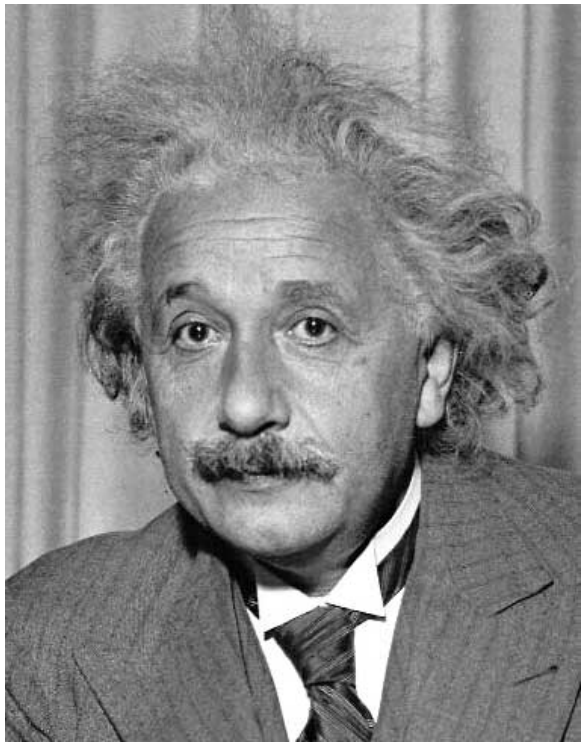
Thresholded Image

Matching with filters

Goal: find  in image

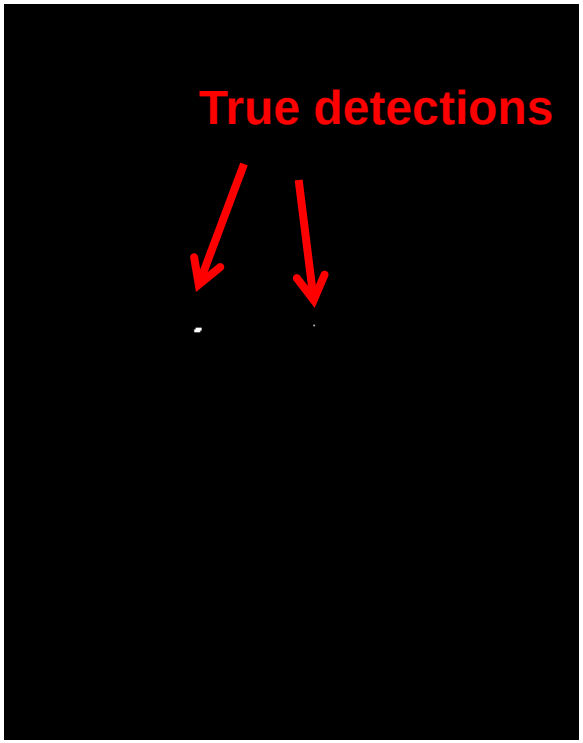
Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input

1- sqrt(SSD)



Thresholded Image

Matching with filters

Can SSD be implemented with linear filters?

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$

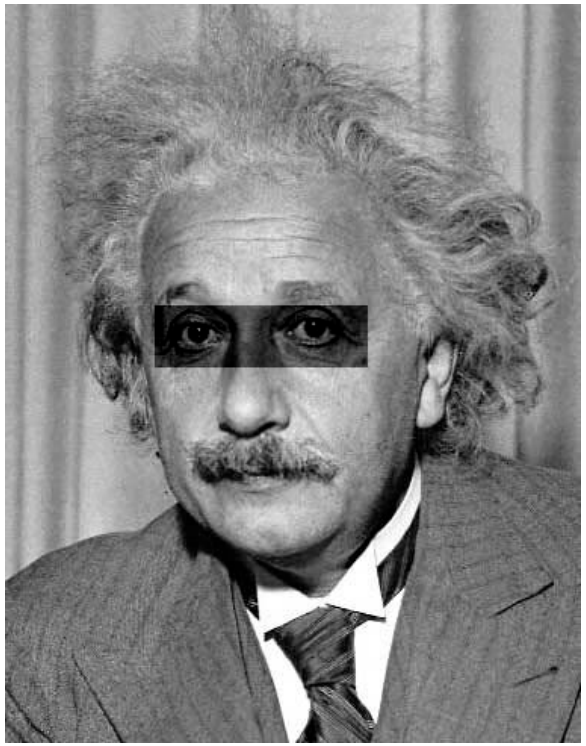
Matching with filters

Goal: find  in image

What's the potential downside of SSD?

Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2$$



Input

1- sqrt(SSD)

Side by Derek Hoiem

Matching with filters

Goal: find  in image

Method 3: Normalized cross-correlation

mean template

mean image patch

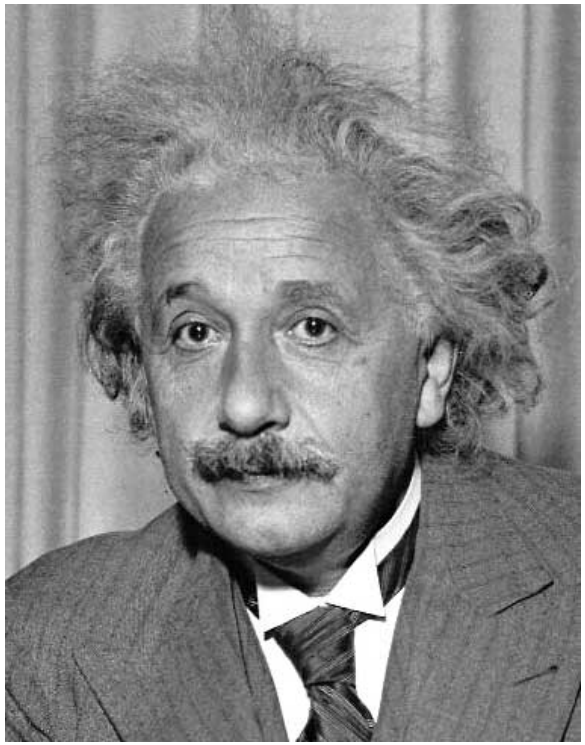
$$h[m,n] = \frac{\sum_{k,l}(g[k,l]-\bar{g})(f[m+k,n+l]-\bar{f}_{m,n})}{\left(\sum_{k,l}(g[k,l]-\bar{g})^2 \sum_{k,l}(f[m+k,n+l]-\bar{f}_{m,n})^2\right)^{0.5}}$$

Side by Derek Hoiem

Matching with filters

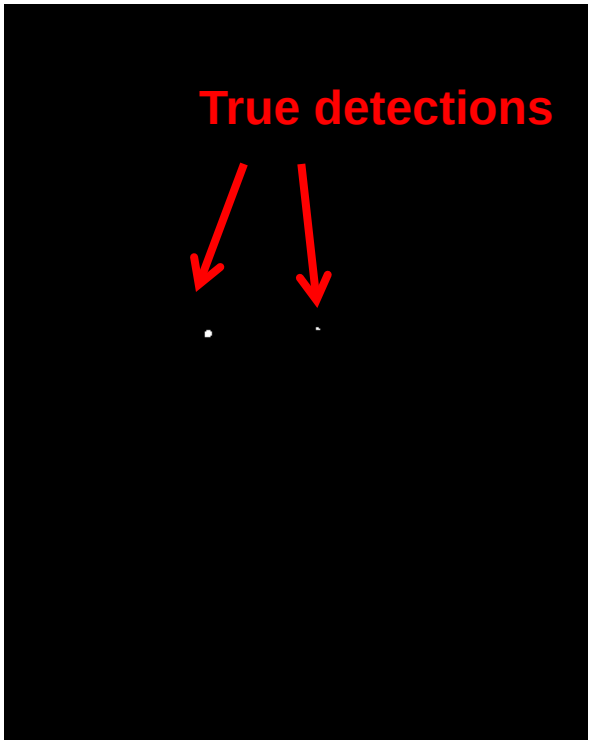
Goal: find  in image

Method 3: Normalized cross-correlation



Input

Normalized X-Correlation

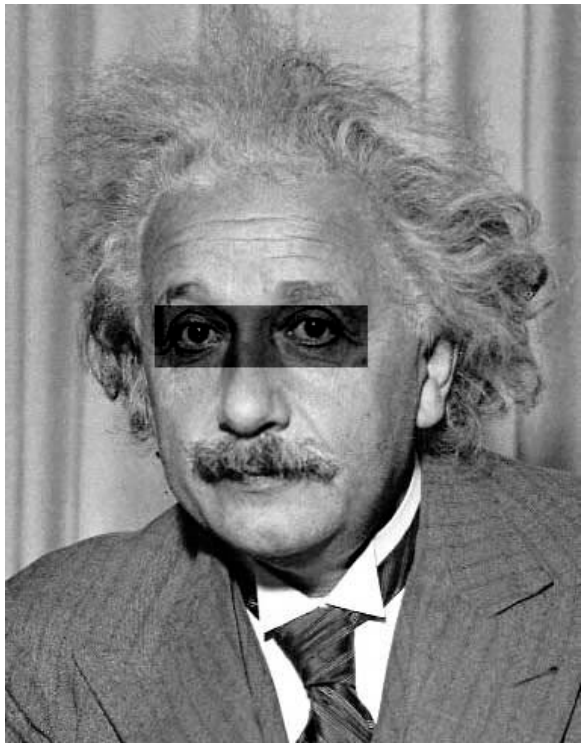


Thresholded Image

Matching with filters

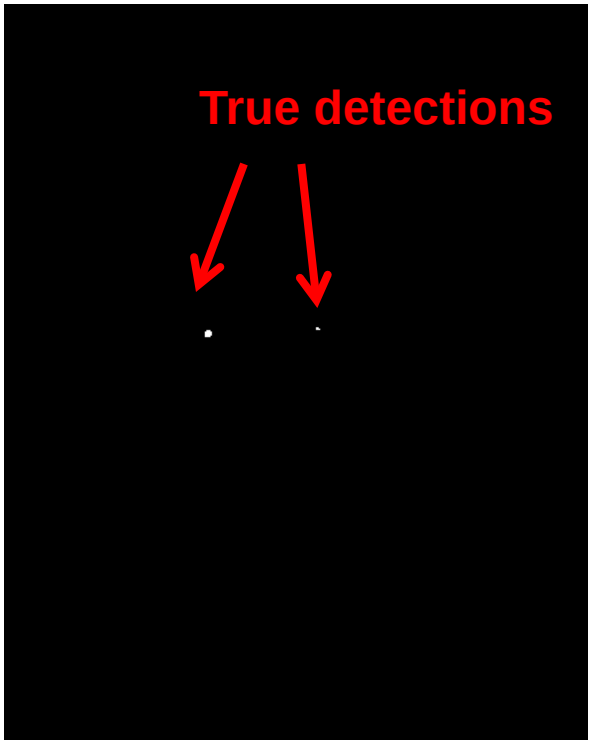
Goal: find  in image

Method 3: Normalized cross-correlation



Input

Normalized X-Correlation



Thresholded Image

Q: What is the best method to use?

A: Depends

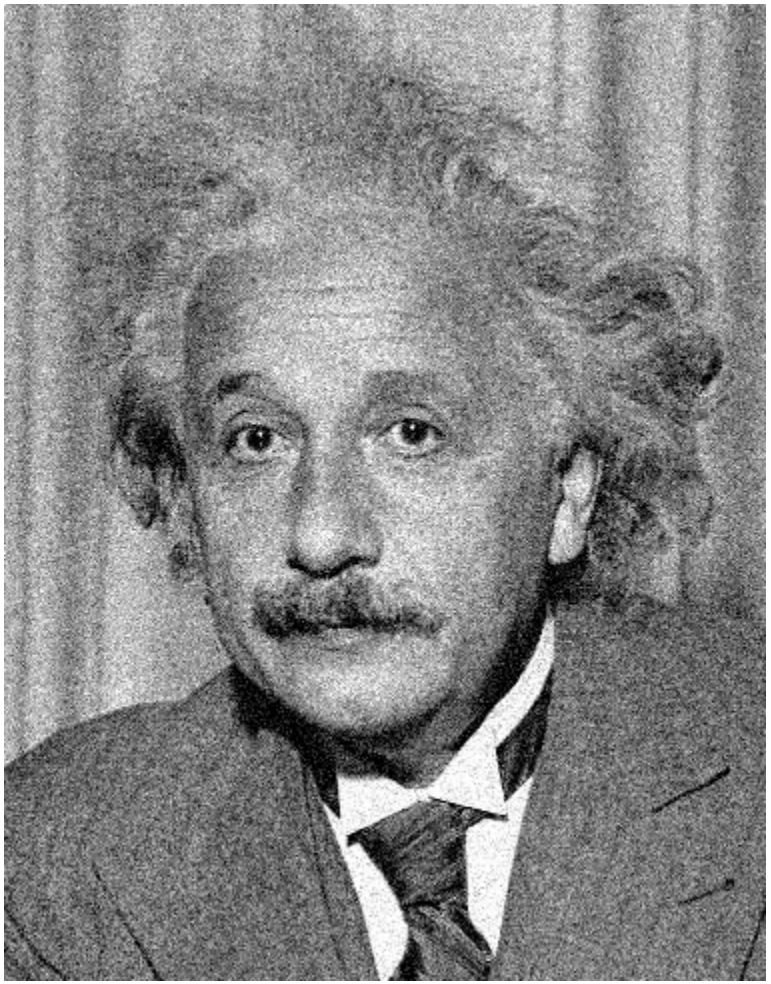
Zero-mean filter: fastest but not a great matcher

SSD: next fastest, sensitive to overall intensity

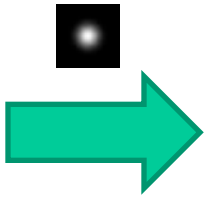
Normalized cross-correlation: slowest, invariant to local average intensity and contrast

Side by Derek Hoiem

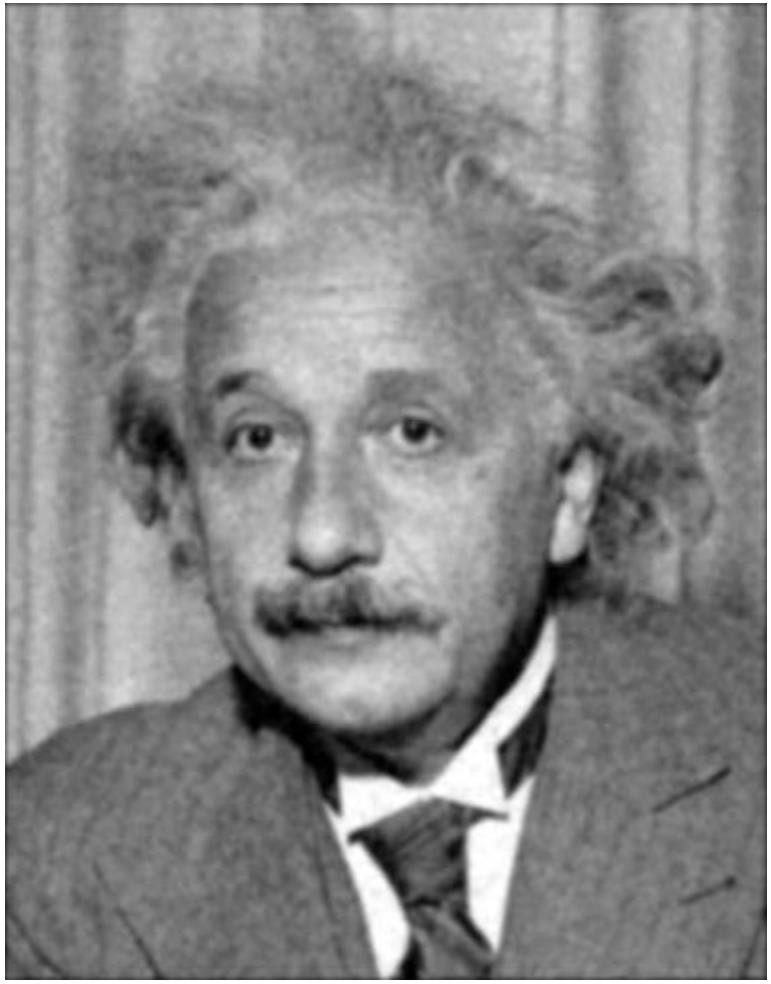
Denoising



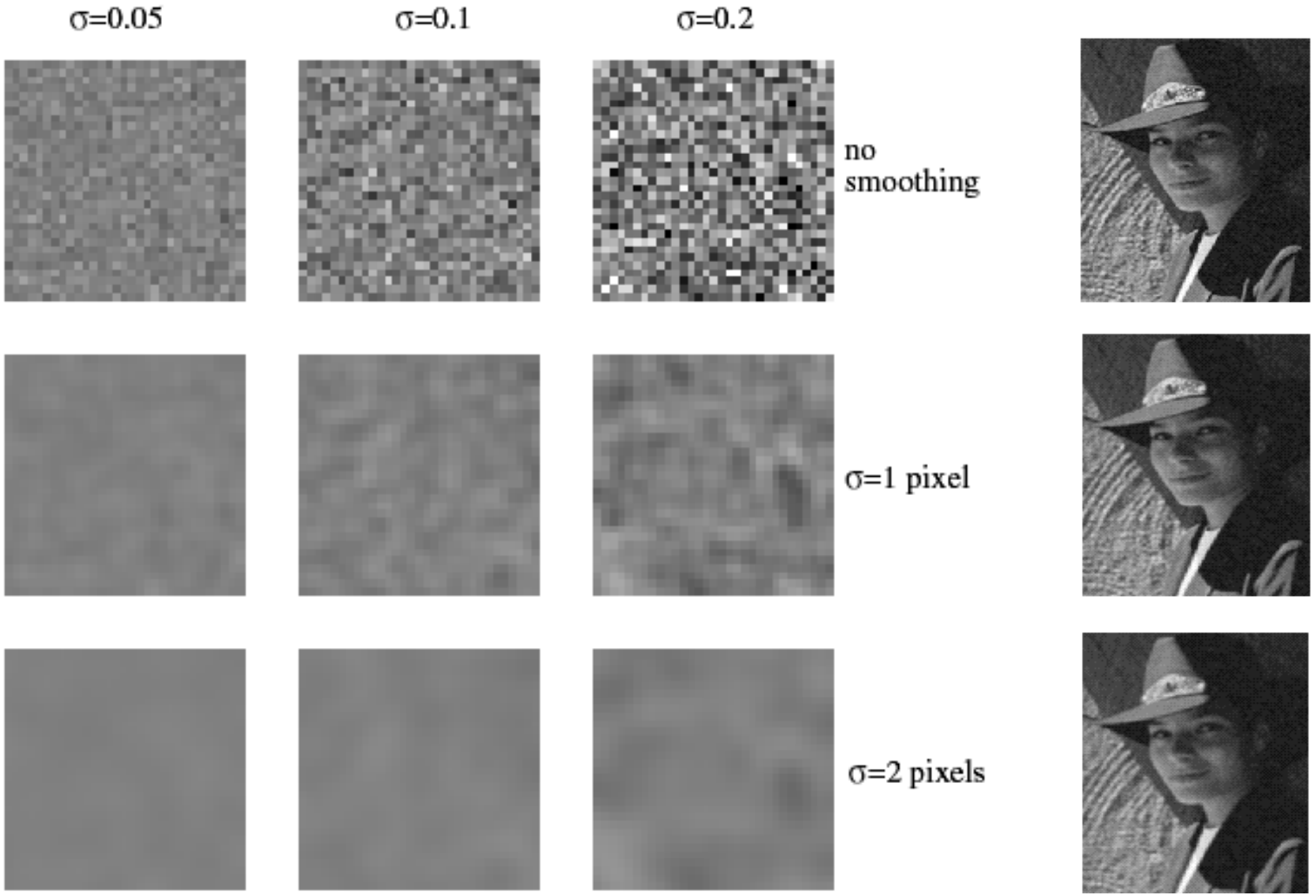
Additive Gaussian Noise



Gaussian
Filter



Reducing Gaussian noise



Smoothing with larger standard deviations suppresses noise, but also blurs the image

Source: S. Lazebnik

Reducing salt-and-pepper noise by Gaussian smoothing

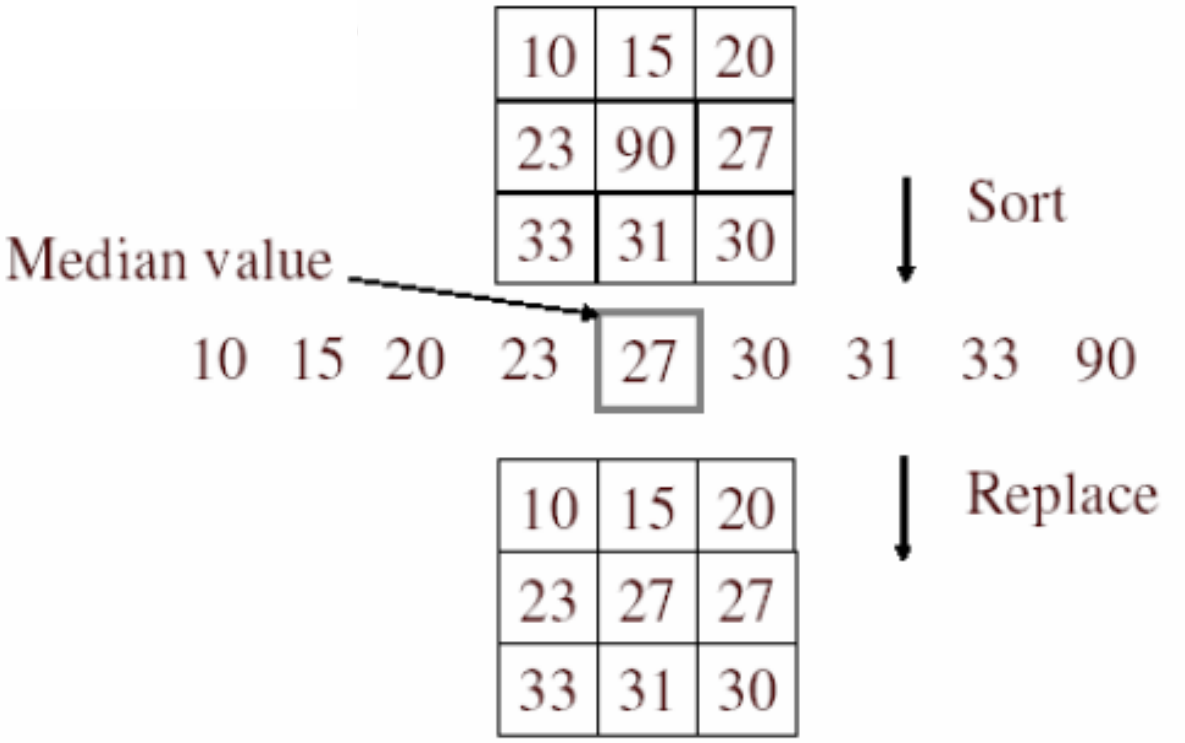
3x3

5x5

7x7

Alternative idea: Median filtering

A **median filter** operates over a window by selecting the median intensity in the window



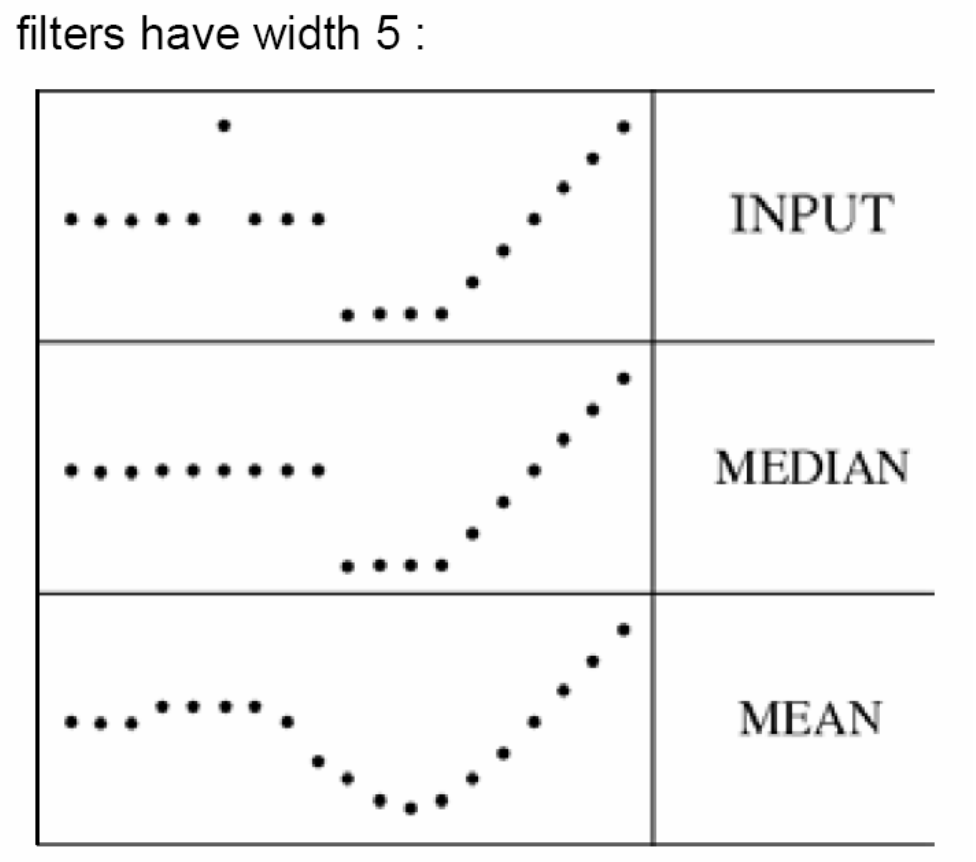
- Is median filtering linear?

Source: K. Grauman

Median filter

What advantage does median filtering have over Gaussian filtering?

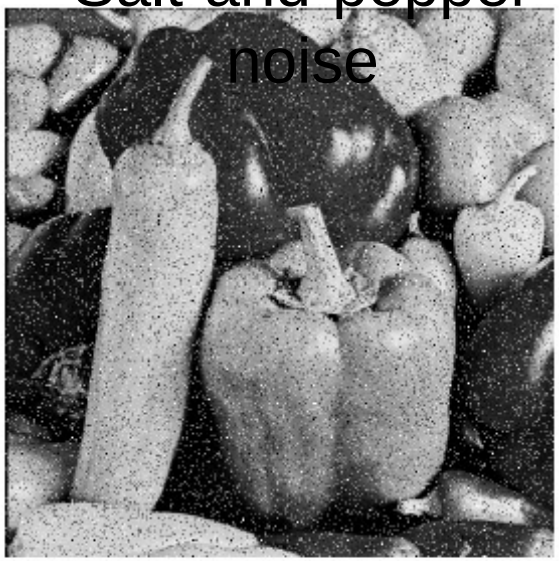
- Robustness to outliers



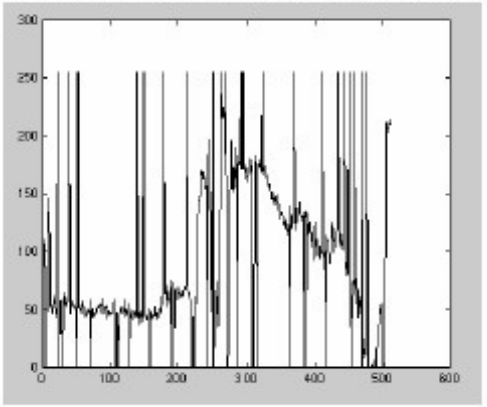
Source: K. Grauman

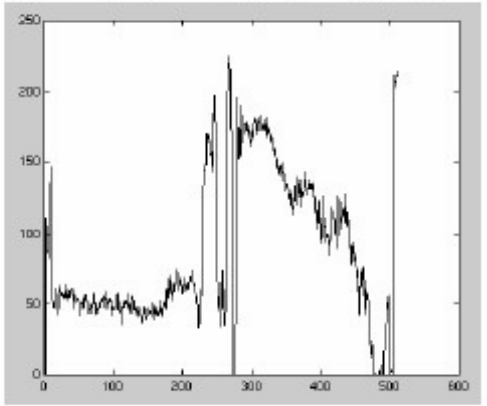
Median filter

Salt-and-pepper noise



Median filtered

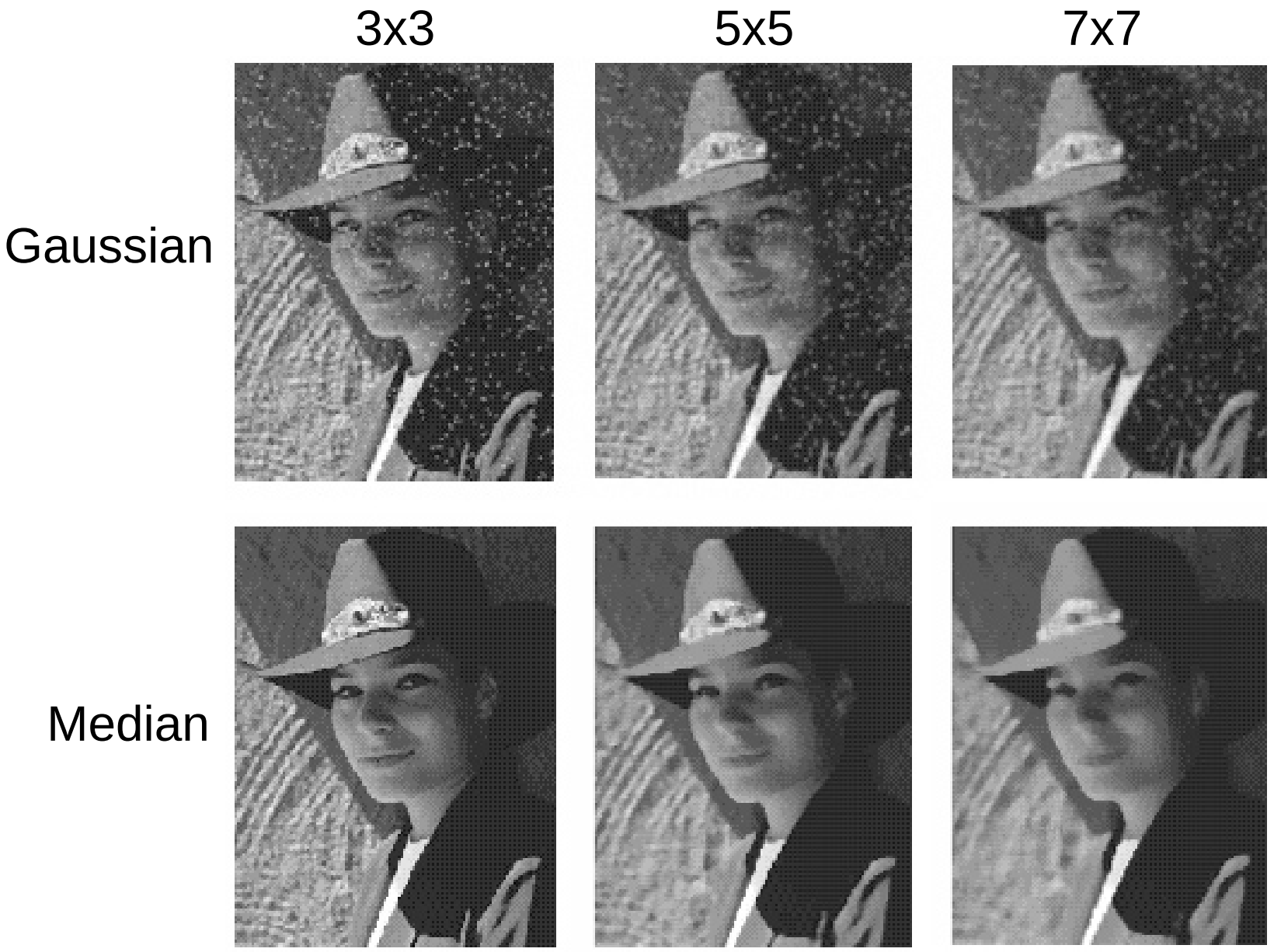




MATLAB: medfilt2(image, [h w])

Source: M. Hebert

Median vs. Gaussian filtering



EXTRA SLIDES

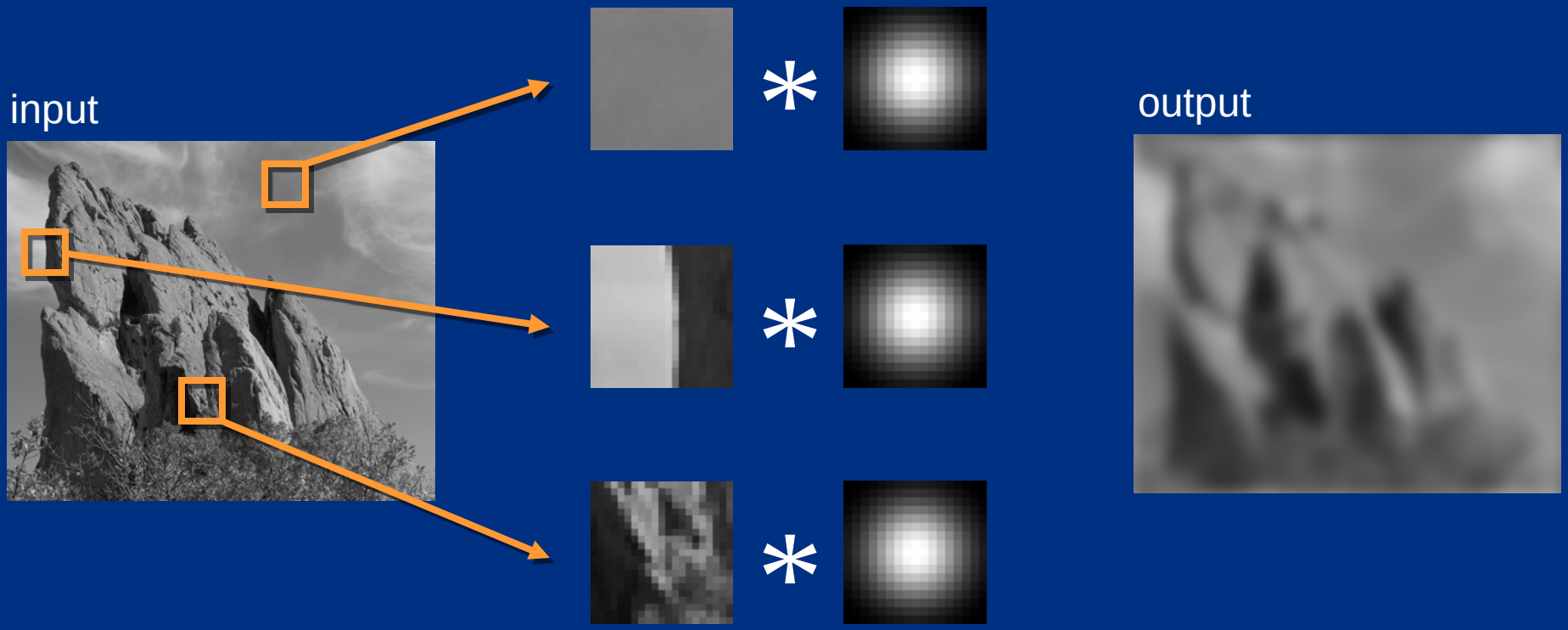
A Gentle Introduction
to Bilateral Filtering
and its Applications

SIGGRAPH2007

“Fixing the Gaussian Blur”: the Bilateral Filter

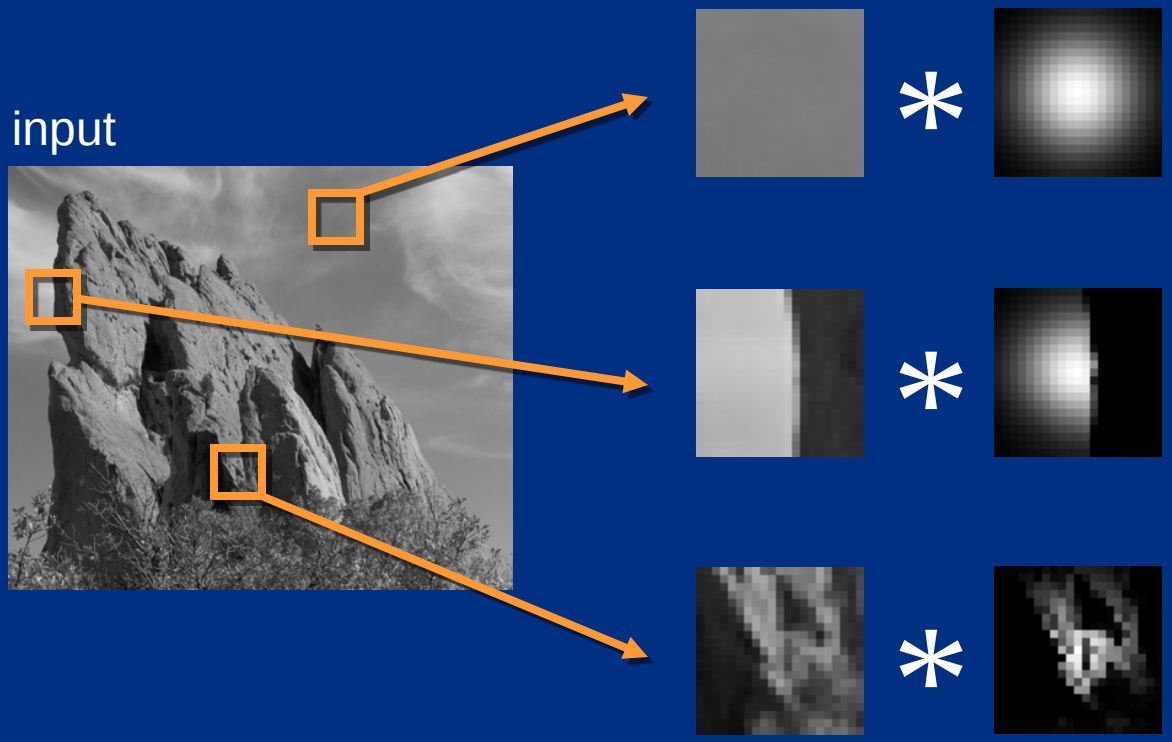
Sylvain Paris – MIT CSAIL

Blur Comes from Averaging across Edges



Same Gaussian kernel everywhere.

Bilateral Filter [Aurich 95, Smith 97, Tomasi 98]
No Averaging across Edges



The kernel shape depends on the image content.

Bilateral Filter Definition: an Additional Edge Term

Same idea: **weighted average of pixels.**

new

not new

new

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}}-I_{\mathbf{q}}|\right) I_{\mathbf{q}}$$

normalization factor

space weight

range weight

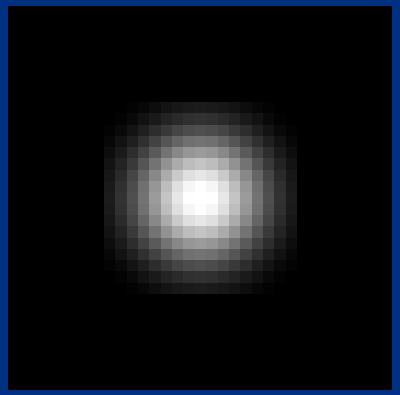
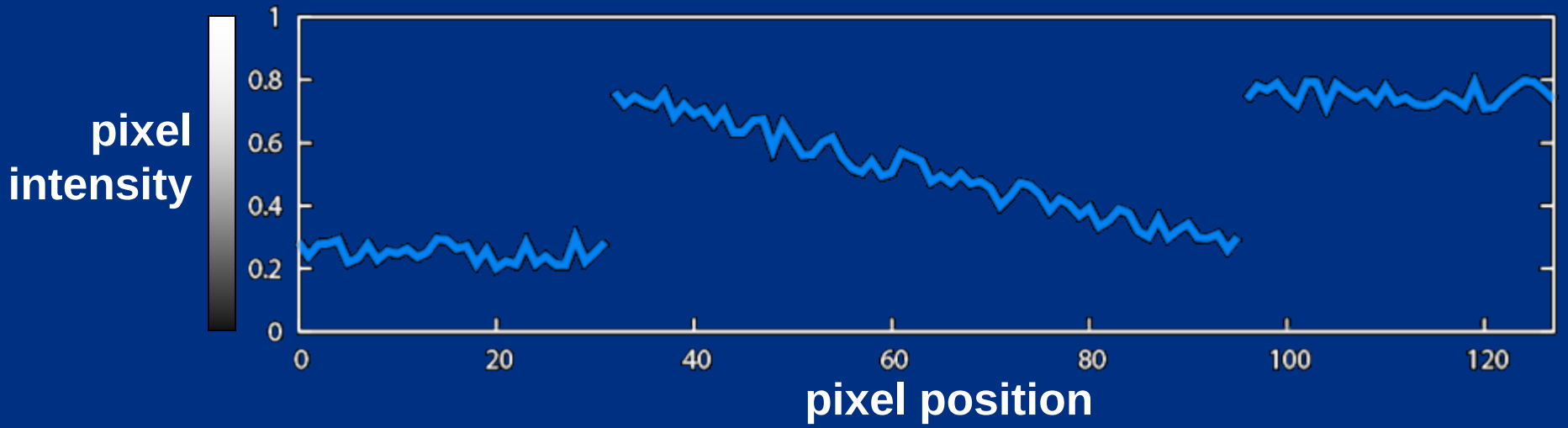


Illustration a 1D Image

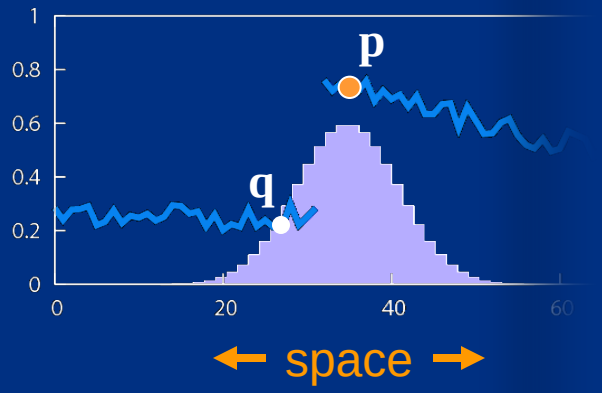
- 1D image = line of pixels

- Better visualized as a plot



Gaussian Blur and Bilateral Filter

Gaussian blur

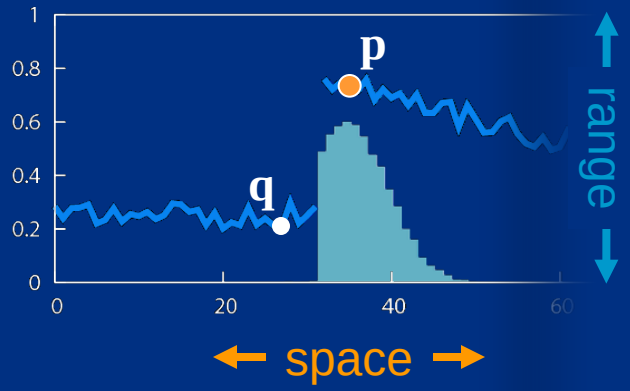


$$GB[I]_{\mathbf{p}} = \sum_{\mathbf{q} \in S} G_{\sigma}(\|\mathbf{p}-\mathbf{q}\|) I_{\mathbf{q}}$$

space

Bilateral filter

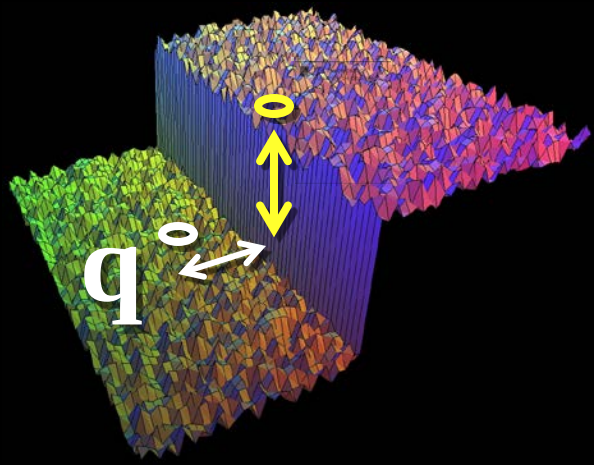
[Aurich 95, Smith 97, Tomasi 98]



$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}(\|\mathbf{p}-\mathbf{q}\|) G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|) I_{\mathbf{q}}$$

space range

normalization


q

Space and Range Parameters

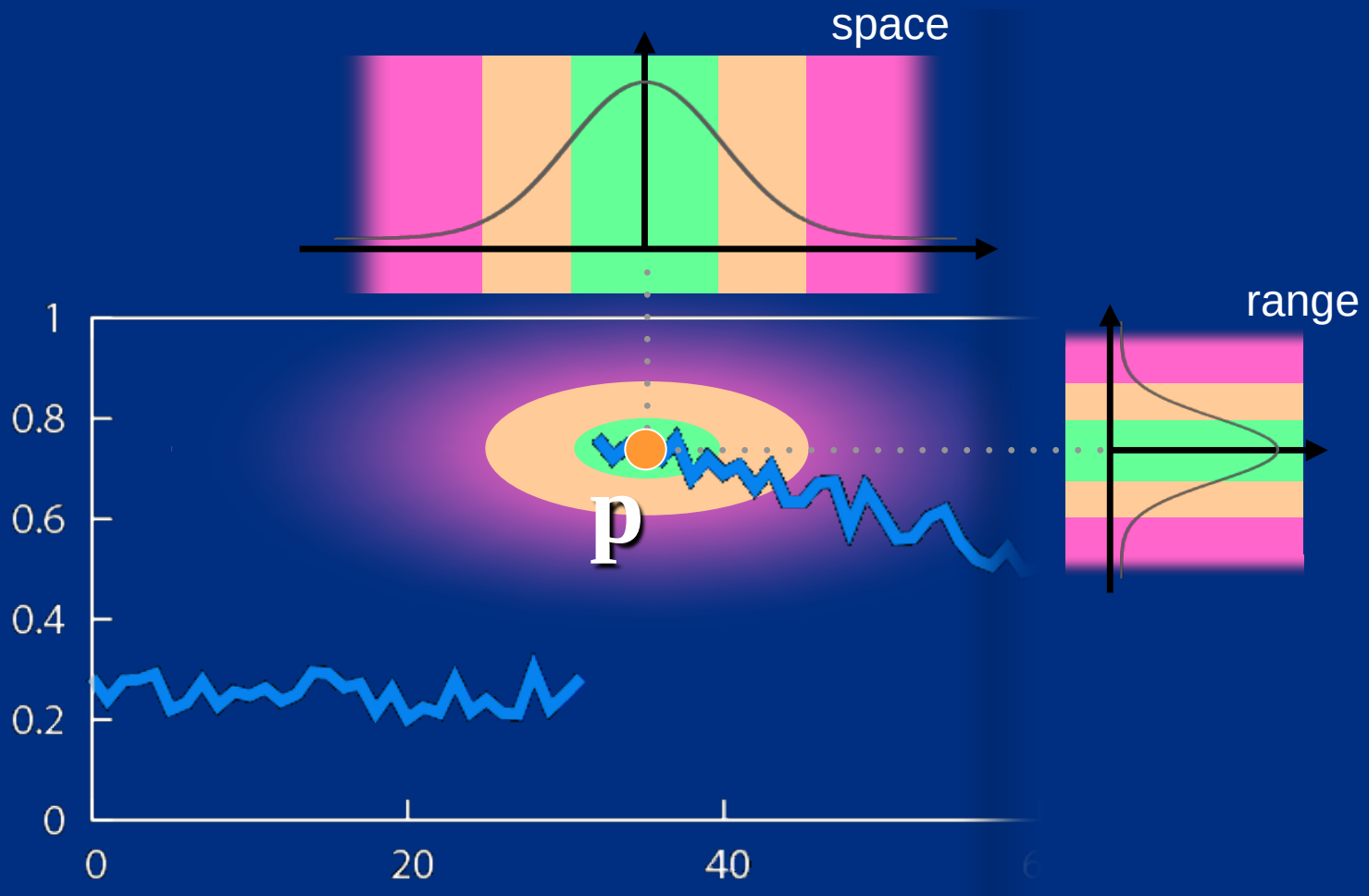
$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_s}\left(\|\mathbf{p}-\mathbf{q}\|\right) G_{\sigma_r}\left(|I_{\mathbf{p}} - I_{\mathbf{q}}|\right) I_{\mathbf{q}}$$

- space σ_s : spatial extent of the kernel, size of the considered neighborhood.

- range σ_r : “minimum” amplitude of an edge

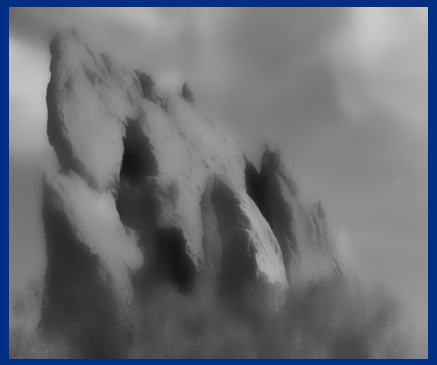
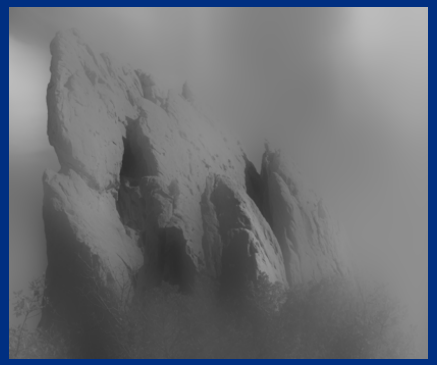
Influence of Pixels

Only pixels close in space and in range are considered.



Exploring the Parameter Space

input

	$\sigma_r = 0.1$	$\sigma_r = 0.25$	$\sigma_r = \infty$ (Gaussian blur)
$\sigma_s = 2$			
$\sigma_s = 6$			
$\sigma_s = 18$			

Varying the Range Parameter

input

$\sigma_r = 0.1$ $\sigma_r = 0.25$ $\sigma_r = \infty$ (Gaussian blur)

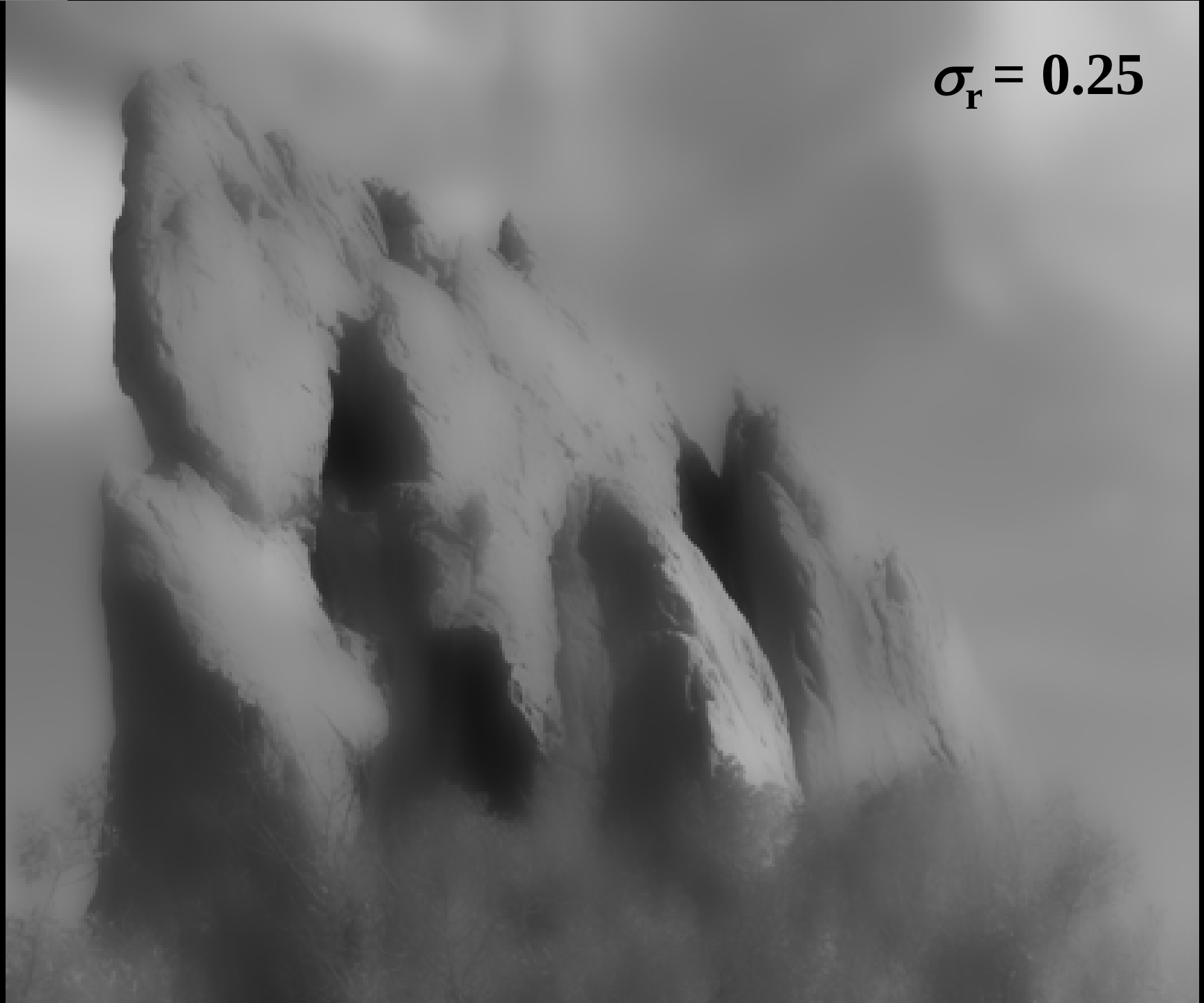
$\sigma_s = 2$

$\sigma_s = 6$

$\sigma_s = 18$


input


$\sigma_r = 0.1$


$\sigma_r = 0.25$


$\sigma_r = \infty$
(Gaussian blur)

Varying the Space Parameter

input

	$\sigma_r = 0.1$	$\sigma_r = 0.25$	$\sigma_r = \infty$ (Gaussian blur)
$\sigma_s = 2$			
$\sigma_s = 6$			
$\sigma_s = 18$			


input


$\sigma_s = 2$


$\sigma_s = 6$


$\sigma_s = 18$