Bottom Up Parsing

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LR(0) Parsing

- Build the derivation tree from the bottom
- First build the children, then connect to the parent
- Can handle left recursion
  - Which is common in real-world grammars

LR(0) Item

An **LR(0) item** is of the form:

- $N \rightarrow \alpha . \beta$

The **dot** gives us the current location (a local view).

LR(0) Item

An LR(0) item with the dot at the end is called **reduce** item:

- $N \rightarrow \alpha\beta.$

Otherwise, it's a **shift** item:

- $N \rightarrow .\alpha\beta$
- $N \rightarrow \alpha.\beta$

LR(0) Item Closure Set

The **LR(0) closure set** of an item is it's ϵ-closure.

For example, given the following CFG:

- $S \rightarrow E\$$
- $E \rightarrow ID = X$
- $E \rightarrow \{ID\}$
- $X \rightarrow INT$

the closure set of the $S \rightarrow .E\$$ contains:

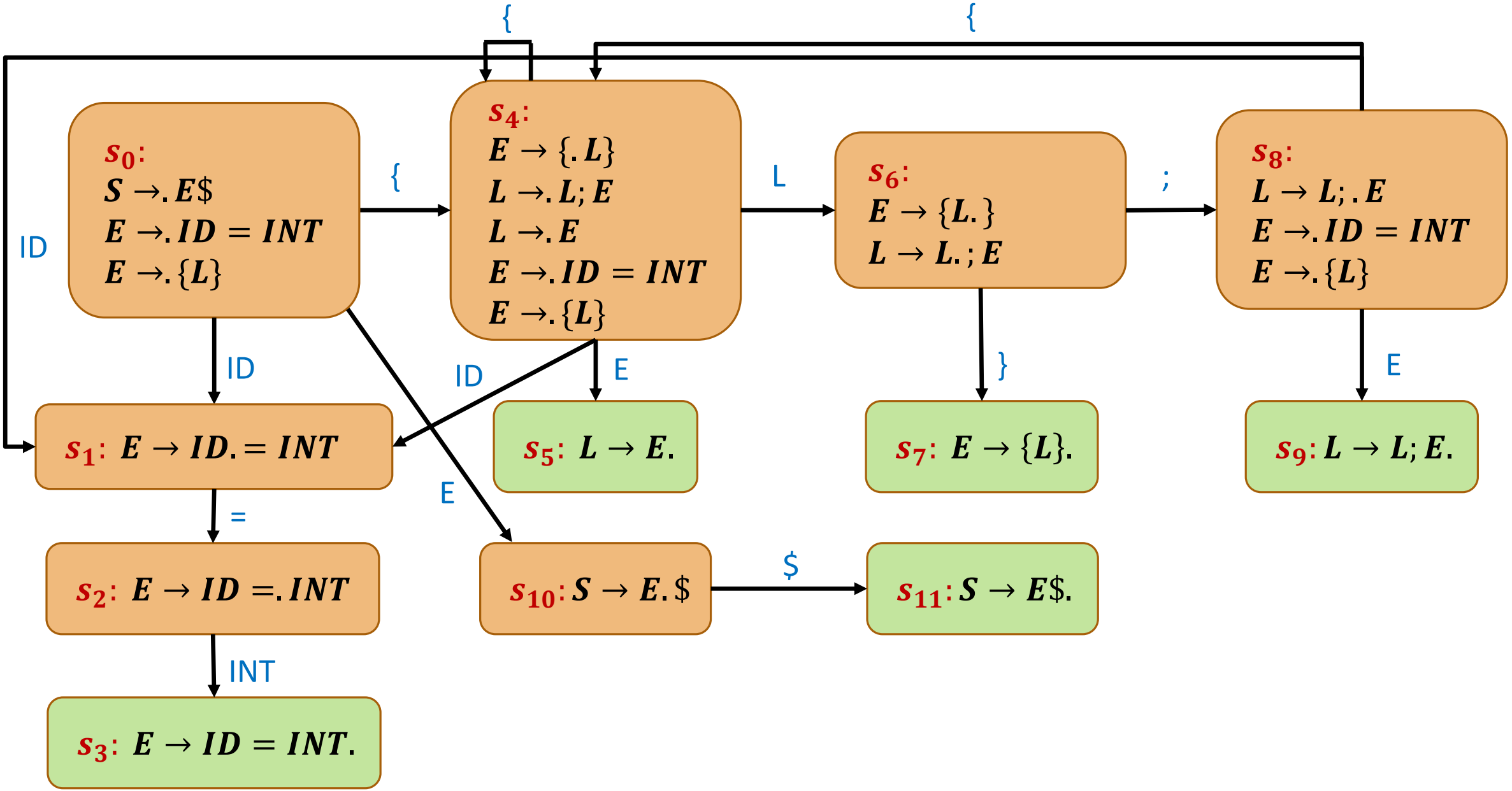
- $S \rightarrow .E\$$
- $E \rightarrow .ID = X$
- $E \rightarrow .\{L\}$

LR(0) Parsing

Consider the following CFG:

- $S \rightarrow E\$$
- $E \rightarrow ID = INT$
- $E \rightarrow \{L\}$
- $L \rightarrow E$
- $L \rightarrow L; E$

What will be the **transition system** of the LR(0) parser for this CFG?


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_1: $E \rightarrow ID. = INT$
s_2: $E \rightarrow ID = .INT$
s_3: $E \rightarrow ID = INT.$
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_5: $L \rightarrow E.$
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
s_7: $E \rightarrow \{L\}.$
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_9: $L \rightarrow L; E.$
s_{10}: $S \rightarrow E.\$$
s_{11}: $S \rightarrow E\$.$
{
{
ID
ID
ID
E
E
L
;
}
E
=
INT
$

LR(0) Parser

We start with the initial LR(0) item (that comes from the initial rule):

- $S \rightarrow .E\$$

The initial state is the ϵ-**closure** of that item, which contains:

- $S \rightarrow .E\$$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

s_0:

$S \rightarrow .E\$$

$E \rightarrow .ID = INT$

$E \rightarrow .\{L\}$

LR(0) Parser

From s_0, if we recognized ID, then the next state will contain:

- $E \rightarrow ID.= INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

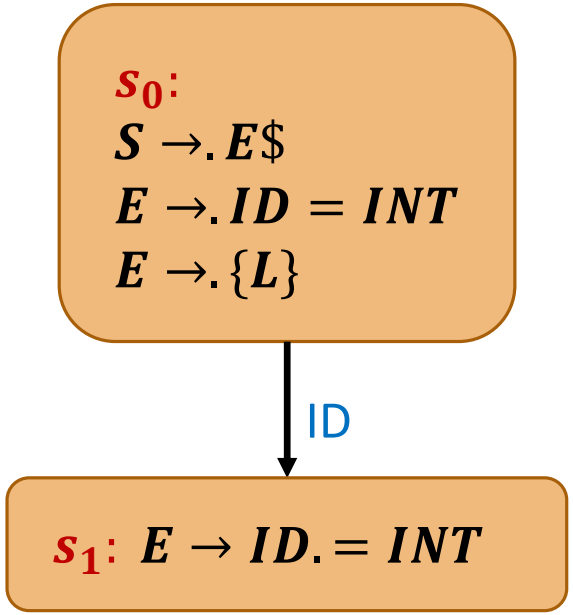
From s_0, if we recognized ID, then the next state will contain:

- $E \rightarrow ID.= INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID.= INT$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
s_1: $E \rightarrow ID. = INT$

LR(0) Parser

From s_1, if we recognized $=$, then the next state will contain:

- $E \rightarrow ID =.INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

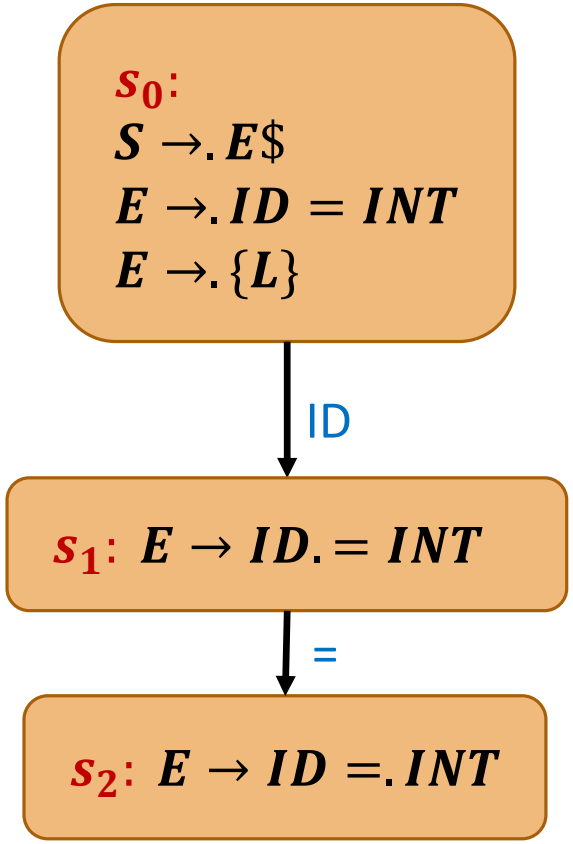
From s_1, if we recognized $=$, then the next state will contain:

- $E \rightarrow ID =.INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID =.INT$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID =. INT$

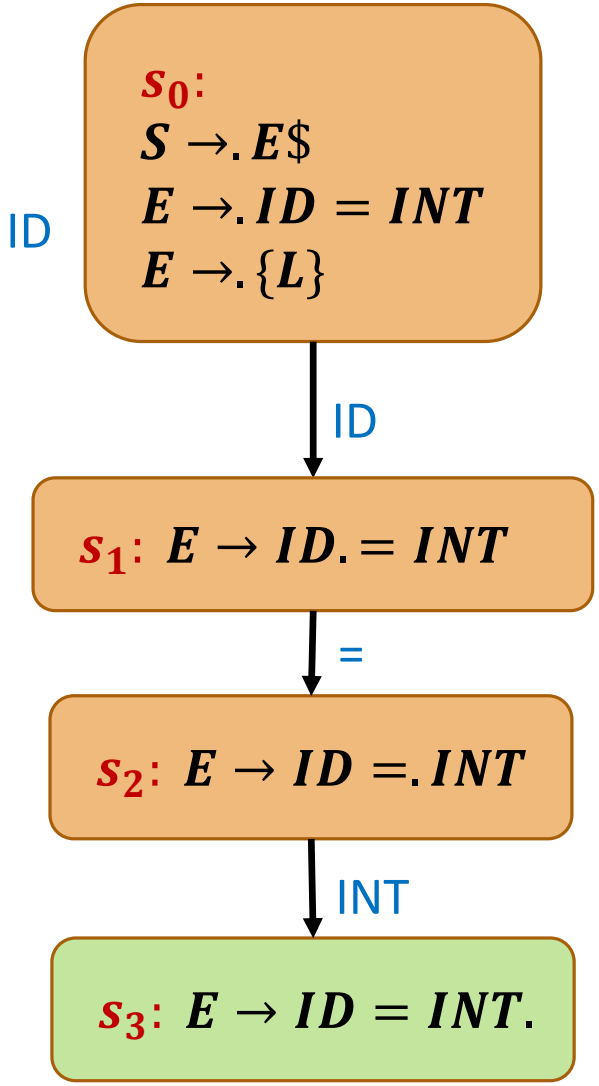
LR(0) Parser

From s_2, if we recognized INT, then the next state will contain:

- $E \rightarrow ID = INT.$

Which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_0, if we recognized {, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

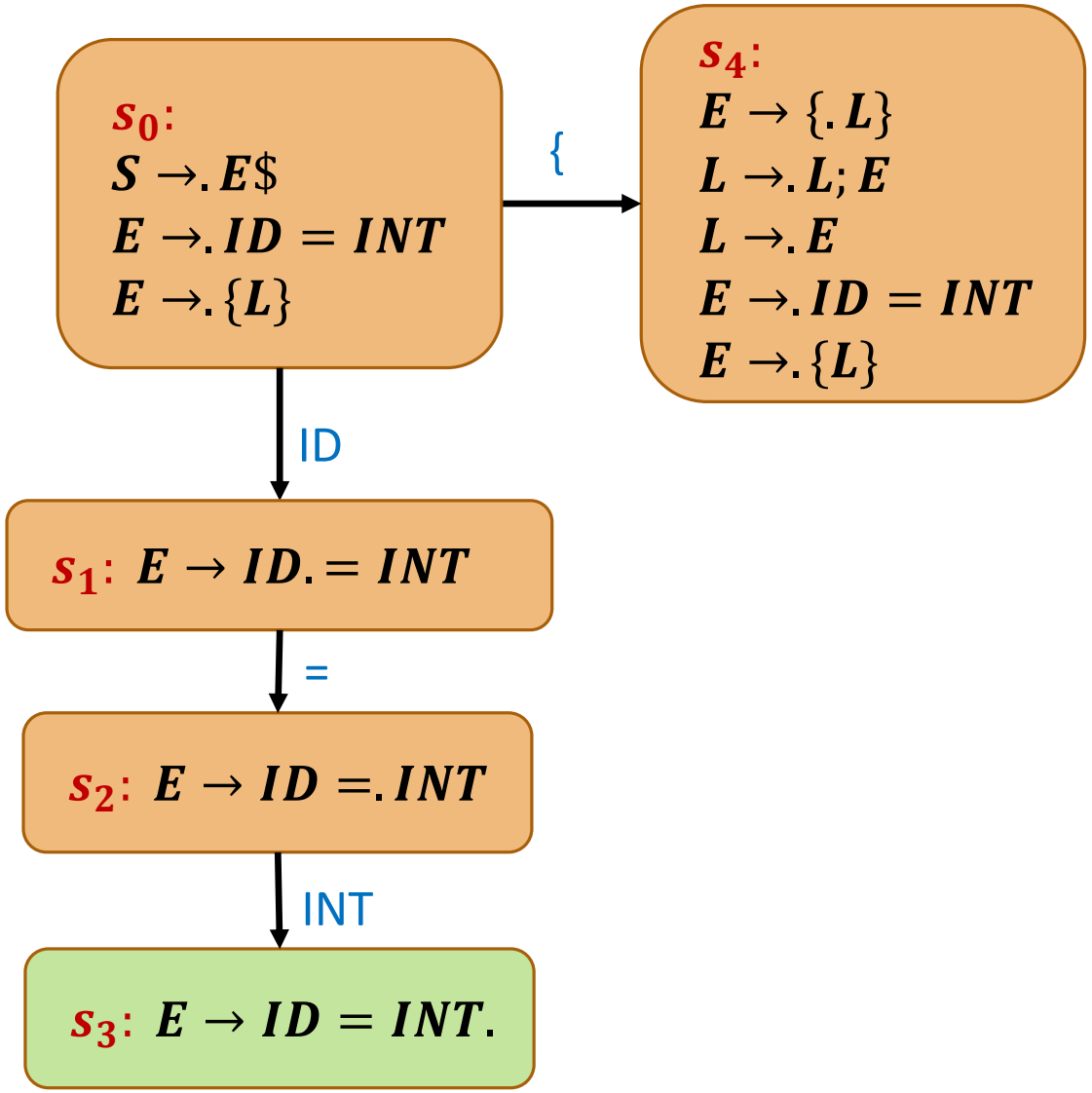
From s_0, if we recognized {, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L;E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_4, if we recognized {, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

From s_4, if we recognized {, then the next state will contain:

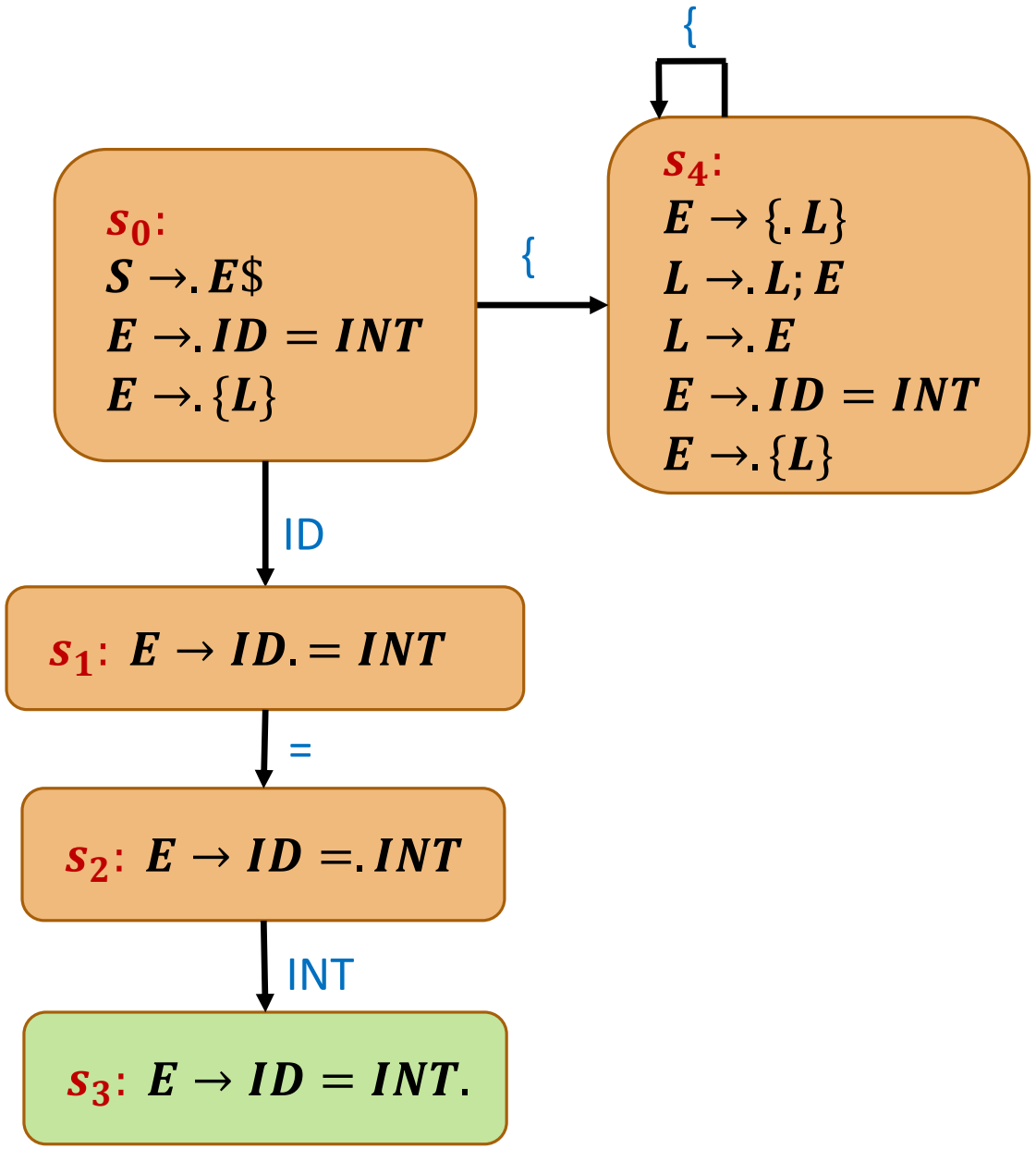
- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L;E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

which was already computed: s_4

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$


{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_4, if we recognized ID, then the next state will contain:

- $E \rightarrow ID.= INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

From s_4, if we recognized ID, then the next state will contain:

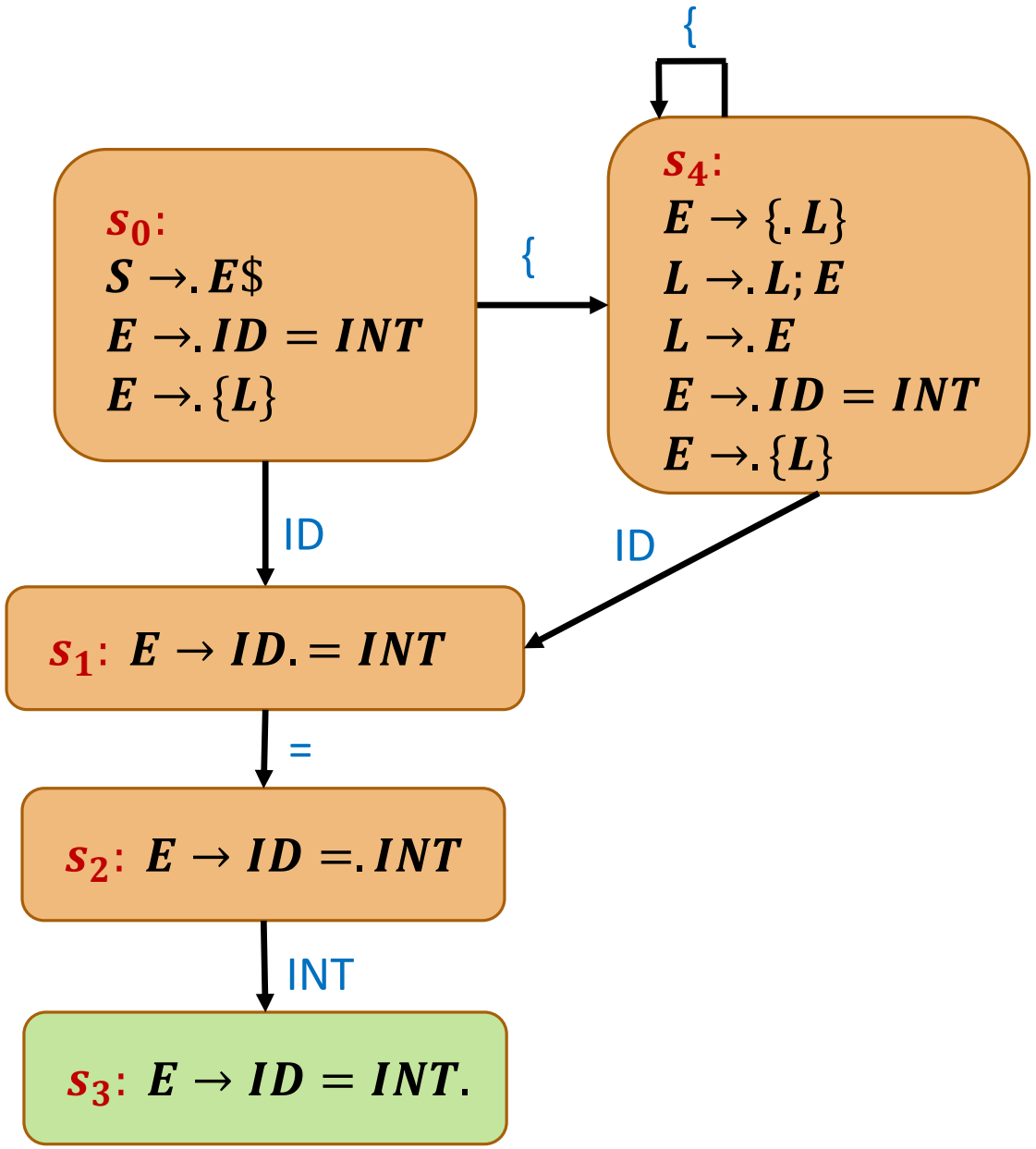
- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID. = INT$

which was already computed: s_1

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$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s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
s_1: $E \rightarrow ID. = INT$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

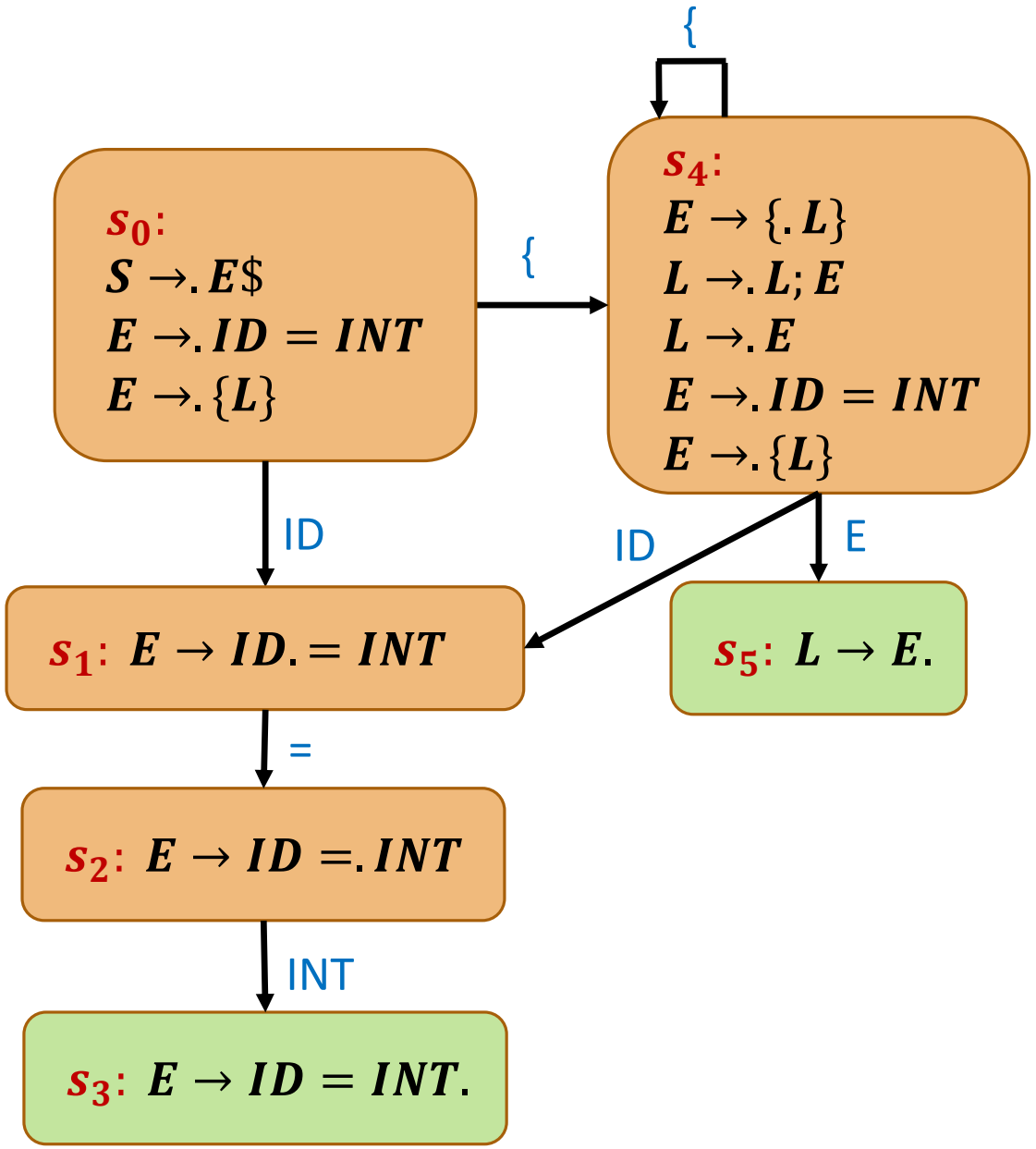
LR(0) Parser

From s_4, if we recognized E, then the next state will contain:

- $L \rightarrow E.$

which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
=
s_2: $E \rightarrow ID = .INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_4, if we recognized L, then the next state will contain:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.;E$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$

LR(0) Parser

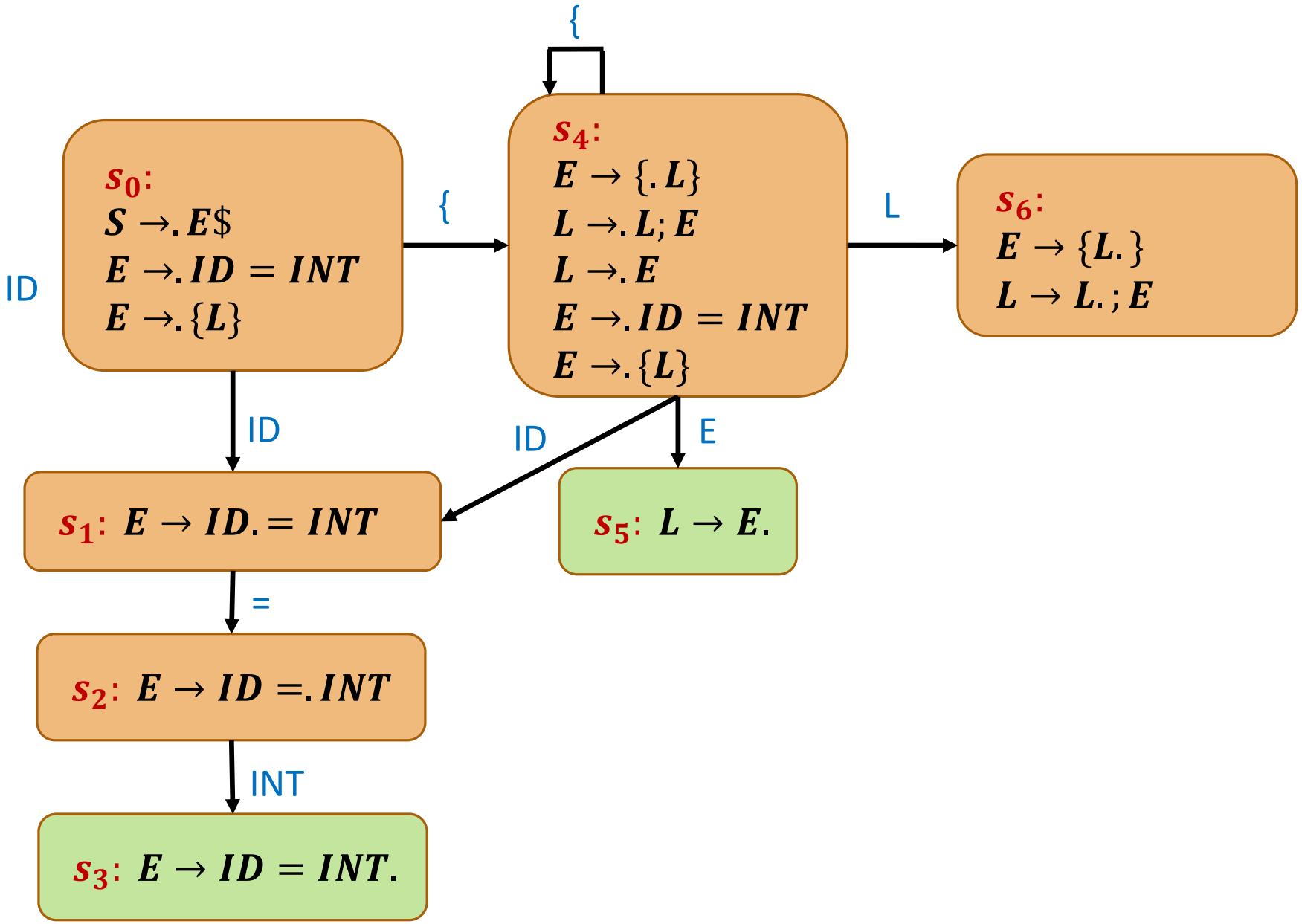
From s_4, if we recognized L, then the next state will contain:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.;E$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{L.\}$
- $L \rightarrow L.;E$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$


{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
ID
ID
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

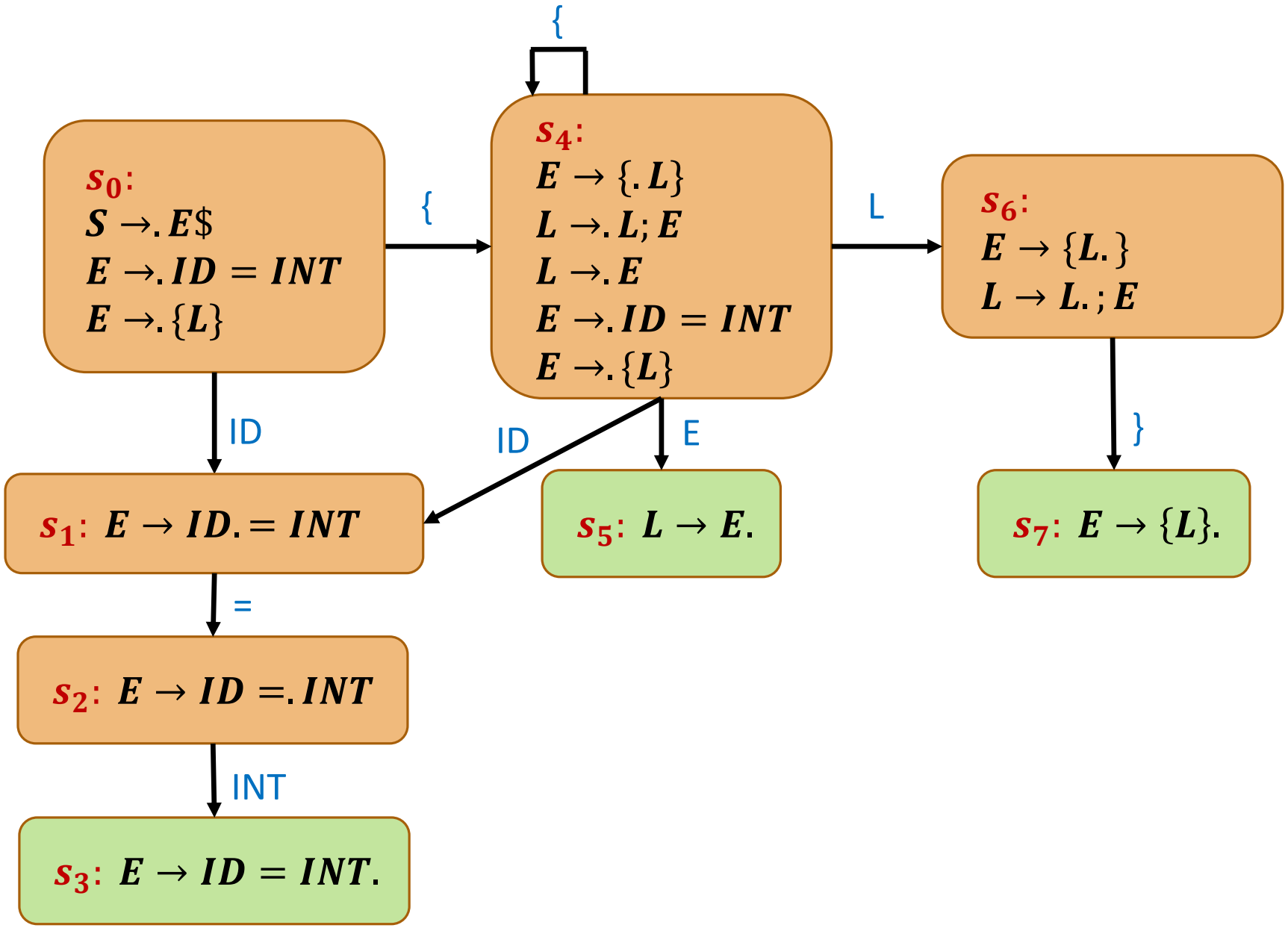
LR(0) Parser

From s_6, if we recognized }, then the next state will contain:

- $E \rightarrow \{L\}.$

Which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
ID
ID
E
}
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
=
s_2: $E \rightarrow ID =. INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_6, if we recognized ;, then the next state will contain:

- $L \rightarrow L; . E$

So the next state (the ϵ-closure) contains:

$$S \rightarrow E\$$$
$$E \rightarrow ID = INT$$
$$E \rightarrow \{L\}$$
$$L \rightarrow E$$
$$L \rightarrow L; E$$

LR(0) Parser

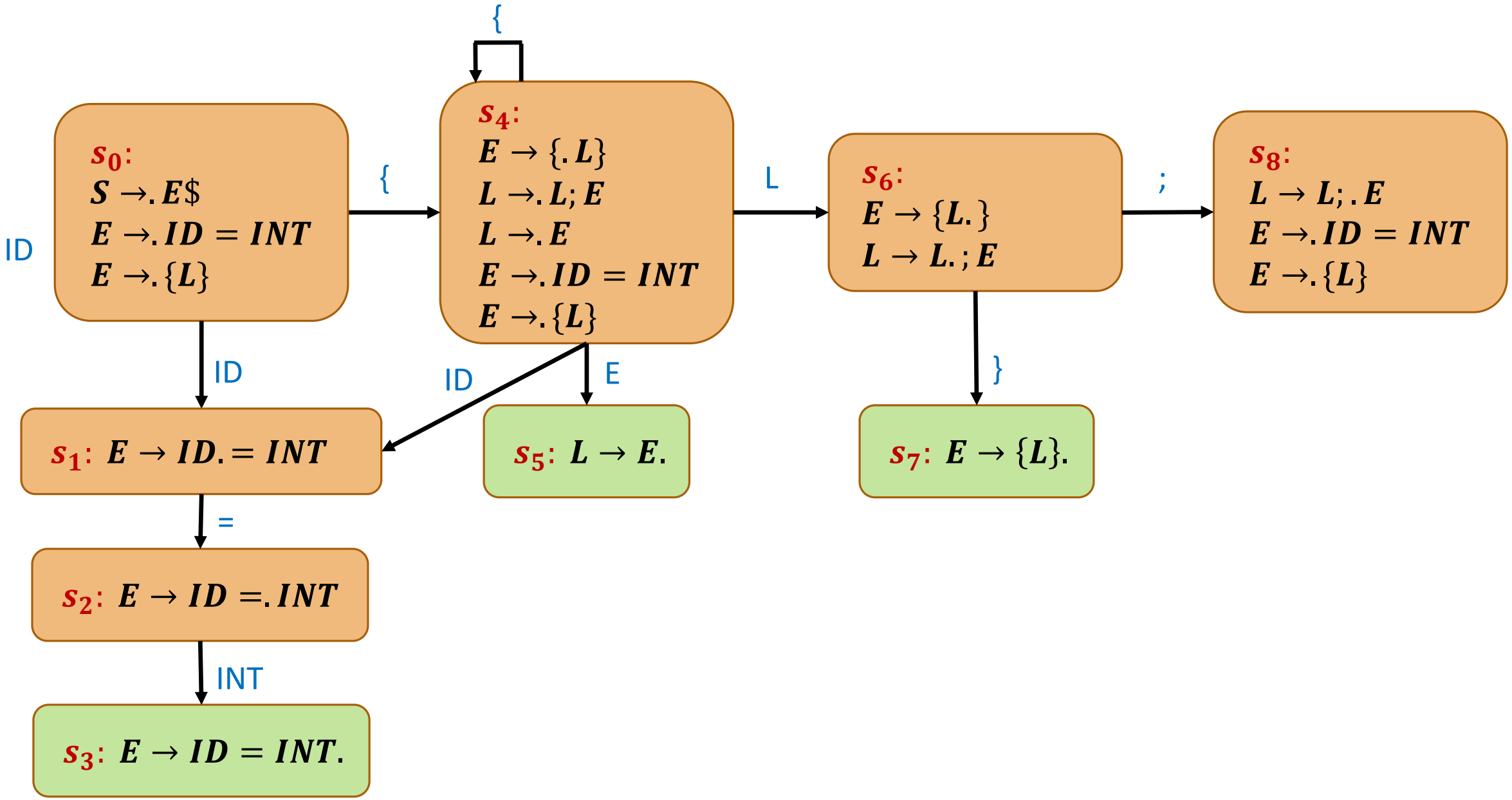
From s_6, if we recognized ;, then the next state will contain:

- $L \rightarrow L; . E$

So the next state (the ϵ-closure) contains:

- $L \rightarrow L; . E$
- $E \rightarrow . ID = INT$
- $E \rightarrow . \{L\}$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


{
s_0:
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s_4:
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$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
=
s_2: $E \rightarrow ID = .INT$
INT
s_3: $E \rightarrow ID = INT.$

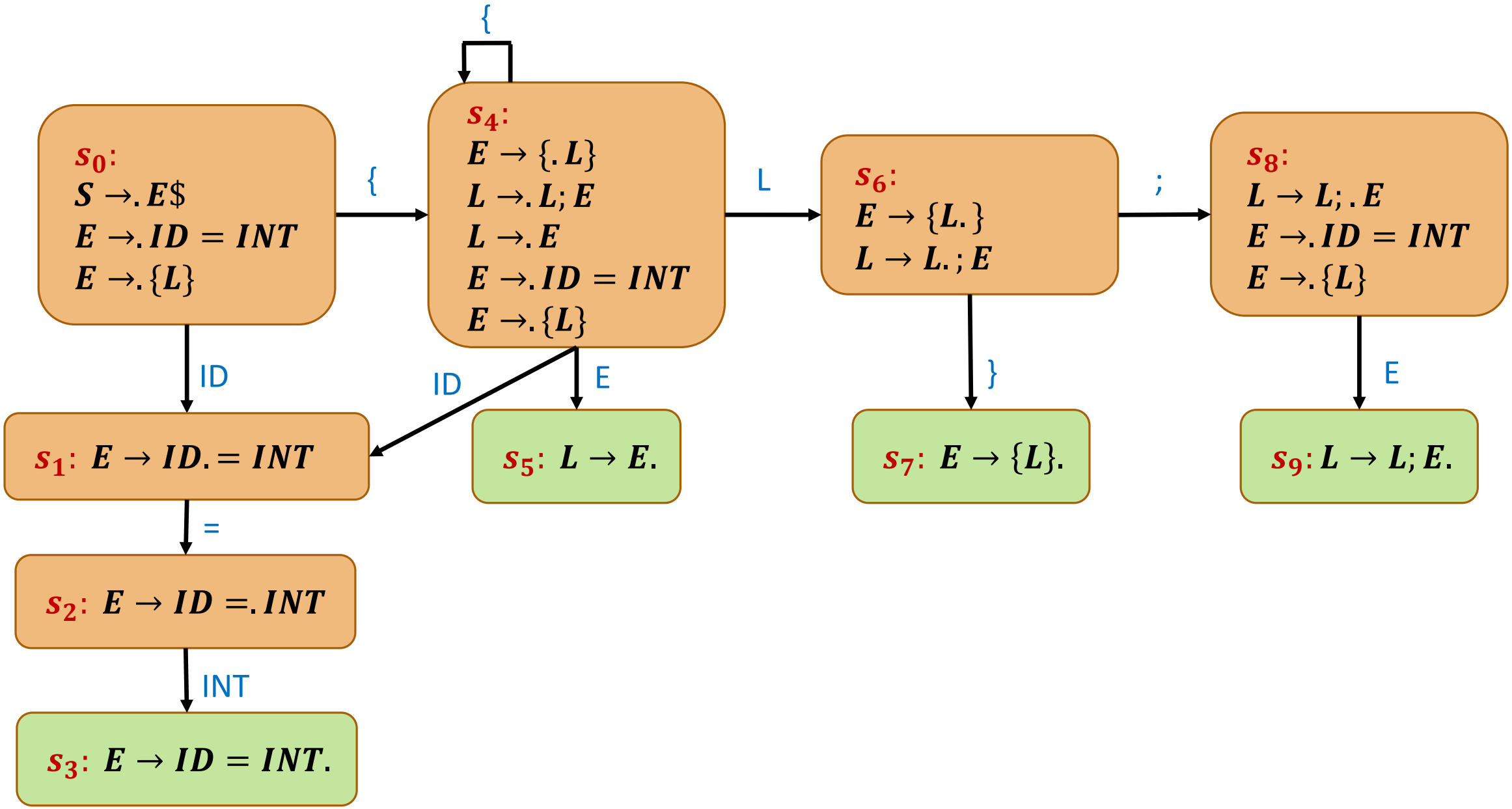
LR(0) Parser

From s_8, if we recognized E, then the next state will contain:

- $E \rightarrow L; E.$

which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


{
s_0:
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$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
=
s_2: $E \rightarrow ID = .INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_8, if we recognized {, then the next state will contain:

- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

From s_8, if we recognized {, then the next state will contain:

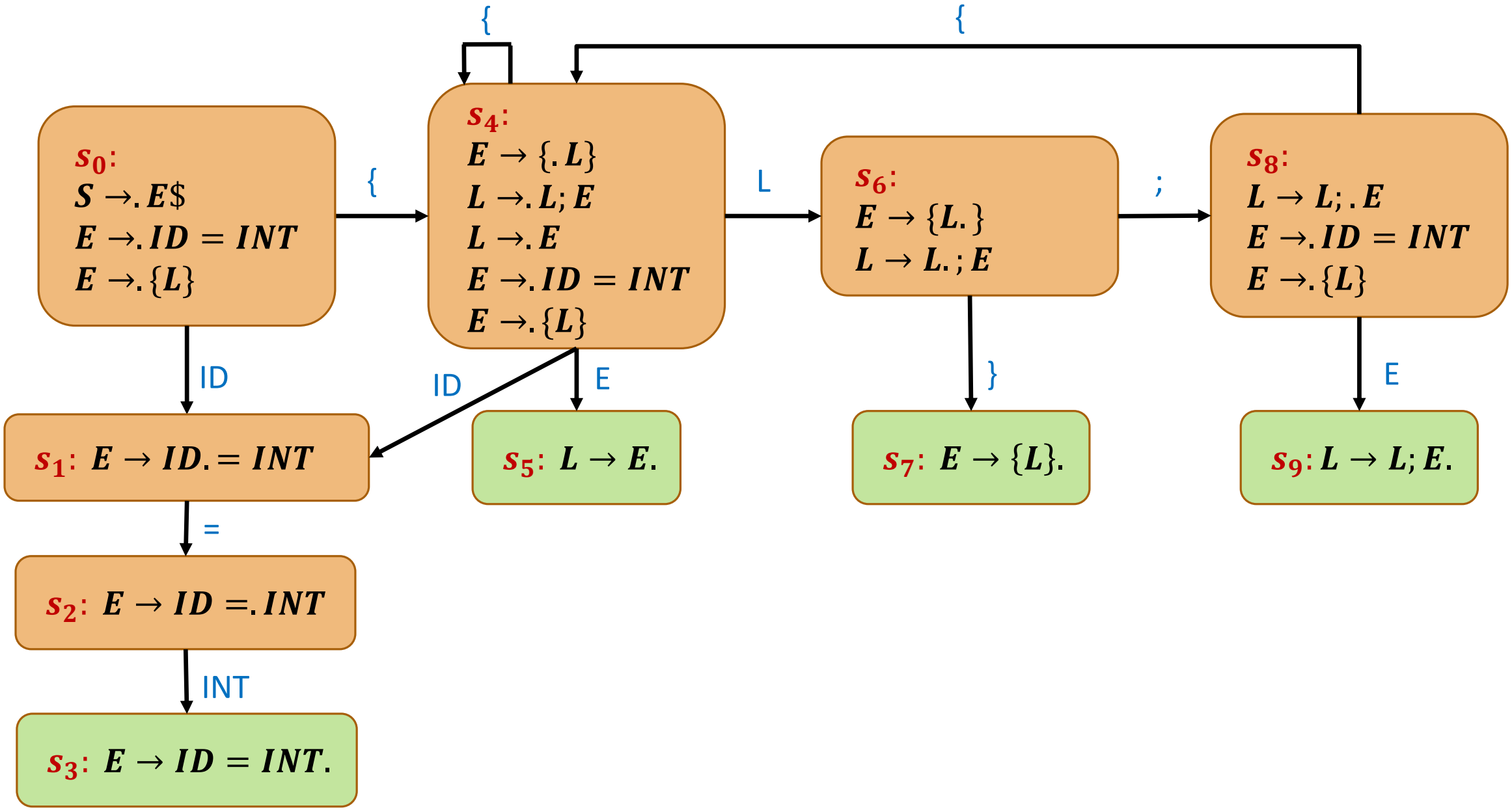
- $E \rightarrow \{.L\}$

So the next state (the ϵ-closure) contains:

- $E \rightarrow \{.L\}$
- $L \rightarrow .L;E$
- $L \rightarrow .E$
- $E \rightarrow .ID = INT$
- $E \rightarrow .\{L\}$

which was already computed: s_4

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L;E$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s_0:
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E
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s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
=
s_2: $E \rightarrow ID = .INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_8, if we recognized ID, then the next state will contain:

- $E \rightarrow ID. = INT$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

From s_8, if we recognized ID, then the next state will contain:

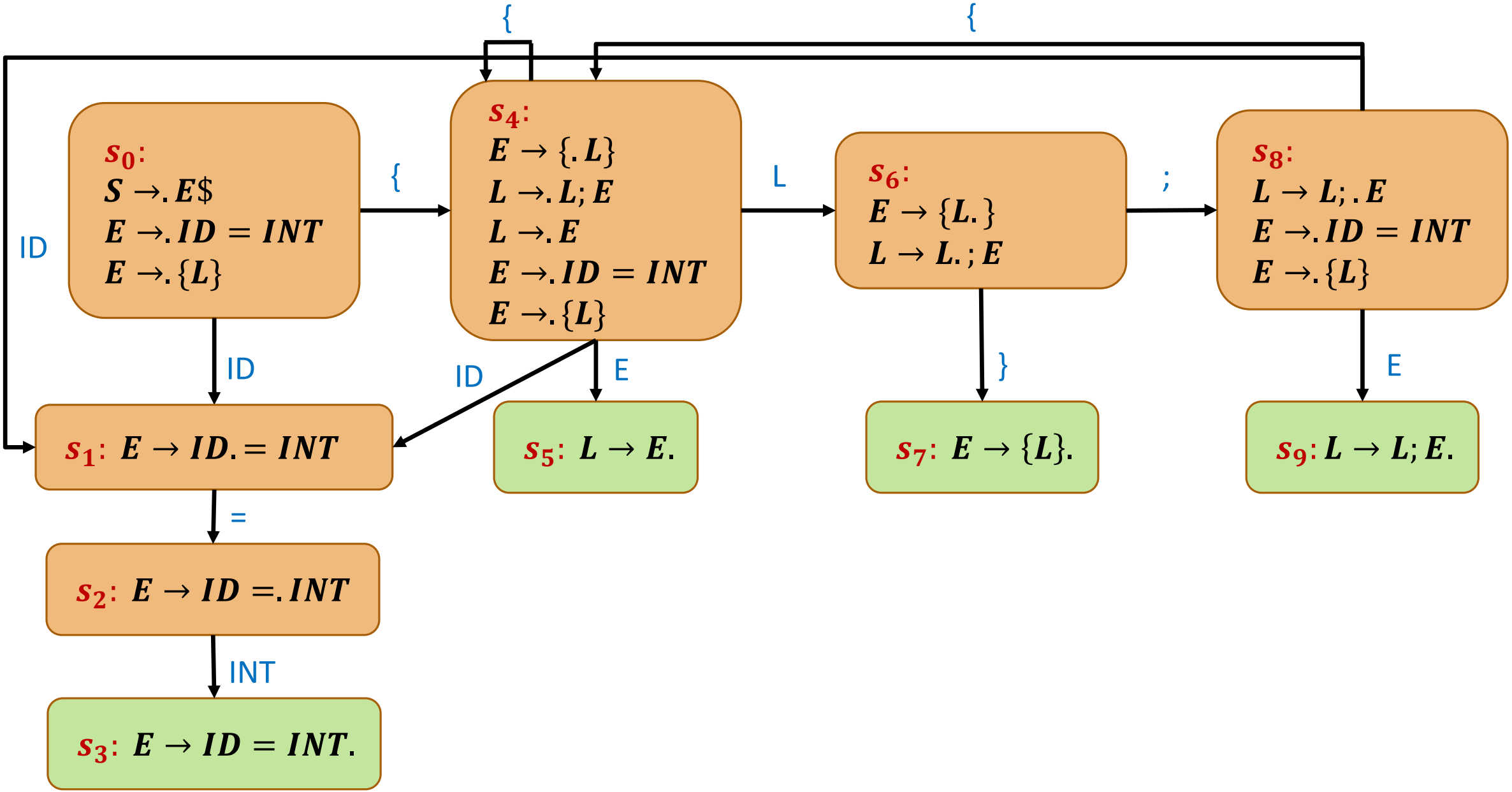
- $E \rightarrow ID.= INT$

So the next state (the ϵ-closure) contains:

- $E \rightarrow ID.= INT$

which was already computed: s_1

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
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s_4:
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$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
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s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
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$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
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s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
=
s_2: $E \rightarrow ID = .INT$
INT
s_3: $E \rightarrow ID = INT.$

LR(0) Parser

From s_0, if we recognized E, then the next state will contain:

- $S \rightarrow E.\$$

So the next state (the ϵ-closure) contains:

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

LR(0) Parser

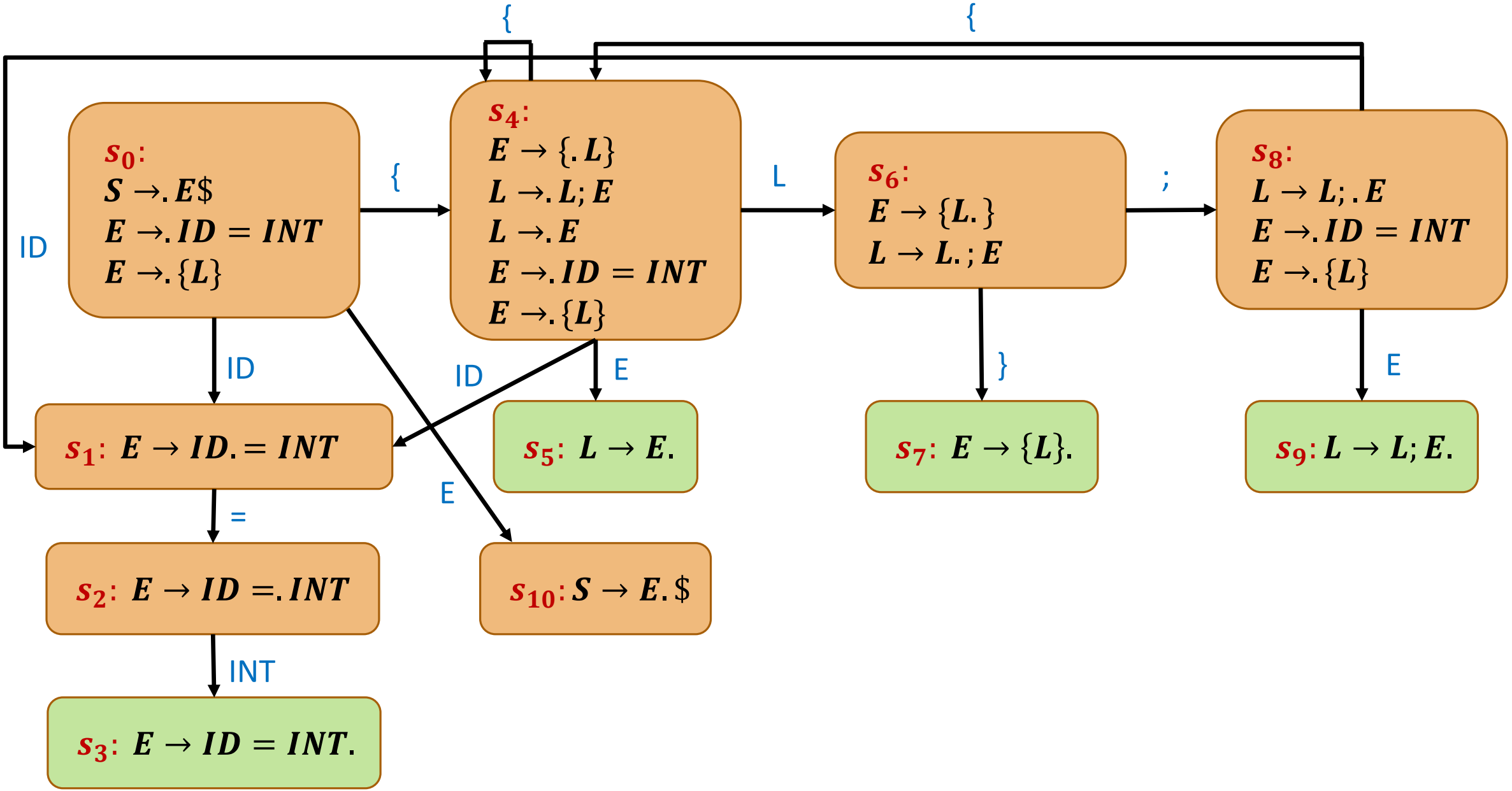
From s_0, if we recognized E, then the next state will contain:

- $S \rightarrow E.\$$

So the next state (the ϵ-closure) contains:

- $S \rightarrow E.\$$

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
{
L
;
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
INT
s_3: $E \rightarrow ID = INT.$

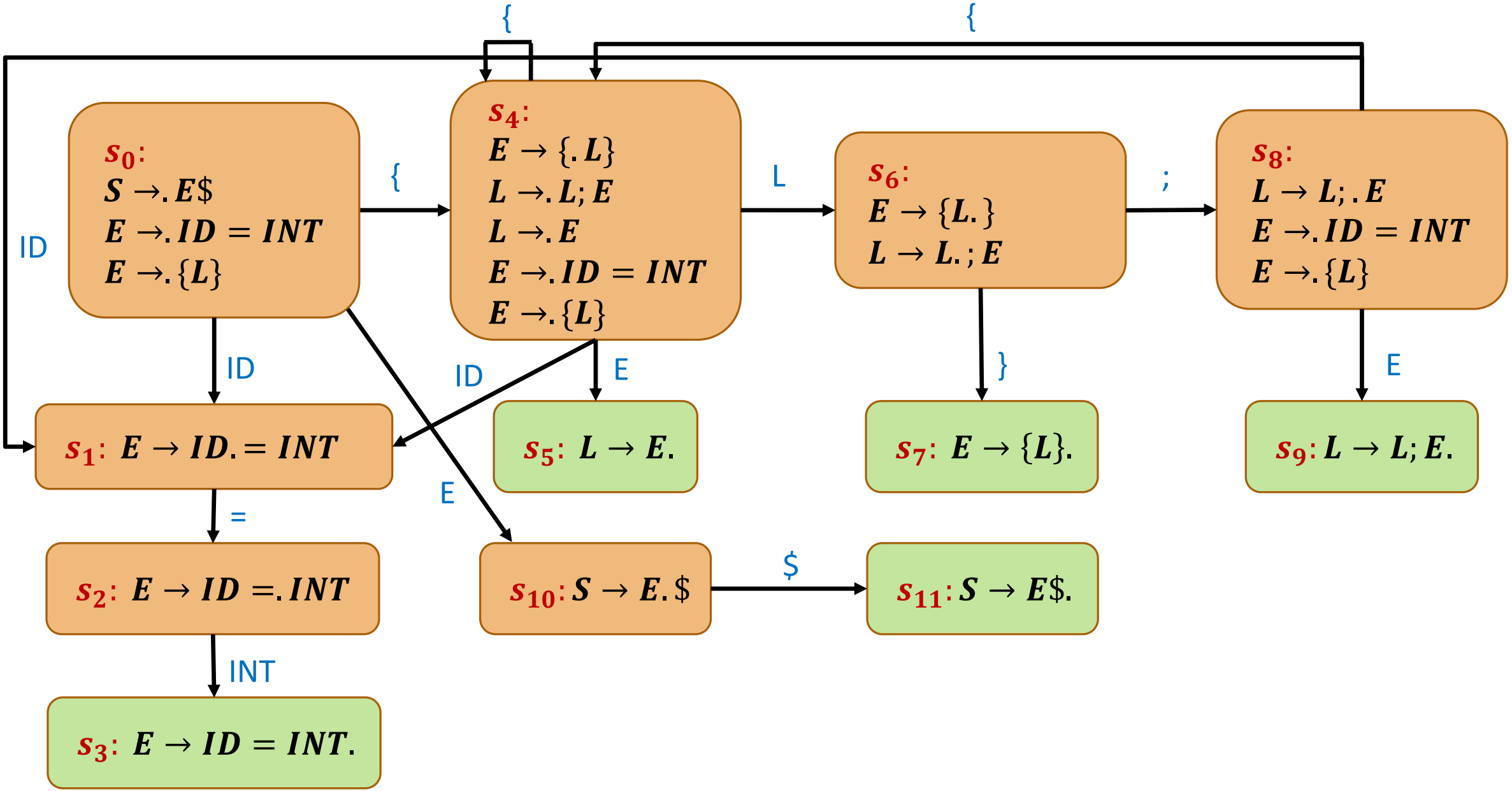
LR(0) Parser

From s_0, if we recognized E, then the next state will contain:

- $S \rightarrow E\$.$

which is a reduce state.

$S \rightarrow E\$$
$E \rightarrow ID = INT$
$E \rightarrow \{L\}$
$L \rightarrow E$
$L \rightarrow L; E$

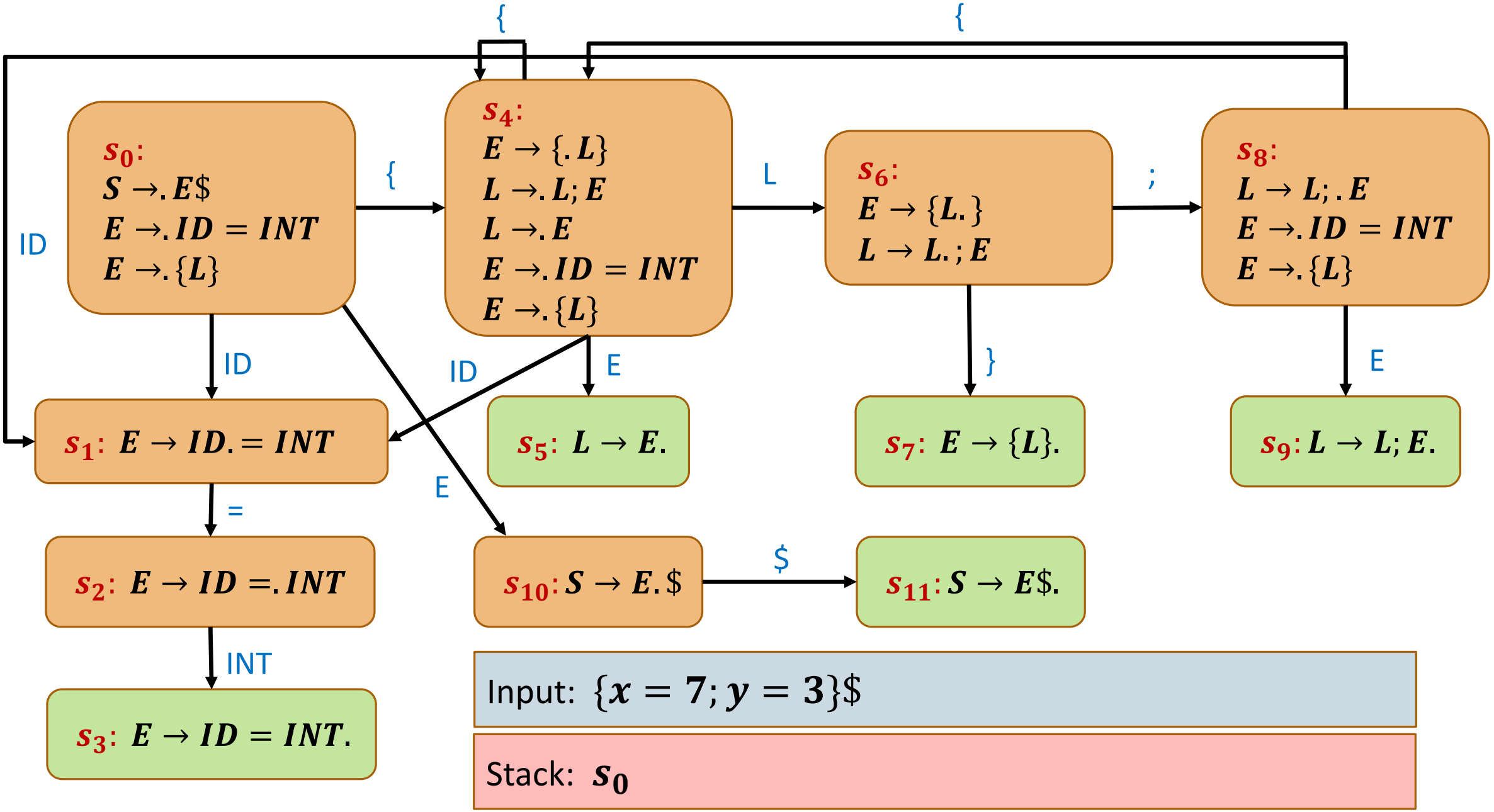


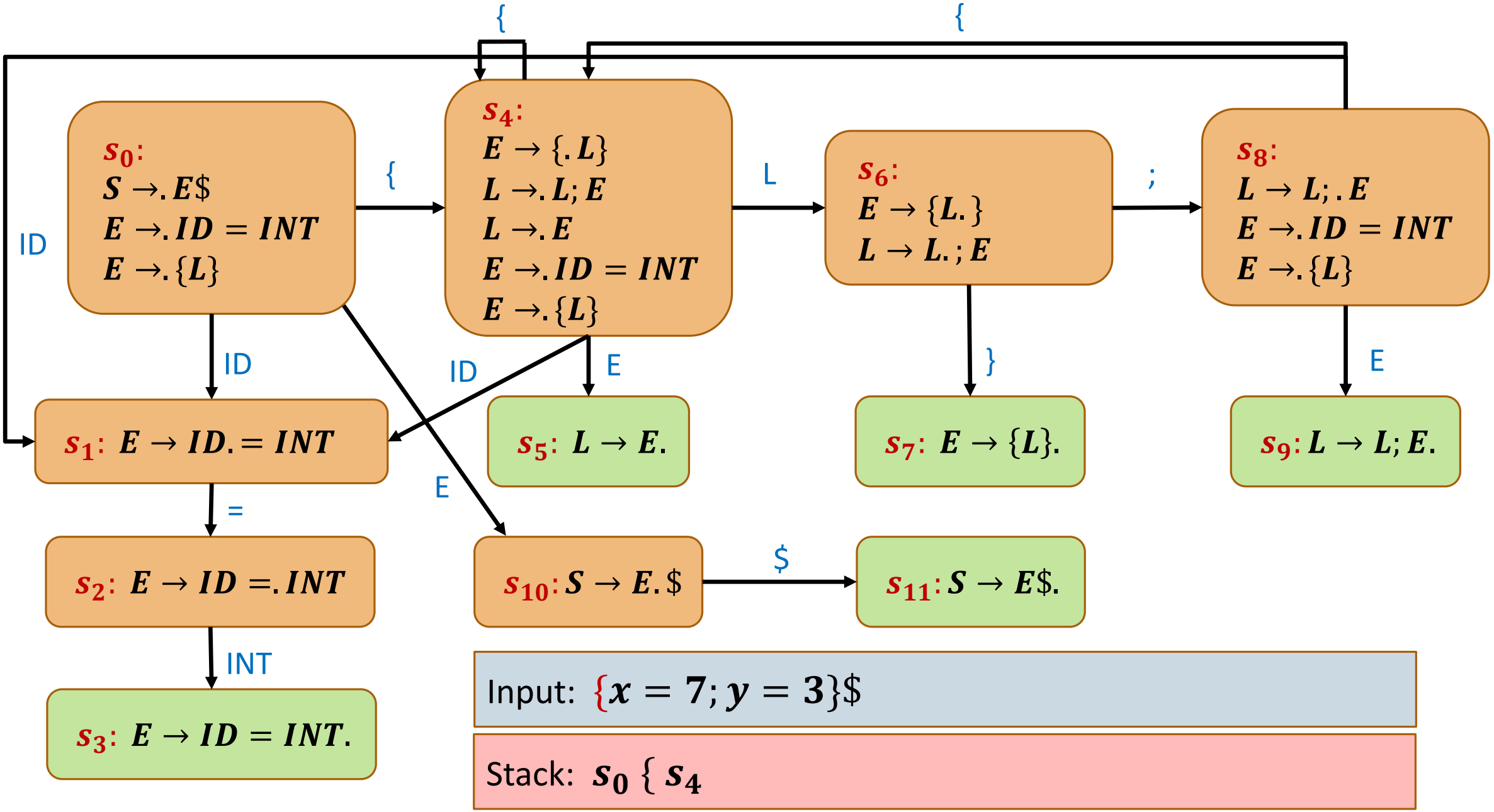
{
{
s_0:
$S \to .E\$$
$E \to .ID = INT$
$E \to .\{L\}$
{
s_4:
$E \to \{.L\}$
$L \to .L; E$
$L \to .E$
$E \to .ID = INT$
$E \to .\{L\}$
L
s_6:
$E \to \{L.\}$
$L \to L.; E$
;
s_8:
$L \to L; .E$
$E \to .ID = INT$
$E \to .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \to ID. = INT$
s_5: $L \to E.$
s_7: $E \to \{L\}.$
s_9: $L \to L; E.$
E
=
s_2: $E \to ID =. INT$
s_{10}: $S \to E.\$$
$
s_{11}: $S \to E\$.$
INT
s_3: $E \to ID = INT.$

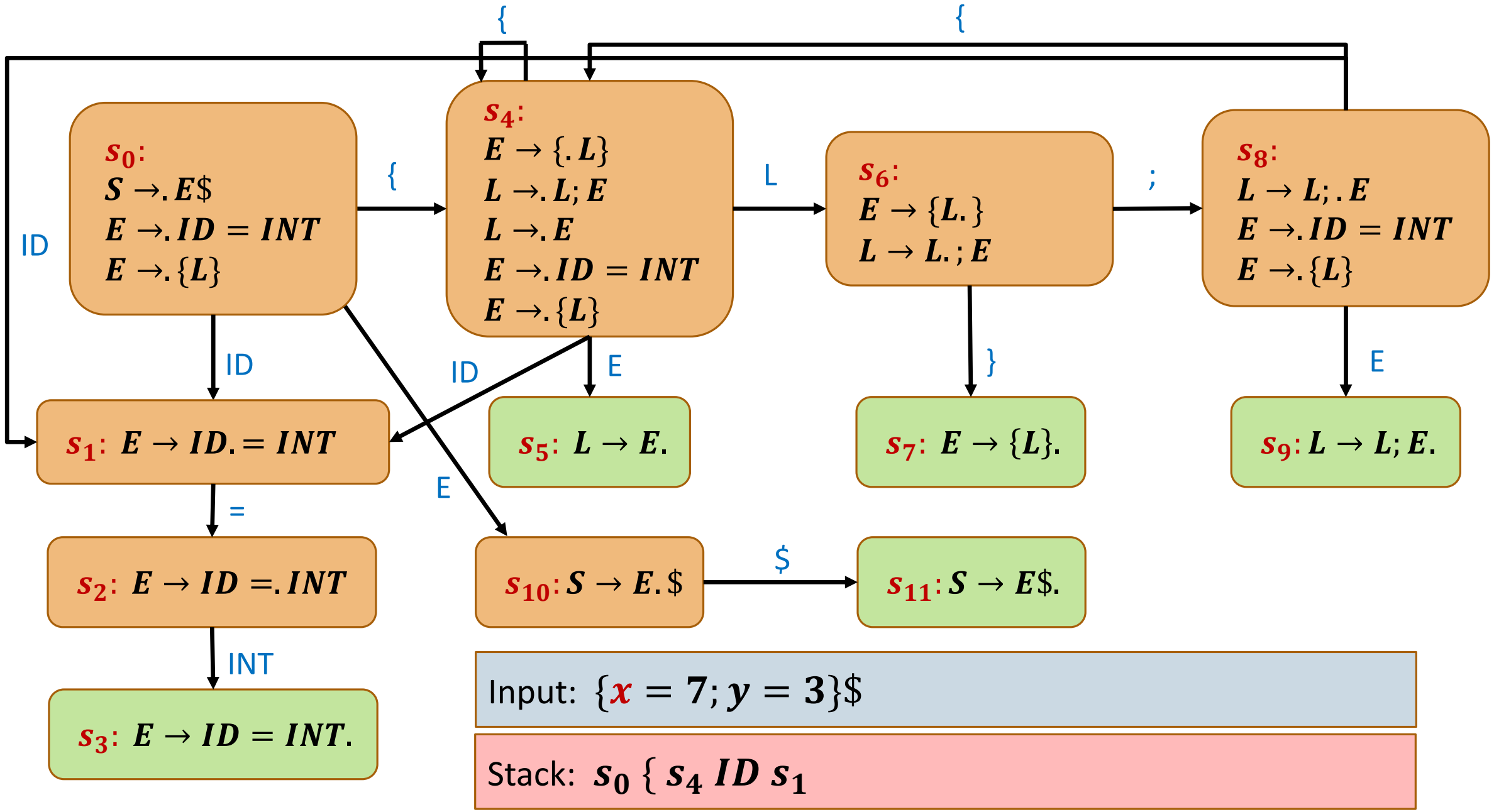
LR(0) Parser: Running Example

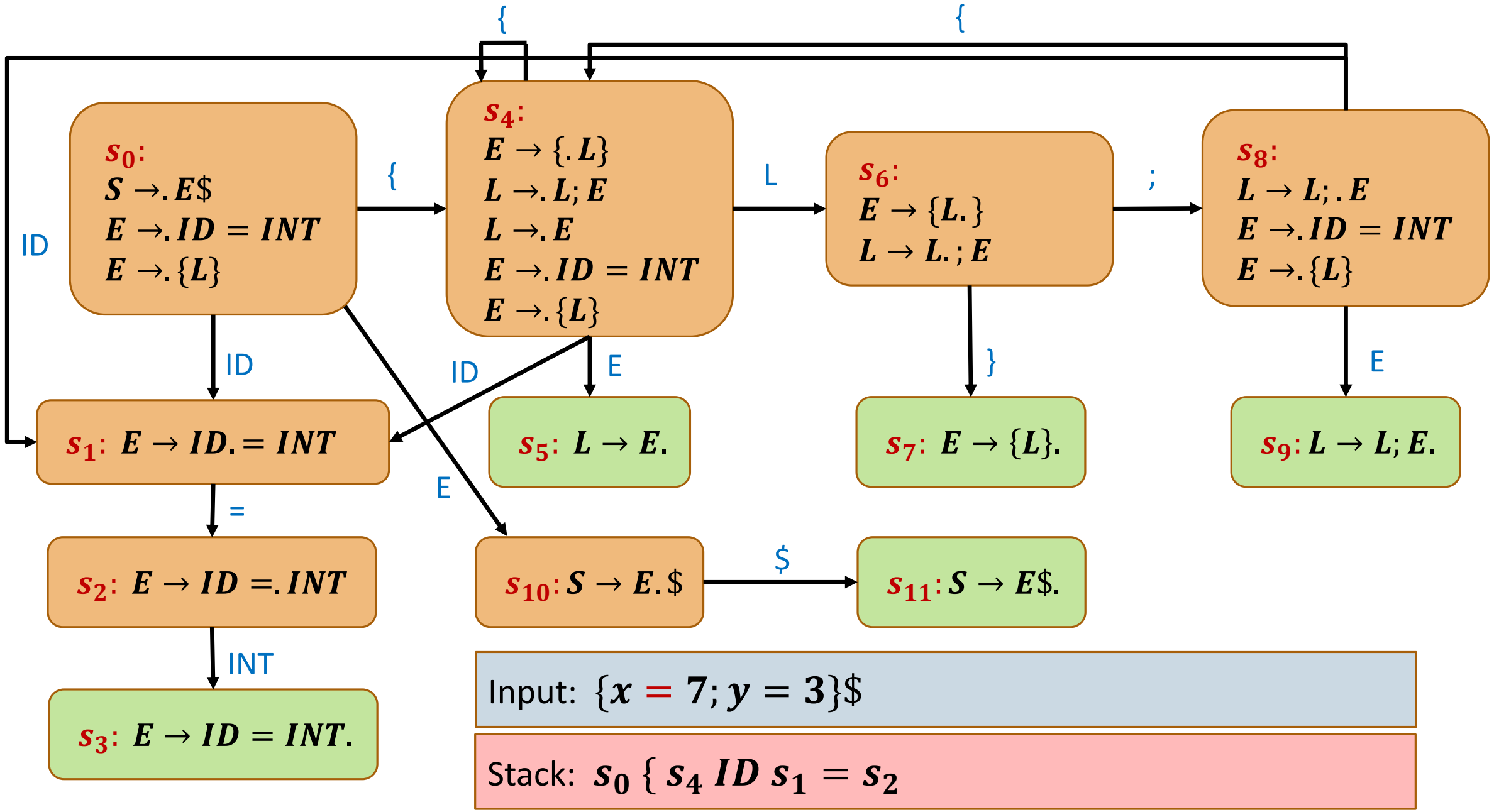
What will happen with the following input:

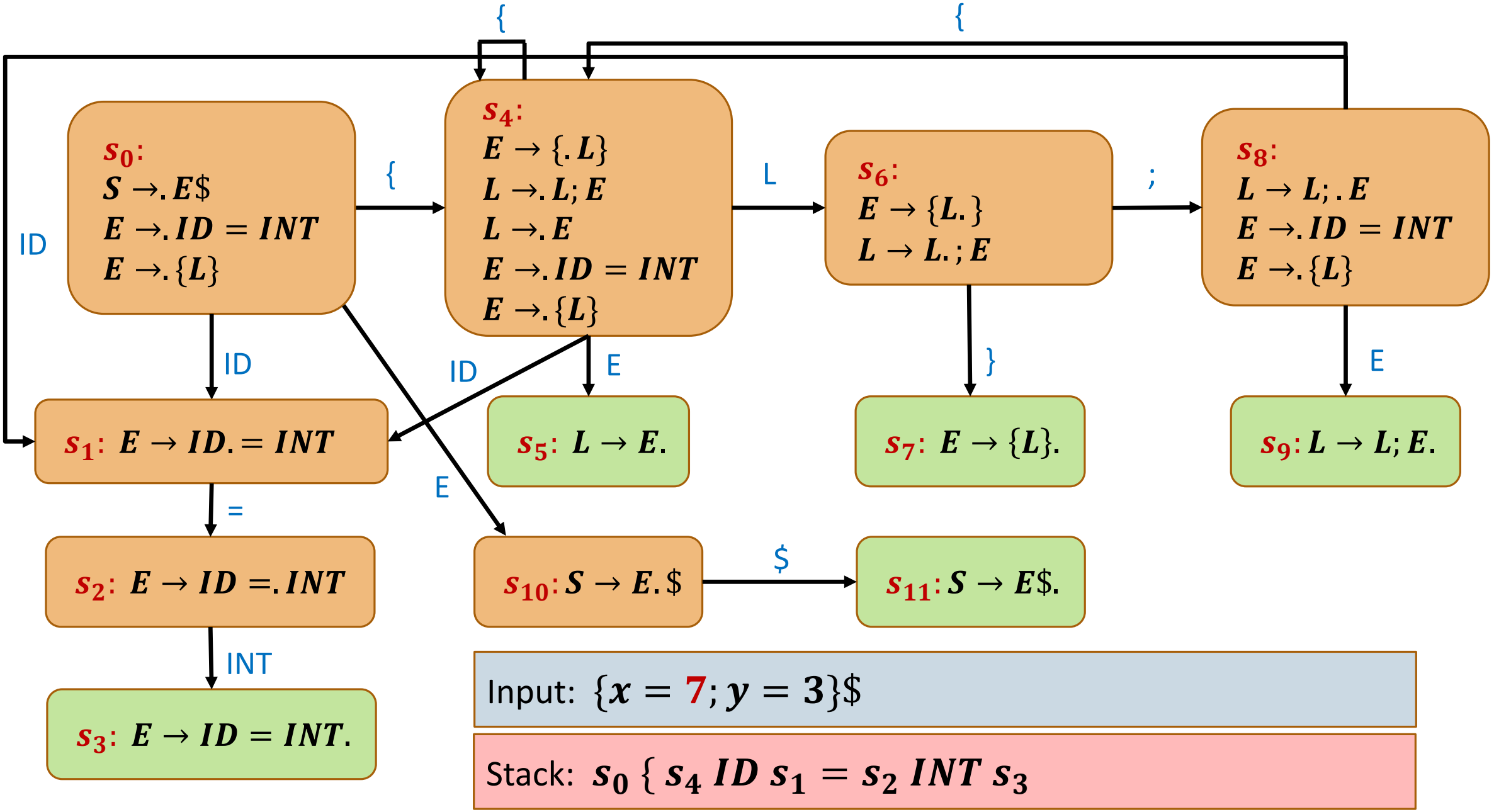
- $\{\mathrm{x} = 7; y = 3\}\$$

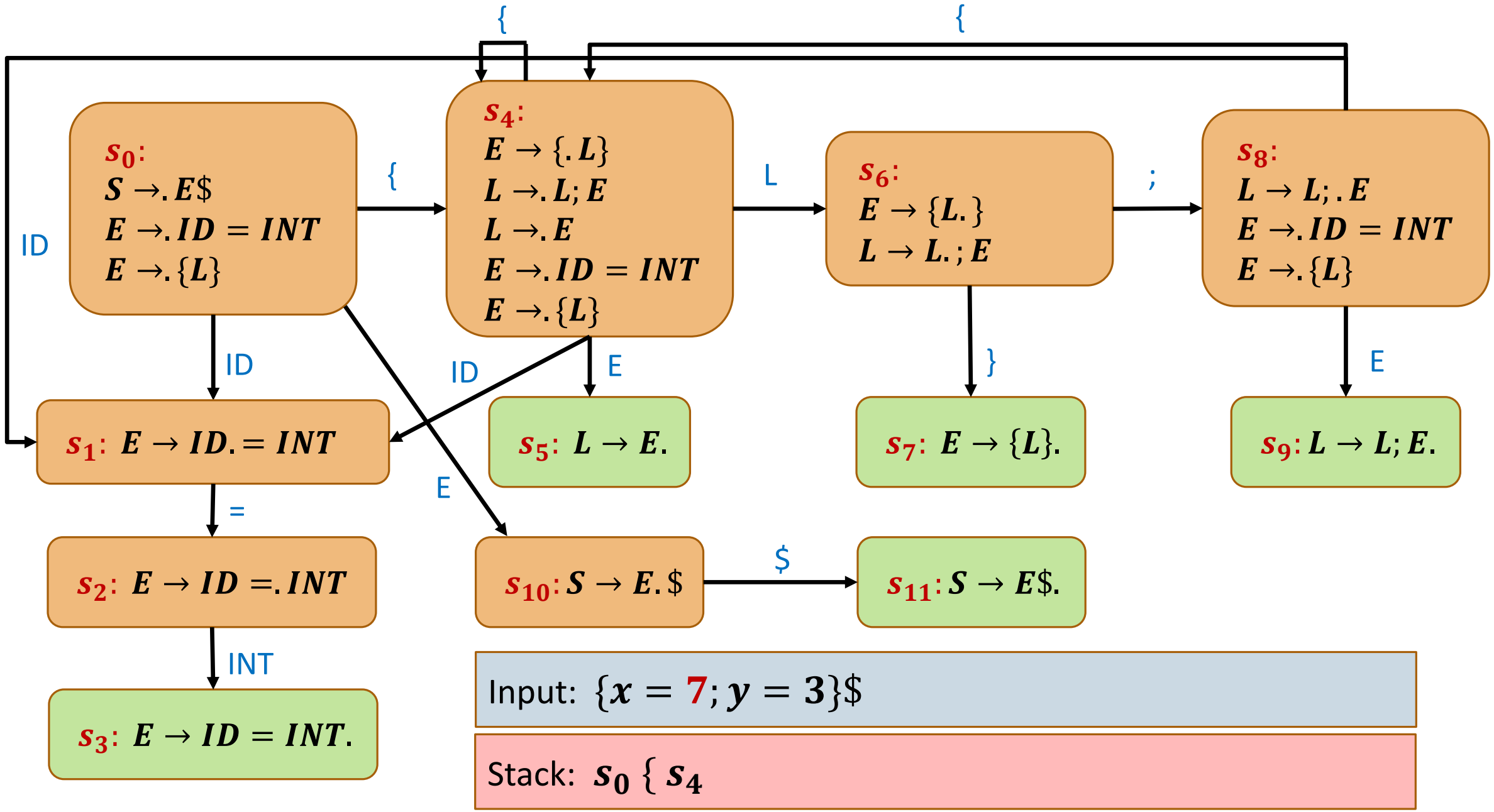

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: s_0

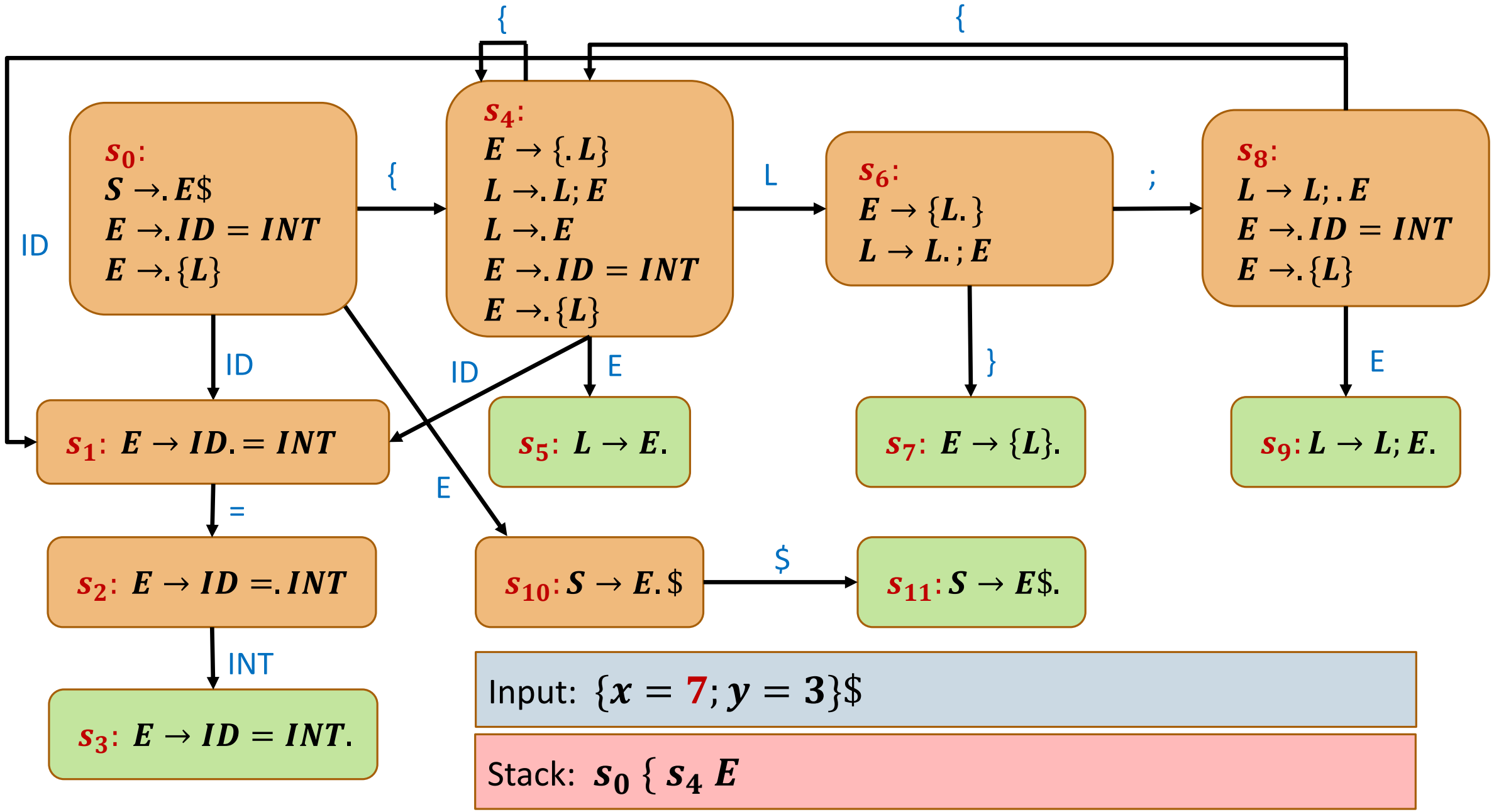

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0\ \{\ s_4$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
=
E
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 ID s_1$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
ID
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 \; ID \; s_1 = s_2$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 \; ID \; s_1 = s_2 \; INT \; s_3$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0\ \{\ s_4$


s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
s_{11}: $S \rightarrow E\$.$
s_3: $E \rightarrow ID = INT.$
{
{
ID
{
L
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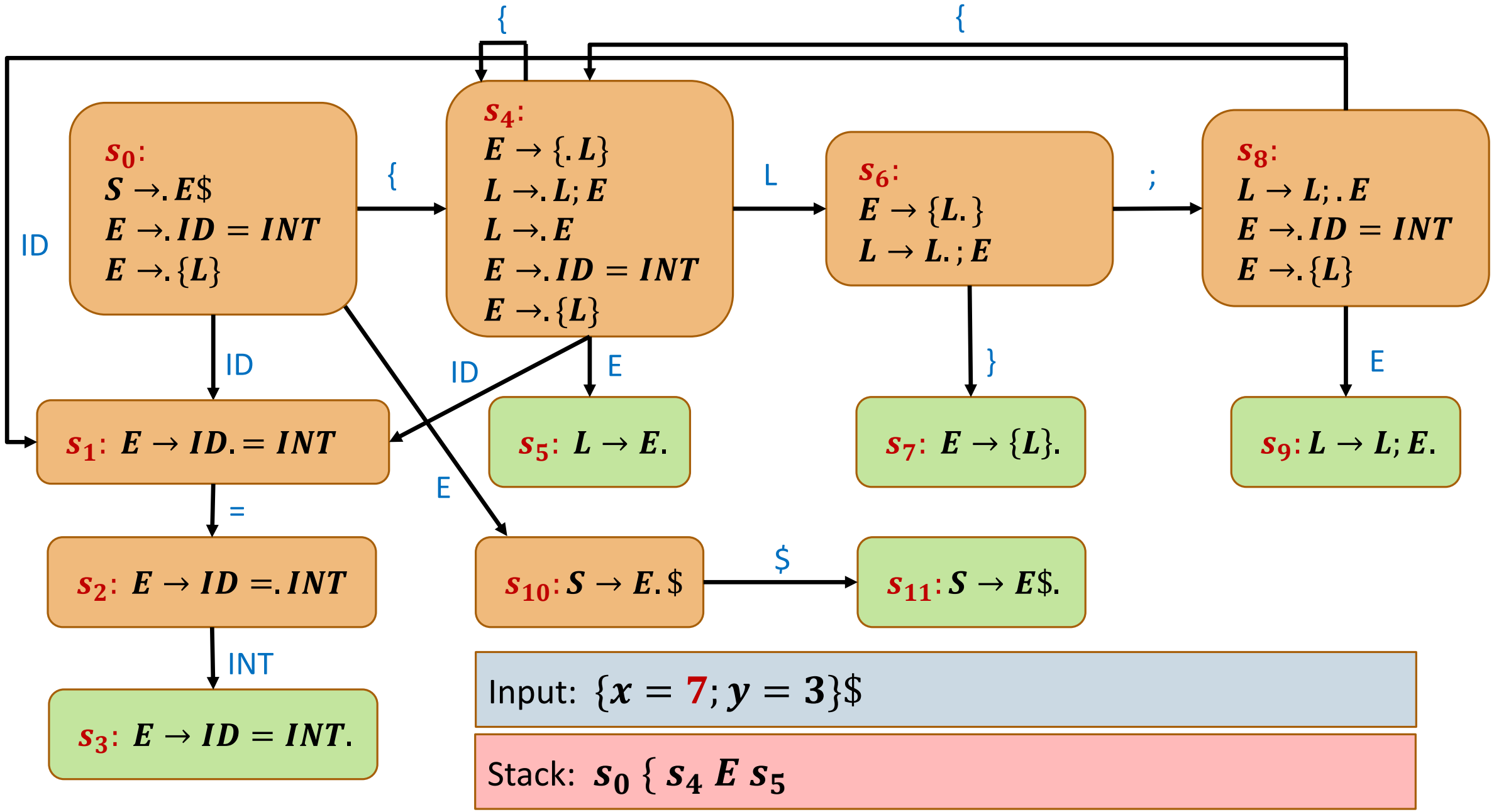
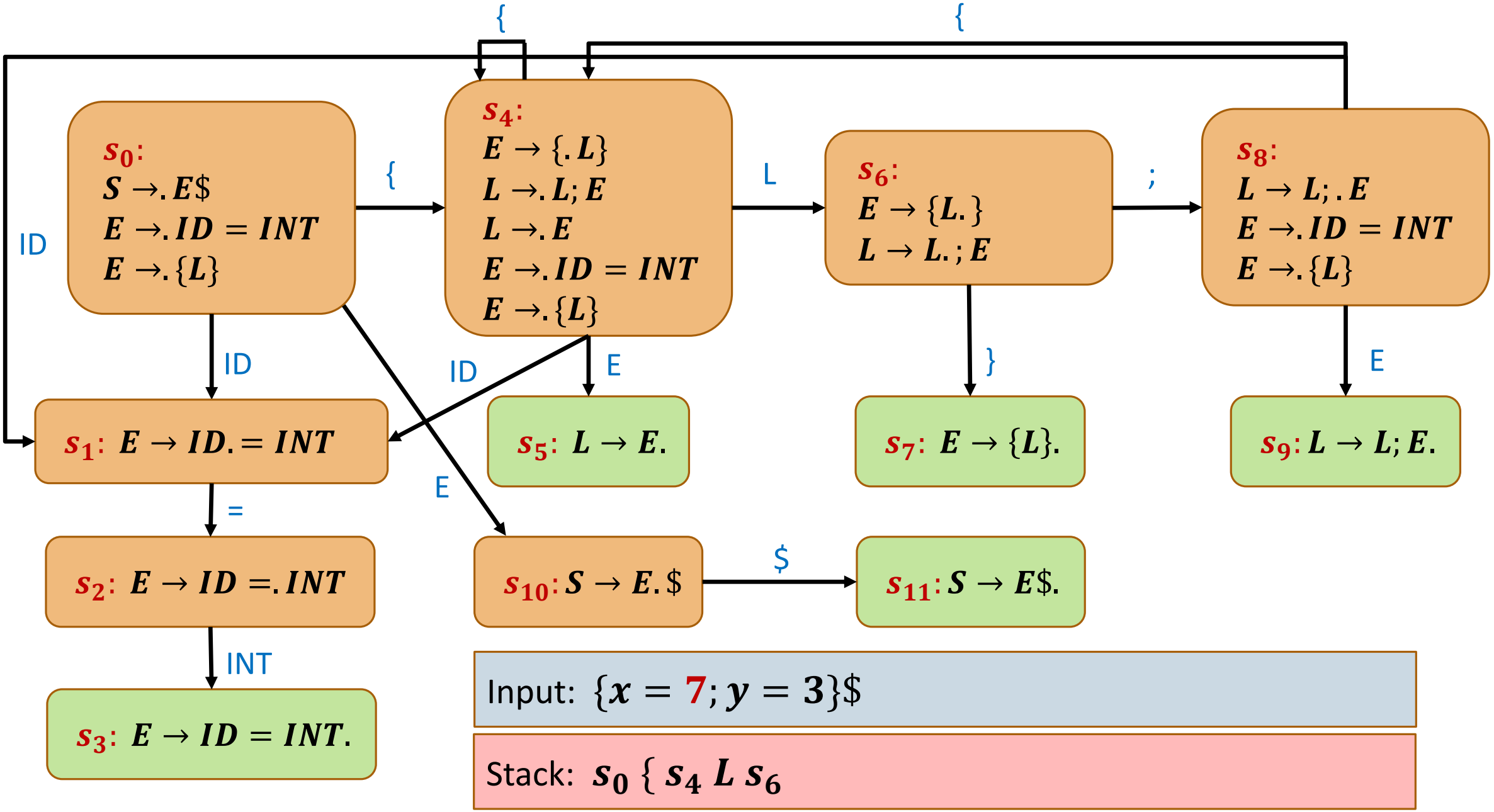
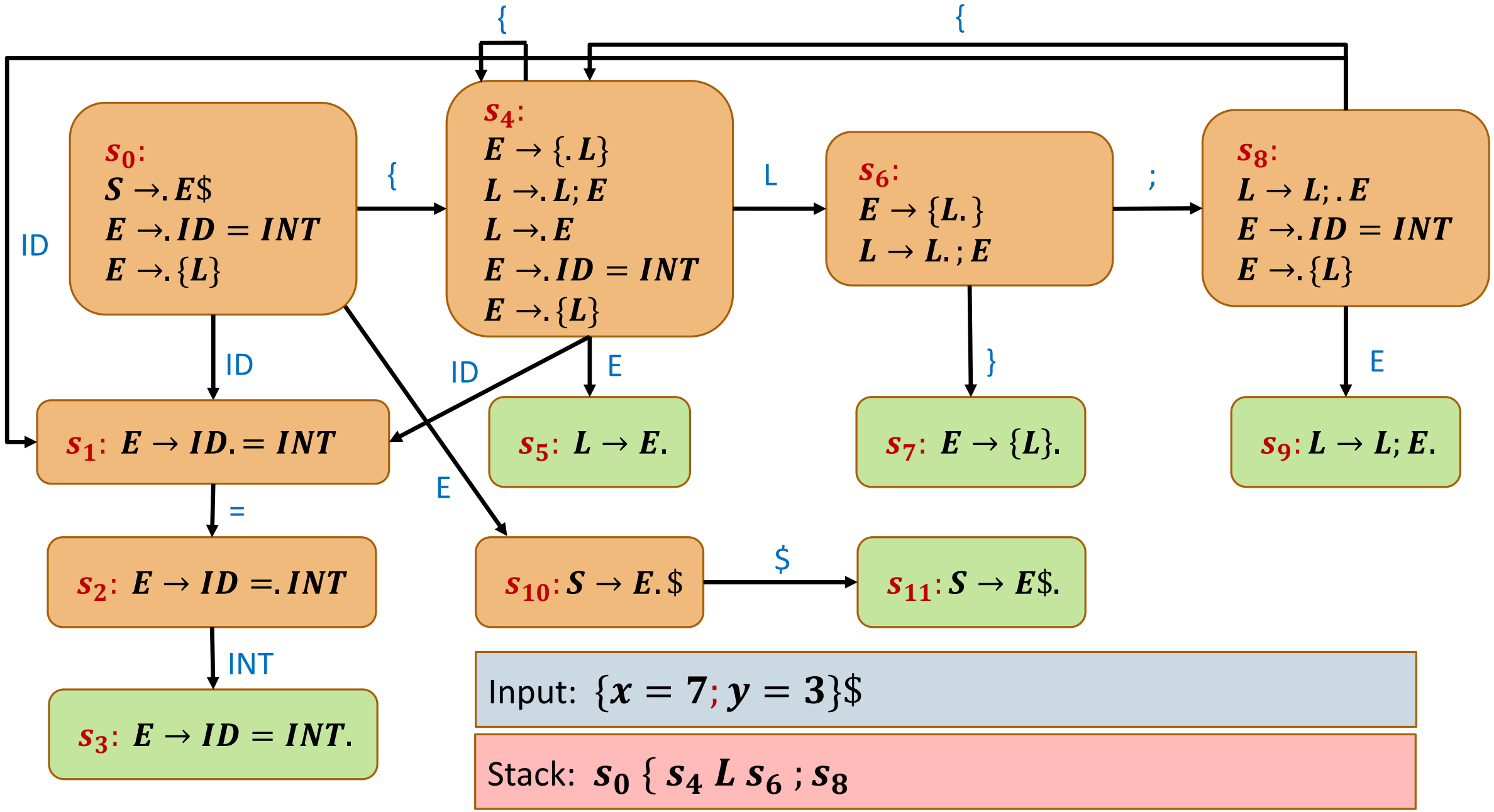
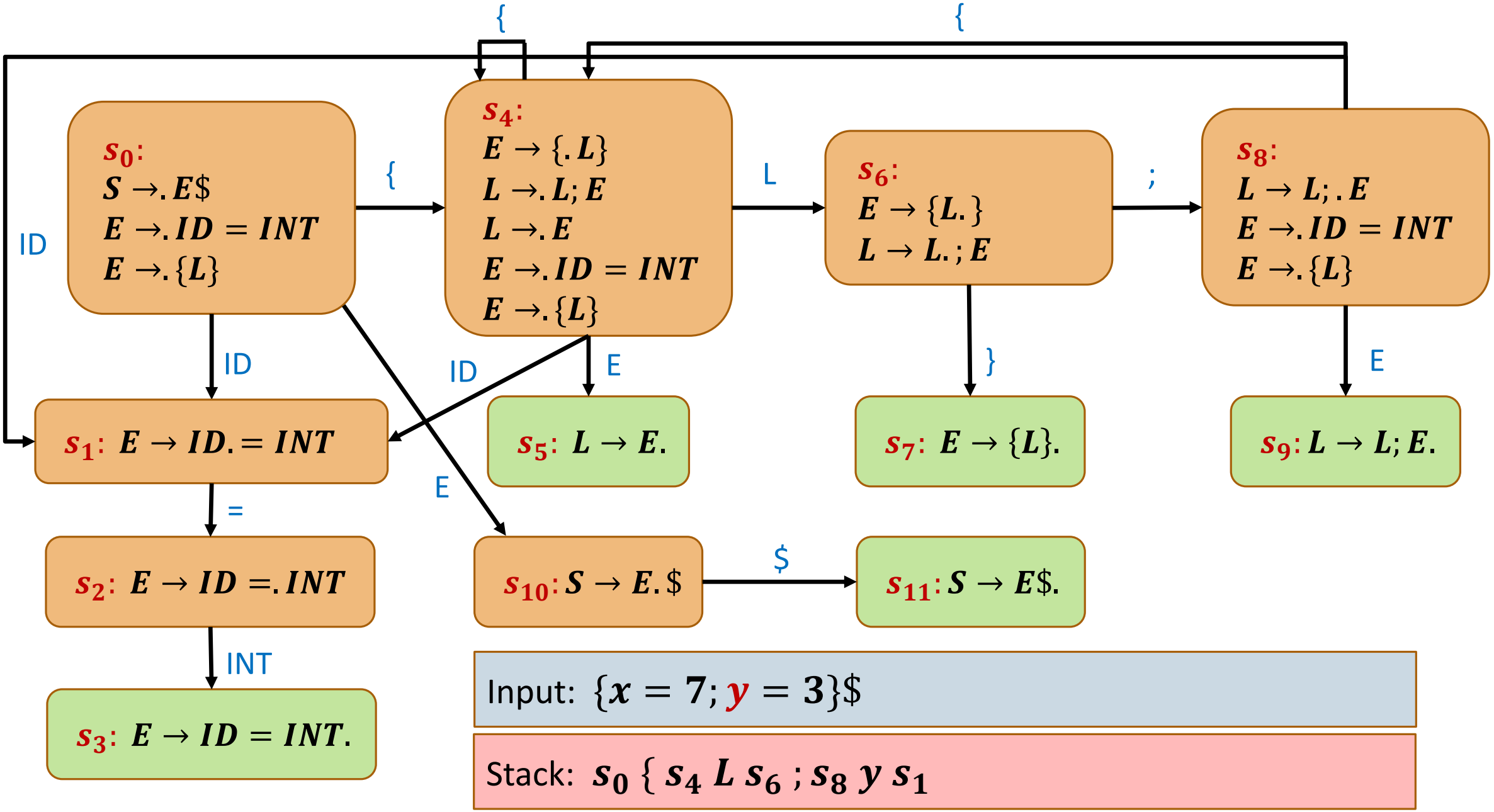
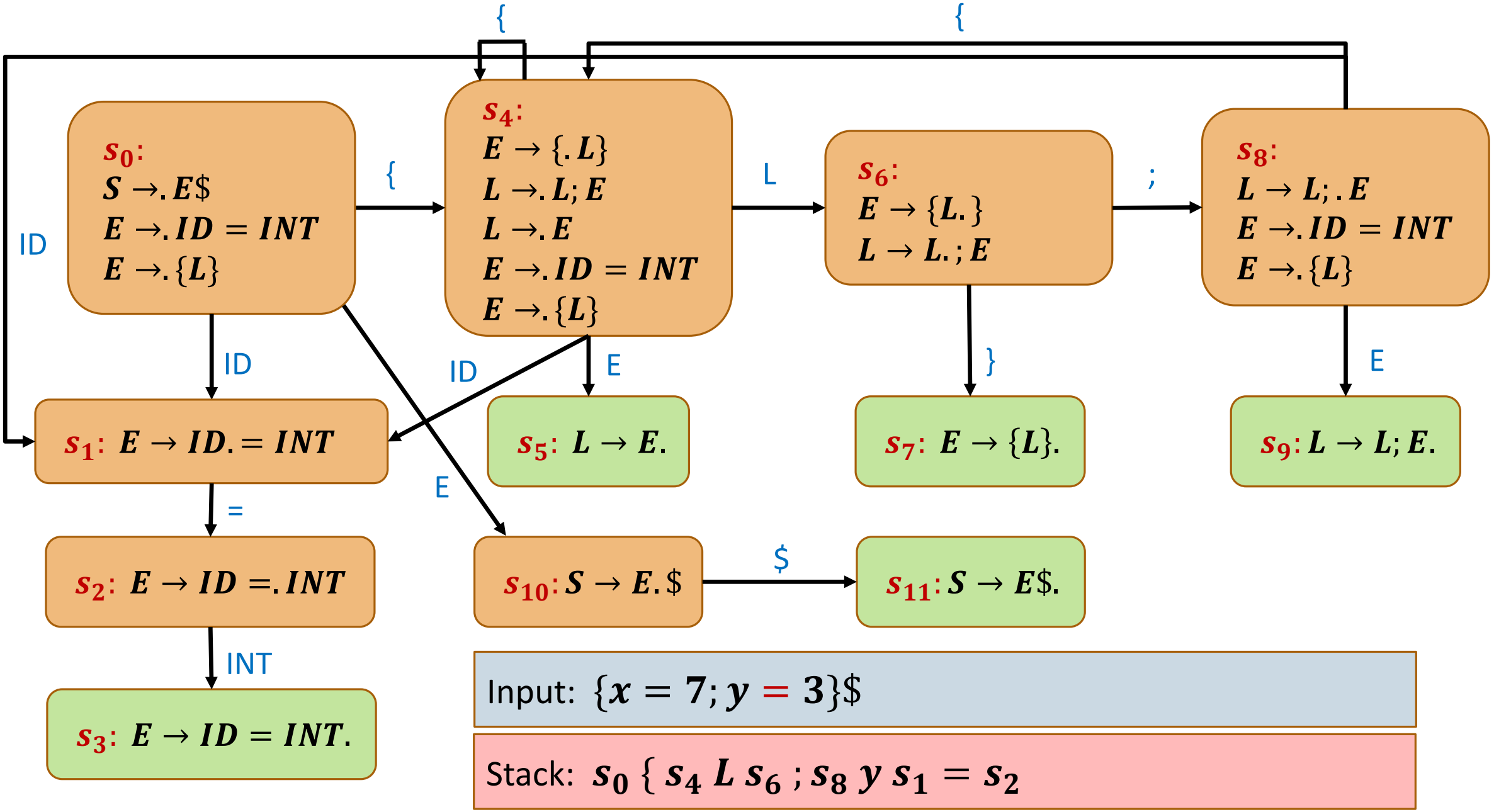
ID
E
}
E
ID
E
=
$
INT
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 E$

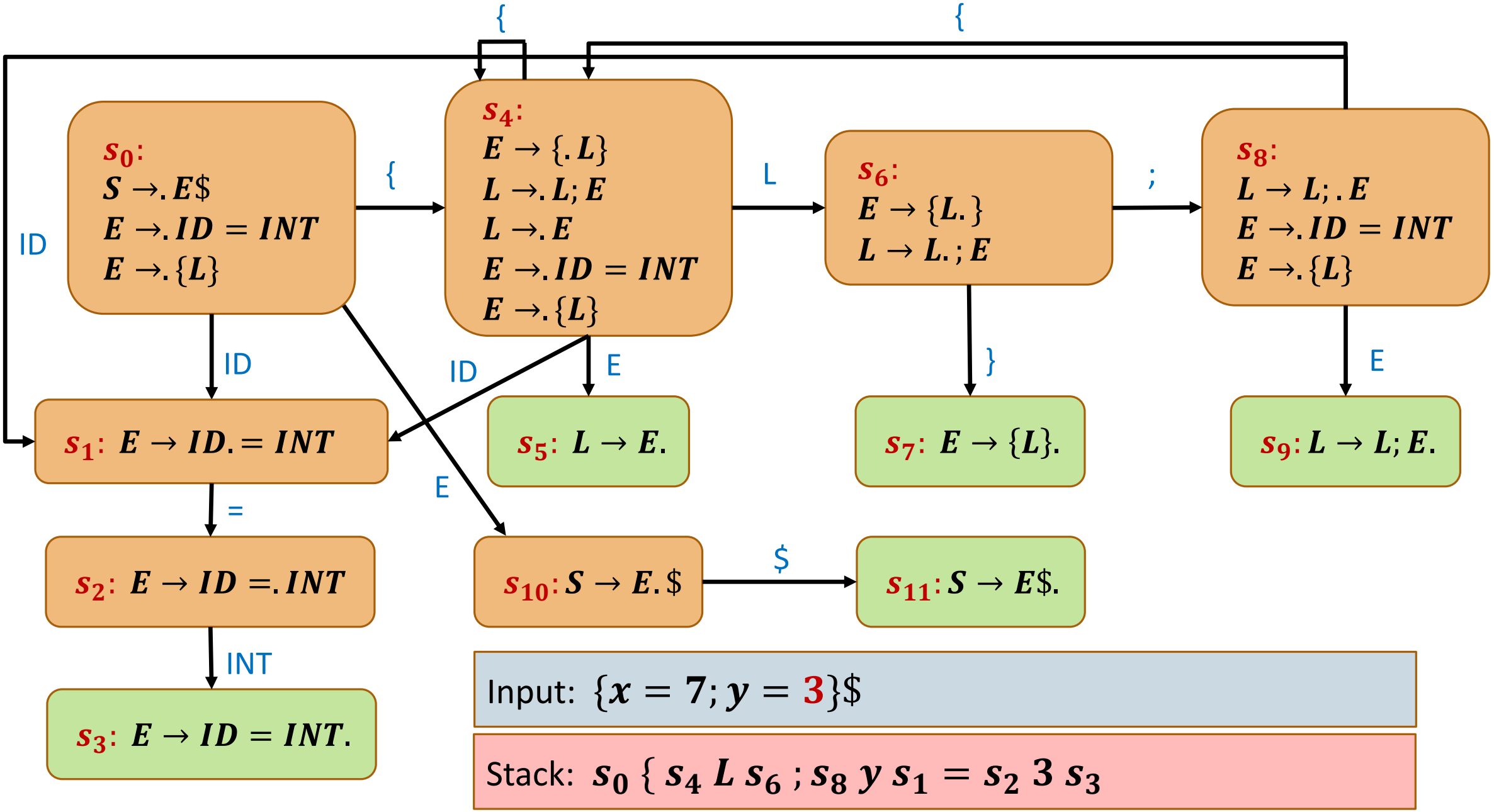

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 E s_5$

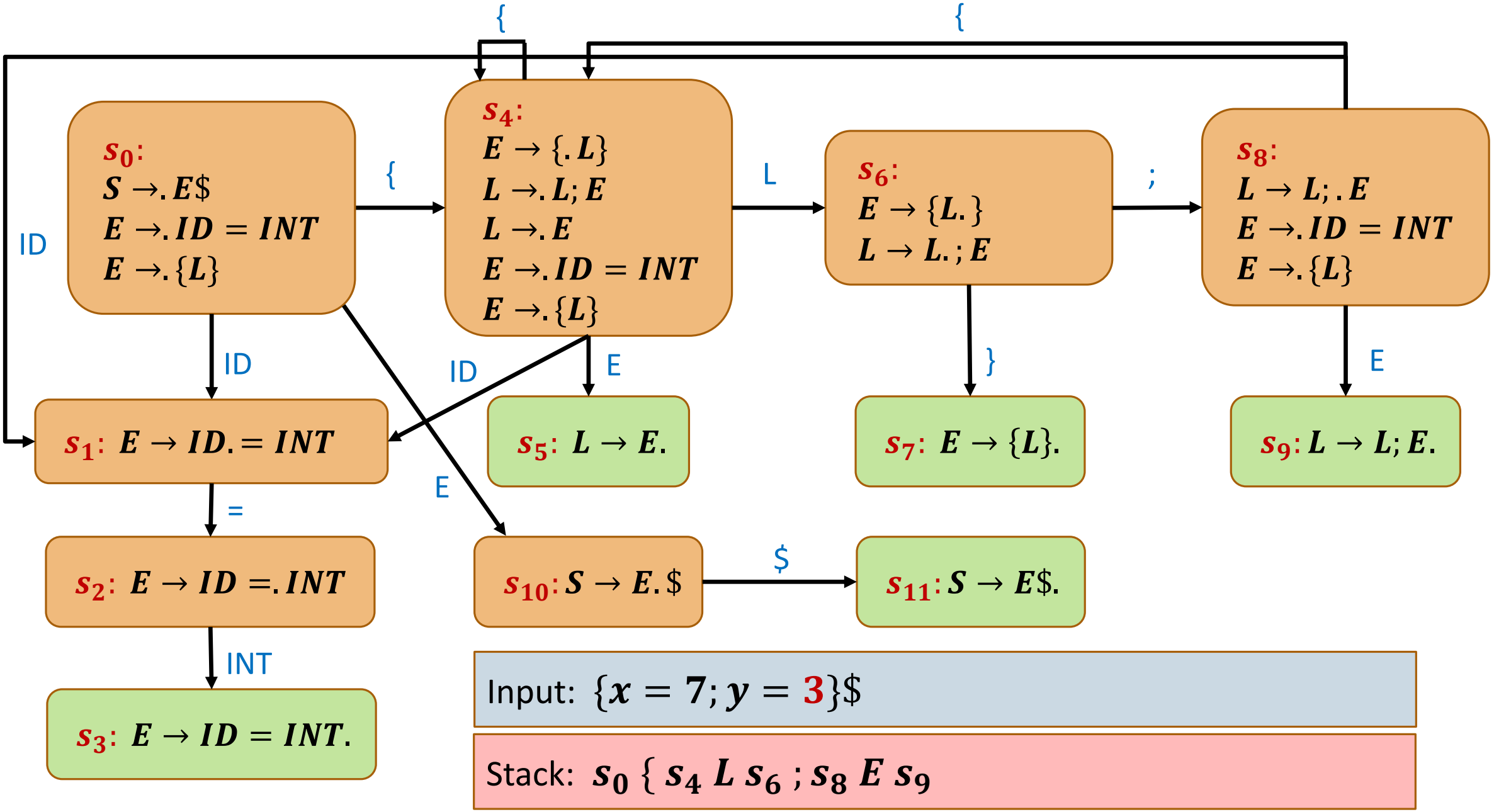

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
}
E
ID
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6$

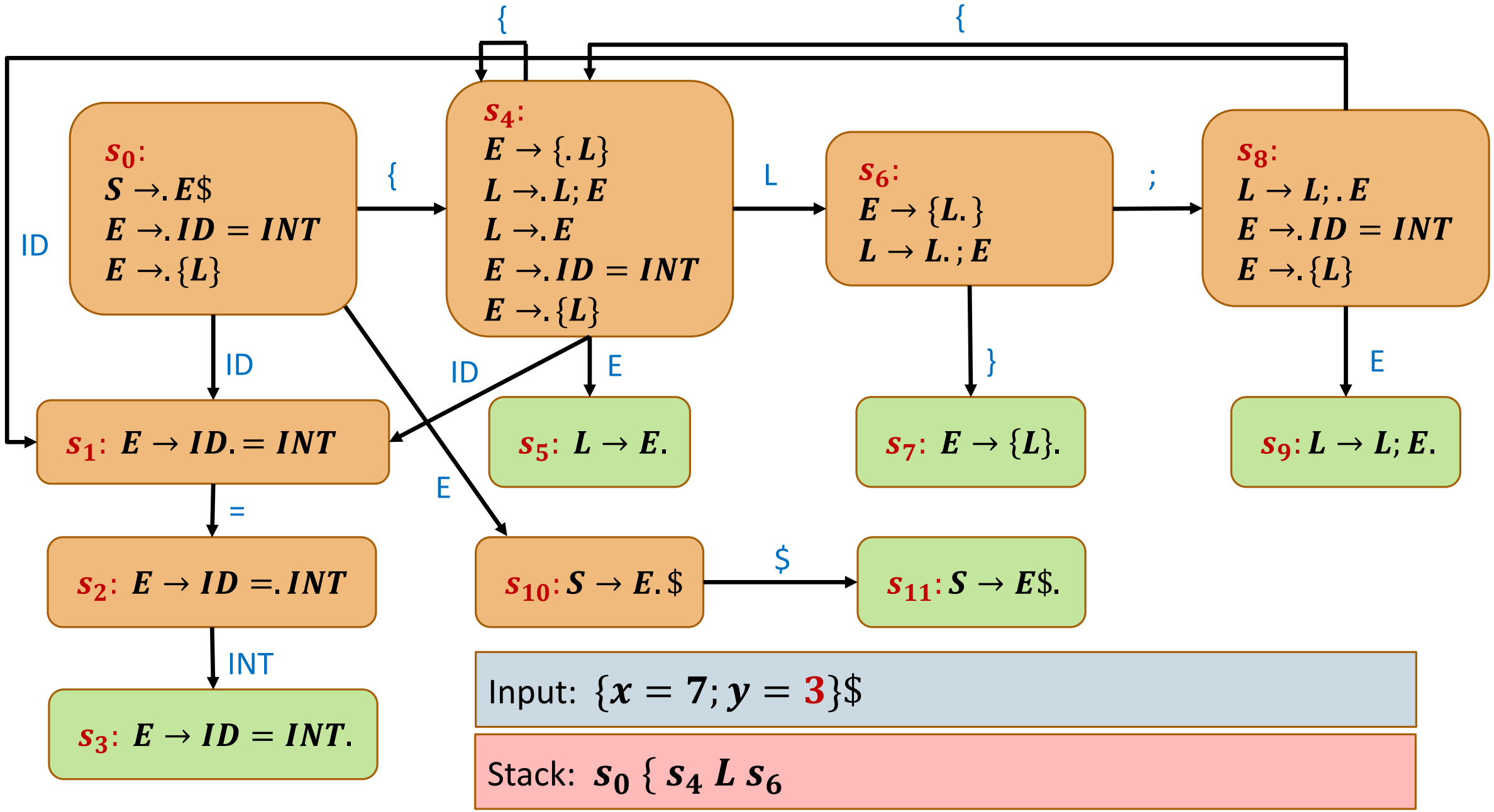

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6 ; s_8$

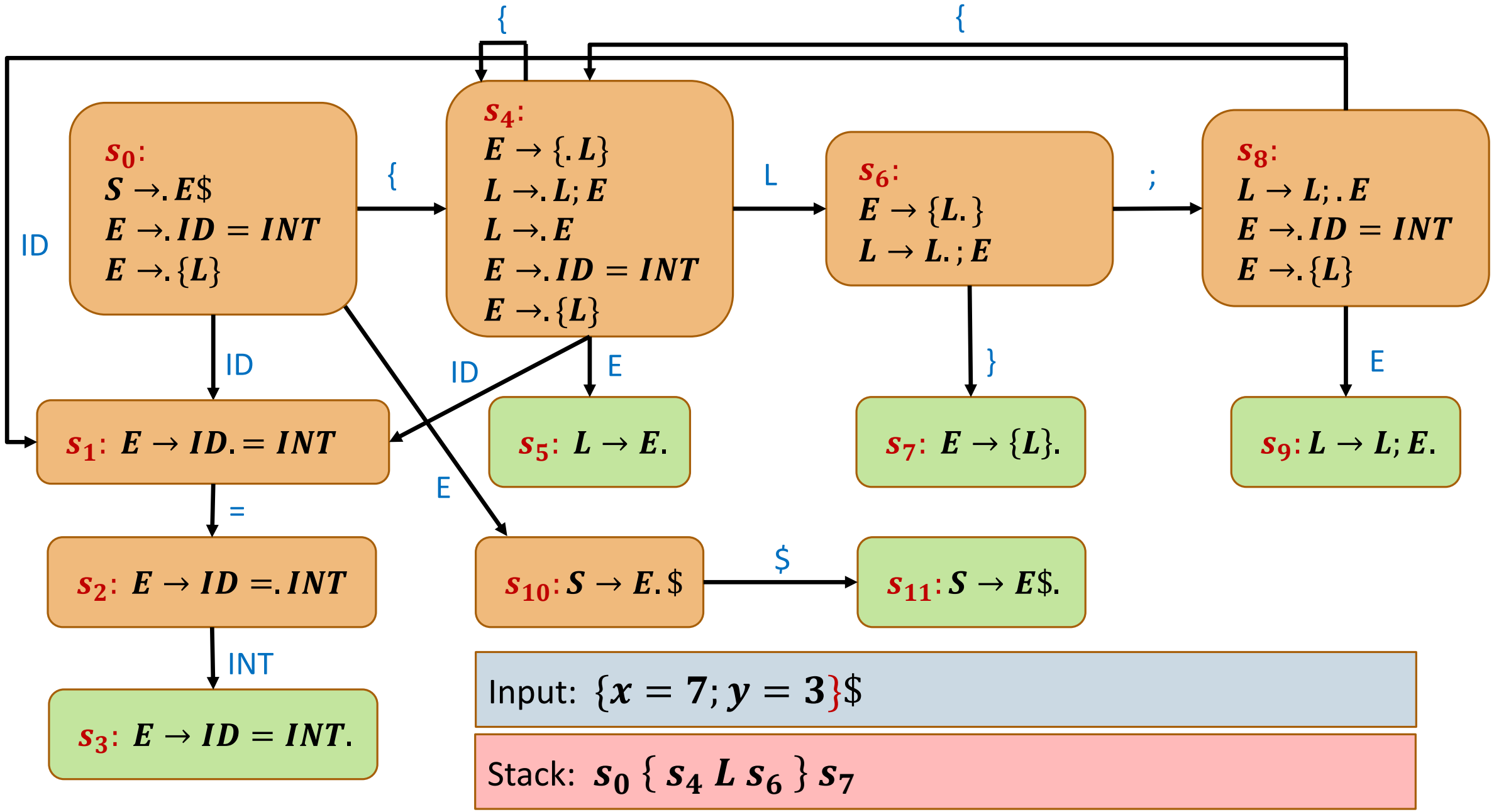

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
}
E
ID
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6 ; s_8 y s_1$

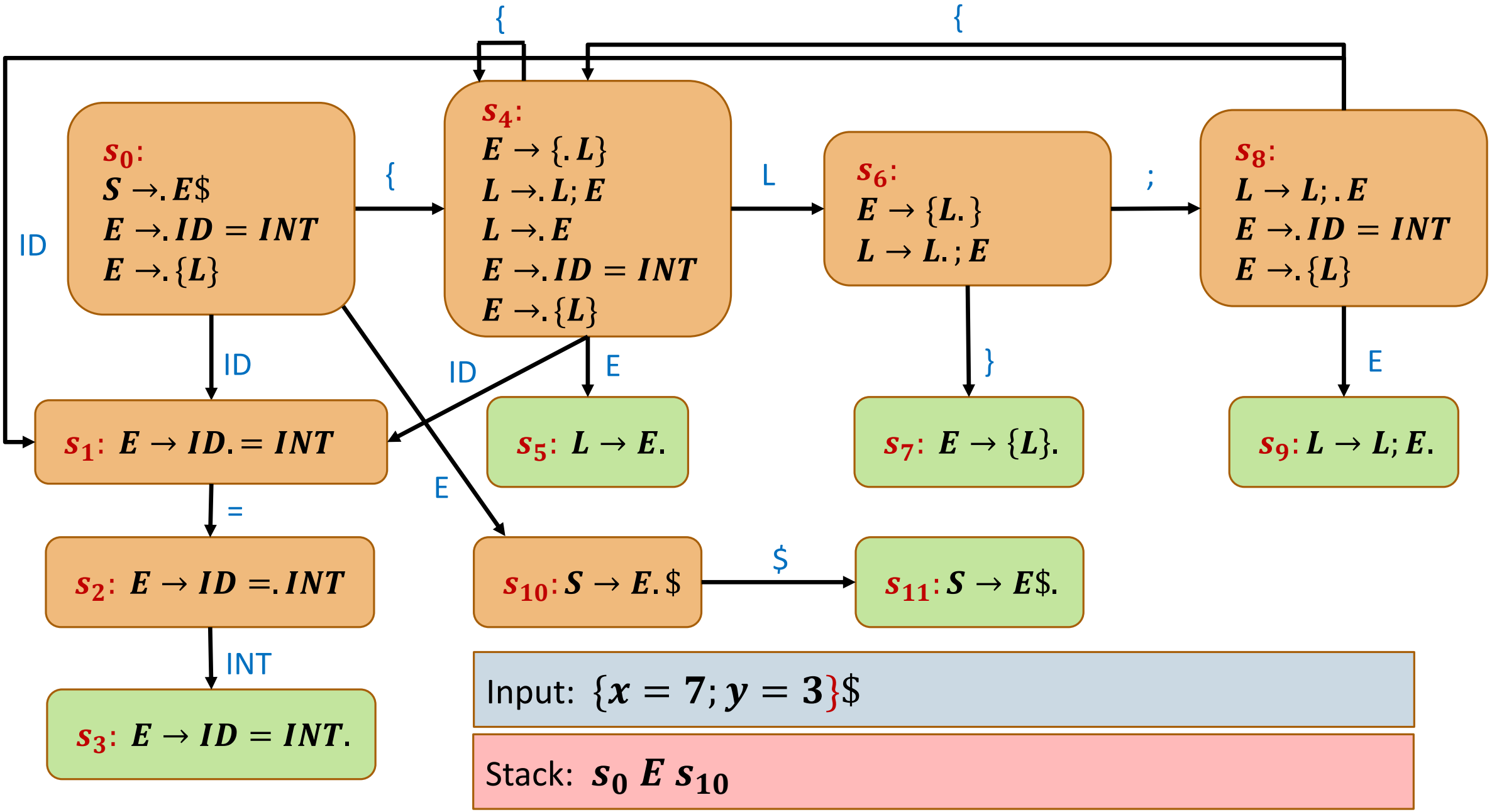

{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6 ; s_8 y s_1 = s_2$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6 ; s_8 y s_1 = s_2 3 s_3$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6 ; s_8 E s_9$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
ID
E
}
E
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6$


{
{
s_0:
$S \rightarrow .E\$$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
{
s_4:
$E \rightarrow \{.L\}$
$L \rightarrow .L; E$
$L \rightarrow .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
L
s_6:
$E \rightarrow \{L.\}$
$L \rightarrow L.; E$
;
s_8:
$L \rightarrow L; .E$
$E \rightarrow .ID = INT$
$E \rightarrow .\{L\}$
ID
ID
E
}
E
ID
s_1: $E \rightarrow ID. = INT$
s_5: $L \rightarrow E.$
s_7: $E \rightarrow \{L\}.$
s_9: $L \rightarrow L; E.$
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0 \{ s_4 L s_6 \} s_7$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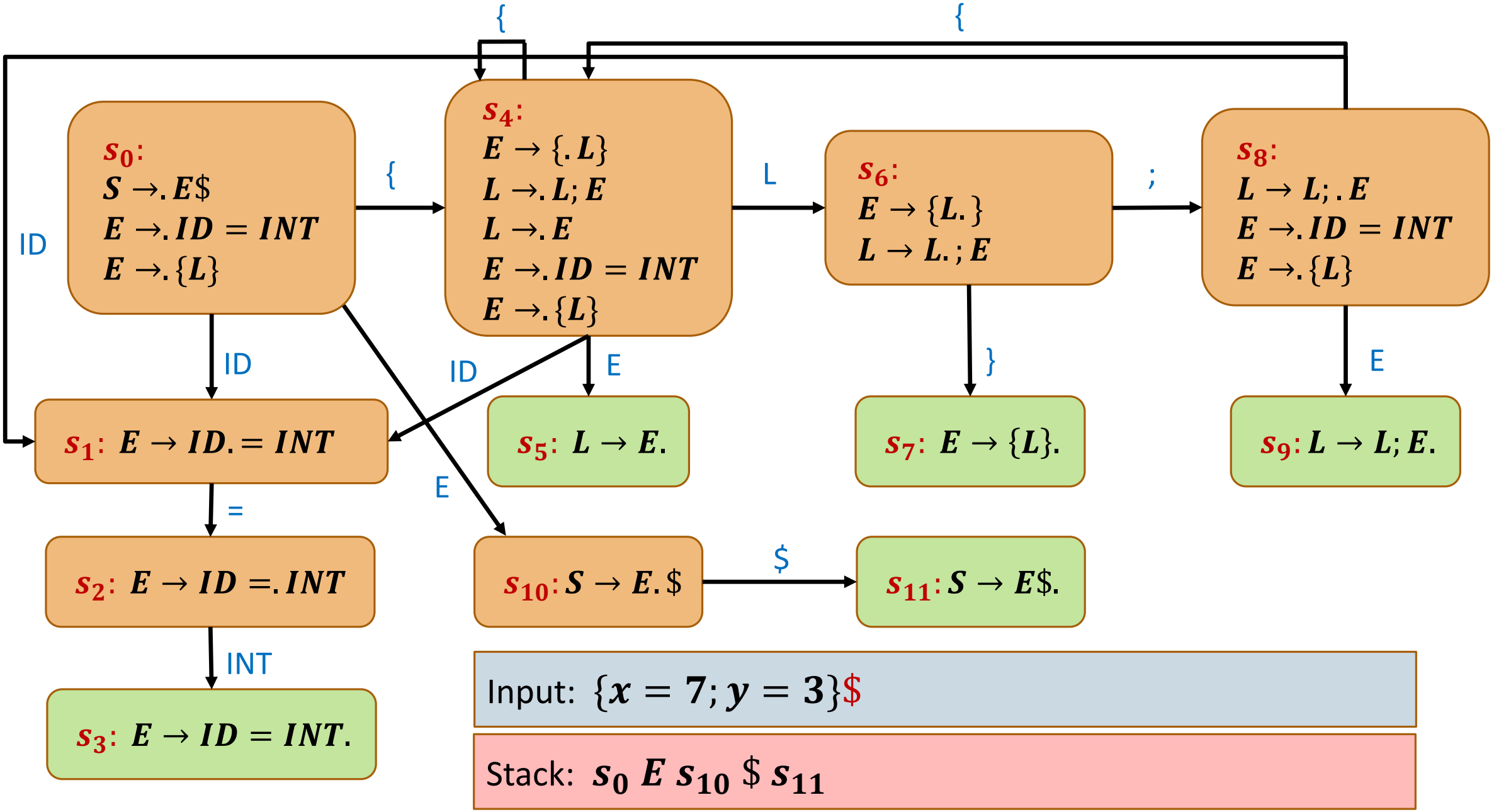
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0\ E\ s_{10}$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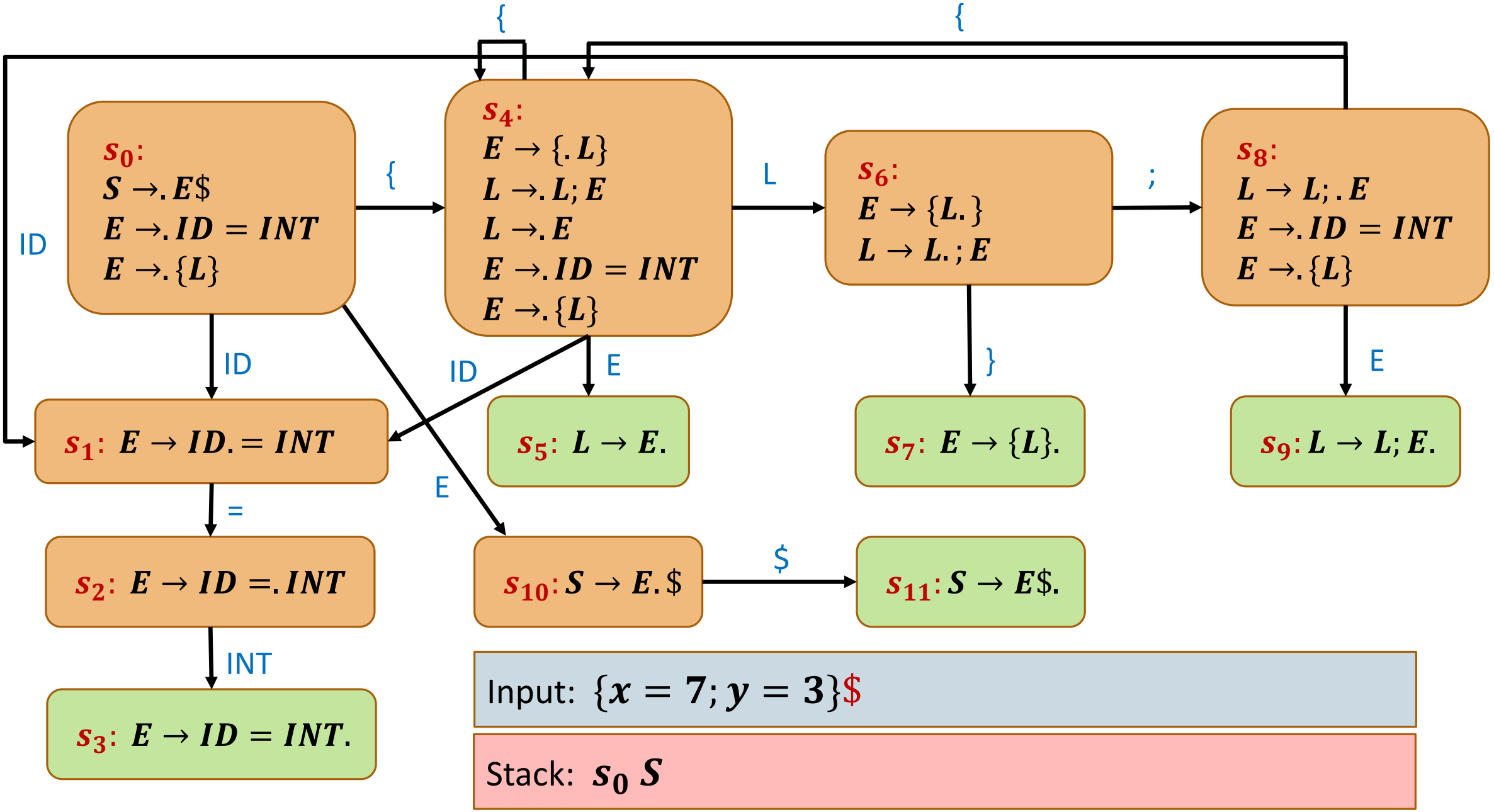
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0\ E\ s_{10}\ \$\ s_{11}$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ID
ID
E
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E
s_1: $E \rightarrow ID. = INT$
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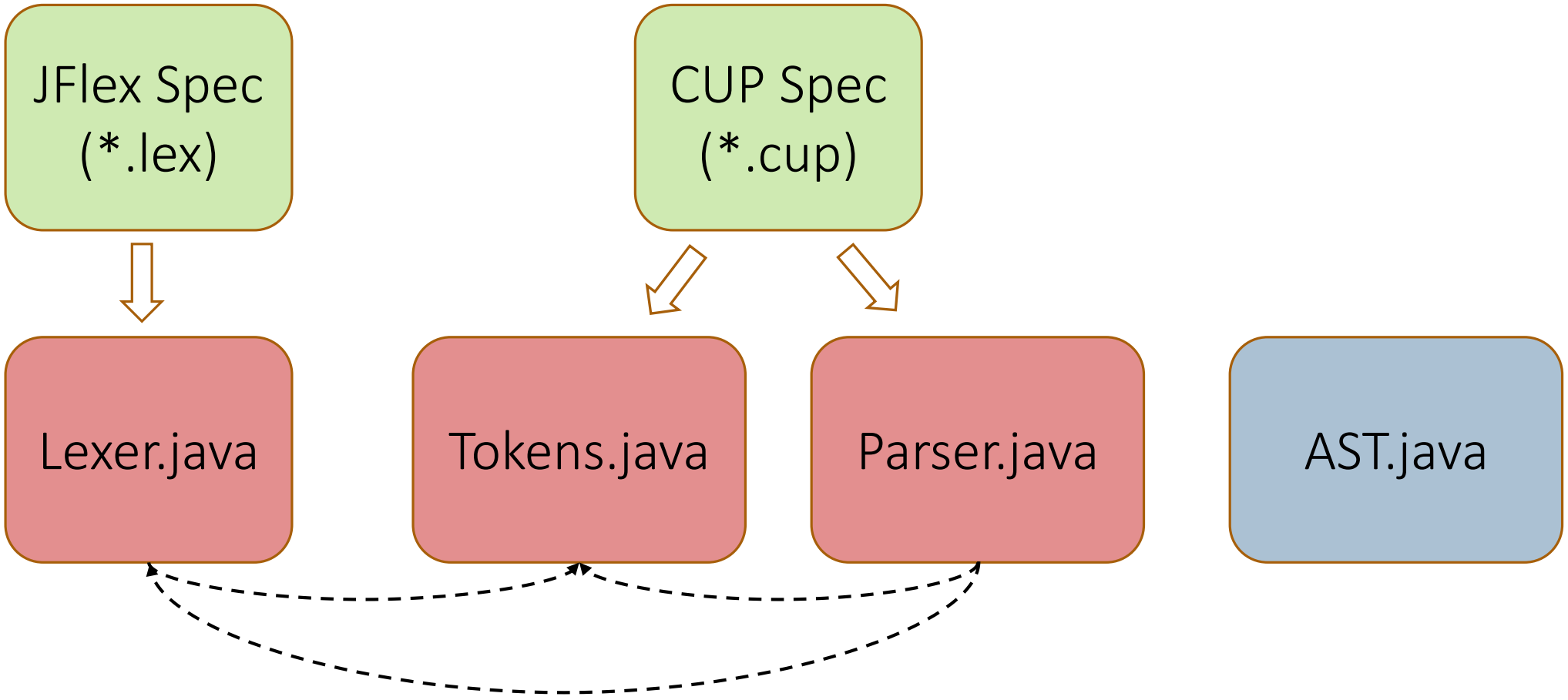
E
=
s_2: $E \rightarrow ID = .INT$
s_{10}: $S \rightarrow E.\$$
$
s_{11}: $S \rightarrow E\$.$
INT
s_3: $E \rightarrow ID = INT.$
Input: $\{x = 7; y = 3\}\$$
Stack: $s_0\ S$

Parsing with CUP

CUP

- Given a user-specified grammar, generates an LALR parser
- Works with JFlex, which provides the parsed tokens
- Other tools:
  - Bison (for C)

CUP/JFlex Workflow



CUP Spec: Parser Setup

```
parser code {:
        public Lexer lexer;
        public Parser(Lexer lexer) {
                super(lexer);
                this.lexer = lexer;
        }
        public void report_error(String message, Object info) {
                System.exit(0);
        }
:}
```

CUP Spec: Lexer Setup

```
scan with {:
        Symbol s;
        s = lexer.next_token();
        // print token…
        return s;
:};
```

CUP Spec: Terminals

```
terminal T1;
terminal T2;
terminal T3;
terminal T4;
…
```

CUP Spec: Non-Terminals

non terminal AST_NODE_1 E1;
non terminal AST_NODE_2 E2;
non terminal AST_NODE_3 E3;
…

CUP Spec: Operator Precedence

precedence left OP1;
precedence left OP2;
precedence left OP3;
precedence left OP4;
…

These are token names…

CUP Spec: Grammar

```
start with S;
S ::=
  E1:e1 E2:e2 {: RESULT = new AST_NODE_1(e1, e2);  :} |
  E3:e3       {: RESULT = new AST_NODE_2(e3);  :} ;

E1 ::= ID:id    {: RESULT = new AST_NODE_3(id);   :}

E2 ::= INT:i    {: RESULT = new AST_NODE_4(i);  :}

E3 ::= …
```

CUP Spec: AST Nodes

- We need to **decide** which node types we have in our AST
- We need to **define** the classes for these AST nodes
- For example, for integers (INT) we can do:

```
public class AST_EXP_INT extends AST_EXP {
        public int value;
        public AST_EXP_INT(int value) {
                this.value = value;
        }
        public void PrintMe() { … }
}
```

CUP Example

Consider the following CFG:

- $E \rightarrow INT$
- $E \rightarrow V$
- $E \rightarrow E + E$
- $E \rightarrow E\ - E$
- $V \rightarrow ID$
- $V \rightarrow V\ .ID$

CUP Example: Terminals

```
terminal Integer INT;
terminal String ID;
terminal PLUS;
terminal MINUS;
Terminal DOT;
```

CUP Example: Non-Terminals

```
non terminal AST_EXP exp;
non terminal AST_VAR var;
```

CUP Example: Operator Precedence

```
precedence left PLUS;
precedence left MINUS;
```

CUP Example: Grammar

```
start with EXP;
EXP ::=
    INT:i {: RESULT = new AST_EXP_INT(i); :} |
    VAR:v {: RESULT = new AST_EXP_VAR(v); :} |
    EXP:e1 PLUS  EXP:e2 {: RESULT = new AST_EXP_BINOP(e1, e2, 0); :} |
    EXP:e1 MINUS EXP:e2 {: RESULT = new AST_EXP_BINOP(e1, e2, 1); :};
VAR ::=
    ID:name {: RESULT = new AST_VAR_SIMPLE(name); :} |
    VAR:v DOT ID:fieldName {: RESULT = new AST_VAR_FIELD(v, fieldName); :};
```

CUP Example: AST Nodes

For the rule ***ID:name***:

```
public class AST_VAR_SIMPLE extends AST_VAR {
        public String name;
        public AST_VAR_SIMPLE(String name) {
                this.name = name;
        }
}
```

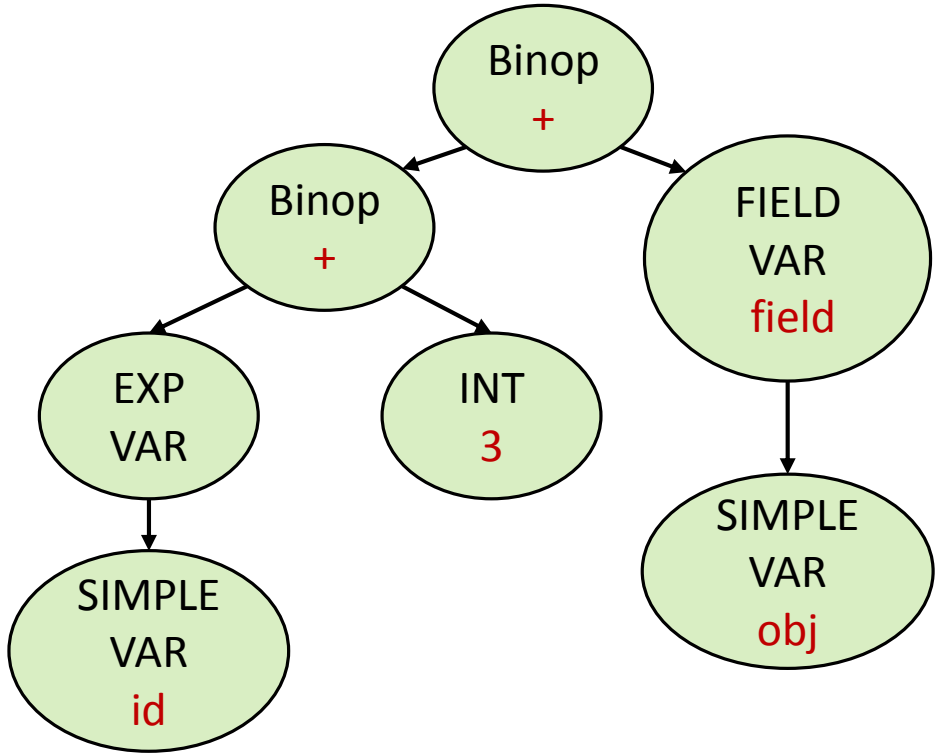
CUP Example: AST Nodes

For the rule ***var:v DOT ID:fieldname***:

```
public class AST_VAR_FIELD extends AST_VAR {
        public AST_VAR var;
        public String fieldName;
        public AST_VAR_FIELD(AST_VAR var, String fieldName) {
                this.var = var;
                this.fieldName = fieldName;
        }
}
```

CUP Example: Debugging

We can generate an image of the AST (using the exercise template)
For the input foo + 3 + obj.field we have:



CUP Example: Debugging

- Can dump he automaton (and goto/action table)
- Can detect conflicts:
  - shift-reduce, …