

Top Down Parsing

TEACHING ASSISTANT: DAVID TRABISH

Examples

```
void f(int a) {  
    if ((8)) {  
  
    }  
}
```

Examples

```
void f(int a) {  
    if ((8)) {  
  
    }  
}
```

Valid

Examples

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void f(int a) {  
    if ((8))) {  
  
    }  
}
```

Examples

```
void f(int a) {  
    if ((8))) {  
  
    }  
}
```

Invalid

Examples

```
if ((8)) {  
  
}
```

Examples

```
if ((8)) {  
  
}
```

Invalid

Examples

```
void f() {  
    int a[];  
}
```


Examples

```
void f() {  
    int a[];  
}
```

Invalid

Examples

```
void f() {  
    int a[10.0];  
}
```

Examples

```
void f() {  
    int a[10.0];  
}
```

Invalid

Examples

```
void f(int a[]) {  
  
}
```

Examples

```
void f(int a[]) {  
  
}
```

Valid

Examples

```
void f() {  
    int i = 0;  
    int j = 1;  
    j + i;  
}
```

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    int j = 1;  
    j + i;  
}
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Valid

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    j + i;  
    int j = 1;  
}
```

Valid

Language of Balanced Parentheses

Contains string of the form:

- 8, (1), (((0))), ...

Disallowing:

- ((1), 8()

Language of Balanced Parentheses

Contains string of the form:

- ϵ , (1) , $((0))$, ...

Disallowing:

- $((1)$, $\epsilon()$

Is there a DFA/NFA that accepts the language?

Is there a regular expression that accepts the language?

Language of Balanced Parentheses

The language is **not regular**

- There is no DFA that accepts it

Proof:

- If it has a DFA, then we have d states
- Consider the input $((\dots))$ that has $d + 1$ left parentheses
- Every time we read $($, we need to change to a new state
 - We need to act differently if we saw 4 parentheses or 10
- But we have only d states...

Context Free Grammar

- A set of terminals T and a set of non-terminals V
- Production rules of the form
 - $A \rightarrow a_1 a_2 \dots a_n$
 - $A \in V, a_i \in T \cup V$
- Starting symbol S :
 - $S \rightarrow a_1 a_2 \dots a_n$

Context Free Grammar

Example:

- $S \rightarrow c$
- $S \rightarrow aSb$

Which words belong to this grammar?

Context Free Grammar

Example:

- $S \rightarrow c$
- $S \rightarrow aSb$

Which words belong to this grammar?

- $c, acb, aacbb, aaacbbb, \dots$

Context Free Grammar

Does the language of **balanced parentheses** have a CFG?

Context Free Grammar

Does the language of **balanced parentheses** have a CFG?

- $S \rightarrow N$
- $S \rightarrow (S)$

Context Free Grammar: Questions

Are there languages which **have** no CFG?

Context Free Grammar: Questions

Are there languages which have no CFG? Yes

Context Free Grammar: Questions

Are there languages which **have no CFG?** Yes

Can we have **multiple** CFG's describing the same language?

Context Free Grammar: Questions

Are there languages which **have no CFG**? **Yes**

Can we have **multiple** CFG's describing the same language? **Yes**

Predictive Parser: Definition

Some languages has a predictive parser:

- We determine the production rule according to the current token
- We begin we the start symbol
 - From the top...

Predictive Parser: Example

The language of balanced parentheses:

- $S \rightarrow N$
- $S \rightarrow (S)$

has a predictive parser.

Predictive Parser: Example

```
void parse_S() {
    switch (token) {
    case N:
        parse_token(N);
        break;
    case L_PAREN:
        parse_token(L_PAREN);
        parse_S();
        parse_token(R_PAREN);
        break;
    default:
        // error
    }
}
```

```
void parse_token(int expected) {
    if (token == expected) {
        token = lexer.next_token();
    } else {
        // error
    }
}

void parse() {
    parse_S();
    if (token != EOF)
        // error
}
```


Predictive Parser: Example

What happens for the input (7)?

Call trace:

- parse_S
 - parse_token // match with '('
 - parse_S
 - parse_token // match with '7'
 - parse_token // match with ')'

Predictive Parser: Example

What happens for the input ((7)?

Call trace:

- parse_S
 - parse_token // match with '('
 - parse_S
 - parse_token // match with '('
 - parse_S
 - parse_token // match with '7'
 - parse_token // match with ')'
 - parse_token // error, expecting ')'

Language of Balanced Parentheses 2

Find a CFG for a language with the 3 kinds of parentheses:

- `()`, `[]`, `{}`

Contains string of the form:

- `(([][]){}))[]`
- `[()]`

Not allowing:

- `((())){`

Language of Balanced Parentheses 2

CFG definition:

- $S \rightarrow (S)S$
- $S \rightarrow [S]S$
- $S \rightarrow \{S\}S$
- $S \rightarrow \epsilon$

Language of Balanced Parentheses 2

```
void parse_S() {  
    switch (token) {  
        case L_PAREN:  
            parse_S1();  
            break;  
        case L_BRACKET:  
            parse_S2();  
            break;  
        case L_BRACE:  
            parse_S3();  
            break;  
        default:  
            break;  
    }  
}
```

```
void parse_S1() {  
    parse_token(L_PAREN);  
    parse_S();  
    parse_token(R_PAREN);  
    parse_S();  
}  
void parse_S2() {  
    parse_token(L_BRACKET);  
    parse_S();  
    parse_token(R_BRACKET);  
    parse_S();  
}  
void parse_S3() {  
    parse_token(L_BRACE);  
    parse_S();  
    parse_token(R_BRACE);  
    parse_S();  
}
```

Calculator Language

A language with binary operators (+, -, *, /) and numbers:

- 1
- 1+1
- $(1+1)*(7/2)$
- 2+1-7

Calculator Language

A (possible) CFG for that language:

- $S \rightarrow N$
- $S \rightarrow S + S$
- $S \rightarrow S - S$
- $S \rightarrow S * S$
- $S \rightarrow S / S$
- $S \rightarrow (S)$

Calculator Language

A (possible) CFG for that language:

- $S \rightarrow N$
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Will predictive parsing work here?

Left Recursion

There is no predictive parser which can handle the previous CFG

Why?

- If the first token was 5, we can't predict the right rule
- It can be 5 ($S \rightarrow N$)
- But also can be 5+8 ($S \rightarrow S + S$)

Left Recursion

Why it happens?

In the rule $S \rightarrow S + S$:

- S itself appears on the **left side** of the alternative

If we still want a predictive parser

- Need to **eliminate** left recursion

Left Recursion Elimination

If we have:

- $X \rightarrow a$
- $X \rightarrow Xb$

Then the language contains:

- $a, ab, abb, abbb, \dots$

Define an alternative CFG:

- $X \rightarrow aY$
- $Y \rightarrow bY \mid \epsilon$

Left Recursion Elimination

In general, if we have:

- $X \rightarrow a_1 \mid a_2 \mid \dots$
- $X \rightarrow Xb_1 \mid Xb_2 \mid \dots$

We will rewrite as follows:

- $X \rightarrow a_1Y \mid a_2Y \mid \dots$
- $Y \rightarrow b_1Y \mid b_2Y \mid \dots \mid \epsilon$

Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S / S$

What are our a_i, b_i ?

Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S / S$

What are our a_i, b_i ?

- $a_1 = N, a_2 = (S)$
- $b_1 = +S, b_2 = -S, b_3 = * S, b_4 = /S$

Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S / S$

The resulting CFG:

- $S \rightarrow NT \mid (S)T$
- $T \rightarrow +ST \mid -ST \mid *ST \mid /ST \mid \epsilon$

LL(1)

Definitions:

- A grammar that has a predictive parser is called LL(1)
- A language that has LL(1) grammar is called LL(1)

CFG vs Language

- A language may have more than one CFG
- We might have a language which 2 CFG's where:
 - One of them is LL(1)
 - The other one isn't...

Derivation Tree

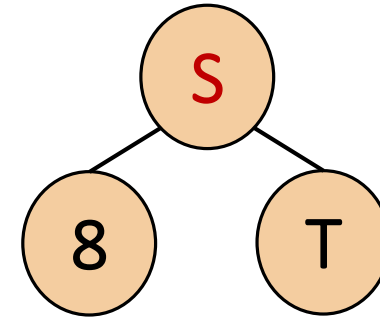
Which rules are applied for the expression $8 * 4 + 3$?

- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow * ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$

Derivation Tree

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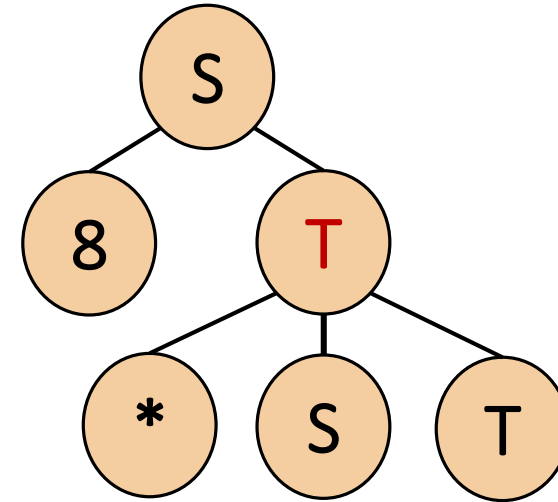
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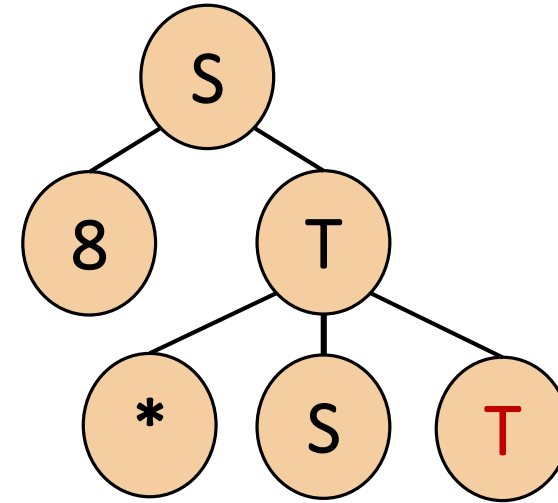
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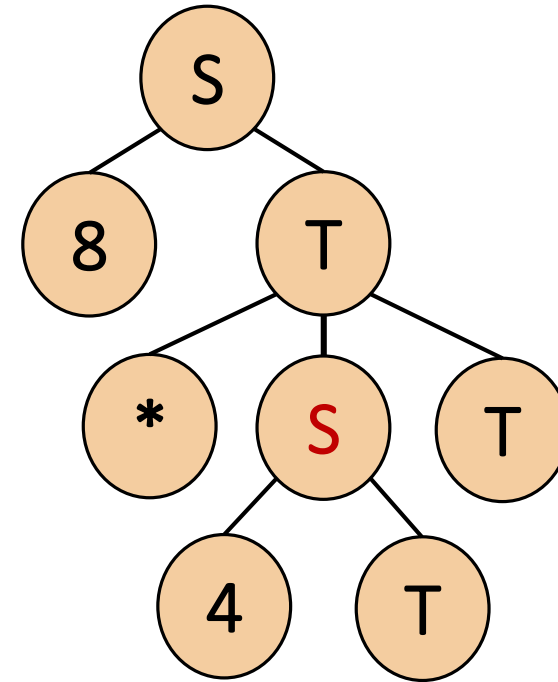
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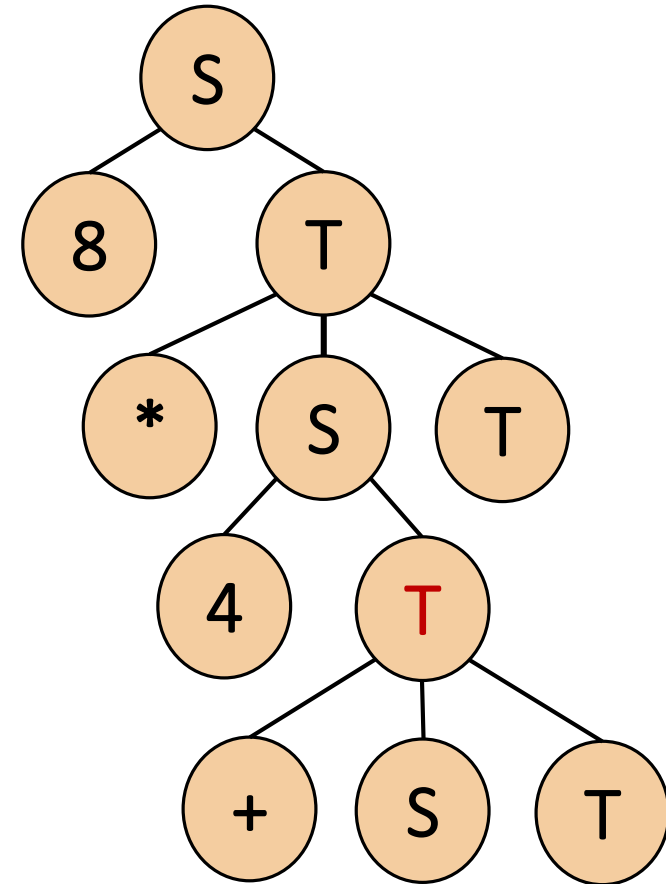
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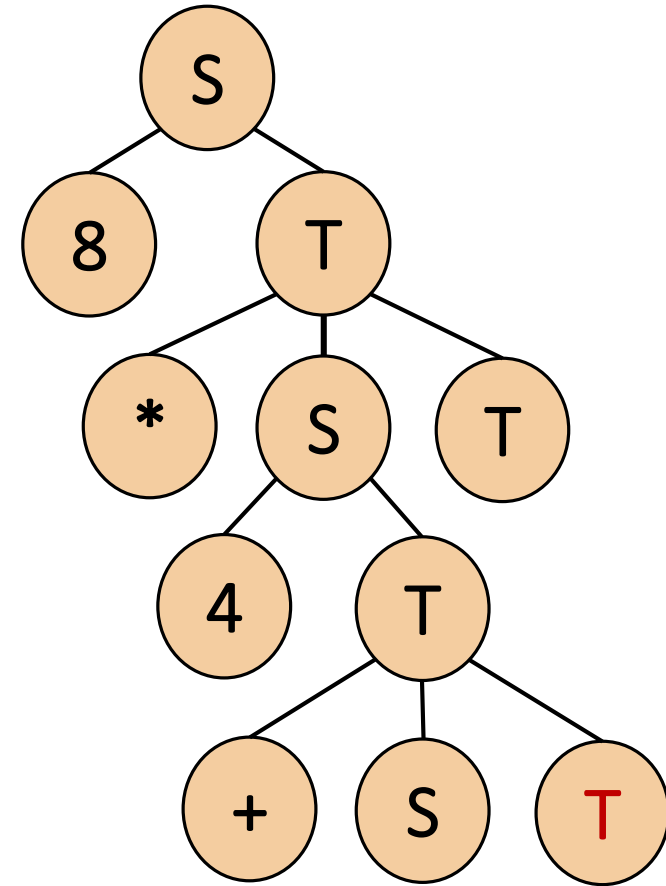
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Derivation Tree

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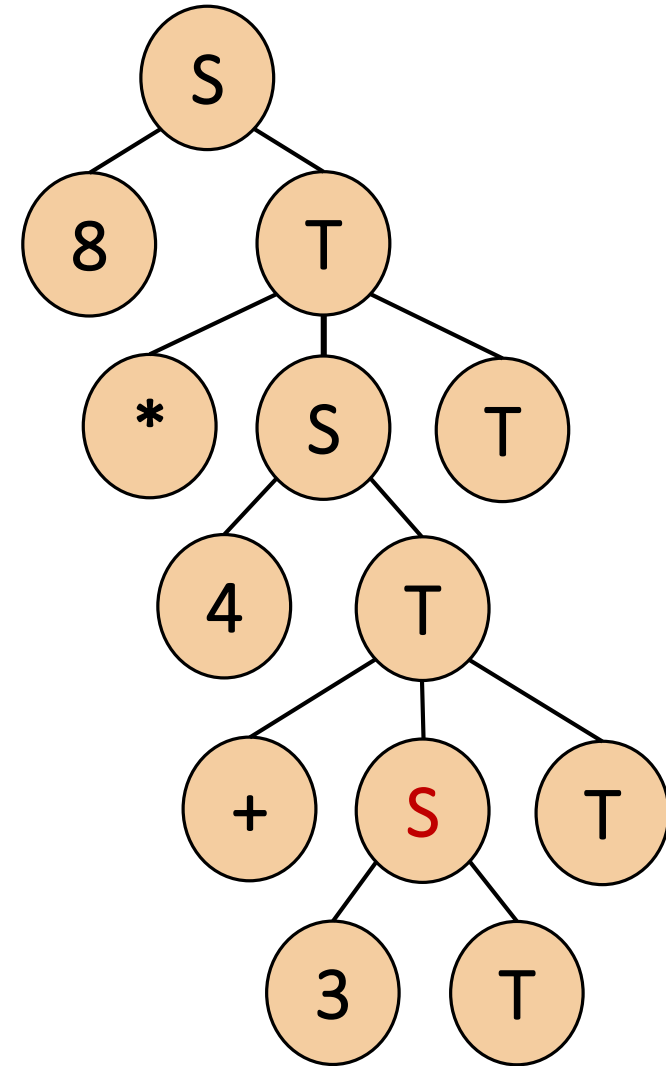
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Derivation Tree

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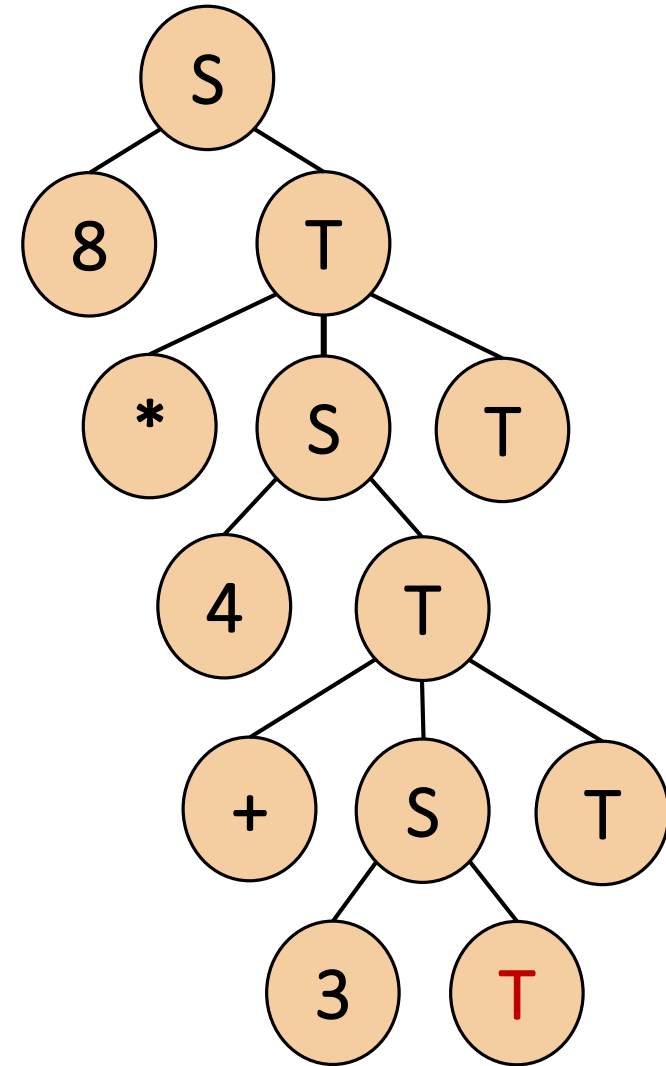
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Derivation Tree

Which rules are applied for the expression $8 * 4 + 3$?

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- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow *ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



Operator Precedence

Our CFG does not contain information about **operator precedence**!

- The expression $8 * 4 + 3$ is interpreted as $8 * (4 + 3)$
- We need to find another grammar...

Operator Precedence

A CFG with operator precedence:

- $S \rightarrow S + T \mid S - T \mid T$
- $T \rightarrow T * F \mid T / F \mid F$
- $F \rightarrow N \mid (S)$

Operator Precedence

A CFG with **operator precedence**:

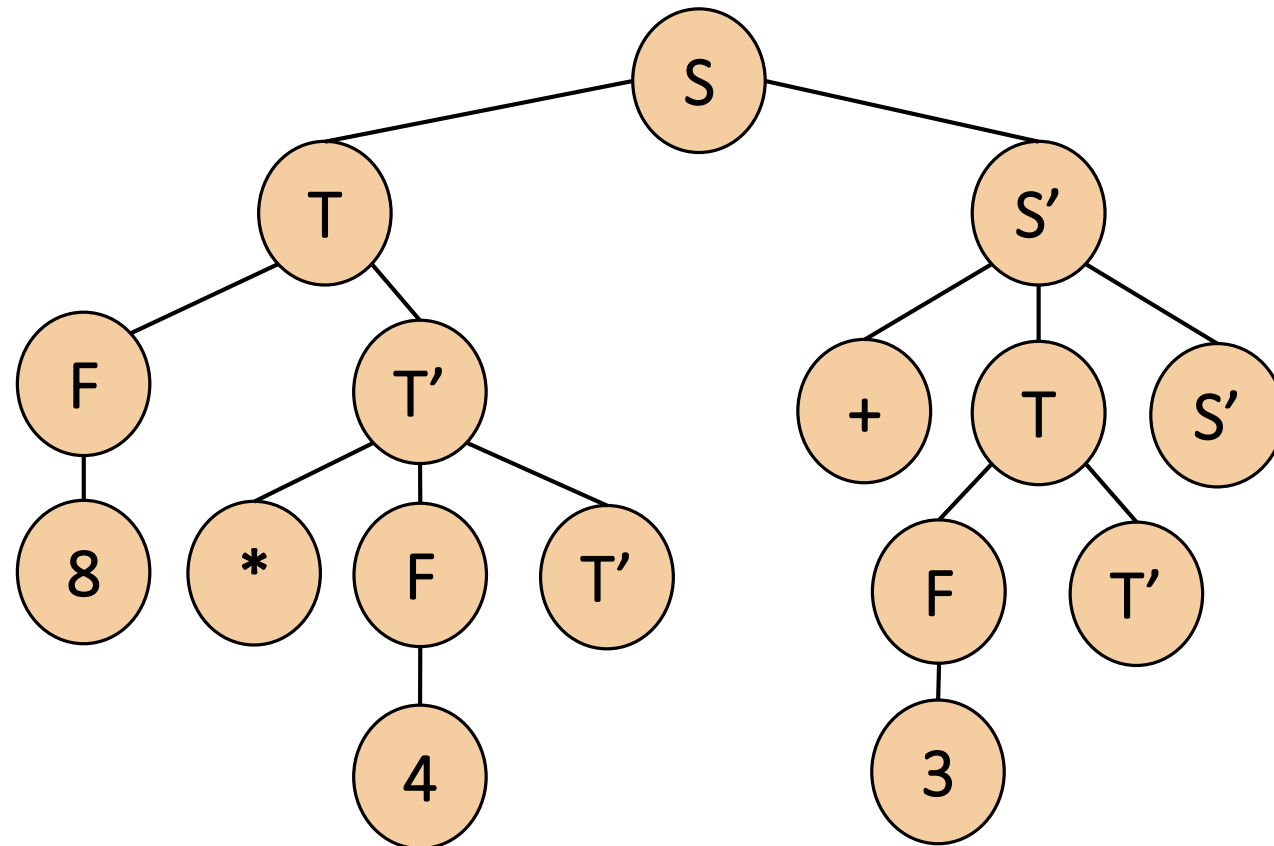
- $S \rightarrow S + T \mid S - T \mid T$
- $T \rightarrow T * F \mid T / F \mid F$
- $F \rightarrow N \mid (S)$

After eliminating **left recursion**:

- $S \rightarrow TS'$
- $S' \rightarrow +TS' \mid -TS' \mid \epsilon$
- $T \rightarrow FT'$
- $T' \rightarrow *FT' \mid /FT' \mid \epsilon$
- $F \rightarrow N \mid (S)$

Derivation Tree

With the new CFG, the derivation tree for $8 * 4 + 3$:



Left Factoring

Left recursion was an issue, are there other issues?

What about the following grammar:

- $E \rightarrow \text{if } (E) \text{ then } E$
- $E \rightarrow \text{if } (E) \text{ then } E \text{ else } E$
- $E \rightarrow \text{int}$

Left Factoring

Rewrite the original CFG:

- $E \rightarrow \text{if } (E) \text{ then } E$
- $E \rightarrow \text{if } (E) \text{ then } E \text{ else } E$
- $E \rightarrow \text{int}$

To the following:

- $E \rightarrow \text{if } (E) \text{ then } EX$
- $X \rightarrow \epsilon$
- $X \rightarrow \text{else } E$
- $E \rightarrow \text{int}$

Nullable Rules

Consider the following grammar:

- $S \rightarrow Tab$
- $T \rightarrow a \mid \epsilon$

No left recursion, no left factoring...

But can we build a predictive parser for it?

Nullable Rules

Consider the following grammar:

- $S \rightarrow Tab$
- $T \rightarrow a \mid \epsilon$

No left recursion, no left factoring...

But can we build a predictive parser for it?

No!

Nullable Rules

Consider the following grammar:

- $S \rightarrow Tab$
- $T \rightarrow a \mid \epsilon$

If the first symbol is a , we can't predict the right rule:

- If we choose $A \rightarrow a$, then it will fail to parse the input ab
- If we choose $A \rightarrow \epsilon$, then it will fail to parse the input aab

Nullable Rules

We can substitute T with it's possible alternatives.

The original grammar:

- $S \rightarrow Tab$
- $T \rightarrow a \mid \epsilon$

After substitution:

- $S \rightarrow ab$
- $S \rightarrow aab$

Are we done?

Nullable Rules

We need to perform left factoring:

- $S \rightarrow ab$
- $S \rightarrow aab$

After left factoring:

- $S \rightarrow aX$
- $X \rightarrow b \mid ab$

Building an LL(1) Parser

Some of the common issues:

- Left recursion
- Left factoring
- Nullable rules

LL(1) Parsing is not always possible

The following grammar can't be fixed:

- $S \rightarrow A$
- $S \rightarrow B$
- $A \rightarrow aAb$
- $A \rightarrow \epsilon$
- $B \rightarrow aBbb$
- $B \rightarrow \epsilon$

LL(1) Parsing: is it always desirable?

Grammars of real languages are overloaded with

- Left recursion
- Left factoring
- Nullable rules

Even if we can fix it, the resulting grammar may be unreadable...