

# Top Down Parsing

---

TEACHING ASSISTANT: DAVID TRABISH

# Examples

```
void f(int a) {  
    (((8))) ;  
}
```

# Examples

```
void f(int a) {  
    (((8))) ;  
}
```

Valid

# Examples

```
void f(int a) {  
    (((8);  
}
```

# Examples

```
void f(int a) {  
    (((8);  
}
```

Invalid

# Examples

```
void f(int a) {  
    5; ; ; ;  
}
```

# Examples

```
void f(int a) {  
    5;;;;;  
}
```

Valid

# Examples

```
void f() {  
    int a[];  
}
```



# Examples

```
void f() {  
    int a[];  
}
```

Invalid

# Examples

```
void f() {  
    int a[10.0];  
}
```

# Examples

```
void f() {  
    int a[10.0];  
}
```

Invalid

# Examples

```
void f(int a[]) {  
  
}
```

# Examples

```
void f(int a[]) {  
}
```

Valid

# Examples

```
void f() {  
    int i = 0;  
    i = 8;  
    int j = i;  
}
```

# Examples

```
void f() {  
    int i = 0;  
    i = 8;  
    int j = i;  
}
```

Valid

# Examples

```
void f() {  
    int i = 0;  
    i = 8;  
    int j = z;  
}
```



# Examples

```
void f() {  
    int i = 0;  
    i = 8;  
    int j = z;  
}
```

Valid

# Examples

```
void f() {  
    int i = 0;  
    i = 8;  
    int j = i  
}
```

# Examples

```
void f() {  
    int i = 0;  
    i = 8;  
    int j = i  
}
```

Invalid

# Language of Balanced Parentheses

Contains string of the form:

- 8, (1), (((03))), ...

Disallowing:

- ((1), 8()

# Language of Balanced Parentheses

Contains string of the form:

- 8, (1), (((03))), ...

Disallowing:

- ((1), 8()

Is there a DFA/NFA that accepts the language?

Is there a regular expression the accepts the language?

# Language of Balanced Parentheses

The language is **not regular**

- There is no DFA that accepts it

Proof:

- If it has a DFA, then we have  $d$  states
- Consider the input  $((\dots))$  that has  $d + 1$  left parentheses
- Every time we read  $($ , we need to change to a new state
  - We need to act differently if we saw 4 parentheses or 10
- But we have only  $d$  states...

# Context Free Grammar

- A set of terminals  $T$  and a set of non-terminals  $V$
- Production rules of the form
  - $A \rightarrow a_1 a_2 \dots a_n$
  - $A \in V, a_i \in T \cup V$
- Starting symbol  $S$  :
  - $S \rightarrow a_1 a_2 \dots a_n$

# Context Free Grammar

Example:

- $S \rightarrow c$
- $S \rightarrow aSb$

Which words belong to this grammar?



# Context Free Grammar

Example:

- $S \rightarrow c$
- $S \rightarrow aSb$

Which words belong to this grammar?

- $c, acb, aacbb, aaacbbb, \dots$

# Context Free Grammar

Does the language of **balanced parentheses** have a CFG?

# Context Free Grammar

Does the language of **balanced parentheses** have a CFG?

- $S \rightarrow INT$
- $S \rightarrow (S)$

# Context Free Grammar: Questions

Are there languages which **have no CFG**?

Can we have **multiple** CFG's describing the same language?

# Context Free Grammar: Questions

Are there languages which **have no CFG?** **Yes**

Can we have **multiple** CFG's describing the same language?

# Context Free Grammar: Questions

Are there languages which **have no CFG**? **Yes**

Can we have **multiple** CFG's describing the same language? **Yes**

# Predictive Parser: Definition

Some languages has a predictive parser:

- We determine the production rule according to the current token
- We begin we the start symbol
  - From the top...

# Predictive Parser: Example

The language of balanced parentheses:

- $S \rightarrow INT$
- $S \rightarrow (S)$

**has** a predictive parser.



# Predictive Parser: Example

```
void parse_S() {  
    switch (token) {  
        case INT:  
            parse_token(INT);  
            break;  
        case L_PAREN:  
            parse_token(L_PAREN);  
            parse_S();  
            parse_token(R_PAREN);  
            break;  
        default:  
            // error  
    }  
}
```

```
void parse_token(int expected) {  
    if (token == expected) {  
        token = lexer.next_token();  
    } else {  
        // error  
    }  
}
```

# Predictive Parser: Example

What happens for the input (7)?

Call trace:

- parse\_S
  - parse\_token // match with '('
  - parse\_S
    - parse\_token // match with '7'
  - parse\_token // match with ')'

# Predictive Parser: Example

What happens for the input ((7)?

Call trace:

- parse\_S
  - parse\_token // match with '('
  - parse\_S
    - parse\_token // match with '('
    - parse\_S
      - parse\_token // match with '7'
      - parse\_token // match with ')'
    - parse\_token // error, expecting ')'

# Language of Balanced Parentheses 2

Find a CFG for a language with the 3 kinds of parentheses:

- `()`, `[]`, `{}`

Contains string of the form:

- `(([][]){}))[]`
- `[()]`

Not allowing:

- `((())){`

# Language of Balanced Parentheses 2

CFG definition:

- $S \rightarrow (S)S$
- $S \rightarrow [S]S$
- $S \rightarrow \{S\}S$
- $S \rightarrow \epsilon$

# Language of Balanced Parentheses 2

```
void parse_S() {  
    switch (token) {  
        case L_PAREN:  
            parse_S1();  
            break;  
        case L_BRACKET:  
            parse_S2();  
            break;  
        case L_BRACE:  
            parse_S3();  
            break;  
        default:  
            break;  
    }  
}
```

```
void parse_S1() {  
    parse_token(L_PAREN);  
    parse_S();  
    parse_token(R_PAREN);  
    parse_S();  
}  
void parse_S2() {  
    parse_token(L_BRACKET);  
    parse_S();  
    parse_token(R_BRACKET);  
    parse_S();  
}  
void parse_S3() {  
    parse_token(L_BRACE);  
    parse_S();  
    parse_token(R_BRACE);  
    parse_S();  
}
```

# Calculator Language

A language with binary operators (+, -, \*, /) and numbers:

- 1
- 1+1
- $(1+1)*(7/2)$
- 2+1-7

# Calculator Language

A (possible) CFG for that language:

- $S \rightarrow N$
- $S \rightarrow S + S$
- $S \rightarrow S - S$
- $S \rightarrow S * S$
- $S \rightarrow S / S$
- $S \rightarrow (S)$



# Calculator Language

A (possible) CFG for that language:

- $S \rightarrow N$
- $S \rightarrow S + S$
- $S \rightarrow S - S$
- $S \rightarrow S * S$
- $S \rightarrow S / S$
- $S \rightarrow (S)$

Will predictive parsing work here?

# Left Recursion

There is no predictive parser which can handle the previous CFG

Why?

- If the first token was 5, we can't predict the right rule
- It can be 5 ( $S \rightarrow N$ )
- But also can be 5+8 ( $S \rightarrow S + S$ )

# Left Recursion

Why it happens?

In the rule  $S \rightarrow S + S$ :

- $S$  itself appears on the **left side** of the alternative

If we still want a predictive parser

- Need to **eliminate** left recursion

# Left Recursion Elimination

If we have:

- $X \rightarrow a$
- $X \rightarrow Xb$

Then the language contains:

- $a, ab, abb, abbb, \dots$

Define an alternative CFG:

- $X \rightarrow aY$
- $Y \rightarrow bY \mid \epsilon$

# Left Recursion Elimination

In general, if we have:

- $X \rightarrow a_1 \mid a_2 \mid \dots$
- $X \rightarrow Xb_1 \mid Xb_2 \mid \dots$

We will rewrite as follows:

- $X \rightarrow a_1Y \mid a_2Y \mid \dots$
- $Y \rightarrow b_1Y \mid b_2Y \mid \dots \mid \epsilon$

# Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S / S$

What are our  $a_i, b_i$  ?

# Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S / S$

What are our  $a_i, b_i$  ?

- $a_1 = N, a_2 = (S)$
- $b_1 = +S, b_2 = -S, b_3 = * S, b_4 = /S$

# Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S / S$

The resulting CFG:

- $S \rightarrow NT \mid (S)T$
- $T \rightarrow +ST \mid -ST \mid *ST \mid /ST \mid \epsilon$



# CFG vs Language

- A language may have more than one CFG
- We might have a language which 2 CFG's where:
  - One has a predictive parser
  - The other one doesn't...

# LL(1)

Definitions:

- A grammar that has a predictive parser is called LL(1)
- A language that has LL(1) grammar is called LL(1)

# Derivation Tree

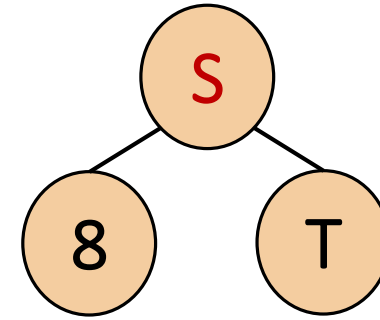
Which rules are applied for the expression  $8 * 4 + 3$ ?

- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow * ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$

# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

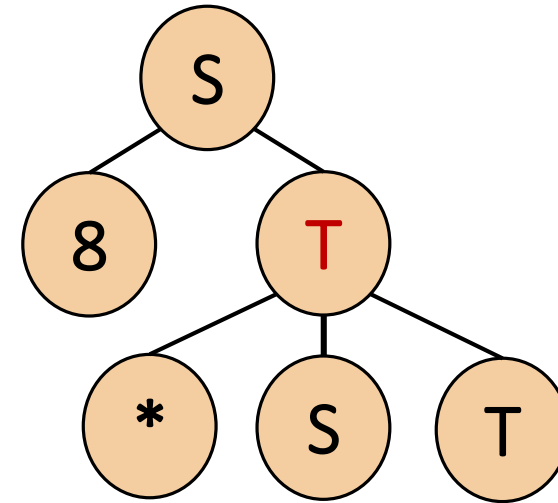
- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow * ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

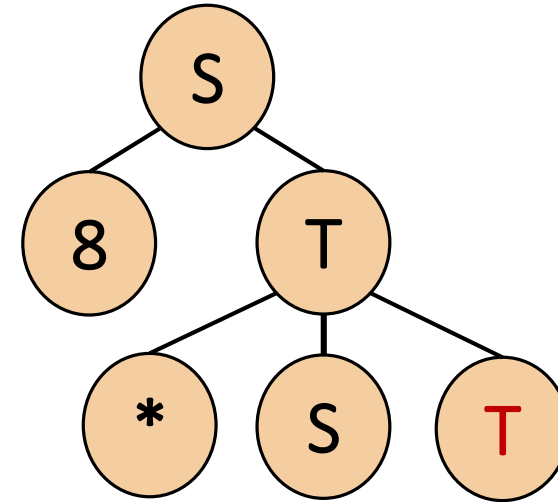
- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow * ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

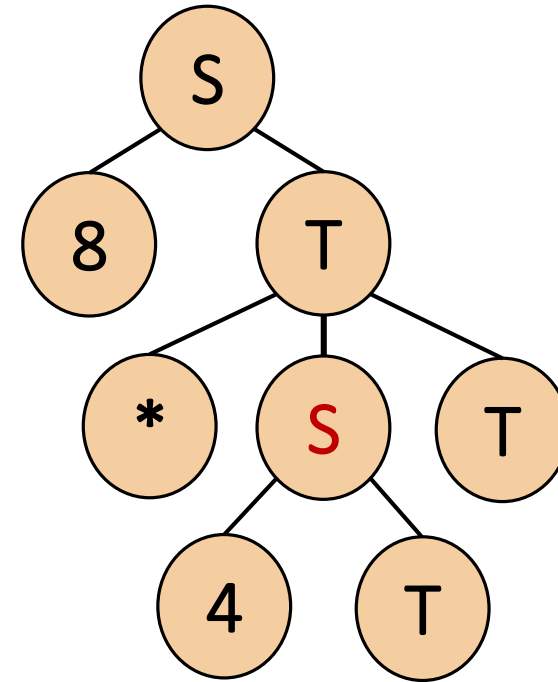
- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow * ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

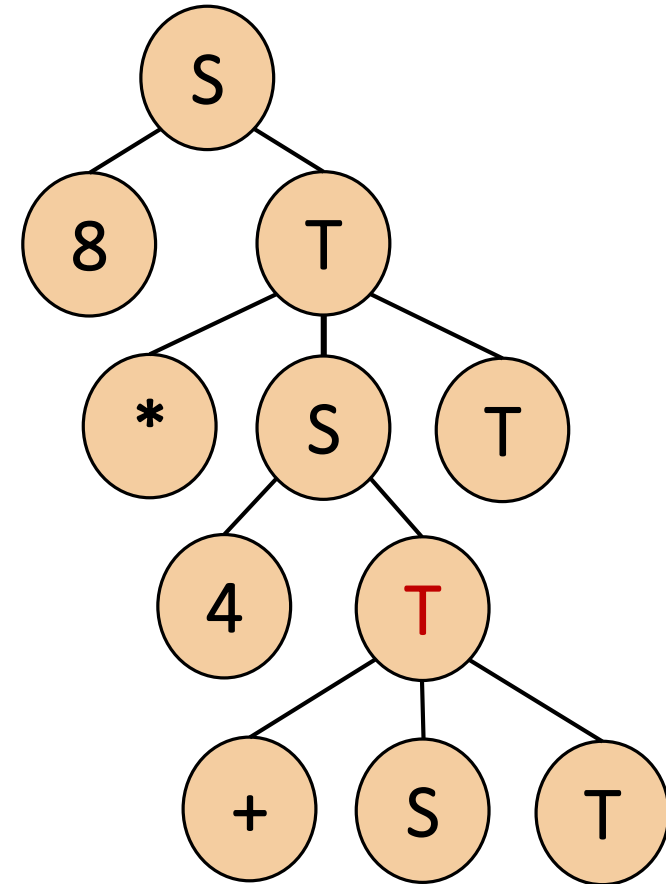
- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow * ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow *ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$

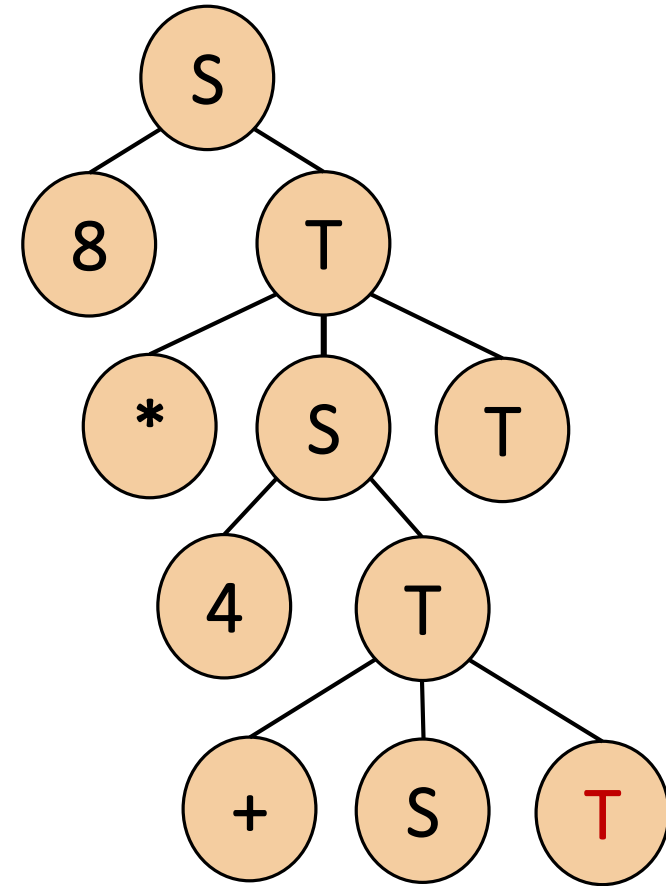




# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

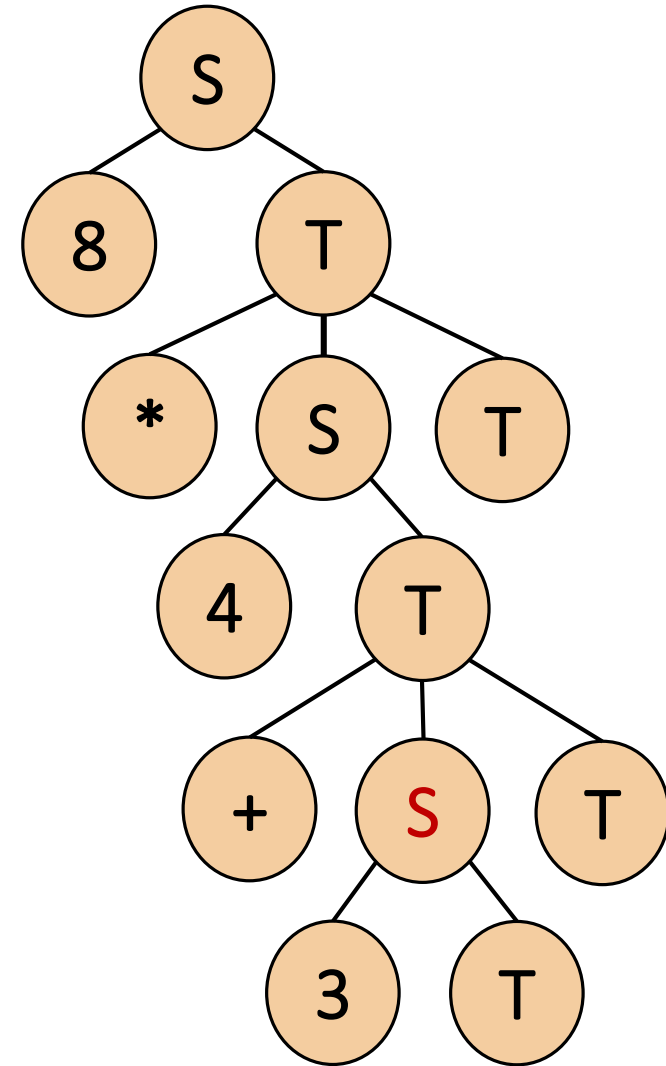
- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow *ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

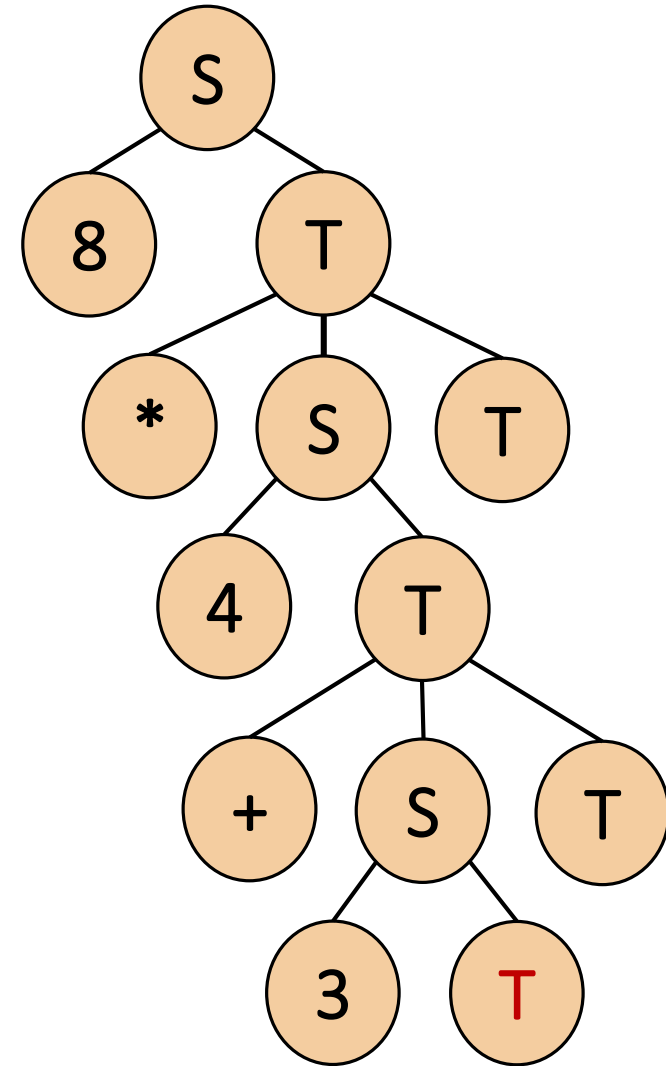
- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow *ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Derivation Tree

Which rules are applied for the expression  $8 * 4 + 3$ ?

- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow *ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$



# Operator Precedence

Our CFG does not contain information about **operator precedence**!

- The expression  $8 * 4 + 3$  is interpreted as  $8 * (4 + 3)$
- We need to find another grammar...

# Operator Precedence

A CFG with operator precedence:

- $S \rightarrow S + T \mid S - T \mid T$
- $T \rightarrow T * F \mid T / F \mid F$
- $F \rightarrow N \mid (S)$

# Operator Precedence

A CFG with **operator precedence**:

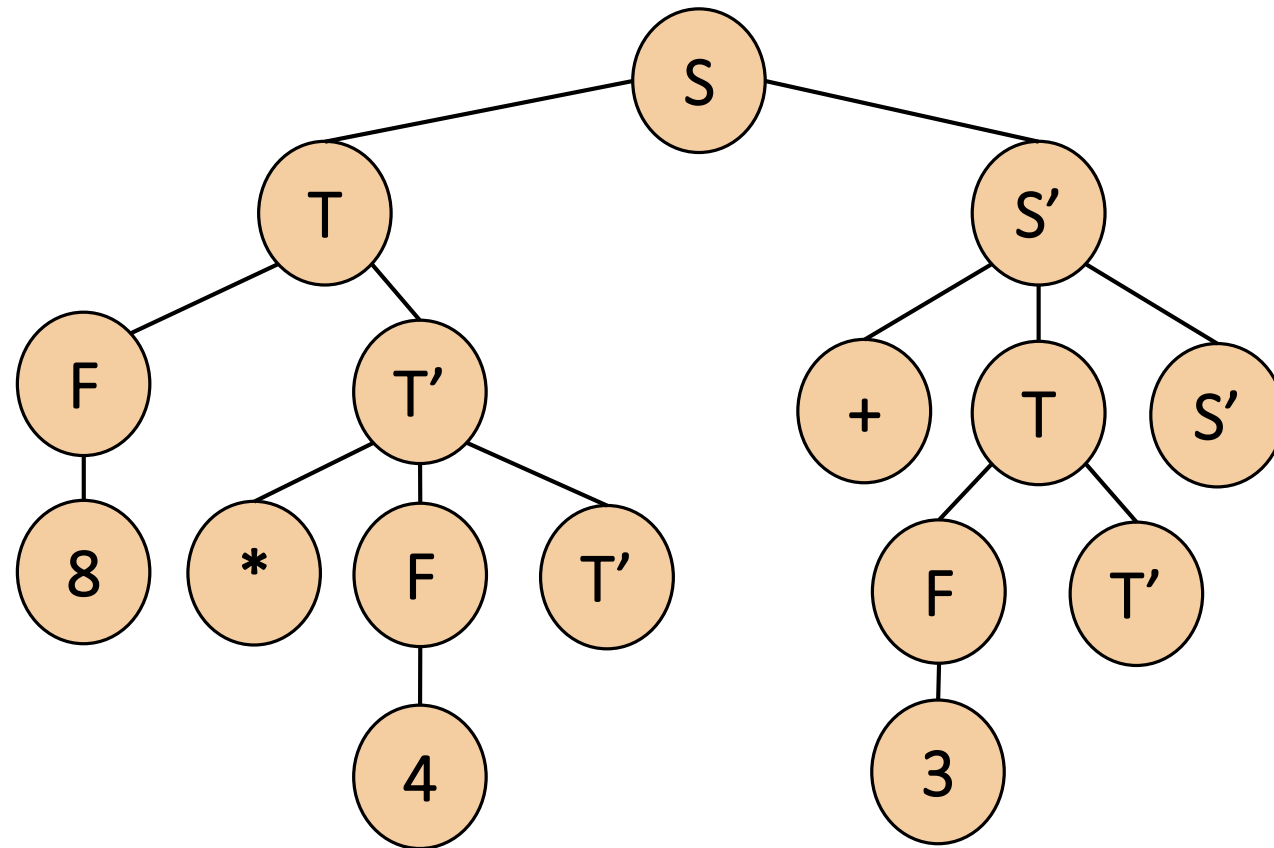
- $S \rightarrow S + T \mid S - T \mid T$
- $T \rightarrow T * F \mid T / F \mid F$
- $F \rightarrow N \mid (S)$

After eliminating **left recursion**:

- $S \rightarrow TS'$
- $S' \rightarrow +TS' \mid -TS' \mid \epsilon$
- $T \rightarrow FT'$
- $T' \rightarrow *FT' \mid /FT' \mid \epsilon$
- $F \rightarrow N \mid (S)$

# Derivation Tree

With the new CFG, the derivation tree for  $8 * 4 + 3$ :



# Left Factoring

Left recursion was an issue, are there other issues?

What about the following grammar:

- $E \rightarrow \text{if } (E) \text{ then } E$
- $E \rightarrow \text{if } (E) \text{ then } E \text{ else } E$
- $E \rightarrow \text{int}$



# Left Factoring

Rewrite the original CFG:

- $E \rightarrow \text{if } (E) \text{ then } E$
- $E \rightarrow \text{if } (E) \text{ then } E \text{ else } E$
- $E \rightarrow \text{int}$

To the following:

- $E \rightarrow \text{if } (E) \text{ then } EX$
- $X \rightarrow \epsilon$
- $X \rightarrow \text{else } E$
- $E \rightarrow \text{int}$

# Nullable Rules

Consider the following grammar:

- $S \rightarrow Aab$
- $A \rightarrow a$
- $A \rightarrow \epsilon$

No left recursion, no left factoring...

But can we build a predictive parser for it?

# Nullable Rules

Consider the following grammar:

- $S \rightarrow Aab$
- $A \rightarrow a$
- $A \rightarrow \epsilon$

No left recursion, no left factoring...

But can we build a predictive parser for it?

**No!**

# Nullable Rules

Consider the following grammar:

- $S \rightarrow Aab$
- $A \rightarrow a \mid \epsilon$

If the first symbol is  $a$ , we can't predict the right rule:

- If we choose  $A \rightarrow a$ , then it will fail to parse the input  $ab$
- If we choose  $A \rightarrow \epsilon$ , then it will fail to parse the input  $aab$

# Nullable Rules

We can substitute  $A$  with its possible alternatives.

The original grammar:

- $S \rightarrow Aab$
- $A \rightarrow a \mid \epsilon$

After substitution:

- $S \rightarrow ab$
- $S \rightarrow aab$

Are we done?

# Nullable Rules

We need to perform left factoring:

- $S \rightarrow ab$
- $S \rightarrow aab$

After left factoring:

- $S \rightarrow aX$
- $X \rightarrow b \mid ab$

# Building an LL(1) Parser

Some of the common issues:

- Left recursion
- Left factoring
- Nullable rules

# LL(1) Parsing is not always possible

The following grammar can't be fixed:

- $S \rightarrow A$
- $S \rightarrow B$
- $A \rightarrow aAb$
- $A \rightarrow \epsilon$
- $B \rightarrow aBbb$
- $B \rightarrow \epsilon$



# LL(1) Parsing: is it always desirable?

Grammars of real languages are overloaded with

- Left recursion
- Left factoring
- Nullable rules

Even if we can fix it, the resulting grammar may be unreadable...