Top Down Parsing

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Examples

```
void f(int a) {
     ((((8))));
}
```

Examples

```
void f(int a) {
     ((((8))));
}
```

Valid

Examples

```
void f(int a) {
      ((((8);
}
```

Examples

```
void f(int a) {
      ((((8);
}
```

Invalid

Examples

```
void f(int a) {
     5;;;;;
}
```

Examples

```
void f(int a) {
     5;;;;;
}
```

Valid

Examples

```
void f() {
        int a[];
}
```

Examples

```
void f() {
        int a[];
}
```

Invalid

Examples

```
void f() {
        int a[10.0];
}
```

Examples

```
void f() {
        int a[10.0];
}
```

Invalid

Examples

```
void f(int a[]) {

}
```

Examples

```
void f(int a[]) {

}
```

Valid

Examples

```
void f() {
        int i = 0;
        i = 8;
        int j = i;
}
```

Examples

```
void f() {
      int i = 0;
      i = 8;
      int j = i;
}
```

Valid

Examples

```
void f() {
        int i = 0;
        i = 8;
        int j = z;
}
```

Examples

```
void f() {
        int i = 0;
        i = 8;
        int j = z;
}
```

Valid

Examples

```
void f() {
        int i = 0;
        i = 8;
        int j = i
}
```

Examples

```
void f() {
        int i = 0;
        i = 8;
        int j = i
}
```

Invalid

Language of Balanced Parentheses

Contains string of the form:

- 8, (1), (((03))), …

Not allowing:

- ((1), 8()

Language of Balanced Parentheses

Contains string of the form:

- 8, (1), (((03))), …

Not allowing:

- ((1), 8()

Is there a DFA/NFA that accepts the language?

Is there a regular expression the accepts the language?

Language of Balanced Parentheses

The language is **not regular**

- There is no DFA that accepts it

Proof:

- If it has a DFA, the we have d states
- Consider the input $((\ldots 77$ that has $d+1$ left parentheses
- Every time we read (, we need to change to a new state
  - We need to act differently if we saw 4 parentheses or 10
- But we have only d states…

Context Free Grammar

- A set of terminals T and a set of non-terminals V
- Production rules of the form
  - $A \to a_1 a_2 \dots a_n$
  - $A \in V, a_i \in T \cup V$
- Starting symbol S :
  - $S \to a_1 a_2 \dots a_n$

Context Free Grammar

Example:

- $S \rightarrow c$
- $S \rightarrow aSb$

Which words belong to this grammar?

Context Free Grammar

Example:

- $S \rightarrow c$
- $S \rightarrow aSb$

Which words belong to this grammar?

- $c, acb, aacbb, aaacbbb, \dots$

Context Free Grammar

Does the language of **balanced parentheses** have a CFG?

Context Free Grammar

Does the language of **balanced parentheses** have a CFG?

- $S \rightarrow INT$
- $S \rightarrow (S)$

Context Free Grammar: Questions

Are there languages which **have no CFG**? Yes

Can we have **multiple** CFG’s describing the same language? Yes

Predictive Parser: Definition

Some languages has a predictive parser:

- We determine the production rule according to the current token
- We begin we the start symbol
  - From the top…

Predictive Parser: Example

The language of balanced parentheses:

- $S \rightarrow INT$
- $S \rightarrow (S)$

has a predictive parser.

Predictive Parser: Example

```
void parse_S() {
  switch (token) {
  case INT:
    parse_token(INT);
    break;
  case L_PAREN:
    parse_token(L_PAREN);
    parse_S();
    parse_token(R_PAREN);
    break
  default:
    // error
  }
}
```

```
void parse_token(int expected) {
  if (token == expected) {
     token = lexer.next_token();
  } else {
    // error
  }
}
```

Predictive Parser: Example

What happens for the input (7)?

Call trace:

- parse_S
  - parse_token // match with '('
  - parse_S
    - parse_token // match with '7'
  - parse_token // match with ')'

Predictive Parser: Example

What happens for the input ((7)?

Call trace:

- parse_S
  - parse_token // match with ‘(‘
  - parse_S
    - parse_token // match with ‘(‘
    - parse_S
      - parse_token // match with ‘7’
    - parse_token // match with ‘)’
  - parse_token // error, expecting ‘)’

Language of Balanced Parentheses 2

Find a CFG for a language with the 3 kinds of parentheses:

- (), [], {}

Contains string of the form:

- (([][]{}))[]
- [()]

Not allowing:

- (()){

Language of Balanced Parentheses 2

CFG definition:

- $S \rightarrow (S)S$
- $S \rightarrow [S]S$
- $S \rightarrow \{S\}S$
- $S \rightarrow \epsilon$

Language of Balanced Parentheses 2

```
void parse_S() {
  switch (token) {
  case L_PAREN:
    parse_S1();
    break;
  case L_BRACKET:
    parse_S2();
    break;
  case L_BRACE:
    parse_S3();
    break;
  default:
    break;
  }
}
```

```
void parse_S1() {
  parse_token(L_PAREN);
  parse_S();
  parse_token(R_PAREN);
  parse_S();
}
void parse_S2() {
  parse_token(L_BRACKET);
  parse_S();
  parse_token(R_BRACKET);
  parse_S();
}
void parse_S3() {
  parse_token(L_BRACE);
  parse_S();
  parse_token(R_BRACE);
  parse_S();
}
```

Calculator Language

A language with binary operators (+,-,*,/) and numbers:

- 1
- 1+1
- (1+1)*(7/2)
- 2+1-7

Calculator Language

A (possible) CFG for that language:

- $S \to N$
- $S \to S + S$
- $S \to S - S$
- $S \to S * S$
- $S \to S \,/\, S$
- $S \to (S)$

Calculator Language

A (possible) CFG for that language:

- $S \rightarrow N$
- $S \rightarrow S + S$
- $S \rightarrow S - S$
- $S \rightarrow S * S$
- $S \rightarrow S \mathbin{/} S$
- $S \rightarrow (S)$

Will predictive parsing work here?

Left Recursion

There is no predictive parser which can handle the previous CFG

Why?

- If the first token was 5, we can’t predict the right rule
- It can be 5 ($S \rightarrow N$)
- But also can be 5+8 ($S \rightarrow S + S$)

Left Recursion

Why it happens?

In the rule $S \to S + S$:

- S itself appears on the **left side** of the alternative

If we still want a predictive parser

- Need to **eliminate** left recursion

Left Recursion Elimination

If we have:

- $X \to a$
- $X \to Xb$

Then the language contains:

- $a, ab, abb, abbb, \ldots$

Define an alternative CFG:

- $X \to aY$
- $Y \to bY \mid \epsilon$

Left Recursion Elimination

In general, if we have:

- $X \to a_1 \mid a_2 \mid \ldots$
- $X \to Xb_1 \mid Xb_2 \mid \ \ldots$

We will rewrite as follows:

- $X \to a_1Y|a_2Y| \ldots$
- $Y \to b_1Y|b_2Y| \ldots | \, \epsilon$

Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S \ * S \mid S \ / \ S$

What are our a_i, b_i ?

Calculator Language

Before left recursion elimination:

- $S \rightarrow N$
- $S \rightarrow (S) \mid S + S \mid S - S \mid S * S \mid S \,/\, S$

What are our a_i, b_i ?

- $a_1 = N, a_2 = (S)$
- $b_1 = +S, b_2 = -S, b_3 = * \, S, b_4 = /S$

Calculator Language

Before left recursion elimination:

- $S \to N$
- $S \to (S) \mid S + S \mid S - S \mid S * S \mid S \,/\, S$

The resulting CFG:

- $S \to NT \mid (S)T$
- $T \to +ST \mid -ST \mid * \, ST \mid /ST \mid \epsilon$

CFG vs Language

- A language may hove more the one CFG
- We might have a language which 2 CFG’s where:
  - One has a predictive parser
  - The other one doesn’t…

LL(1)

Definitions:

- A grammar that has a predictive parser is called LL(1)
- A language that has LL(1) grammar is called LL(1)

Derivation Tree

Which rules are applied for the expression 8 * 4 + 3?

- $S \rightarrow NT$
- $S \rightarrow (S)T$
- $T \rightarrow +ST$
- $T \rightarrow -ST$
- $T \rightarrow *ST$
- $T \rightarrow /ST$
- $T \rightarrow \epsilon$

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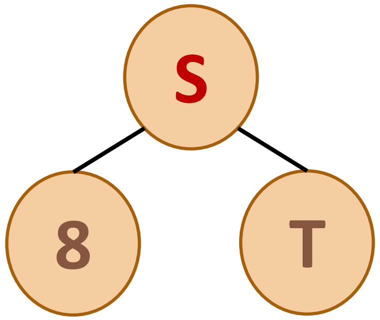

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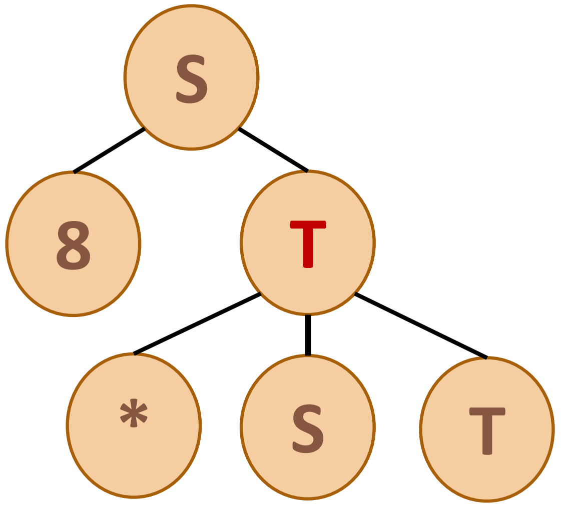

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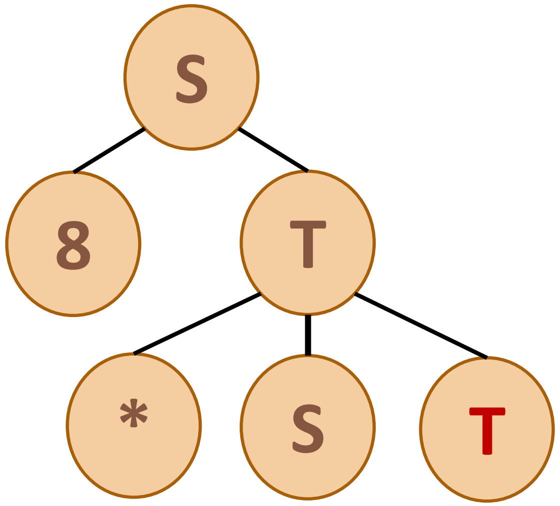

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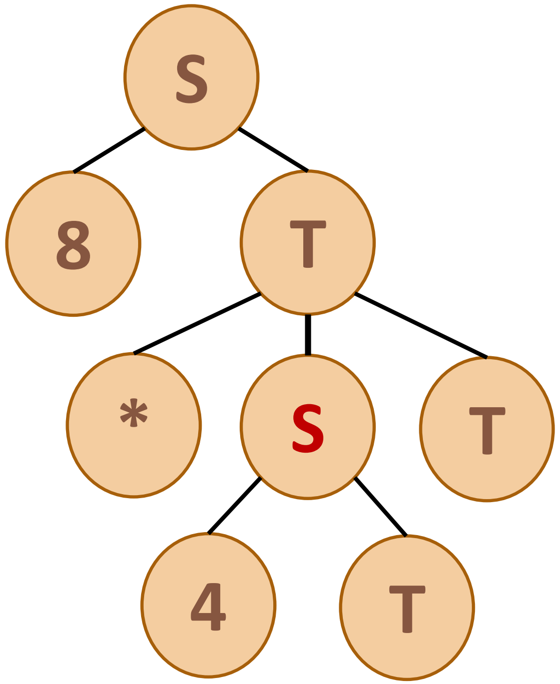

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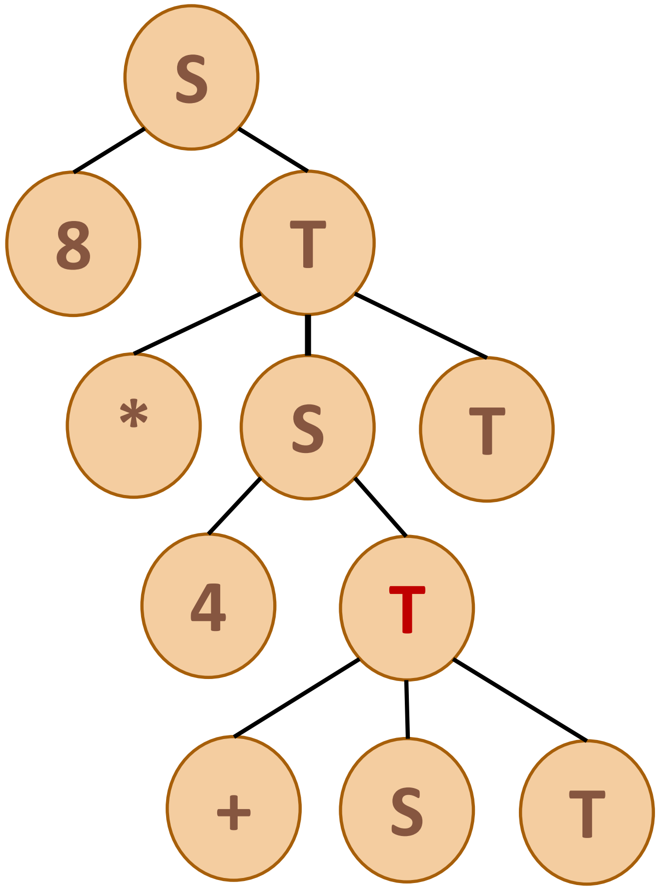

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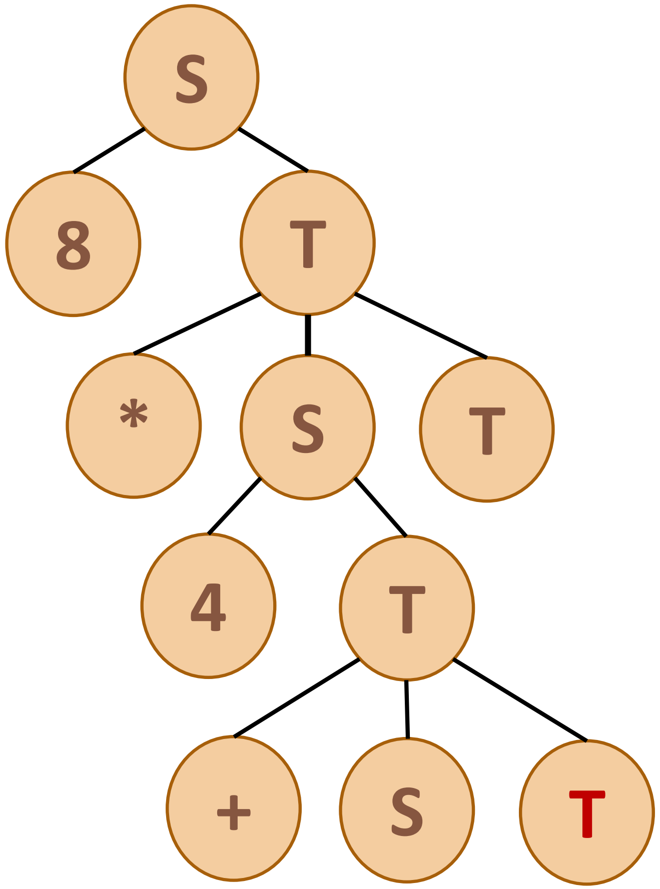

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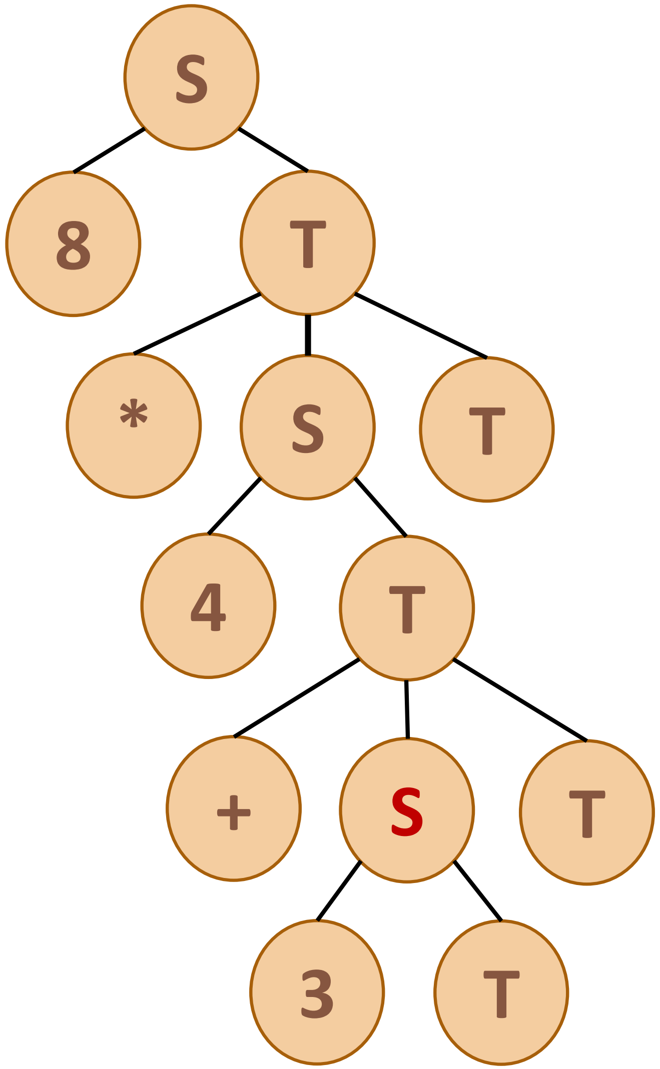

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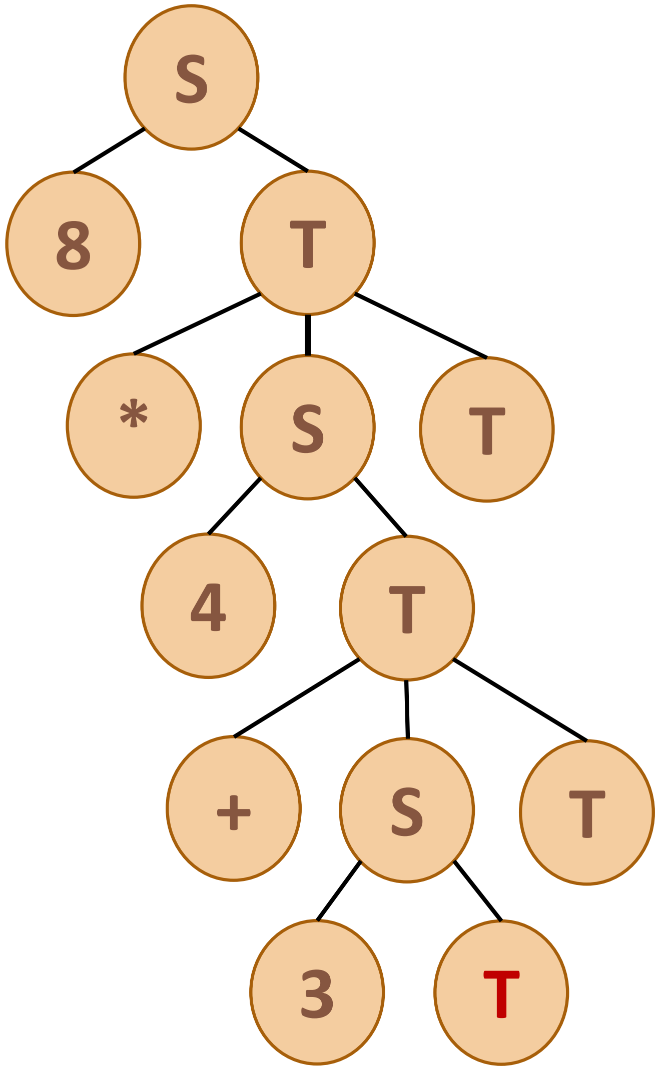


Operator Precedence

Our CFG does not contain information about **operator precedence!**

- The expression 8 * 4 + 3 is interpreted as 8 * (4 + 3)
- We need to find another grammar…

Operator Precedence

A CFG with **operator precedence:**

- $S \rightarrow S + T \mid S - T \mid T$
- $T \rightarrow T * F \mid T \,/\, F \mid F$
- $F \rightarrow N \mid (S)$

Operator Precedence

A CFG with **operator precedence:**

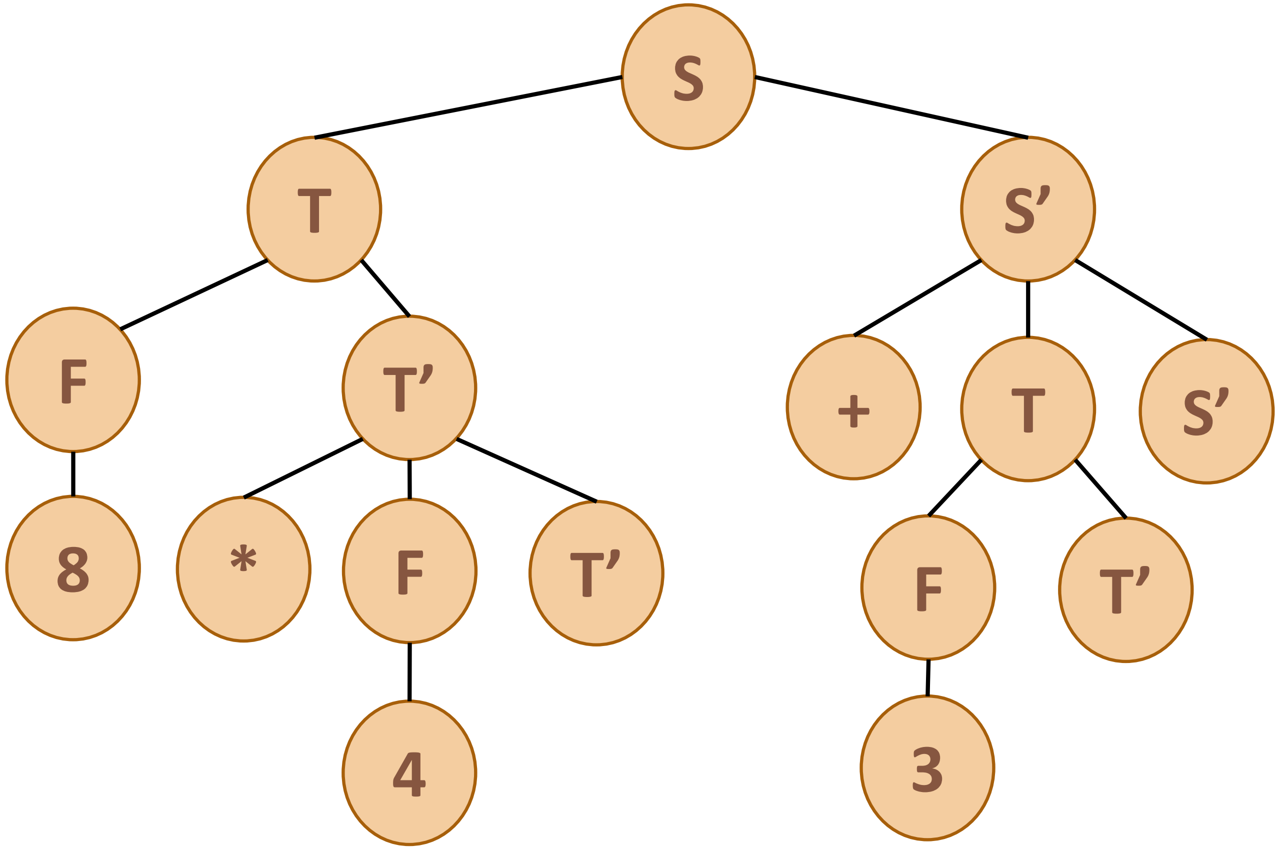
- $S \rightarrow S + T \mid S - T \mid T$
- $T \rightarrow T * F \mid T \,/\, F \mid F$
- $F \rightarrow N \mid (S)$

After eliminating **left recursion**:

- $S \rightarrow TS'$
- $S' \rightarrow +TS' \mid -TS' \mid \epsilon$
- $T \rightarrow FT'$
- $T' \rightarrow *FT' \mid /FT' \mid \epsilon$
- $F \rightarrow N \mid (S)$

Derivation Tree

With the new CFG, the derivation tree for 8 * 4 + 3:



Left Factoring

Left recursion was an issue, are there other issues?

What about the following grammar:

- $E \rightarrow if\ (E)\ then\ E$
- $E \rightarrow if\ (E)\ then\ E\ else\ E$
- $E \rightarrow int$

Left Factoring

Rewrite the original CFG:

- $E \rightarrow if\ (E)\ then\ E$
- $E \rightarrow if\ (E)\ then\ E\ else\ E$
- $E \rightarrow int$

To the following:

- $E \rightarrow if\ (E)\ then\ EX$
- $X \rightarrow \epsilon$
- $X \rightarrow else\ E$
- $E \rightarrow int$

Nullable Rules

Consider the following grammar:

- $S \rightarrow Aab$
- $A \rightarrow a$
- $A \rightarrow \epsilon$

No left recursion, no left factoring…

But can we build a predictive parser for it?

Nullable Rules

Consider the following grammar:

- $S \rightarrow Aab$
- $A \rightarrow a$
- $A \rightarrow \epsilon$

No left recursion, no left factoring...

But can we build a predictive parser for it?

No!

Nullable Rules

Consider the following grammar:

- $S \to Aab$
- $A \to a \mid \epsilon$

If the first symbol is a, we can't predict the right rule:

- If we choose $A \to a$, then it will fail to parse the input ab
- If we choose $A \to \epsilon$, then it will fail to parse the input aab

Nullable Rules

We can substitute A with it's possible alternatives.

The original grammar:

- $S \rightarrow Aab$
- $A \rightarrow a \mid \epsilon$

After substitution:

- $S \rightarrow ab$
- $S \rightarrow aab$

Are we done?

Nullable Rules

We need to perform left factoring:

- $S \rightarrow ab$
- $S \rightarrow aab$

After left factoring:

- $S \rightarrow aX$
- $X \rightarrow b \mid ab$

Building an LL(1) Parser

Some of the common issues:

- Left recursion
- Left factoring
- Nullable rules

LL(1) Parsing is not always possible

The following grammar can't be fixed:

- $S \rightarrow A$
- $S \rightarrow B$
- $A \rightarrow aAb$
- $A \rightarrow \epsilon$
- $B \rightarrow aBbb$
- $B \rightarrow \epsilon$

LL(1) Parsing: is it always desirable?

Grammars of real languages are overloaded with

- Left recursion
- Left factoring
- Nullable rules

Even if we can fix it, the resulting grammar may be unreadable…