Today's Content:

- → Modular operator
- → Modular arithmetics
- → 1 Hard problem

int range → $\{-2*10^9, 2*10^9\}$

long range → $\{-8*10^{18}, 8*10^{18}\}$

% → Modulus / Remainder

Divident = divisor * quotient + Remainder

$10 \% 4 = 2 = 4 * \{10/4\} + r = 8 + r = 10, \quad r = 2$

$13 \% 5 = 3 = 5 * \{13/5\} + r = 10 + r = 13, \quad r = 3$

$100 \% 7 = \quad = 7 * \{100/7\} + r = 98 + r = 100 \quad r = 2$

$150 \% 7 = \quad 7 * \{150/7\} + r = 147 + r = 150, \quad r = 3$

integer division

$-60 \% 9 = 9 * \{-60/9\} + r \longrightarrow r = -6$

$[0 \quad 8]$

$\Rightarrow 9 * (-6) + r = -60 \Rightarrow -54 + r = -60,$

$\{-\infty \ldots +\infty\}$ % $\{10\}$ $\rightarrow$ $\{0, \ldots 9\}$

Min   Max

//

$\{-\infty \ldots +\infty\} \rightarrow \{\% M\} = \{0 \ldots M-1\}$

min   max

limit your range

$\{794, 841\} \rightarrow \%10 = \{0, \ldots 9\}$

limit our range

%
- Consistent Hashing
- Hashmap / dict
- Cryptography

Conceptually

Remainder = Divident − $\underbrace{\text{divisor} * \text{Quotient}}_{\text{greater multiple of divisor} < = \text{divided}}$

10 % 4 = 10 − 8 ⇒ 2

13 % 5 = 13 − 10 ⇒ 2

100 % 7 ⇒ 100 − 98 ⇒ 2

150 % 7 ⇒ 150 − 147 ⇒ 3

−60 % 9 ⇒ −60 − { greater multiple of 9 < = −60 }

−63 < = −60

⇒ −60 − { −63 }

⇒ −60 + 63 ⇒ 3

−40 % 7 ⇒ −40 − { greater multiple of 7 < = −40 }

−42 < = −40

⇒ −40 − { −42 }

⇒ −40 + 42 ⇒ 2

As per your language:

$-80 \% 9 =$
- language: -8
- Concept: $-80 - \{-81\}$ → greater mult ≤ -80 → $-80 + 81$ → 1

$+9$

$-40 \% 7 =$
- language: -5
- Concept: 2

$+7$

$-60 \% 9 =$
- lang: -6
- Conc: 3

$+3$

```
if ( n < 0 ){
    { n%p + p }
}
```

→ only by adding p we can get expected number

Modular Arithmetic

$[0, M-1] + [0\ M-1] \geq M \% M \Rightarrow [0\ M-1]$

$$\boxed{(a+b) \% M = \{\underbrace{a \% M} + \underbrace{b \% M}\} \% M}$$

$a = 6,\ b = 13,\ M = 7$ $= \{\underline{6 \% 7} + \underline{13 \% 7}\} \% 7$

$\underbrace{(19) \% 7}$ → $6 + 6 \Rightarrow (12) \% 7 \Rightarrow 5$

↓

5

$a = 4,\ b = 5,\ M = 6$ $= \{4 \% 6 + 5 \% 6\} \% 6$

$\underline{(9) \% 6}$ $= (4 + 5) \% 6 \Rightarrow 3$

3

$\{0, M-1\} * \{0, M-1\} > M \% M$

// $$\boxed{(a * b) \% M = \{a \% m * b \% m\} \% m}$$

$a = 6,\ b = 7,\ M = 4$ $=$ $\underline{6 \% 4}$ $\underline{7 \% 4}$

$\underline{(42) \% 4}$ → 2 $(2 * 3)$ → $\underline{\underline{6}}$

$(a-b) \% M =$ }

$(a/b) \% M =$ } advanced batch

Problem:

given a, n, p → Calculate $a^n \% p$

No inbuilt functions

a	n	p
3	4	7

→ $(3^4) \% 7$ → $(81) \% 7$ → 4

```
power(int a, int n, int p){   a^n%p
    → // no overflow

    for(int i=1; i<=N; i++){   // wrong
        a = a*a
    }
    return a%p
}
```

given a, $n=5$, $a^5 \% p$

i	// a	
1	a = a*a	a^2
2	a = a*a	a^4
3	a = a*a	a^8
4	a = a*a	a^{16}
5	a = a*a	a^{32}

↓

$a^{32} \% p$

```
power(int a, int n, int p){   a^n%p
    Even if no-overflow code fails

    int ans=1
    for(int i=1; i<=N; i++){   → wrong
        ans = a*a
    }
    return a%p
}
```

// given a, n, p = 4

i	ans
1	a2
2	a2
3	a2
4	a2

```
long power(int a, int n, int p){ // a^n % p
    long ans = 1
    for(int i = 1; i <= N; i++){
        ans = (ans * a) % p
        ans = (ans%p * a%p) % p
    }
    return ans%p
}
```

$\{ p = 10^9 \}$

$\underbrace{ans\%p}_{p-1} * \underbrace{a\%p}_{p-1} \approx p^2 \rightarrow 10^{18}$

$\{ long \rightarrow \{-8*10^{18}, 8*10^{18}\} \}$

$a = 10, n = 14, p = 25$

$a^n = (10^{14}) \% 25$

$a = 10, n = 100, p = 30$

$a^n = (10^{100}) \% 30$

overflow :

Constraints:

$1 <= N <= 10^5$

$1 <= a <= 10^9$

$1 <= p <= 10^9 \rightarrow \}$ Max $p = 10^9$

// $(a^n) \% p = (a^{n\%p}) \% p$ ✗

$a=2, n=5, p=3$

$(32)\%3 = 2$

$\{2^{5\%3}\}\%3$

$\{2^2\}\%3$

$1 <= N <= 10^5$ $1 <= P <= 10^9$

$1 <= a <= 10^9$

```
long power(int a, int n, int p){   a^n % p

    long ans = 1

    for(int i = 1; i <= N; i++){
        ans = (ans%p * a%p) %p
    }

    return ans%p
}
```

$p-1 * p-1 \approx$

TC: O(N)

SC: O(1)

Combine break

9:15AM

$\{a * b\} \% p = (a\%p * b\%p)\%p$

// Dry run: { a, n = 5, p, } → {ans = 1}

i	ans = (ans*a)%p	ans
1	ans = (1 * a) %p	a %p
2	ans = (a%p * a) %p	a^2 %p
3	ans = (a^2%p * a) %p	a^3%p
4	ans = (a^3%p * a) %p	a^4 %p
5	ans = (a^4%p * a) %p	a^5%p

(a%p * a) %p

= {(a%p) * a%p } %p

= {a * a}%p

= (a^2) %p

Expected output

// (a %p) %p = a%p ?

Divisibility rule of 3 : Sum of digits are divisible by 3 ?

= $(789)\%3$ ✓ $(8215)\%3$ ✗ $(3458)\%3 =$ ✗

$10\%3 \Rightarrow 1$

$10^2\%3 \Rightarrow 1$

$10^3\%3 \Rightarrow 1$

$10^n\%3 \rightarrow 1$

$(3458) = 3*10^3 + 4*10^2 + 5*10^1 + 8*10^0$

$(3458)\%3 = (3*10^3 + 4*10^2 + 5*10^1 + 8*10^0)\%3$

$(3*10^3 + 4*10^2 + 5*10^1 + 8*10^0)\%3$

$= \{(3*\underbrace{10^3)\%3}_{1} + (4*\underbrace{10^2)\%3}_{1} + (5*\underbrace{10^1)\%3}_{1} + (8*\underbrace{10^0)\%3}_{1})\%3$

$= \{3 + 4 + 5 + 8\}\%3$

$= \{20\}\%3$

$=$ 2

Divisibility of 4 → { last 2 digits %4} ?

$\{a_3\ a_2\ a_1\ a_0\}\ \%4 = \{a_3 * 10^3 + a_2 * 10^2 + a_1 a_0\}\ \%4$

$\Rightarrow \{a_3 * 10^3 + a_2 * 10^2 + a_1\ a_0\}\ \%4$

$\Rightarrow \{\underbrace{(a_3 * 10^3)\ \%4}_{0} + \underbrace{(a_2 * 10^2)\ \%4}_{0} + (a_1 a_0)\ \%4\}\ \%4$

if last 2 digits are non-zero

Divisibility rule of 9 = {sum of digits %9} ⇒ {TODO}

$$\left\{\begin{aligned} 10 \% 9 &= 1 \\ 10^2 \% 9 &= 1 \\ 10^3 \% 9 &= 1 \\ &\downarrow \\ 10^n \% 9 &= 1 \end{aligned}\right\}$$

5Q) Given 1 number in ar[], calculate number % p

Each ar[i] contains single digit of number

number of digits

$N=5$

$ar[i] = \{0-9\}$

78962

	0	1	2	3	4
ar[5] =	7	8	9	6	2

Constraint :

$1 <= N <= 10^5$

$1 <= p <= 10^9$

$p=5$

Calculate, number % p ⇒ 2

$N=3$: 9 9 9 : $10^3 - 1$

$N=5$: 9 9 9 9 9 : $10^5 - 1$

$N=10$: $10^{10} - 1$

$N=20$: $10^{20} - 1$

$N=10^5$: $10^{10^5} - 1$ %p

number can be this big

$N=5$

	0	1	2	3	4
ar[5] =	3	2	6	4	9

$p=5$

$9*10^0$

$4*10^1$

$6*10^2$

$2*10^3$

$3*10^4$

$(3*10^4 + 2*10^3 + 6*10^2 + 4*10^1 + 9*10^0)$ %p

$$(3*10^4 + 2*10^3 + 6*10^2 + 4*10^1 + 9*10^0)\ \%\ p$$

$$\{(3*10^4)\ \%\ p + (2*10^3)\ \%\ p + (6*10^2)\ \%\ p + (4*10^1)\ \%\ p + (9*10^0)\ \%\ p\}\ \%\ p$$

$$(3\ \%\ p * 10^4\ \%\ p)\ \%\ p$$

$$+ (2\ \%\ p * 10^3\ \%\ p)\ \%\ p$$

$$+ (6\ \%\ p * 10^2\ \%\ p)\ \%\ p$$

$$+ (4\ \%\ p * 10\ \%\ p)\ \%\ p$$

$$+ (9\ \%\ p * 1\ \%\ p)\ \%\ p$$

// ar[6] = ar[0] ar[1] ar[2] ar[3] ar[4] ar[5]

$ar[5] * 10^0$

$ar[4] * 10^1$

$ar[3] * 10^2$

$ar[2] * 10^3$

$ar[1] * 10^4$

$ar[0] * 10^5$

$$(ar[0]*10^5 + ar[1]*10^4 + ar[2]*10^3 + ar[3]*10^2 + ar[4]*10^1 + ar[5]*10^0)\ \%p$$

$$(ar[0]\%p * 10^5\%p)\ \%p$$

$+$

$$(ar[1]\%p * (10^4\%p))\ \%p$$

$+$

$$(ar[2]\%p * (10^3\%p))\ \%p$$

$+$

$$(ar[3]\%p * (10^2\%p))\ \%p$$

$+$

$$(ar[4]\%p * (10\%p))\ \%p$$

$+$

$$(ar[5]\%p * (1\%p))\ \%p$$

$ans = 1$

$ans = (1 * 10)\ \%p = 10\%p$

$ans = (ans*10)\ \%p = (10^2)\ \%p$

$ans = (ans*10)\ \%p = (10^3)\ \%p$

overall ans =

Pseudocode :

```
// given ar[N] & p

long fans = 0
long ans = 1

i = N-1; i >= 0; i--) {

    fans = fans + {(ar[i] % p) * (ans % p)} % p
    fans = fans % p
    ans = (ans * 10) % p
}
return fans;
```

TC: O(N)

SC: O(1)