

Today's Content:

→ Majority Element

→ Josephus Problem

108) Given N elements find majority element? size  
 An element with freq  $> N/2$  ↑  
occurrence of element

Ex<sub>1</sub>: ar[6] = { 1 2 1 6 1 1 } : freq(1) = 4 > 6/2  
 ↳ ans = 1, majority element

Ex<sub>2</sub>: ar[9] = { 3 4 4 8 4 9 4 3 4 } :  
 ↳ ans = 4, majority element freq(4) = 5 > 9/2

Ex<sub>3</sub>: ar[11] = { 3 3 4 6 1 3 2 5 3 3 3 }  
 ↳ ans = 3, majority element freq(3) = 6 > 11/2

Ex<sub>4</sub>: ar[10] = { 4 6 5 3 4 5 4 4 4 8 }  
 ↳ no majority element freq(4) = 5 > 10/2  
 ↳ not majority

Solutions:

1) For every element iterate & get frequency

$$\left. \begin{array}{l} \text{TC: } O(N^2) \\ \text{SC: } O(1) \end{array} \right\} N > \log N \rightarrow \left\{ \begin{array}{l} \text{for large values} \\ \text{of } N \end{array} \right.$$

2) Ex:  $arr[10] \rightarrow \{ 3 \ 2 \ 4 \ 6 \ 6 \ 3 \ 4 \ 4 \ 2 \ 3 \}$

$\uparrow$   
Sort  $\rightarrow \{ N \text{ Elements} \} \rightarrow \text{TC: } O(N \log_2 N)$

$arr[10] = \{ 2 \ 2 \ 3 \ 3 \ 3 \ 4 \ 4 \ 4 \ 6 \ 6 \}$   
 $\uparrow \ 2 \quad \uparrow \ 3 \quad \uparrow \ 3 \quad \uparrow \ 2$

Iterate & get frequency of every element:  $O(N)$

$\rightarrow$  Overall TC:  $N \log_2 N + N \rightarrow \text{TC: } O(N \log_2 N)$

3) Given  $N$  array, Say 2 majority are present  $ele_1, ele_2$ ,

$$\left\{ \begin{array}{l} \text{freq}(ele_1) > N/2 \\ \text{freq}(ele_2) > N/2 \end{array} \right.$$

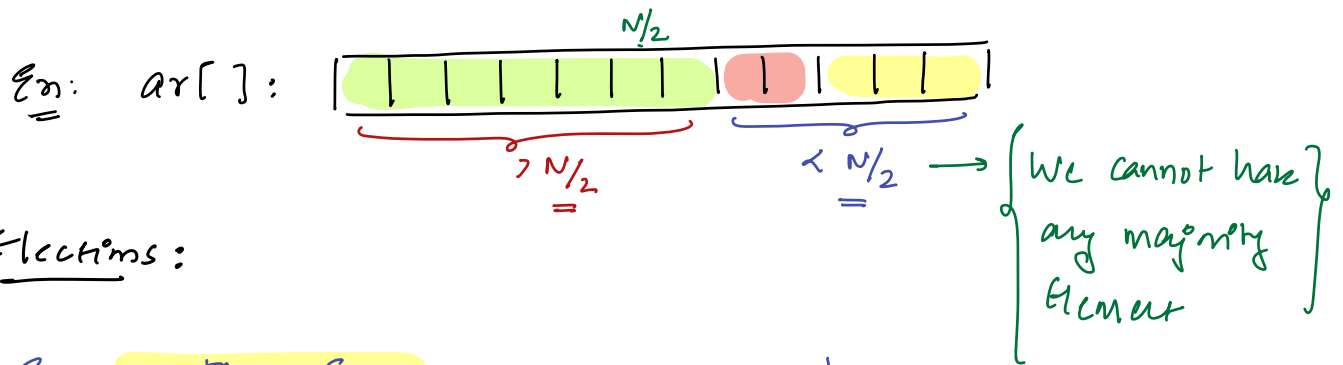
$$\text{freq}(ele_1) + \text{freq}(ele_2) > N$$

not possible?

Because total elements are  $N$

$\rightarrow$  At max we can only have 1 single majority

// Say we bring all majority to left



Elections:

Say 15 MLA Seats

Saurabh:

Ayush:

Cheran:

// Saurabh's  $> 15/2 : \checkmark$

$\downarrow$   $\rightarrow$  We remove 2 tickets

$8 \text{ MLA} > 13/2 : \checkmark$

$\downarrow$   $\rightarrow$  We remove 2 tickets

$7 \text{ MLA} > 11/2 : \checkmark$

$\downarrow$   $\rightarrow$  We remove 2 tickets

$7 \text{ MLA} > 9/2 : \checkmark$

observations:

If we remove 2 different elements, majority element will still remain same

Ex2: 7 MLA:

Nanya:

Satish:

Kulay:

Nanya MLA  $\rightarrow$

$4 > 7/2 : \checkmark$

$\downarrow$

$2 > 5/2 : \times$

Ex1: { ~~0~~ 1 } { ~~1~~ 2 } { ~~2~~ 3 } { ~~3~~ 4 } { ~~4~~ 5 } { ~~5~~ 6 } { ~~6~~ 7 } { 8 } }  $\text{majority} = 4$

freq(u)

5

$> 9/2 :$

↓

4

$> 7/2 :$

↓

3

$> 5/2 :$

↓

2

$> 3/2 :$

↓

1

$> 1/2 :$

finally we are leftout with majority element

→ delete 2 distinct elements?

→

Moore's Voting Algorithm

Ex2:

0 1 2 3 4 5 6 7 8 9 10 }  $\text{majority} = 3$   
 3 3 4 6 1 3 2 5 3 3 3  
 ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓  
 R: 4  
 R: 3

ele: ~~3~~ ~~1~~ ~~2~~ 3

majority element

freq: ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ 3

This is not correct  
frequency of 3

Private q get  $\text{freq}(3) > 11/2$  ✓

Ans: 

0	1	2	3	4	5	6	7	8	9
4	6	5	3	4	5	6	4	4	4

 } → no majority

ele: ~~4~~ ~~6~~ ~~5~~ ~~3~~ 4 → majority element

freq: ~~1~~ ~~0~~ ~~1~~ ~~0~~ 1 ~~0~~ ~~1~~ ~~0~~ 1 2

check if 4 is majority: { iterate & freq of 4 >  $N/2$  }

Ans: 

0	1	2	3	4	5	6
3	4	3	3	6	3	7

ele: ~~3~~ 3 → { check if 3 is majority  
iterate & get  $\text{freq}(3) > 7/2$  }  
freq: ~~1~~ ~~0~~ 1 ~~1~~ ~~1~~ ~~1~~ 1  
 $7 \times 1/2 \rightarrow 7/2 = 3.5$

Ans: 

0	1	2	3	4	5	6
3	3	3	4	4	4	1

ele: ~~3~~ 1 → if 1 is majority:

freq: ~~1~~ ~~1~~ 3 1 1 1 ~~1~~  
iterate & get  $\text{freq}(1) > 7/2$  \*  
no majority element

Pseudocode:

```
int majority(int arr[]) {
```

```
    int N = arr.size();
```

```
    int ele = arr[0], freq = 1;
```

TL:  $O(N)$

```
    for (int i = 1; i < N; i++) {
```

SL:  $O(1)$

```
        if (freq == 0) {
```

```
            ele = arr[i], freq = 1;
```

→  
lower

9:15 am

```
        } else if (ele == arr[i]) {
```

```
            freq = freq + 1;
```

```
        } else { // 2 different elements
```

```
            freq = freq - 1;
```

<      C is decre  
3 → 2

```
    }  
    int C = 0;
```

```
    for (int i = 0; i < N; i++) {
```

```
        if (arr[i] == ele) { C = C + 1; }
```

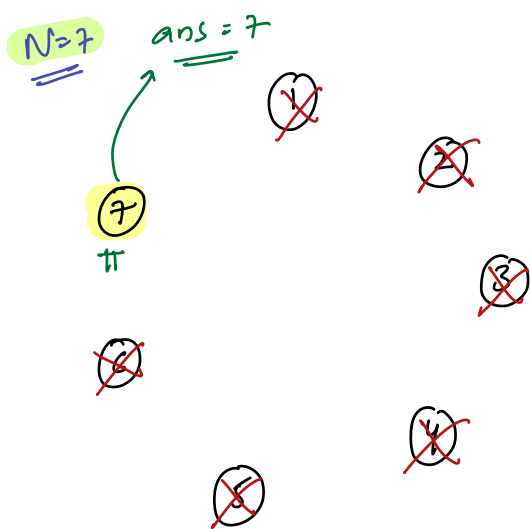
```
    }  
    if (C > N/2) { return ele; }
```

```
    else { // no majority element is present  
        return as per asked in Question  
    }
```

```
}
```

## 28) Josephus Problem:

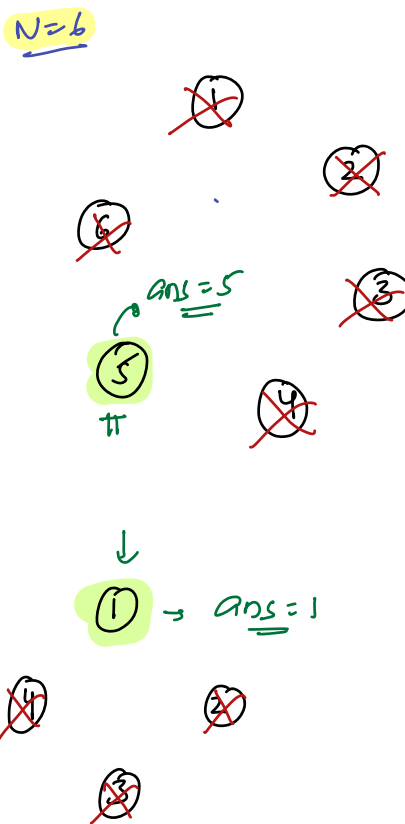
- Initially  $N$  people are in circle, Person 1 has knife, he kills his adjacent clockwise person, and passes the knife to next adjacent clockwise person.
- Repeat until only a single person stands, who is last person standing?



obs:

- If  $n$  is odd:  $n$
- If  $n$  is even:  $n-1$

above won't work





$$N=1 : 2^0 \text{ (1)} \rightarrow \underline{\underline{\text{ans} = 1}}$$

$$N=2 : 2^1 \text{ (1)} \leftarrow$$

~~(2)~~

$$N=4 : 2^2 \text{ (1)} \leftarrow$$

~~(2)~~

~~(3)~~

~~(4)~~

$$N=8 : 2^3 \text{ (1)} \rightarrow \underline{\underline{\text{ans} = 1}}$$

~~(2)~~

~~(3)~~

~~(4)~~

~~(5)~~

~~(6)~~

~~(7)~~

(8)

obs: If  $N$  is power of 2,  
Who ever starts the game  
will win the game

$$N=16 : 2^4$$

$\downarrow$

(1)

~~(2)~~

(3)

~~(4)~~

(5)

~~(6)~~

(7)

~~(8)~~

(9)

~~(10)~~

(11)

~~(12)~~

(13)

~~(14)~~

(15)

$2^5$

$N=32$  : 1 starts game  $\rightarrow$  1 wins

obs: If  $N$  is power of 2,  
1 is winner

$$N=8$$

~~(1)~~

~~(2)~~

$\underline{\underline{\text{ans} = 3}}$

$\uparrow$

(3)  $\leftarrow$

~~(4)~~

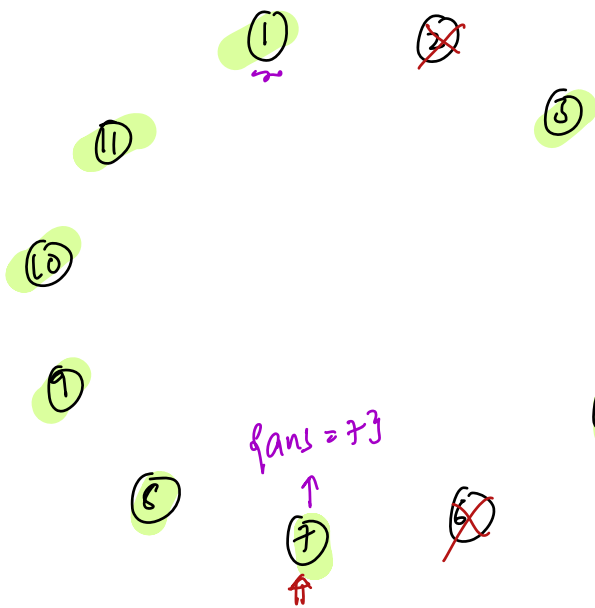
~~(5)~~

~~(6)~~

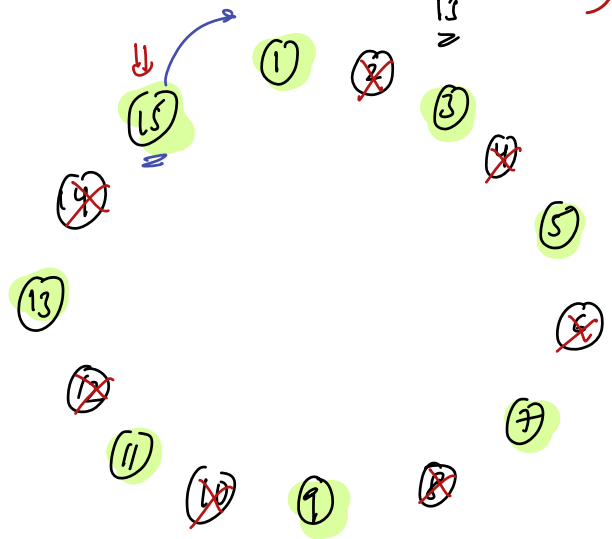
~~(7)~~

~~(8)~~

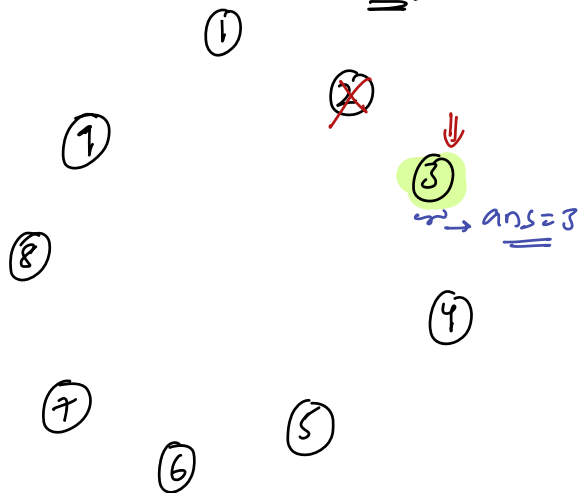
//  $N=11$ : {3 kills}  $\rightarrow 1 + 2(3)$  8



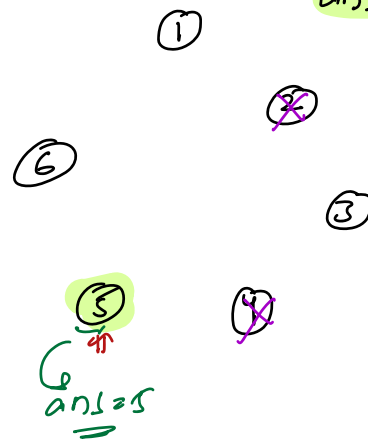
$N=15$ : {7 kills}  $\rightarrow 1 + 2(7)$  15  $\rightarrow$  8



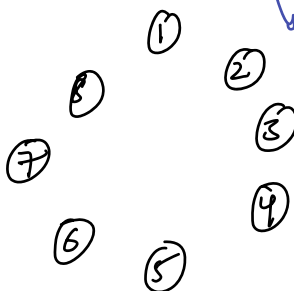
$N=9$ : 1 kills  $\rightarrow 1 + 2(1)$  8  $\rightarrow$  3



$N=6$ : 2 kills  $\rightarrow 1 + 2(2)$  4  $\rightarrow$  5



$N=8$  0 kills  $\rightarrow 1 + 2(0)$  8  $\rightarrow$  1



$N=100$  {36 kills}  $\rightarrow 1 + 36 \times 2$  64

1 starts

nearest power of 2  $\leq 100$

$\boxed{ans = 73}$

// generalize:

```
int lastPerm(N) {
```

→ nearest power of 2  $\leq N$

```
    y = nearestPower(N)
```

How to get nearest of 2

```
    kPills = N - y
```

TODO: { Try with loops on your own }

```
    return 2 * (kPills + 1)
```

```
}
```

→ { Done }

→ { Friday → Sorting }

// Ascii

↳ convert: { char → int }    '0' → '0':  
              { int → char }    '1' → '0':

↳ strings:

$$\left\{ \begin{array}{l} (a+b) \% m = (a \% m + b \% m) \% m \\ (a \times b) \% m = (a \% m \times b \% m) \% m \end{array} \right\} \quad \begin{array}{l} \text{Concept} \\ -ve \% m \rightarrow +ve \\ \rightarrow -ve \text{ lag} \end{array}$$

$$(a-b) \% m = (a \% m - b \% m + m) \% m$$

$$// \quad a \% m = b \% m$$

$$a \% m - b \% m = 0$$

$$a \% m - b \% m + m = m$$

$$(a \% m - b \% m + m) \% m = 0$$

$$(a-b) \% m = 0$$

$m$  should be a factor of  $a-b$