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Ordered Dictionaries :-

- In addition to the dictionary functionality, We want to support following Operations :-
 - $\text{Min}()$
 - $\text{Max}()$
 - $\text{Predecessor}(S, k)$
 - $\text{Successor}(S, k)$

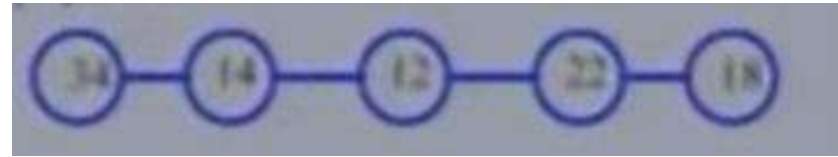
A List-Based Implementation :-

- Unordered List

Searching takes $O(n)$ times.

Inserting takes $O(1)$ times.

Successor takes $O(n)$ times.



- Ordered List

Searching takes $O(n)$ times.

Inserting takes $O(n)$ times.

min, max takes $O(1)$ times.



- Using Array we can improve the Searching time.

Binary Search :-

- findElement(23)

Binary Search										
Search 23	0	1	2	3	4	5	6	7	8	9
	2	5	8	12	16	23	38	56	72	91
23 > 16 take 2 nd half	L=0	1	2	3	M=4	5	6	7	8	H=9
	2	5	8	12	16	23	38	56	72	91
23 < 56 take 1 st half	0	1	2	3	4	L=5	6	M=7	8	H=9
	2	5	8	12	16	23	38	56	72	91
Found 23, Return 5	0	1	2	3	4	L=5, M=5	H=6	7	8	9
	2	5	8	12	16	23	38	56	72	91

- What about insertion and deletion ?

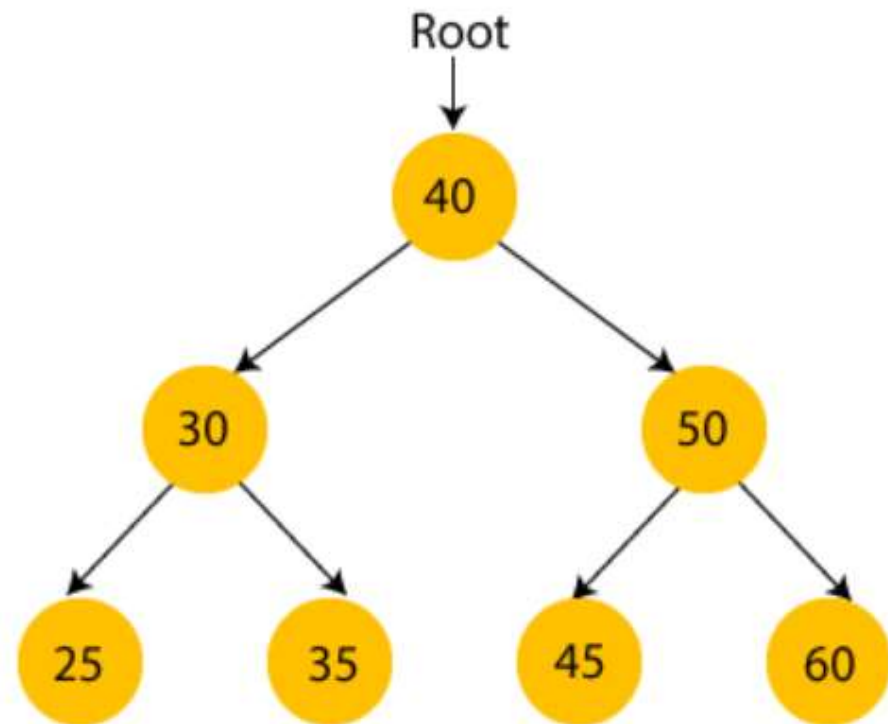
Binary Search Tree :-

- A Binary Search tree is a binary tree with searching ability.
- A binary search tree follows some order to arrange the elements.
- The value of left node must be smaller than the parent node.
- The value of right node must be greater than the parent node.
- This rule is applied recursively to the left and right subtrees of the root.



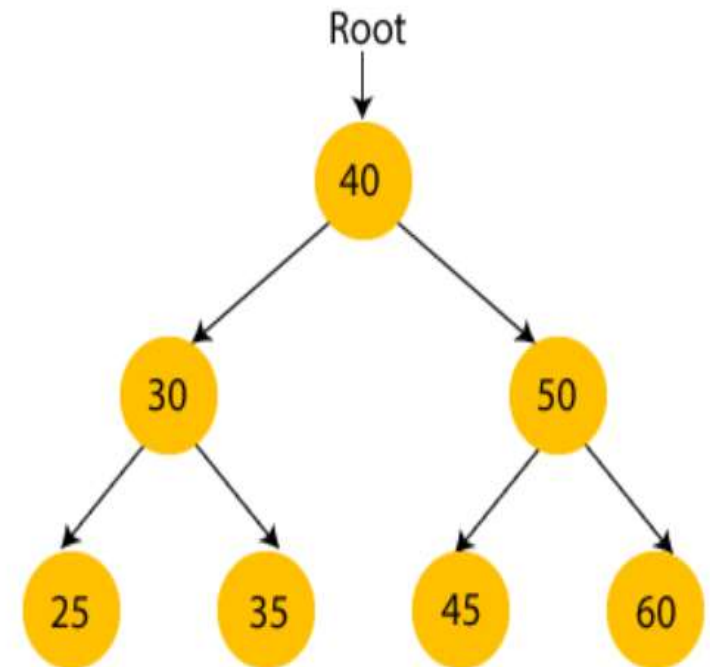
Binary Search Tree:-

- Example sequence
40,30,50,25,35,45,60



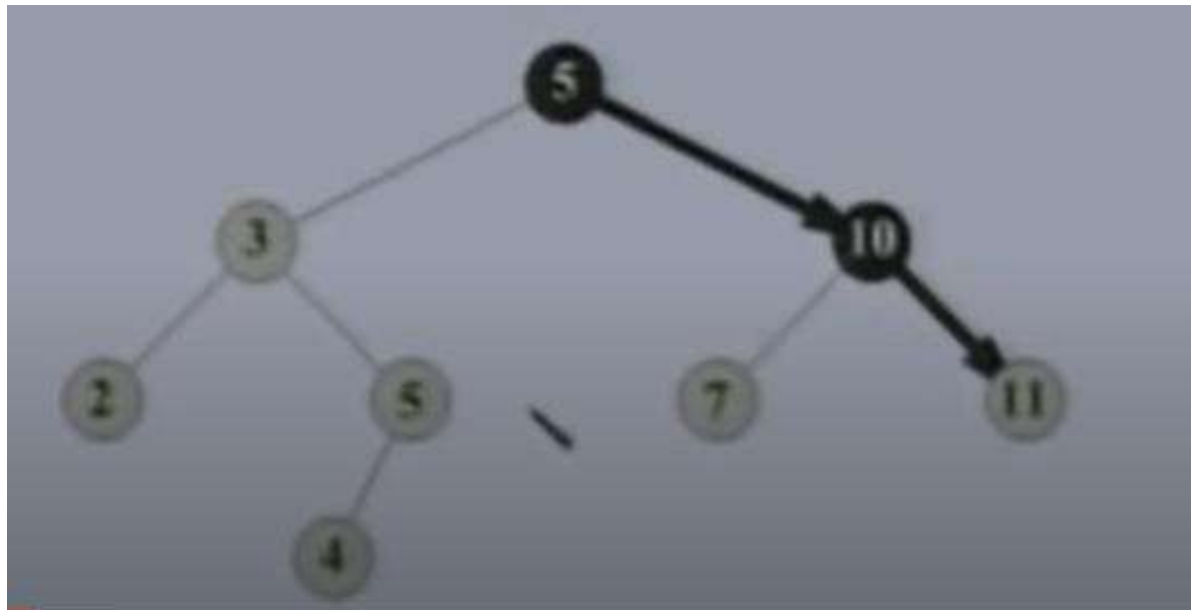
Searching in BST :-

- To find element with key k in a Tree T
- Compare k with $\text{key}[\text{root}[T]]$
- If $k < \text{key}[\text{root}[T]]$, Search for k in $\text{left}[\text{root}[T]]$
- Otherwise , Search for k in $\text{right}[\text{root}[T]]$
- Searching takes $O(h)$, Where h is the height of the binary tree.



Search Example :-

- Search(T,11)



Algorithm for BST Search:-

Search (root, item)

Step 1 - if (item = root → data) or (root = NULL)

return root

else if (item < root → data)

return Search(root → left, item)

else

return Search(root → right, item)

END if

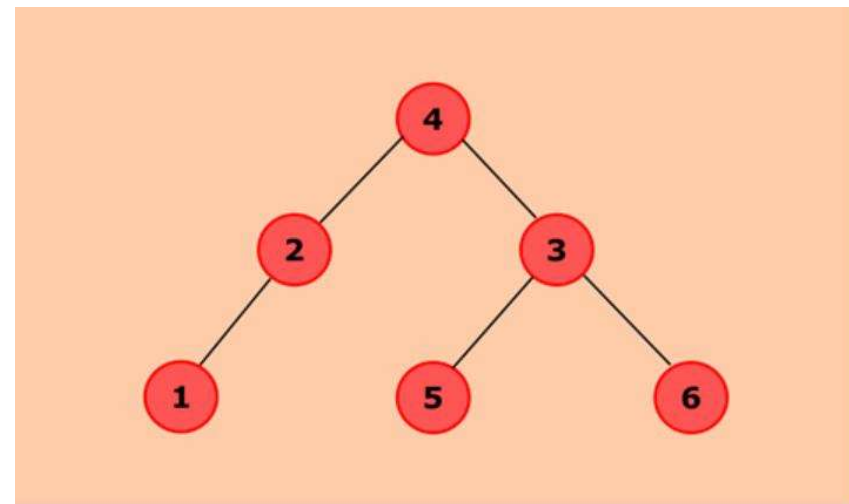
Step 2 - END

BST Minimum:-

- How to find min element in BST ?

```
TreeMinimum(x)  
while left[x] != NULL  
do x <- left[x]  
return x ;
```

- Running Time $O(h)$, i.e. proportional to the height of tree.

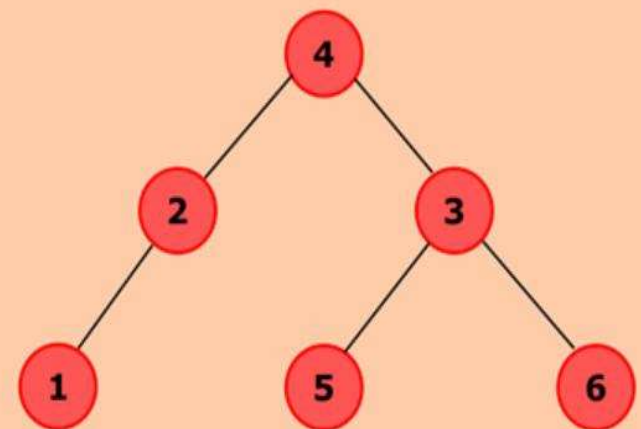


BST maximum :-

- How to find min element in BST ?

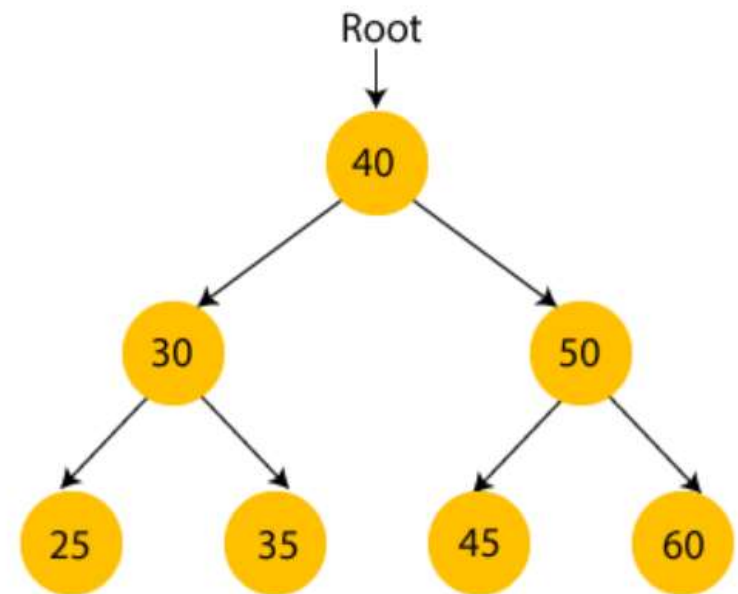
```
TreeMaximum(x)  
  while right[x] != NULL  
  do x <- right[x]  
  return x ;
```

- Running Time $O(h)$, i.e. proportional to the height of tree.



Successor:-

- Given x , find the node with the smallest key greater than $\text{key}[x]$
- We can distinguish two cases, depending on the right subtree of x
- Case -1
right subtree of x is non empty.
Successor will be left most node of the right subtree.

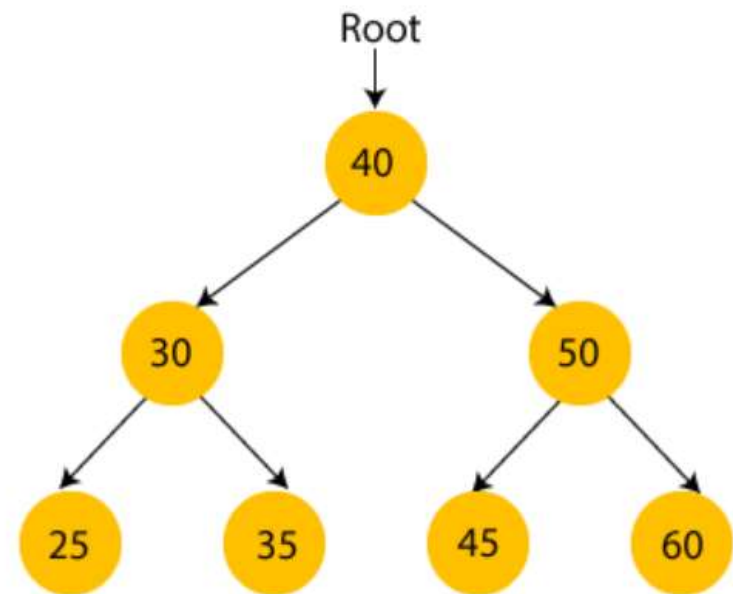


Successor :-

- Case -2

The right sub tree of x is empty.

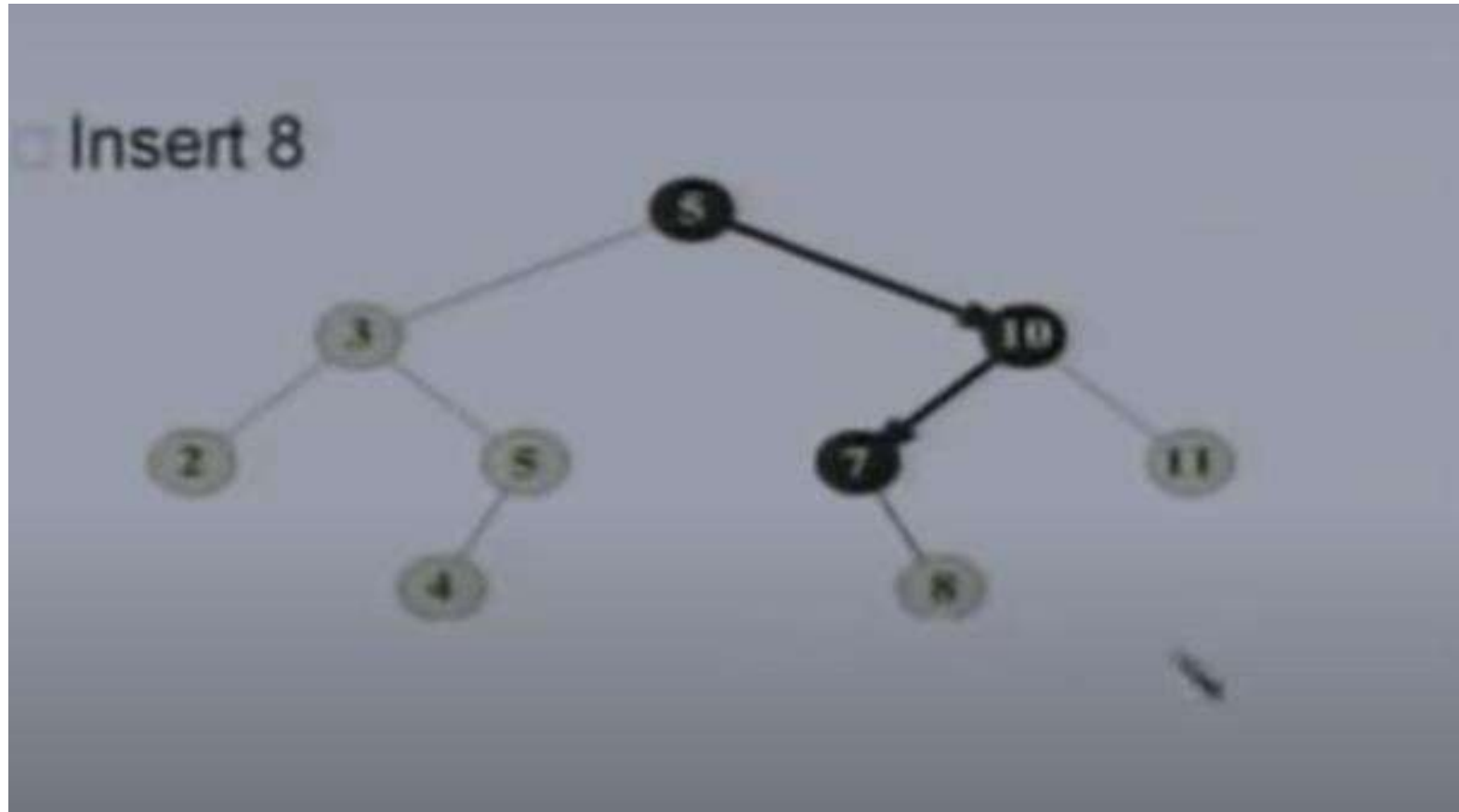
Successor is the lowest ancestor of x whose left child is also an ancestor of x
(Why ?)



BST Insertion:-

- The basic idea is similar to searching.
- take an element k (whose left and right child are null).
- Find place in T where k belongs to just like searching for k .
- And k there.
- The Running on a tree of height h is $O(h)$.
- It is proportional to the height of tree.

BST Insert :-



BST Insertion Algorithm:-

```
insert (Node n, int x)
{
    if (n==null)
    {
        n = new Node(x)
    }
    if(n.data < x)
        insert(n.right, x)
    else if(n.data > x)
        insert(n.left, x)
}
```




Thank you