CSE512 Fall 2018 - Machine Learning - Homework 7

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1. Manual calculation of one round of EM for a GMM.

Ans. M step

1. Write down the likelihood function you are trying to optimize.

Ans.

$$Q(\theta, \theta^{t-1}) = \sum_i \sum_c r_{ic} \log \pi_c + \sum_i \sum_c r_{ic} \log P(x_i | \theta_c)$$

2. After performing the M step for the mixing weights π_1, π_2, what are the new values.

Ans. $\pi_c = \frac{1}{N} \sum_i r_{ic}$

$$\pi_1 = \frac{1}{N} \sum_i r_{i1} = \frac{1}{3}(1 + 0.3 + 0) = \frac{1.3}{3} = 0.433$$

$$\pi_2 = \frac{1}{N} \sum_i r_{i2} = \frac{1}{3}(0 + 0.7 + 1) = \frac{1.7}{3} = 0.567$$

3. After performing the M step for the ~~mixing weights π_1, π_2~~ means μ_1 and μ_2, what are the new values?

Ans. $\mu_c = \frac{\sum_i r_{ic} x_i}{r_c}$

$$\therefore \mu_1 = \frac{\sum_i r_{i1} X_i}{r_1} = \frac{1\times1 + 0.3\times10 + 0\times20}{1.3}$$

$$= \frac{4}{1.3}$$

$$= 3.07$$

~~$\mu_2 = \sum_i$~~

$$\mu_2 = \frac{\sum_i r_{i2} X_i}{r_2} = \frac{0\times1 + 0.7\times10 + 1\times20}{1.7}$$

$$= \frac{27}{1.7}$$

$$= 15.88$$

4. After performing the M step for the standard deviations σ_1 and σ_2, what are the new values?

Ans. $$\sigma_c^2 = \frac{\sum_i r_{ic} X_i^2}{r_c} - \mu_c^2$$

$$\therefore \sigma_1^2 = \frac{(1\times1) + (0.3\times100) + (0\times400)}{1.3} - 9.46$$

$$= 14.38$$

$$\sigma_1 = \sqrt{14.38}$$

$$= 3.79$$

$$\sigma_2^2 = \frac{(0\times1) + (0.7\times100) + (1\times400)}{1.7} - 252.17$$

$$= 24.30$$

$$\sigma_2 = \sqrt{24.30} = 4.92$$

E step

1. Write down the formula for the probability of observation x_i belonging to cluster c.

Ans. $$r_{ic} = \frac{\pi_c P(x_i|\mu_c^{(t-1)})}{\sum_c \pi_c P(x_i|\mu_c^{(t-1)})}$$

$$= \frac{\pi_c \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(x_i - \mu_c^{t-1})^2}{2\sigma_c^2}\right)}{\sum_c \left[\pi_c \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(x_i - \mu_c^{t-1})^2}{2\sigma_c^2}\right)\right]}$$

2. After performing the E step, what is the new value of R^2?

Ans. (i) $$r_{i1} = \frac{\frac{\pi_1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x_1 - \mu_1^{t-1})^2}{2\sigma_1^2}\right)}{\frac{\pi_1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x_1 - \mu_1^{t-1})^2}{2\sigma_1^2}\right) + \frac{\pi_2}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(x_1 - \mu_2^{t-1})^2}{2\sigma_2^2}\right)}$$

$$= \frac{\left(\frac{0.433}{3.79}\right) \exp\left(\frac{-(1-3.07)^2}{2\times 14.38}\right)}{\left(\frac{0.433}{3.79}\right) \exp\left(\frac{-(1-3.07)^2}{2\times 14.38}\right) + \left(\frac{0.567}{4.92}\right) \exp\left(\frac{-(1-15.88)^2}{2\times 24.3}\right)}$$

$r_{11} = 0.987$

(ii) $r_{12} = 1 - r_{11} = 1 - 0.987 = 0.013$

(iii) $$r_{21} = \frac{\frac{\pi_1}{\sqrt{2\pi}\sigma_1} \exp\left(\frac{-(x_2 - \mu_1^{t-1})^2}{2\sigma_1^2}\right)}{\frac{\pi_1}{\sqrt{2\pi}\sigma_1} \exp\left(\frac{-(x_2 - \mu_1^{t-1})^2}{2\sigma_1^2}\right) + \frac{\pi_2}{\sqrt{2\pi}\sigma_2} \exp\left(\frac{-(x_2 - \mu_2^{t-1})^2}{2\sigma_2^2}\right)}$$

$$= \frac{\left(\frac{0.433}{3.79}\right) \exp\left(\frac{-(10-3.07)^2}{2\times 14.38}\right)}{\left(\frac{0.433}{3.79}\right) \exp\left(\frac{-(10-3.07)^2}{2\times 14.38}\right) + \left(\frac{0.567}{4.92}\right) \exp\left(\frac{-(10-15.88)^2}{2\times 24.3}\right)}$$

~~$= 0.272$~~

$= 0.2727$

(iv) $r_{22} = 1 - r_{21} = 1 - 0.2727$

$= 0.7273$

~~(v) $r_{31} = \frac{\pi_1}{\sqrt{2\pi}}$~~

(v) $$r_{31} = \frac{\frac{\pi_1}{\sqrt{2\pi}\,\sigma_1} \exp\left(\frac{-(x_3 - \mu_1^{t-1})^2}{2\sigma_1^2}\right)}{\frac{\pi_1}{\sqrt{2\pi}\,\sigma_1} \exp\left(\frac{-(x_3 - \mu_1^{t-1})^2}{2\sigma_1^2}\right) + \frac{\pi_2}{\sqrt{2\pi}\,\sigma_2} \exp\left(\frac{-(x_3 - \mu_2^{t-1})^2}{2\sigma_2^2}\right)}$$

$$= \frac{\left(\frac{0.433}{0.79}\right) \exp\left(\frac{-(20 - 3.07)^2}{2 \times 14.38}\right)}{\left(\frac{0.433}{0.79}\right) \exp\left(\frac{-(20 - 3.07)^2}{2 \times 14.38}\right) + \left(\frac{0.567}{4.92}\right) \exp\left(\frac{-(20 - 15.88)^2}{2 \times 24.3}\right)}$$

$$= 6.6 \times 10^{-5}$$

(vi) $r_{32} = 1 - r_{31} = 1 - 6.6 \times 10^{-5} = 0.999934$

$$R = \begin{bmatrix} 0.987 & 0.013 \\ 0.2727 & 0.7273 \\ 6.6 \times 10^{-5} & 0.999934 \end{bmatrix}$$

Rank: 67
Test accuracy: 0.66600