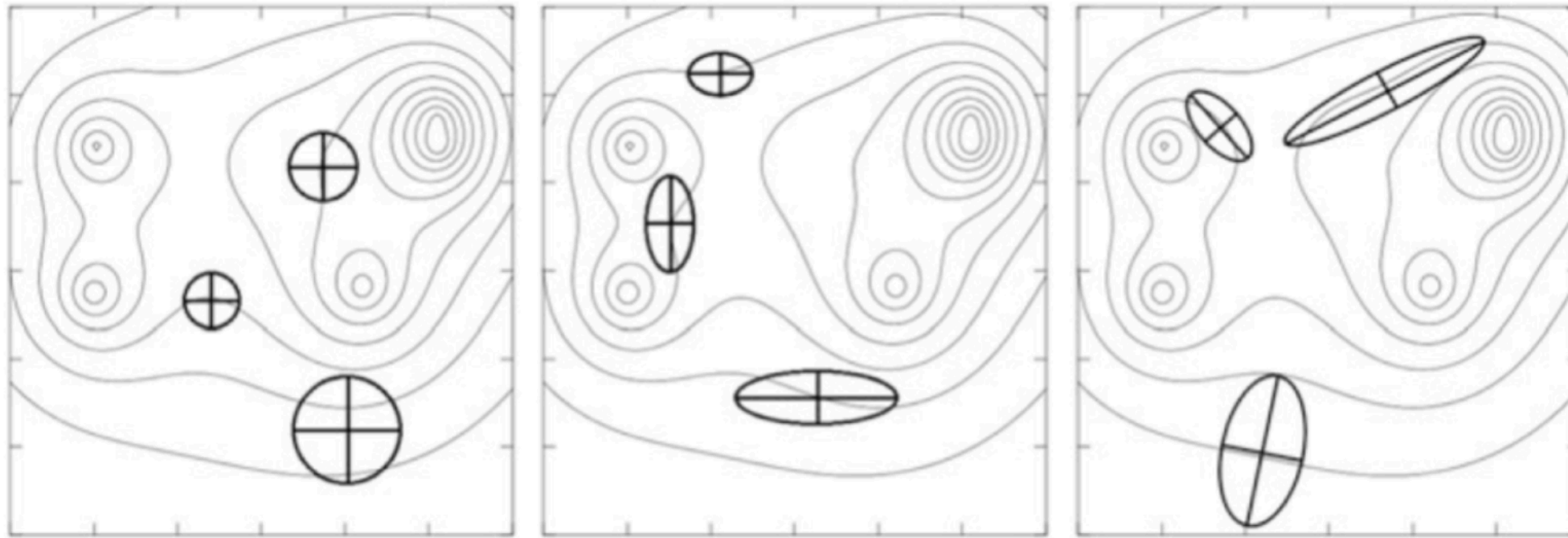


## 11: Evolution Strategies 2

- ES recombination
- Parent selection
- Survivor selection
- Example application
- Textbook Chapter 6.2, 4.4.2

# Self-adaption in ES



constant variance for  
all dimensions

different variance for  
each dimension

covariance matrix  
adaptation (CMA)

$n_\sigma$	$n_\alpha$	Structure of individuals	Remark
1	0	$\langle x_1, \dots, x_n, \sigma \rangle$	Standard mutation
$n$	0	$\langle x_1, \dots, x_n, \sigma_1, \dots, \sigma_n \rangle$	Standard mutations
$n$	$n \cdot (n - 1) / 2$	$\langle x_1, \dots, x_n, \sigma_1, \dots, \sigma_n, \alpha_1, \dots, \alpha_{n \cdot (n - 1) / 2} \rangle$	Correlated mutations

## ES technical sketch

Representation	Real-valued vectors
Recombination	Discrete or intermediary
Mutation	Gaussian perturbation
Parent selection	Uniform random
Survivor selection	Deterministic elitist replacement by $(\mu, \lambda)$ or $(\mu + \lambda)$
Speciality	Self-adaptation of mutation step sizes

# Recombination

- Create one child per recombination
- How to create an allele for one child
  - discrete recombination
  - intermediary recombination
- How to choose parents
  - global recombination: select two parents for each position

# Recombination

- ES typically uses global recombination
- Discrete recombination for object variables
  - preserves diversity within the phenotype (solution) space
  - allows the trial of very different combinations of variables
- Intermediary recombination for strategy parameters (mutation step size)
  - assures a more cautious adaptation of strategy parameters

# Parent selection

- Parents are selected by uniform random distribution whenever an operator needs one/some
- Thus: ES parent selection is unbiased - every individual has the same probability to be selected
- Note that in ES, “parent” means a population member (in GA, a population member selected to undergo variation)

# Survivor selection

- Applied after creating  $\lambda$  children from the  $\mu$  parents by recombination and mutation
- Deterministically picks the best ones
- Basis of selection is either:
  - the set of children only:  $(\mu, \lambda)$
  - the set of parents and children:  $(\mu + \lambda)$

# Survivor selection

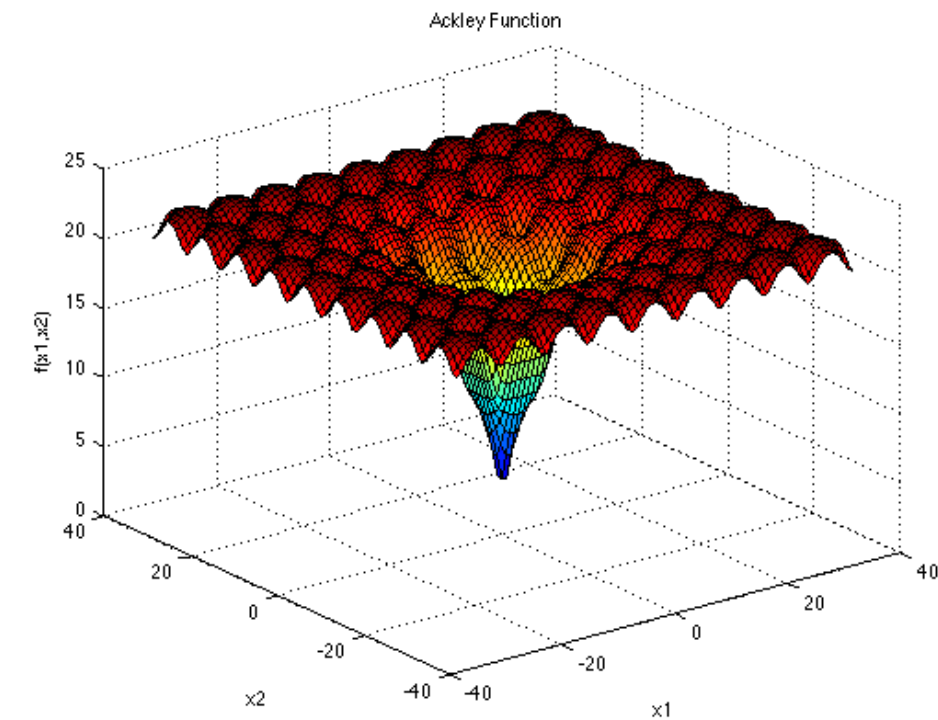
- $(\mu + \lambda)$  - selection is an elitist strategy, monotonic progression
- $(\mu, \lambda)$  - selection can “forget” and more fair, punctuated
- Often  $(\mu, \lambda)$  - selection is preferred by most users:
  - better in leaving local optima, especially working with a small  $\mu$
  - better in following moving optima
  - using the  $+$  strategy bad  $\sigma$  values can survive in  $\langle x, \sigma \rangle$  too long if their host  $x$  is very fit
- Selective pressure in ES is high ( $\lambda \approx 7 \times \mu$  is the common setting)



# Prerequisites for self-adaptation

- $\mu > 1$  to carry different strategies (individuals)
- $\lambda > \mu$  to generate offspring surplus
- Strong selection, e.g.  $\lambda \approx 7 \times \mu$
- $(\mu, \lambda)$  - selection to get rid of mis-adapted  $\sigma$
- Intermediate recombination for strategy parameters while discrete recombination for object variables

# Example application (ES<sub>I</sub>)

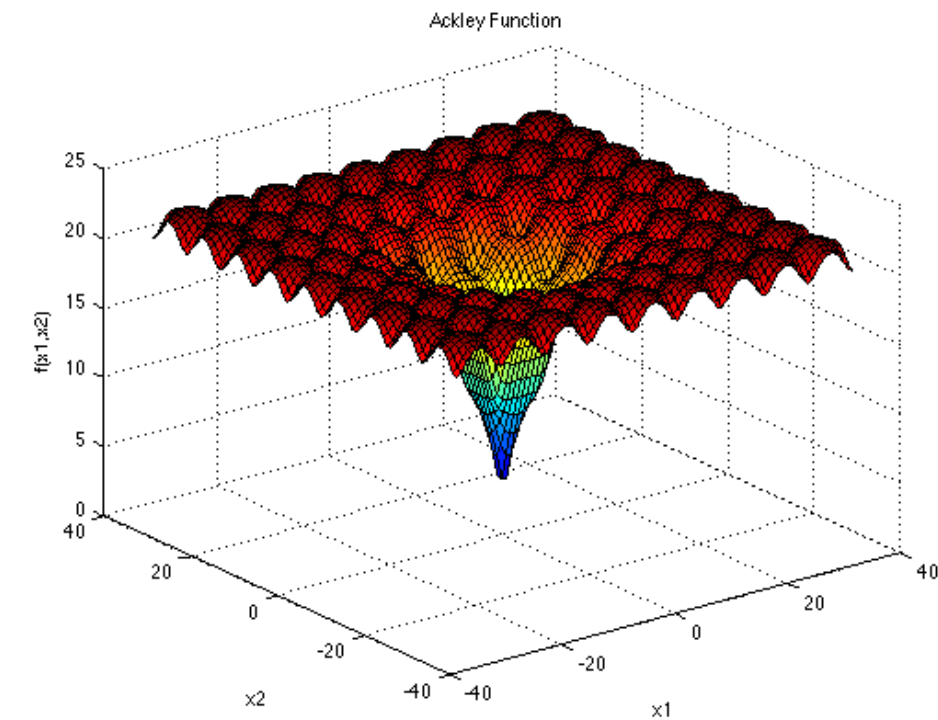


- Task is to minimize the Ackley function with  $n = 30$  (optimal 0 at  $x = (0,0,...,0)$ )

$$f(x) = -20 \cdot \exp\left(-0.2 \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$$

- Representation:  $\langle x_1, x_2, \dots, x_{30}, \sigma \rangle$ ,  $-30 < x_i < 30$
- Selection: (30, 200)
- Termination: after 100k fitness evaluations
- Initial standard deviations:  $\sigma = 3.0$

## Example application (ES<sub>30</sub>)

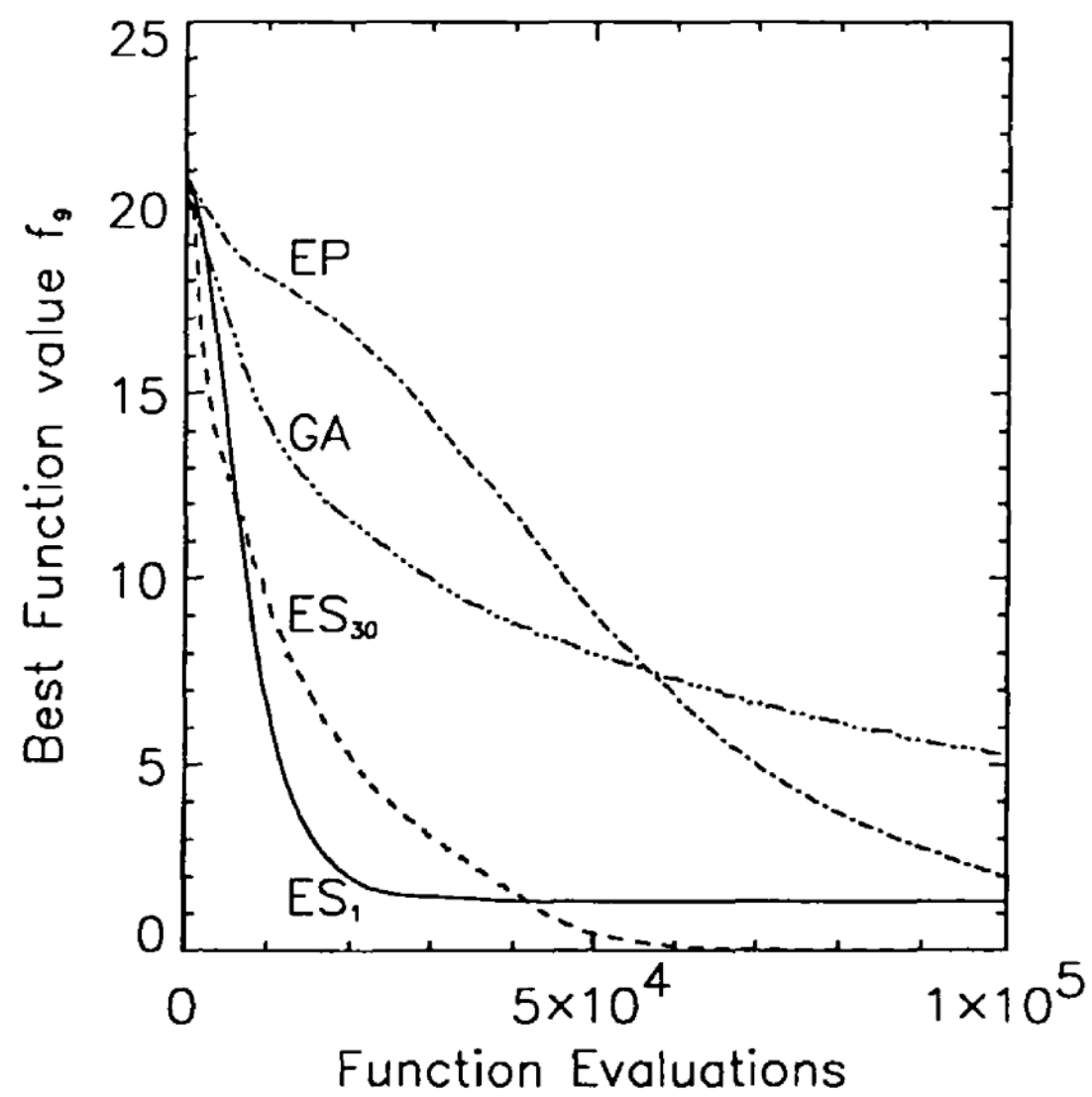


- Task is to minimize the Ackley function with  $n = 30$  (optimal 0 at  $x = (0,0,\dots,0)$ )

$$f(x) = -20 \cdot \exp\left(-0.2 \sqrt{\frac{1}{n} \cdot \sum_{i=1}^n x_i^2}\right) - \exp\left(\frac{1}{n} \sum_{i=1}^n \cos(2\pi x_i)\right) + 20 + e$$

- Representation:  $\langle x_1, x_2, \dots, x_{30}, \sigma_1, \sigma_2, \dots, \sigma_{30} \rangle$ ,  $-30 < x_i < 30$
- Selection: (30, 200)
- Termination: after 100k fitness evaluations
- Initial standard deviations:  $\sigma_i = 3.0$

# Result comparison



20 runs	Mean best	SD
ES <sub>1</sub>	1.326	1.039
ES <sub>30</sub>	$1.618 \times 10^{-3}$	$9.290 \times 10^{-4}$