6: Integer and Real-Valued Representations

- Integer representations
  - mutation (random resetting and creep mutation)
  - crossover (*n*-point and uniform crossover)
- Real-valued representation
  - mutation (uniform and nonuniform mutation)
  - crossover (discrete and arithmetic crossover)
- Textbook Chapter 4.3, 4.4

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Integer representations

- Some problems naturally have integer variables
- Ordinal vs. cardinal attributes
- Examples
  - graph *k*-coloring problem
  - function optimization with integer variables
  - path finding on a square grid
  - combinatorial optimizations

Integer representations - mutation

- Random resetting
  - randomly choose a permissible new value to replace
  - all alleles are equally likely to be chosen
- Creep mutation
  - adding a small value to each gene with a probability *p*
  - sampled from a distribution symmetric about zero
  - more likely to generate small changes than larger ones
  - parameter *mutation_step_size*

Integer representations - crossover

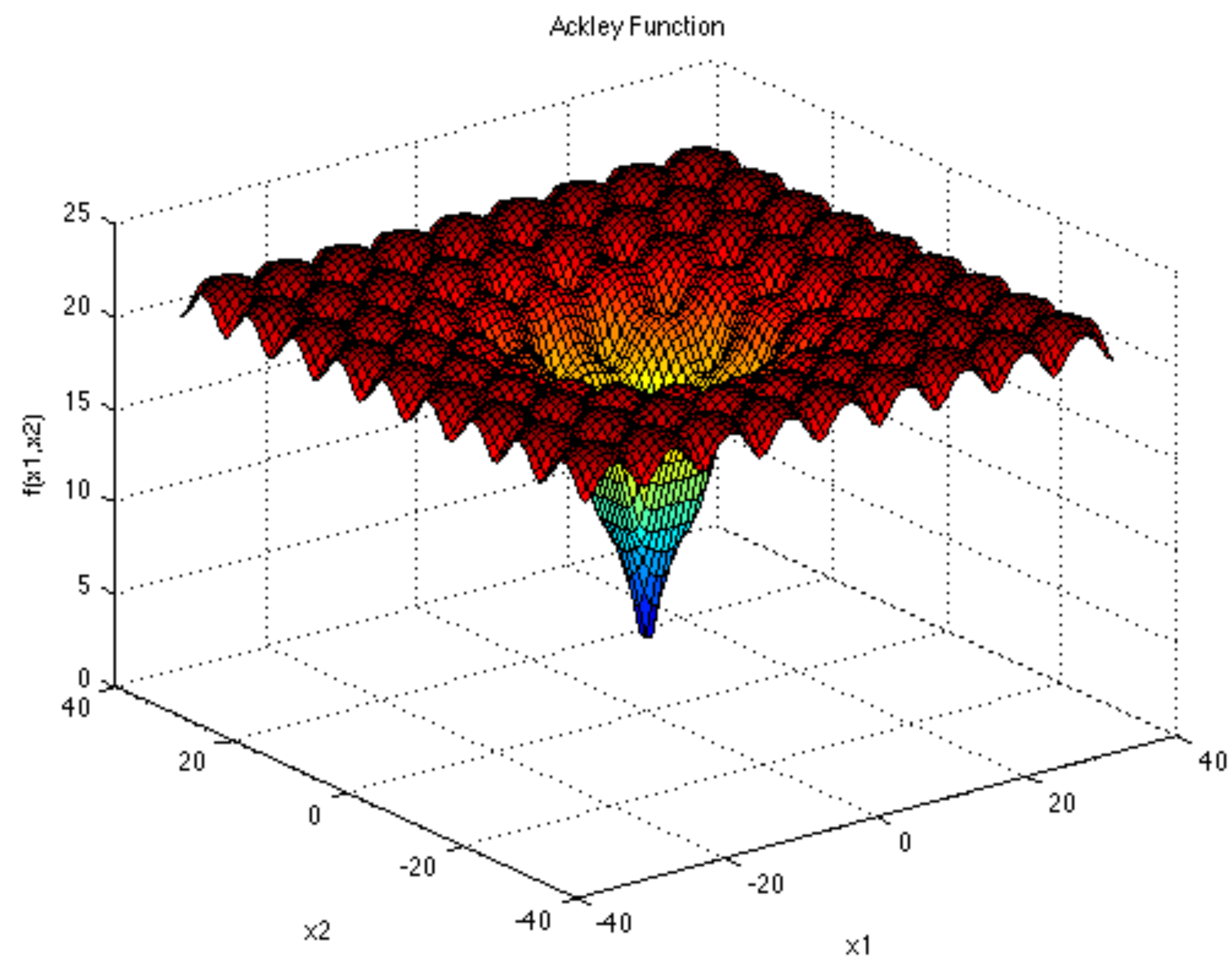
- Similar to binary representations
- *n*-point crossover
  - positional bias
- Uniform crossover
  - distributional bias

Real-valued representations

- Many problems occur as real-valued, e.g. continuous parameter optimization
- Example: Ackley's function

$$f(\overline{x}) = -c_1 \cdot exp\left(-c_2 \cdot \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n} \cdot \sum_{i=1}^{n} cos(c_3 \cdot x_i)\right) + c_1 + 1$$

$$c_1 = 20,\ c_2 = 0.2,\ c_3 = 2\pi$$



Floating-point representations - mutation I

- General scheme of floating point mutations

$$\langle x_1, \ldots, x_n \rangle \to \langle x'_1, \ldots, x'_n \rangle, \quad \text{where} \quad x_i, x'_i \in [L_i, U_i]$$

- Uniform mutation
  - x'_i are drawn uniformly randomly from $[L_i, U_i]$
  - Analogous to bit-flipping (binary) or random resetting (integers)

Floating-point representations - mutation 2

- Non-uniform mutations:
  - Most schemes are probabilistic but usually only make a small change to value
  - Most common method is to add random deviate to each variable separately, taken from a Gaussian distribution (with zero mean)
  - Standard deviation (*mutation_step_size*) controls the magnitude of change
  - Other distributions, e.g., Cauchy, power-law

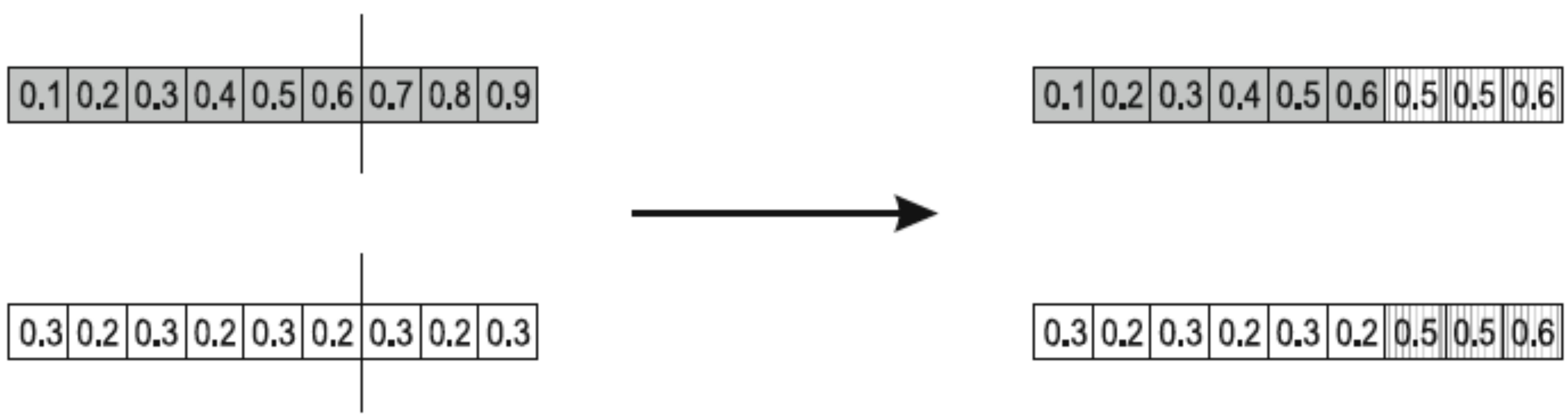
Floating-point representations - crossover

- Discrete:
  - each allele value in offspring *z* comes from one of its parents (*x*,*y*) with equal probability:

    $z_i = x_i$ or y_i
  - could use *n*-point or uniform
- Arithmetic (intermediate):
  - exploits idea of creating children "between" parents
  - $z_i = \alpha x_i + (1 - \alpha) y_i$, where α is between 0 and 1
  - The parameter α can be:
    - constant
    - variable (e.g. depend on the age of the population)
    - picked at random every time

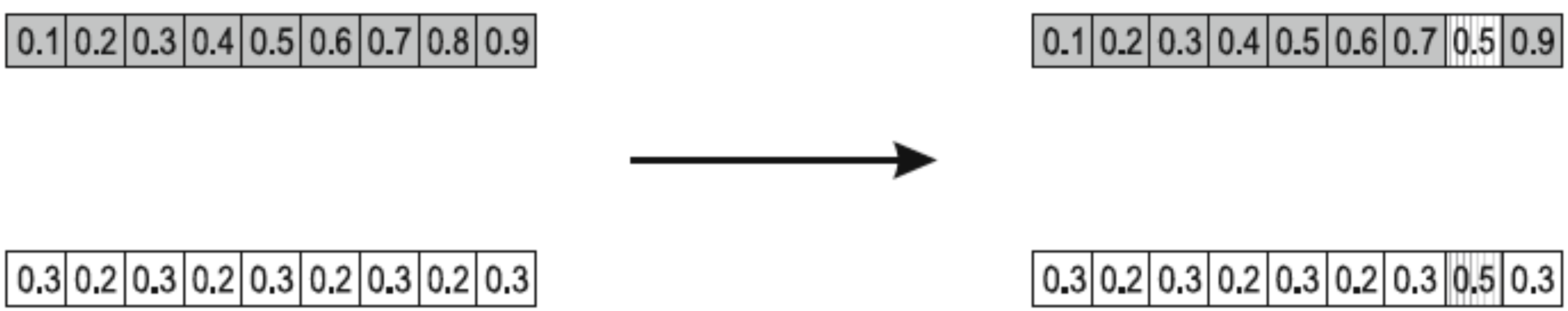
One-point arithmetic crossover

- Pick a recombination point *k*
- Child 1: $\langle x_1, \ldots, x_k, \alpha \cdot y_{k+1} + (1-\alpha) \cdot x_{k+1}, \ldots, \alpha \cdot y_n + (1-\alpha) \cdot x_n \rangle$



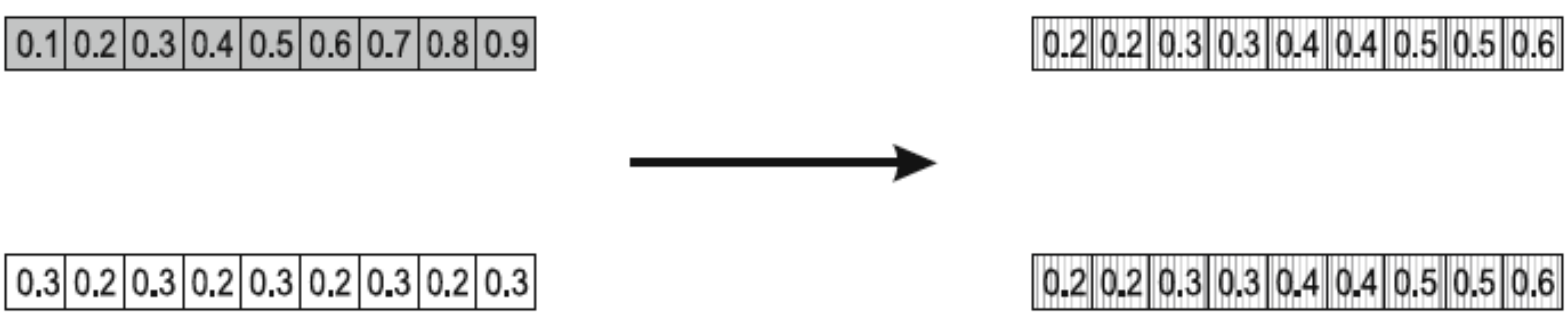
Single arithmetic crossover

- Pick a random locus *k*
- At that position, take the arithmetic average of the two parents
- Child 1: $\langle x_1, \ldots, x_{k-1}, \alpha \cdot y_k + (1-\alpha) \cdot x_k, x_{k+1}, \ldots, x_n \rangle$



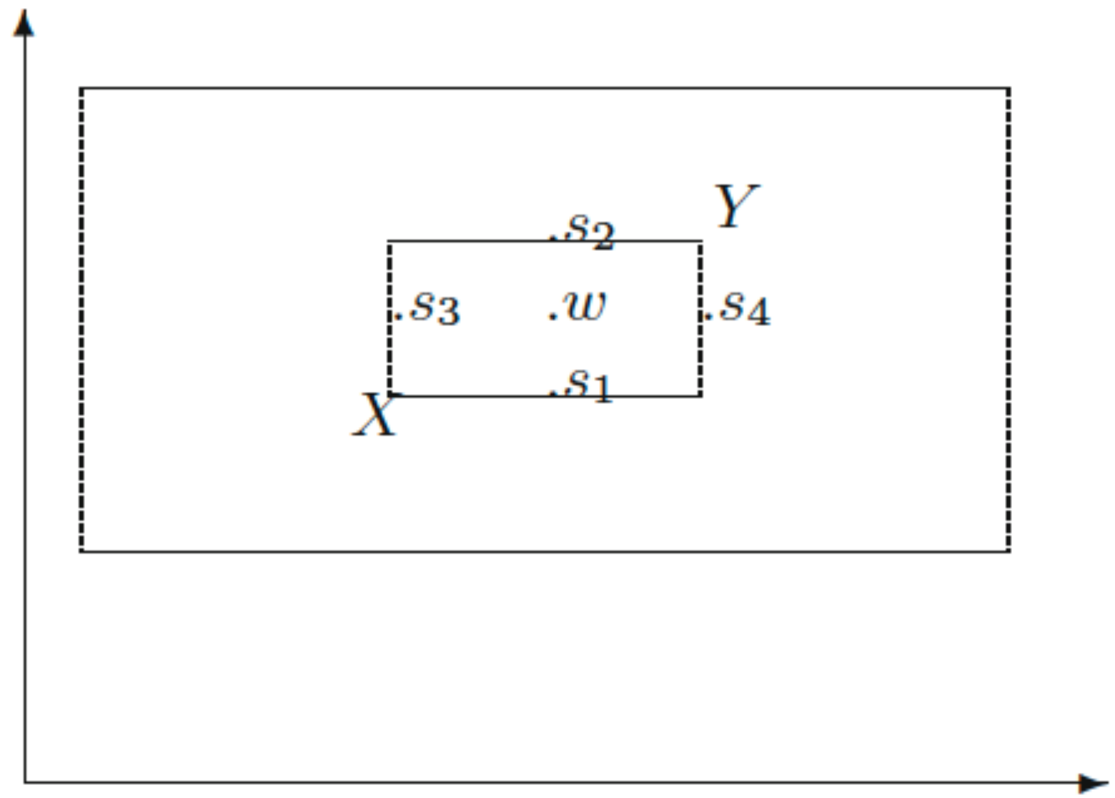
Whole arithmetic crossover

- Most commonly used operator
- Taking the weighted sum of the two parental allele for each gene
- Child 1: $\alpha \cdot \bar{x} + (1-\alpha) \cdot \bar{y}$
- Chile 2: $\alpha \cdot \bar{y} + (1-\alpha) \cdot \bar{x}$



Blend crossover

- Create offspring in a region that is bigger than the (n-dimensional) rectangle spanned by the parents
- Two parents x and y and assume that $x_i < y_i$ at position i , $d_i = y_i - x_i$
- The *i*-th value in the child z is in the range $[x_i - \alpha \times d_i, y_i + \alpha \times d_i]$



Multi-parent recombination

- Not constricted by the practicalities of nature
- Been around since 1960s, still rare but shown useful
- Three main types:
  - Based on allele frequencies, e.g. voting uniform crossover
  - Based on segmentation and recombination of the parents, e.g. generalizing *n*-point crossover
  - Based on numerical operations on real-valued alleles, e.g., center of mass crossover, generalizing arithmetic recombination operators