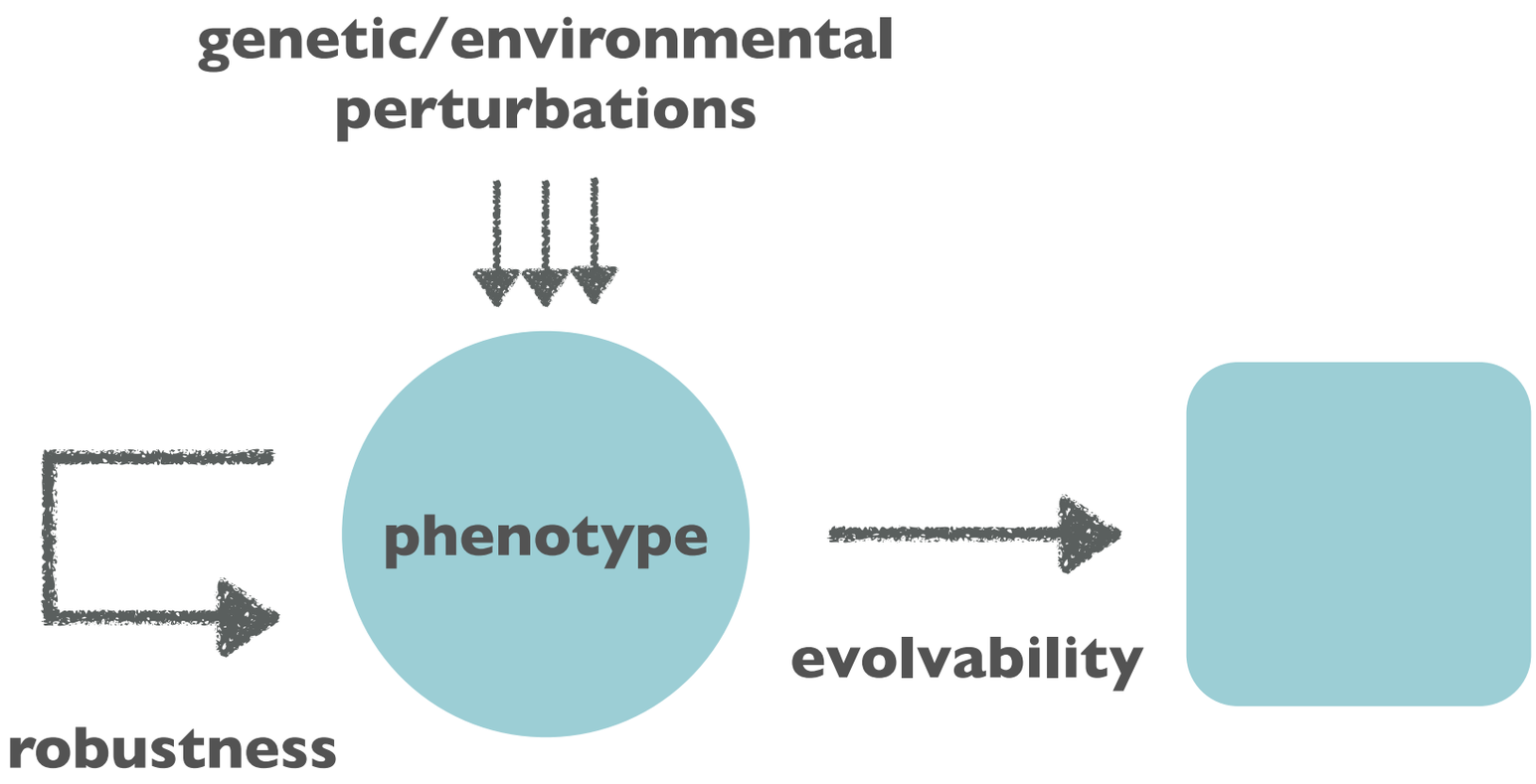
22: Genotype-Phenotype Map

- Robustness and evolvability
- Genotype to phenotype map
- Genotype network
- Quantification of robustness and evolvability
- An example study

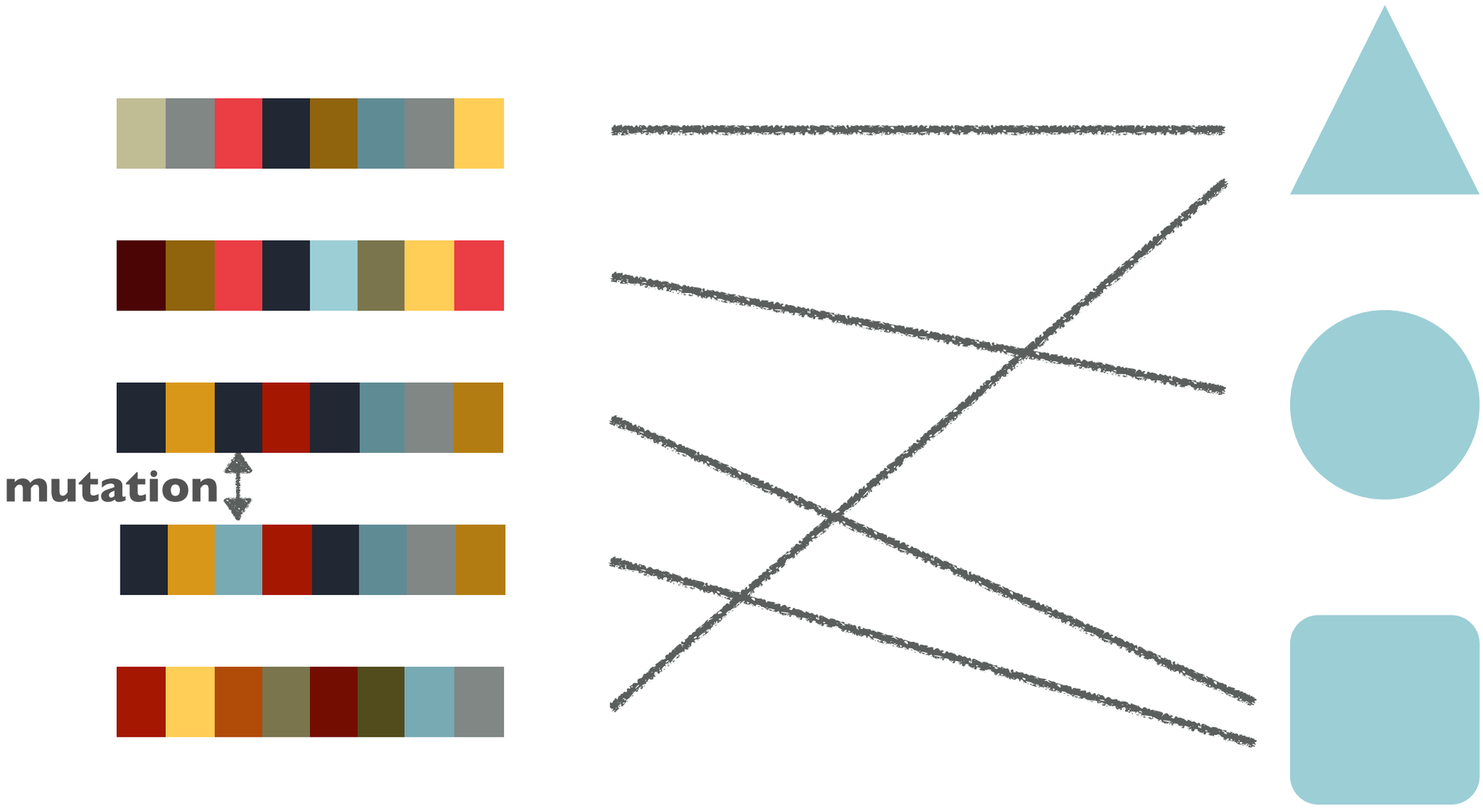
Genotype and phenotype

- In biological science
  - *phenotype*: an organism's observable characteristics
  - governs its interaction with the environment and ultimately determine its fitness
  - *genotype*: the heritable information that shapes the phenotype displayed by the individual
  - *development*: the process an organism's genotype and environment interact to determine its phenotype

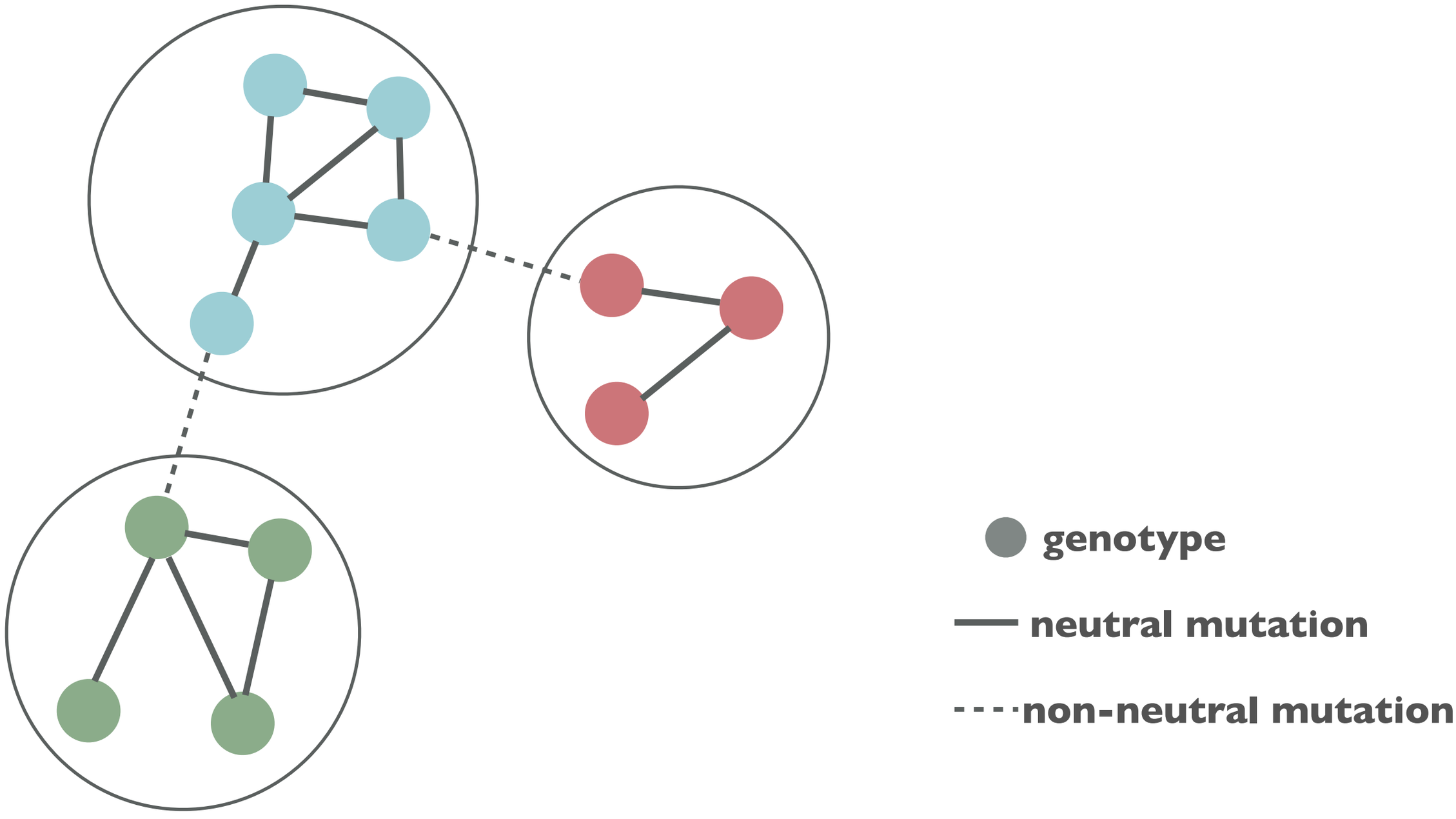
Robustness and evolvability

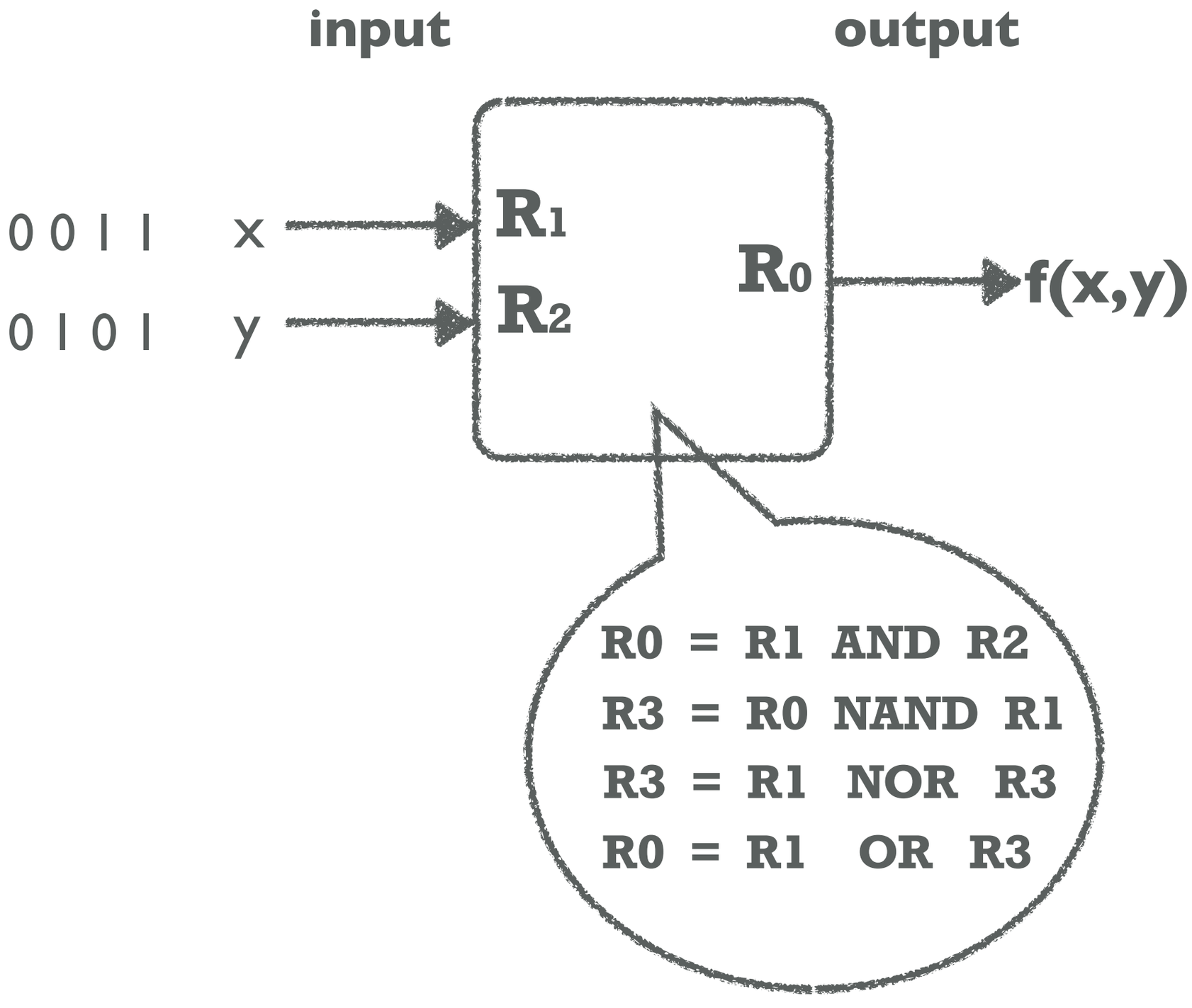


Redundancy and neutrality



Genotype network (a.k.a., neutral network)





0 0 0 0 FALSE
0 0 0 1 x AND y
...
0 0 1 1 x
...
1 0 0 1 x == y
...
1 1 1 1 TRUE

Genotype space

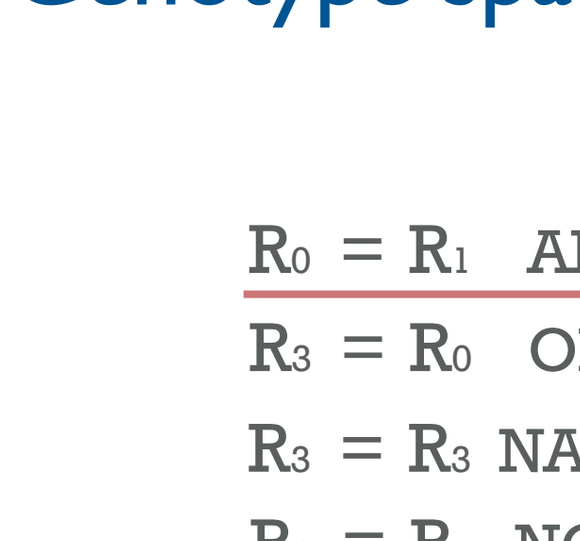
$R_0 = R_1 \ \text{AND} \ R_2$

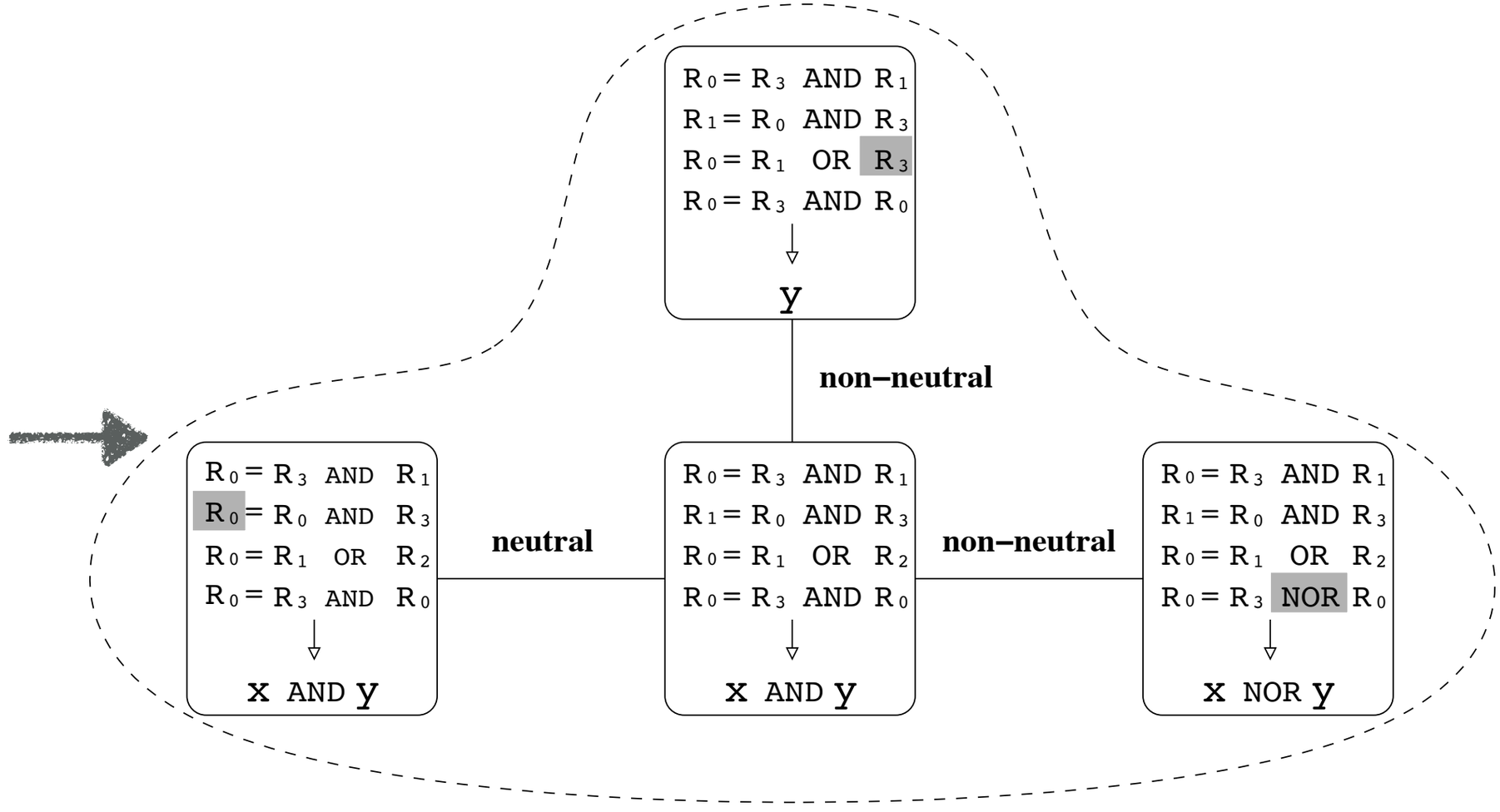
$R_3 = R_0 \ \text{OR} \ R_1$

$R_3 = R_3 \ \text{NAND} \ R_2$

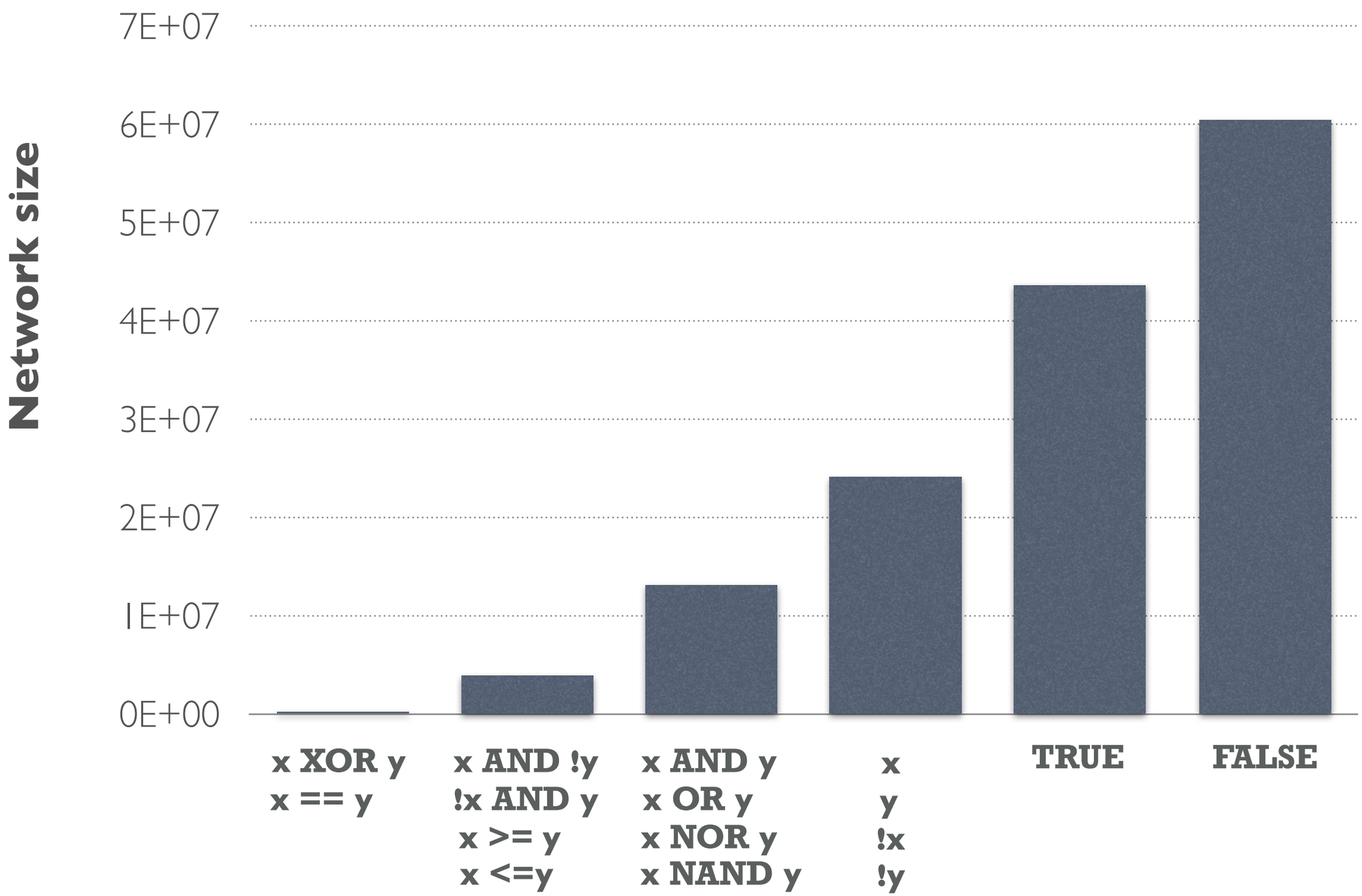
$R_0 = R_1 \ \text{NOR} \ R_3$

$(2 \times 4 \times 4 \times 4)^4 = 2^{28}$ genotypes

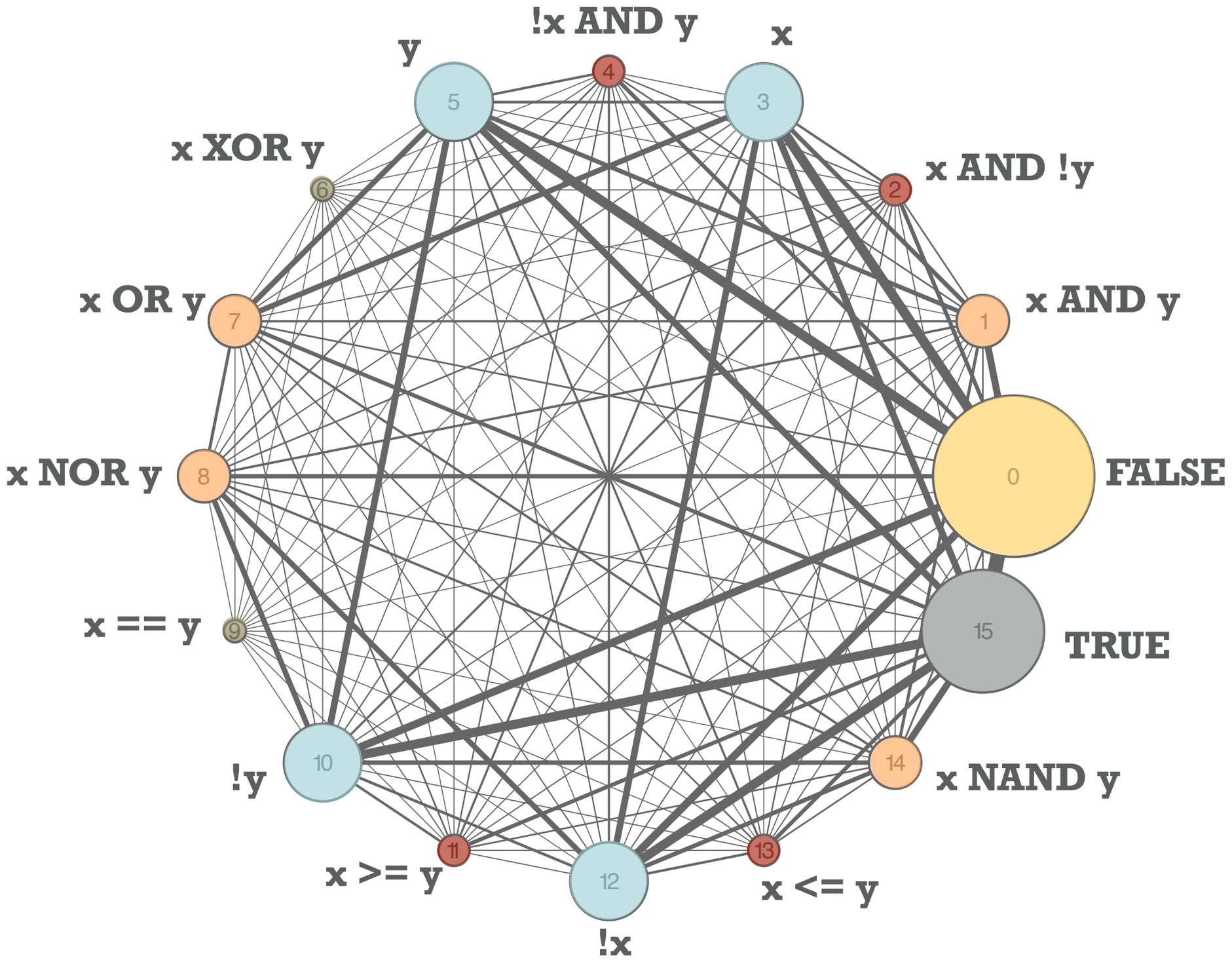




Genotype networks



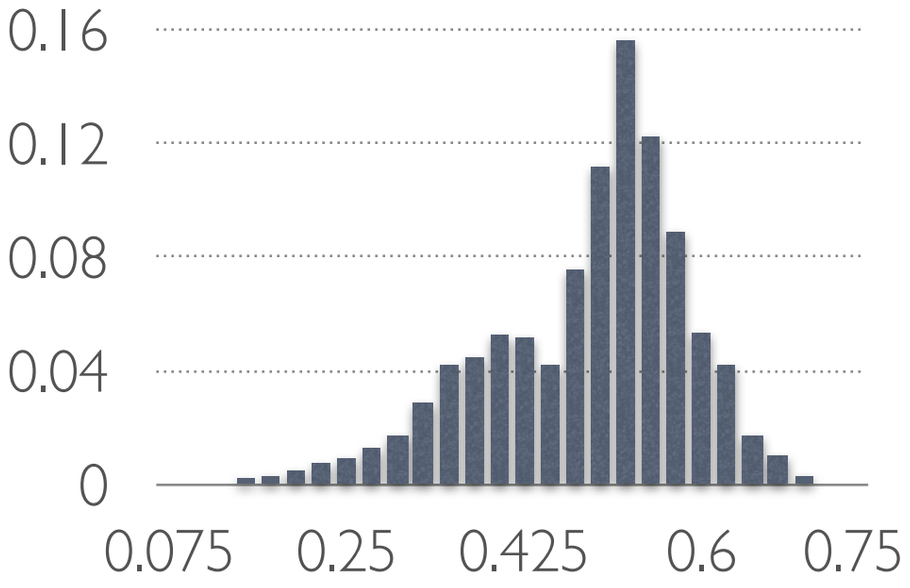
Phenotype network

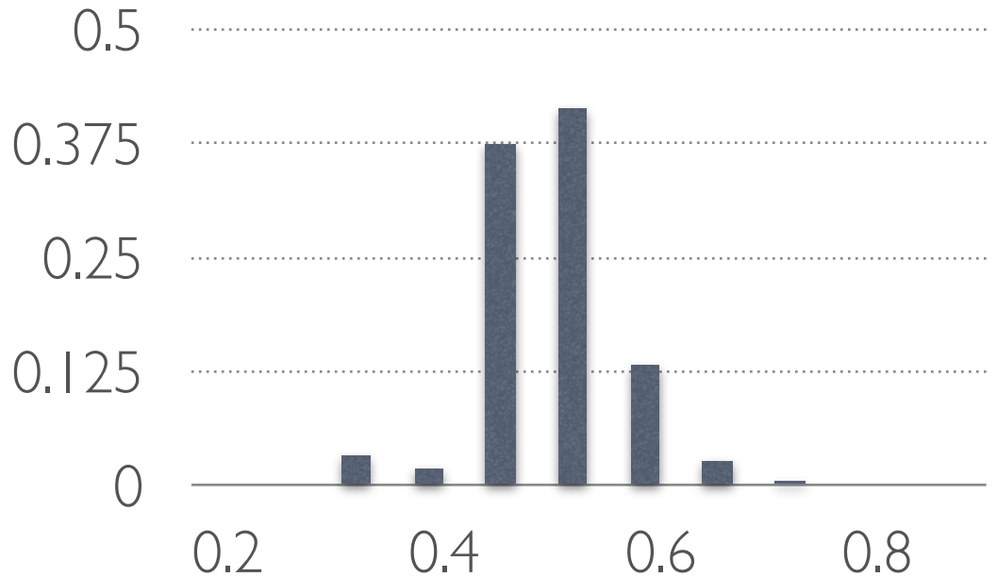


Quantification of robustness and evolvability

	Evolvability	Robustness	Accessibility
Genotype	**reachable phenotypes**	**node degree in genotype network**	
Phenotype	$E_i = 1 - \sum_j f_{ij}^2$ $f_{ij} = \frac{v_{ij}}{\sum_{k \neq i} v_{ik}}$	**genotype network size**	$A_i = \sum_j f_{ji}$

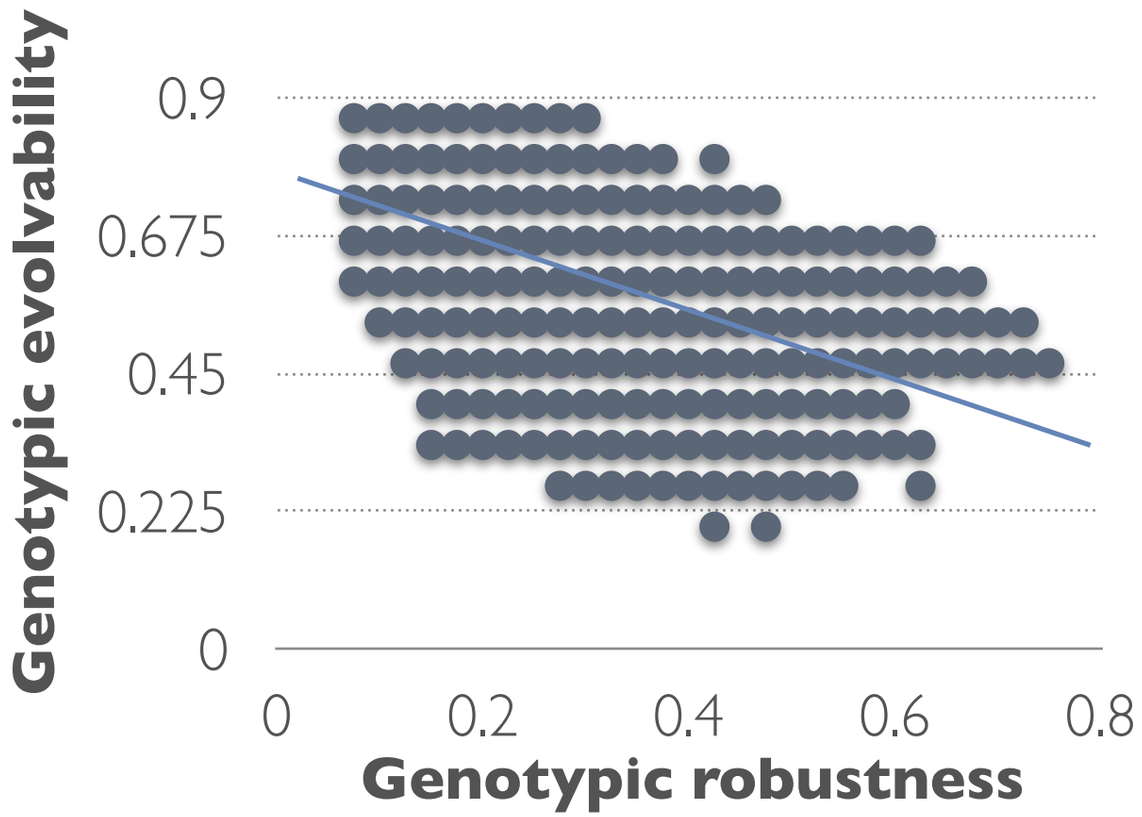
Example phenotype x >= y

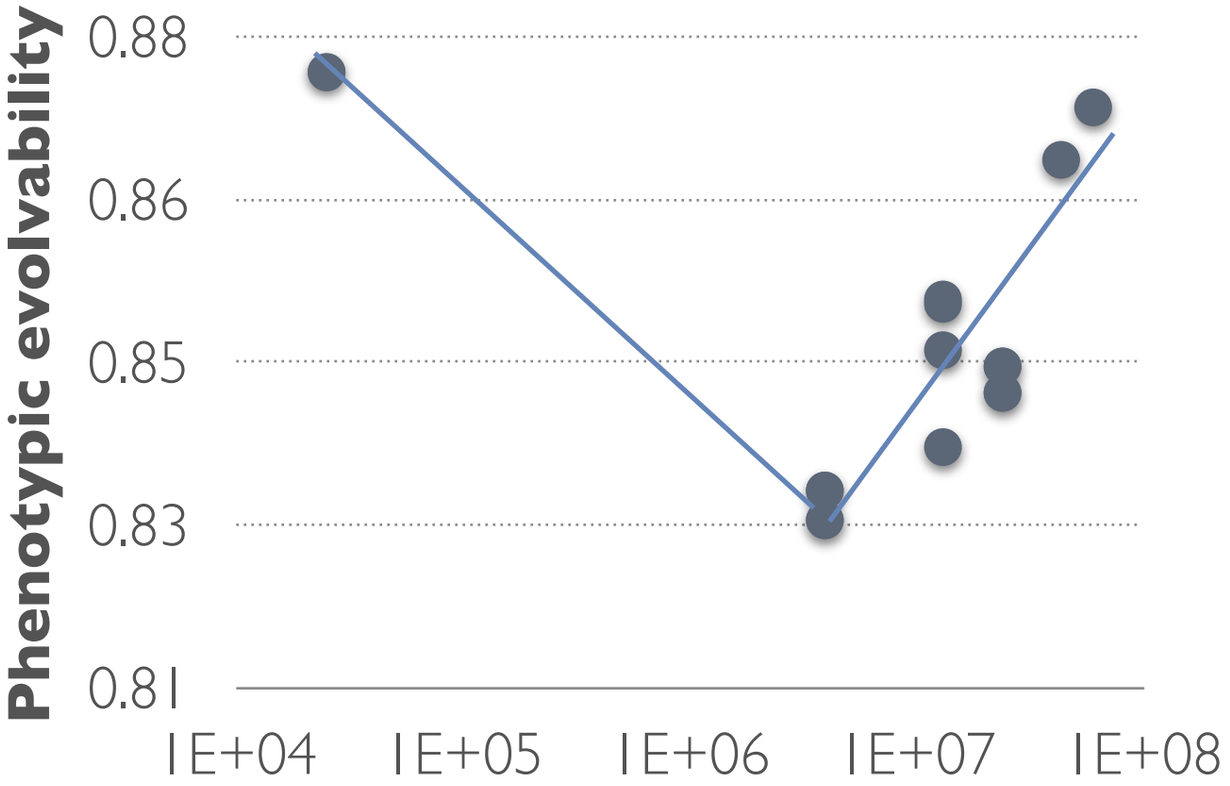


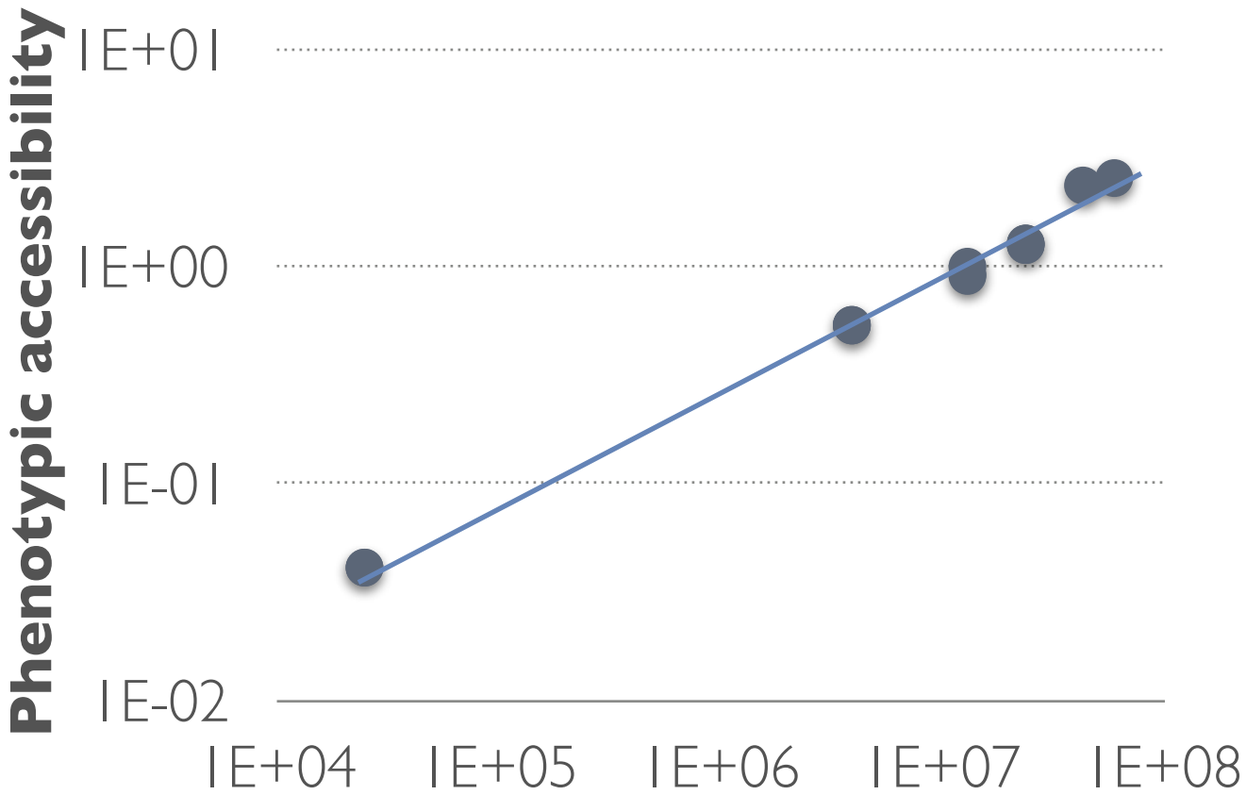


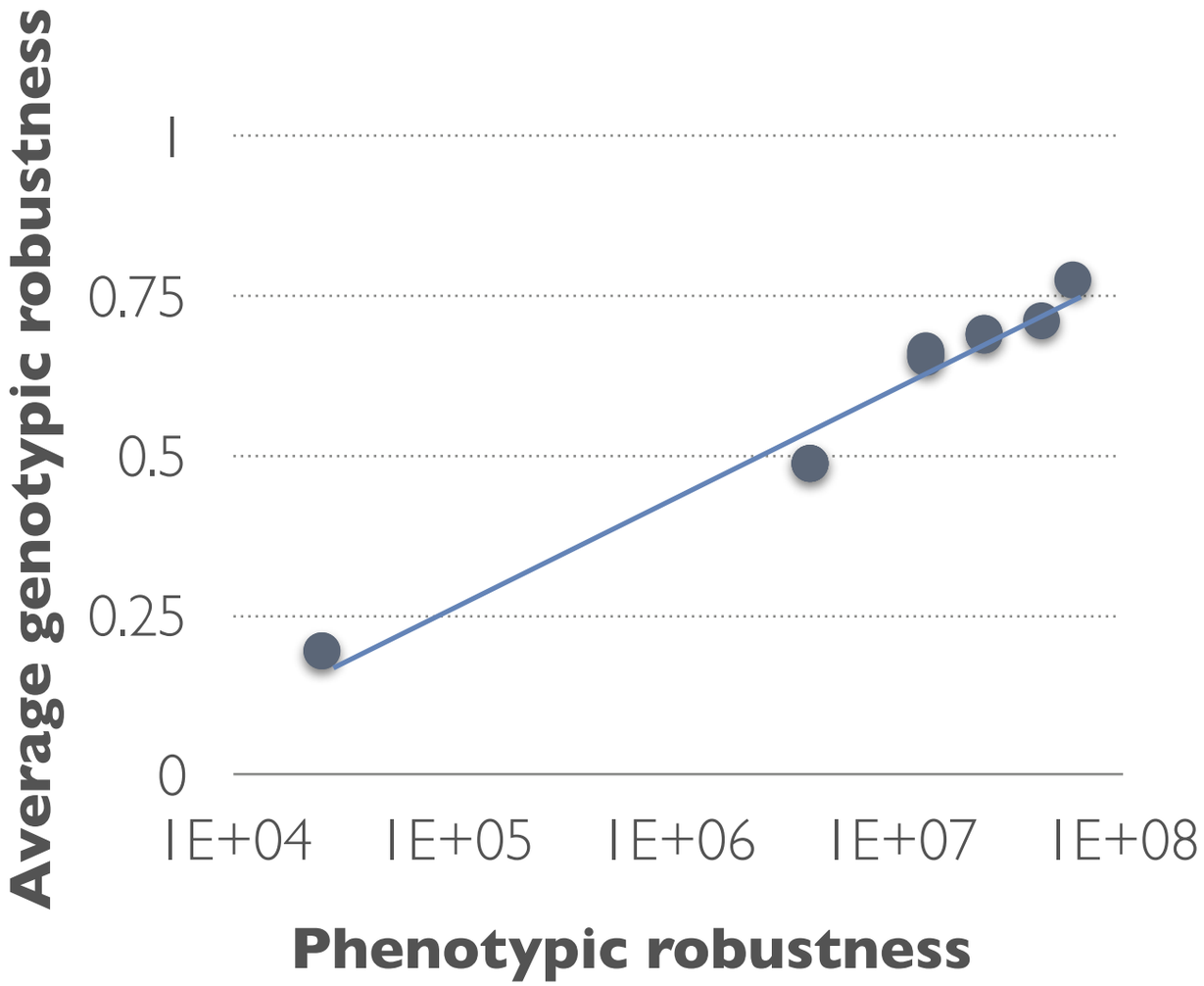
Genotypic robustness

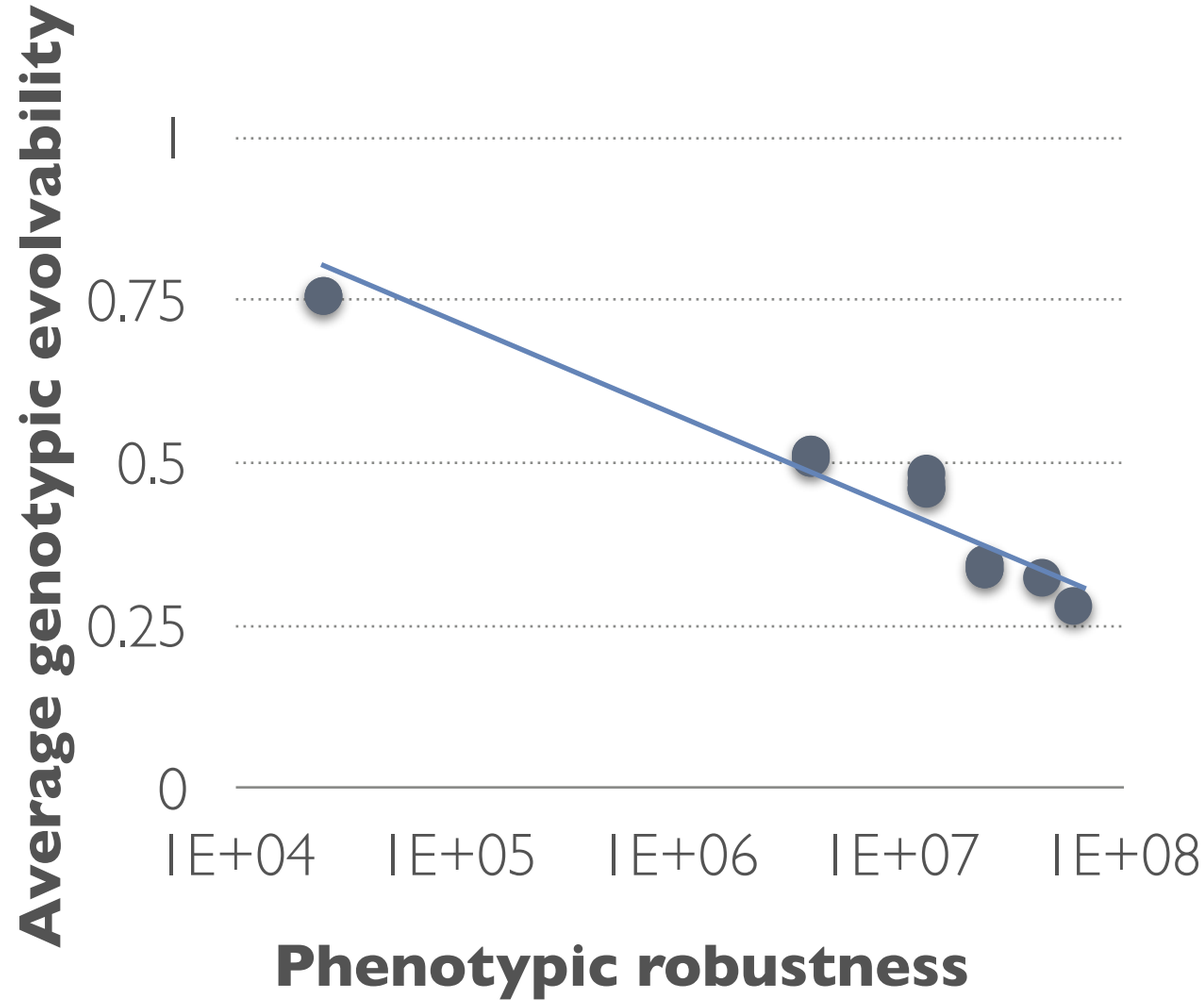
Genotypic evolvability




Phenotypic evolvability
0.88
0.86
0.85
0.83
0.81
IE+04
IE+05
IE+06
IE+07
IE+08
Phenotypic robustness


Phenotypic accessibility
IE+01
IE+00
IE-01
IE-02
IE+04
IE+05
IE+06
IE+07
IE+08
Phenotypic robustness

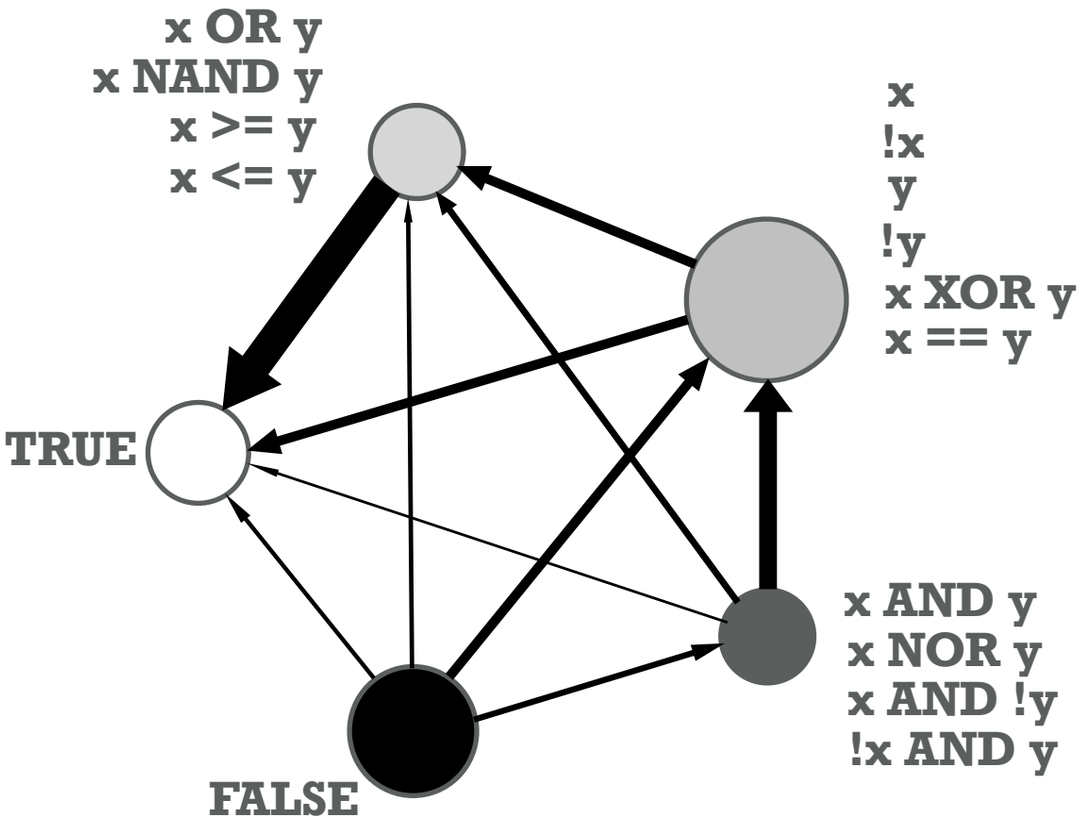

Average genotypic robustness
1
0.75
0.5
0.25
0
1E+04
1E+05
1E+06
1E+07
1E+08
Phenotypic robustness

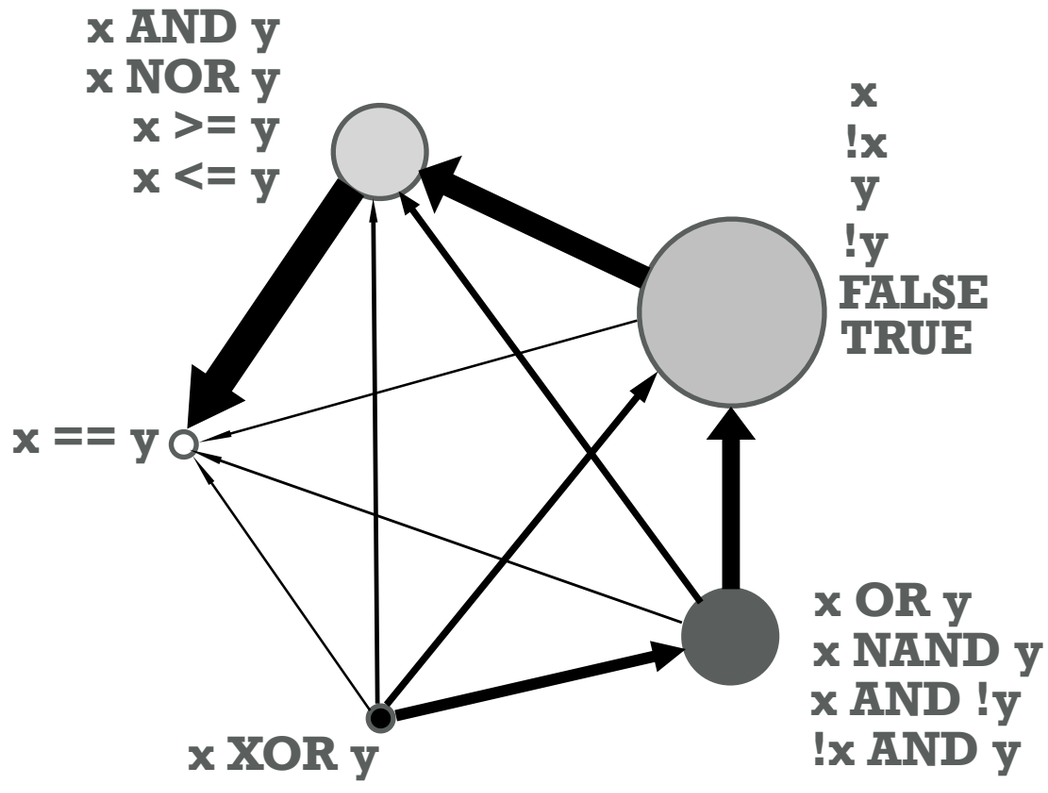

Average genotypic evolvability
1
0.75
0.5
0.25
0
1E+04
1E+05
1E+06
1E+07
1E+08
Phenotypic robustness

Genotype, phenotype, and fitness

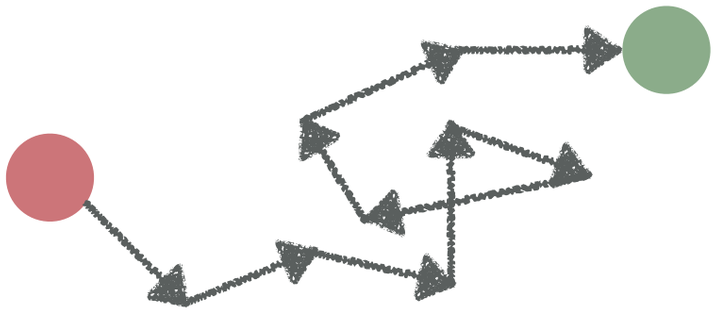
Fitness network

Target: **TRUE** **x == y**



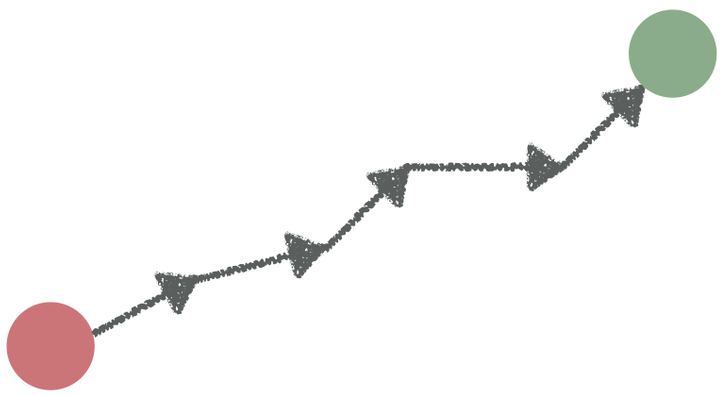


Search



Random walk

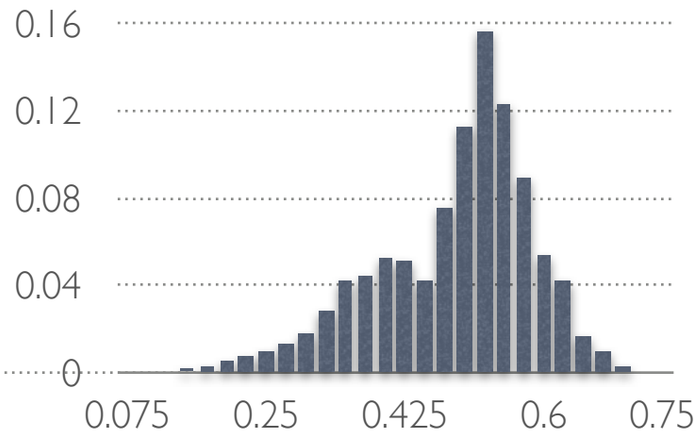
absence of fitness selection

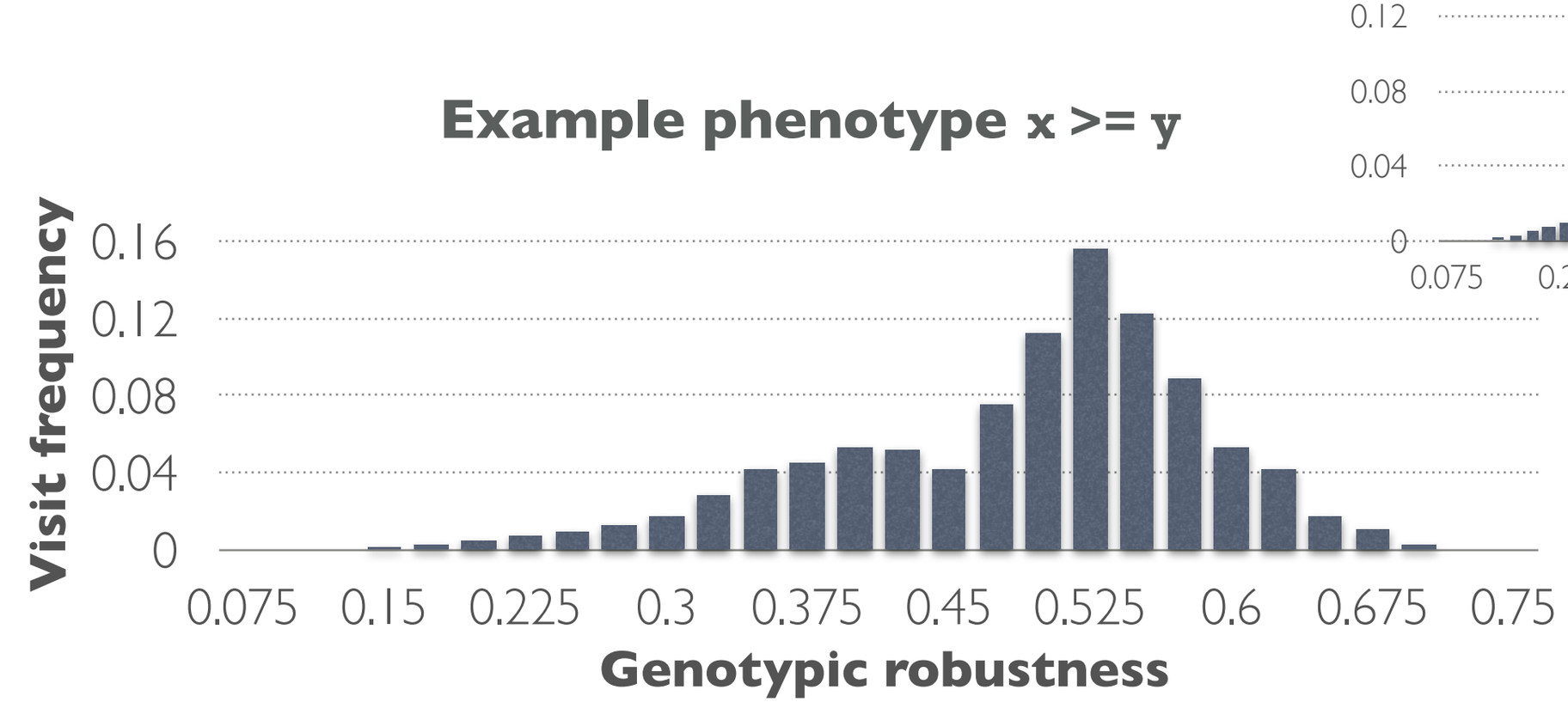


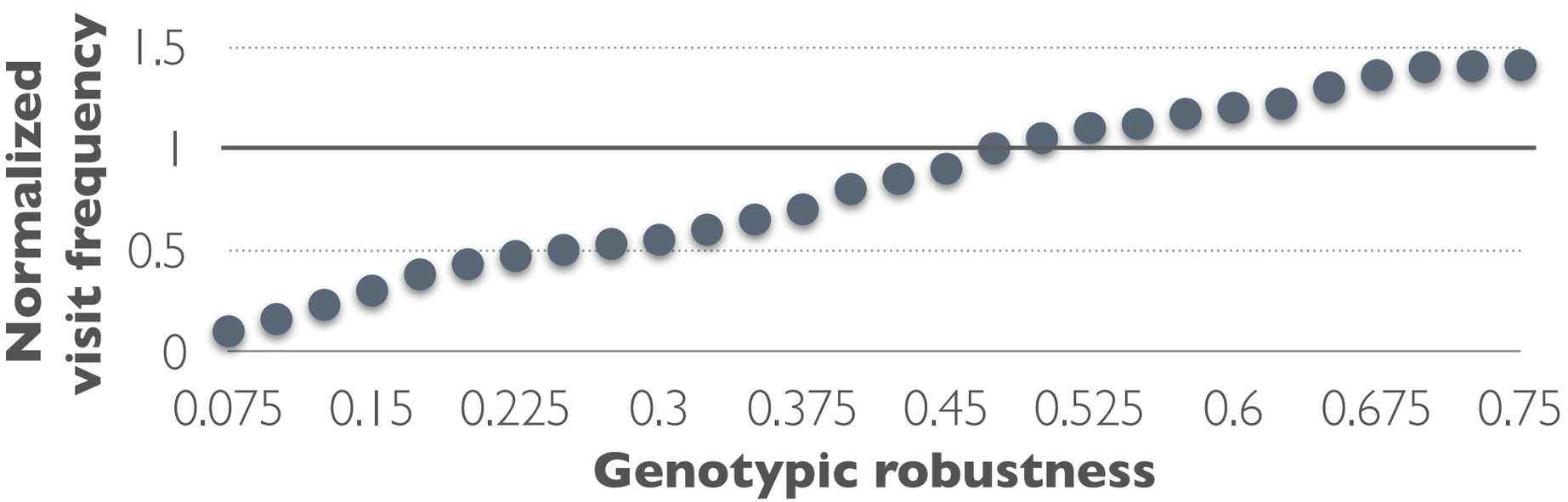
Hill climbing

replace if better or equal

Random walk within genotype network



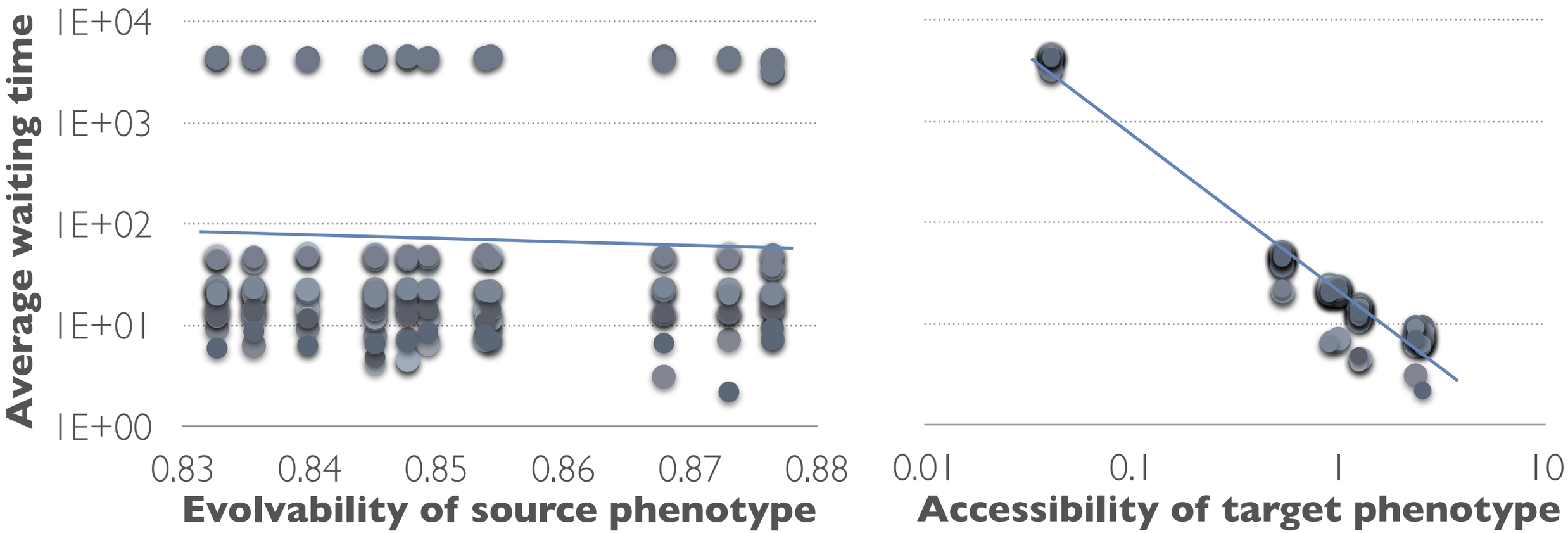




Random walk in phenotype space

$$T = \begin{matrix} \\ 16\ \textbf{source} \end{matrix} \overset{\overset{15}{\textbf{target}}}{\begin{pmatrix} \searrow & t_{0\text{-}1} & t_{0\text{-}2} & \dots & t_{0\text{-}15} \\ t_{1\text{-}0} & \searrow & t_{1\text{-}2} & \dots & t_{1\text{-}15} \\ \dots & & & & \\ t_{15\text{-}0} & t_{15\text{-}1} & t_{15\text{-}2} & \dots & \searrow \end{pmatrix}}$$

$t_{i\text{-}j}$ average waiting time



Hill climbing

$$A = \begin{matrix} \\ 16 \\ \textbf{source} \end{matrix} \overset{\overset{15}{\textbf{target}}}{\begin{pmatrix} \diagdown & a_{0\text{-}1} & a_{0\text{-}2} & \dots & a_{0\text{-}15} \\ a_{1\text{-}0} & \diagdown & a_{1\text{-}2} & \dots & a_{1\text{-}15} \\ \dots & & & & \\ a_{15\text{-}0} & a_{15\text{-}1} & a_{15\text{-}2} & \dots & \diagdown \end{pmatrix}}$$

$a_{i\text{-}j}$ average adaptation time