INDIAN INSTITUTE OF TECHNOLOGY, PATNA

End Semester Examination 2021

Time: 3 hours	Simulation Lab(MC503)	Full Marks : 50

Instructions

1. All questions are compulsory.
2. Here you are not supposed to use any R packages.

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1. Write a program to find the sum of following series upto 100 terms **(5)**

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\ldots}}}}$$

2. Plot the curve of probability when throw a dice of four faces and the probability of each faces is given by 0.1, 0.2, 0.3, 0.4 respectively, and total number of throw is 1000 times. **(5)**

3. Consider a real dataset, which are the number of revolutions in millions before for each of the 23 ball bearings in the life test and they are-

17.88	28.92	33.00	41.52	42.12	45.60	48.40	51.84	51.96	54.12	55.56	67.80
68.64	68.64	66.88	84.12	93.12	98.64	105.12	105.84	127.92	128.04	173.40	

Consider $\alpha = 0.4341,\ \sigma = 77.3300,\ \delta = 17.8800$ and try to fit this data for Generalized exponential (GE) distributions by applying the Chi-square test and the K-S test. Cumulative density function of GE distribution is given below:

$$F(x) = \begin{cases} 0, & \text{if } x < \delta \\ 1 - \left(1 - \alpha\frac{x-\delta}{\sigma}\right)^{\frac{1}{\alpha}}, & \text{if } \delta \le x \le \delta + \frac{\sigma}{\alpha} \\ 1, & \text{if } x > \delta + \frac{\sigma}{\alpha} \end{cases}$$

where, $x > 0$, $\delta > 0$, $\alpha > 0$ and $\sigma > 0$. **(10)**

4. Use the algorithms below and generate 1000 Gamma$(\alpha, 1)$ random variables.

Algorithm

For $\alpha > 1$; let $a = (2\alpha - 1)^{-1/2},\ b = \alpha - \ln 4,\ q = \alpha + 1/a,\ \theta = 4.5$ and $d = 1 + \ln\theta$;

Step I: Generate $U_1, U_2\ U(0,1)$.

Step II: Let $V = a\ln[U_1/(1-U_1)]$, $Y = \alpha e^v$, $Z = U_1^2 U_2$ and $W = b + qV - Y$.

Step III: If $W + d - \theta Z \ge 0$, return $X = Y$, otherwise go to Step IV.

Step IV If $W \ge \ln Z$, return $X = Y$, otherwise go back to Step I.

Use the following technique, to generate Gamma(α, β). If $X \sim$ Gamma$(\alpha, 1)$, then $\beta X \sim$ Gamma(α, β). Find the mean and variance of generated sample of βX. Consider, $\alpha = 2$ and $\beta = 3$. **(10)**

5. Using the transformation $Y = e^{-X}$, where X follows the two-parameter Weibull distribution with pdf is given by

$$f(x \mid \alpha\beta) = \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, \ x > 0, \ \alpha > 0, \ \beta > 0.$$

Generate 1000 sample for Y using probability integral transformation when $\alpha = 2$ and $\beta = 1.5$. Find the MLE of the density Y. Also, plot the graph of pdf and hazard rate function of Y for the any two sets of parameter in the range of Y and also, add legend. **(20)**

——————— All the Best ———————