Painter's Partition

Aggressive Cows

Holiday 11 - 15th Nov

Next Mon 13th

Next Wed 15th

1. Given N tasks, K workers. Time taken to complete i^{th} task $\rightarrow$ $A[i]$.
Find minimum time to complete all tasks.

- One task can be only done by 1 person.
- A worker can only do continuous tasks.
- Workers work parallely

$A = \langle 10, 10, 10, 10 \rangle \qquad K = 2$

ans $\rightarrow$ 20

No	Time	
1	40	
2	30	$\underbrace{10}_{P_1} , \underbrace{10, 10, 10}_{P_2}$
2	20	$\underbrace{10, 10}_{P_1} \quad \underbrace{10, 10}_{P_2}$

$A = \langle 10, 20, 30, 40 \rangle \qquad K = 2$

ans = 60

No	Time	
1	100	
2	90	$\underset{P_1}{10} \quad \underset{P_2}{20, 30, 40}$

70 10, 20 30, 40

P_1 P_2

60 10, 20, 30 40

P_1 P_2

A = < 10, 20, 30, 40 > K = 2

2 ppl available = 100 amt of work

Idea 1: Divide work equally among all painters

$$\text{work} = \frac{\text{Total work}}{k}$$

$$\text{Work} = \frac{100}{2} = 50$$

A = < 10, 20, 30, 40 > K = 3

Work = 100/3 = 33.33

N = 15

k = 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6
3	8	9	16	25	27	32	35							37

P_1 P_2

✓ 40 mins

P_1 25 P_2 25 P_3 22

✓ ~~25~~ mins

3 ... 7 | 8 ... 3 | 10 ... 4 | 7 → ×

P_1 16 P_2 18 P_3 15

20 mins

k = 3

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

P_1 26 P_2 30 P_3 15

Can we do all task in 30 mins ? (≤ 30)

✓ 30 ✓ 31 ✓ 32 ✓ 33 ✓ 34 ..

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

k = 3

P_1 P_2 P_3

Can we do all task in 10 mins ? (≤ 10)

← × 8 × 9 × 10

X

x

0 . - - - - . ↓ x ✓

x x x 4→ Y

8 9 10 ..... 28 ... 29 ..... 30 31 32 ...

$\boxed{\text{BS on time}}$

Target - Min time to paint all boards

Search space - s e

max of boards[]
lowest time to paint all board
(1 board → each painter)

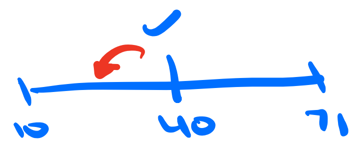
sum of boards[]
highest time to paint all boards
(1 painter has to paint all boards)

P1 P2 P3 P4 P5 . . . .

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

N = 15 K = 4



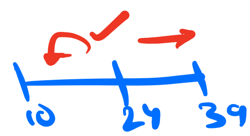
	s	e	mid		
①	10	71	40	ans = 40	left

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

N = 15

34 37

②	10	39	24	ans = 24	left

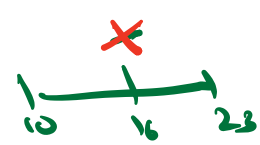


K = 4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

P_1 24 P_2 21 P_3 20 P_4 6

③	10	23	16	right

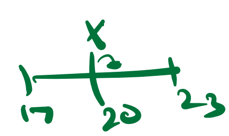


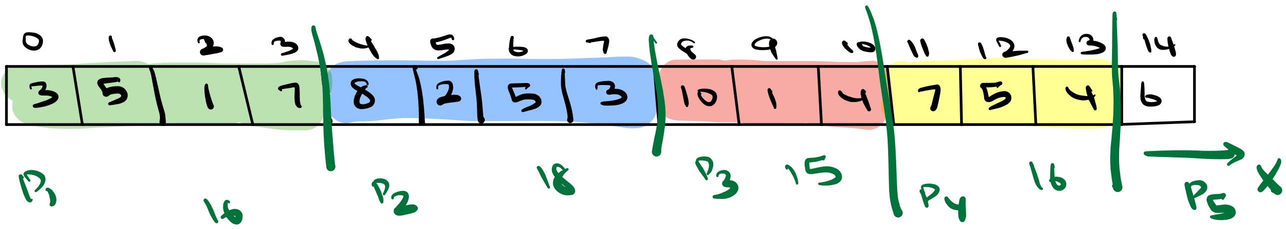
K = 4

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	5	1	7	8	2	5	3	10	1	4	7	5	4	6

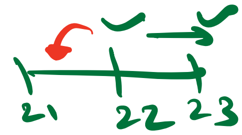
P_1 16 P_2 15 P_3 13 P_4 16 P_5 X

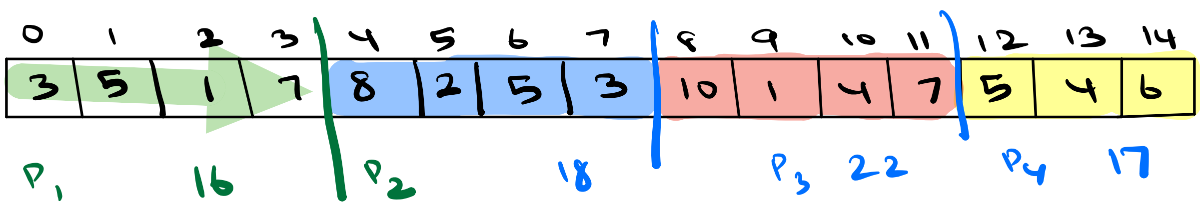
④	17	23	20	X	right



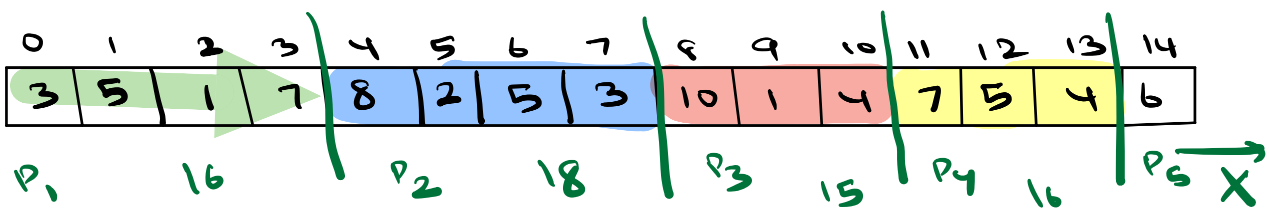


5 21 23 22 ans = 22 left





6 21 21 21 X right



7 22 21 Break

e<s

ans = 22

```
int  time ( int  boards[ ], int  n,  int  k) {
        // BS  on  time
    s =   maxEle (arr)
    e =   sum  (arr)

    while  ( s  < = e) {
        mid  =  (s + e)
                  -----
                    2

        // checking  if  all  jobs  can
           be  completed  in  ≤ mid  time
        if  ( check (mid , boards, n , k)) {
                ans  = mid
                e = mid - 1        // left
        }

        else  {
            s = mid + 1            // right
        }
    }
    return  ans
}
```

```
bool check (int m, int boards[], int n,
                                  int k){

    cur = 1        work = 0

    for (i=0; i<n; i++){
        if (work + board[i] ≤ m){
            work = work + board[i]
        }

        else {
            cur++        work = board[i]
        }
                                        k=4
    }

    if (curr > k)
            return false

    else
            return true
}
```

TC → log (range) × time taken for feasibility check

TC = O(log (sum(ele) - max(ele)) × N)
check fn

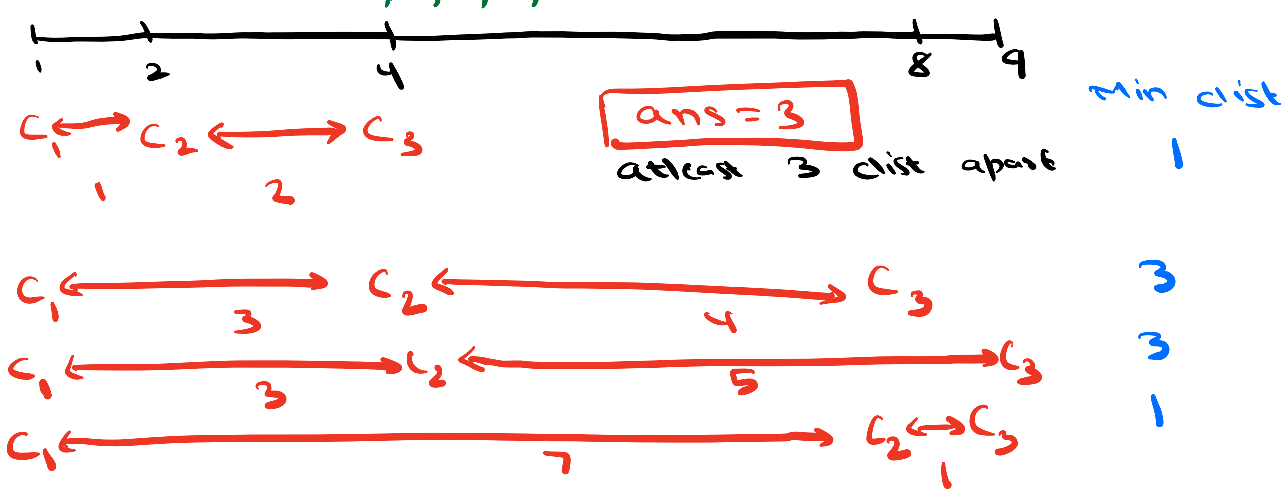
Break → 10:30

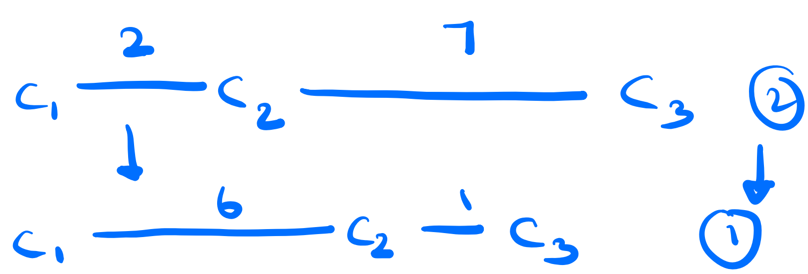
1. Given N stalls, and M cows. $N \geq M$
   Position of each stall is given in
   A[N] in asc order.
   Maximise the closest distance b/w cows.
   min

1 stall → 1 cow

$N = 5$ $M = 3$

<1, 2, 4, 8, 9>





$N = 9, \ M = 4$

$\boxed{ans = 12}$

0	1	2	3	4	5	6	7	8
2	6	11	14	19	25	30	39	43

min dist

$C_1 \xleftrightarrow{4} C_2 \xleftrightarrow{5} C_3 \xleftrightarrow{3} C_4$ — 3

$C_1 \xleftrightarrow[9]{} C_2 \xleftrightarrow[8]{} C_3 \xleftrightarrow[11]{} C_4$ — 8

$C_1 \xleftrightarrow[12]{} C_2 \xleftrightarrow[16]{} C_3 \xleftrightarrow[13]{} C_4$ — 12

✗ $C_1 \xleftrightarrow[17]{} C_2 \xleftrightarrow[20]{} C_3 \xleftrightarrow[<13]{\times} C_4$ — 13

$M = 3$

1 — 2 — 4 — 8 — 9

min dist

cnt = 2 $\quad C_1 \xleftrightarrow[8]{} C_2$ — 8

cnt = 3 $\quad$ 1 — 5 — 9 $\quad C_1 \xleftrightarrow{4} C_3 \xleftrightarrow{4} C_2$ — 4

stall at 5 ?

BF → $\dfrac{\text{Total dist}}{\text{cows}}$ ✗

$\boxed{BS \to dist}$

N = 9   M = 4

0	1	2	3	4	5	6	7	8
2	6	11	14	19	25	30	39	43

$C_1 \xleftrightarrow{23} C_2$

$C_1 \xleftrightarrow{9} C_2 \xleftrightarrow{8} C_3 \xleftrightarrow{6} C_4$

× 20 × 21 × 22 × 23

check if we can place all cows atleast 20 dist apart?

dist ≥ 20

✓ 3 ✓ 4 ✓ 5

check if we can place all cows atleast 5 dist apart?

dist ≥ 5

15 → × 16

✓ 3 ✓ 4 ✓ 5 ✓ 6 ✓ ...... × 18 × 19 × 20 × 21 × 22 ...

Target → Min dist b/w any 2 cows should be max

Max closest dist

Search space →

s ↓ min dist b/w any 2 adjacent stalls

e ↓ Max dist of closest cows

A[N-1] - A[0]

N=M

s=1

e=

1 4 8 9

$C_1 \xleftrightarrow{3} C_2 \xleftrightarrow{4} C_3 \xleftrightarrow{1} C_4$

N=9 M=4

0	1	2	3	4	5	6	7	8
2	6	11	14	19	25	30	39	43

4 5 3 5 6 5 9 4

	s	e	mid	
1	3	41	22	left

3 22 41

can you place cows atleast 22 dist apart?

0	1	2	3	4	5	6	7	8
2	6	11	14	19	25	30	39	43

$C_1 \xleftrightarrow{23} C_2$

M=4

2	3	21	12	ans=12 right

3 12 21

0	1	2	3	4	5	6	7	8
2	6	11	14	19	25	30	39	43

$C_1 \xleftrightarrow{12} C_2 \xleftrightarrow{16} C_3 \xleftrightarrow{13} C_4$

3	13	21	17	X left

13 17 21

0	1	2	3	4	5	6	7	8
2	6	11,	14	19,	25	30,	39,	43

$c_1 \xleftrightarrow[17]{} c_2 \xleftrightarrow[20]{} c_3$

Not able to place 4th cow

(4) 13 16 14 left ✗

dist ≥ 14

✗ ✗ ✗

13 14 16

0	1	2	3	4	5	6	7	8
2	6	11,	14	19,	25	30,	39,	43

$c_1 \xleftrightarrow[17]{} c_2 \xleftrightarrow[20]{} c_3$

(5) 13 13 13 ✗ left

✓

13 13 13

0	1	2	3	4	5	6	7	8
2	6	11,	14	19,	25	30,	39,	43

$c_1 \xleftrightarrow[17]{} c_2 \xleftrightarrow[20]{} c_3$

(6) 13 12 break

$\boxed{\text{ans} = 12}$

NlogN { [Sort stalls]

```
int maxdist (int dist[], int N, int M) {
N {  s = min( dist[i+1] - dist[i]) ∀i
     e = dist[N-1] - dist[0]

     while (s <= e) {
         mid = (s+e)/2        // s + (e-s)/2
         // can we place cows atleast mid dist apart?
         if (check(mid, dist, n, m)) {
                 ans = mid
                 s = mid+1          // right
         }
         else {
                 e = mid-1          // left
         }
     }
     return ans
}
```

```
bool check (int mid, int dist[], int n, int m){
    cur = 1        position = dist[0]
    for (i=1 ; i < n ; i++) {
        if (dist[i] - position >= mid) {
            cur++
            position = dist[i]
            if ( cur == m)
                    return true
        }
    }
    return false
}
```

TC : $O\left(\log \left(\text{max diff b/w ele} - \text{diff b/w any 2 adjacent ele} \right) \times N \right)$

$\log(s-e) + N + N\log N$

int s

int e

$$\text{int mid} = \frac{s+e}{2} \qquad \frac{10^8 + 10^9}{2}$$

overflow

$$\text{mid} = s + \frac{(e-s)}{2}$$

$$s + \frac{e}{2} - \frac{s}{2}$$

$$\frac{s}{2} + \frac{e}{2} = \frac{(s+e)}{2}$$

---

paint all boards)

place all cows

Task to do

1-2 constraint

① K painters

M cows

Minimize or Maximize

(2) BS → Monotonic search space

Monotonicity - come up with a condn where part of the search satisfies condn and other part doesn't satisfy condn, we can use BS to find pivot point.

F F F F F | T T T T T T T T

T T T T T T T | F F F F

(1)

1	2	4	6	8	10	20	30	50
T	T	T	T	F	F	F	F	F

$arr[i] \leq 6$

(2)

4	8	2	1	6	30	20	10	50
T	F	T	T	T	F	F	F	F

$arr[i] \leq 6$

8 9 10 11 12 ... ~~x~~ 30 31 32 32

F F F F F ... T T T T T T

X X X

F F F F F F . F T T T T

Most recent

↓

Least recent

1 Questions Assignment

↓

2 Notes