

Count Sort

Stable & In place sorting

Merge 2 sorted arrays

Merge sort

Inversion Count

Q. Find smallest no. that can be formed by rearranging digits given in an array.

$$A = \langle 6, 3, 4, 2, 7, 2, 1 \rangle$$

$$\begin{matrix} \langle 1, 2, 3 \rangle \\ \downarrow \\ 123 \end{matrix}$$

$$A = \langle 1, 2, 2, 3, 4, 6, 7 \rangle$$

$$A = \langle 4, 2, 7, 3, 9, 0 \rangle$$

$$|A| = N$$

$$A = \langle 0, 2, 3, 4, 7, 9 \rangle$$

Obs \rightarrow sort the digits in array

\downarrow
smallest no.

Approach : Arrays. sort() \rightarrow $TC: O(N \log N)$

\downarrow

$TC: O(N)$

Approach 2 : Digit \rightarrow 0 to 9

$$A = \langle 6, 3, 4, 2, 7, 2, 1 \rangle$$

0000 111 222 3...4...5...

Maintain freq of digits

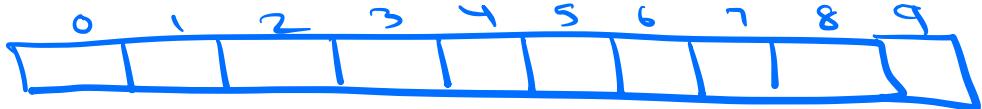
① hm < int, int > hm

\downarrow \downarrow
digit freq

② Fixed sized arr digits 0→9

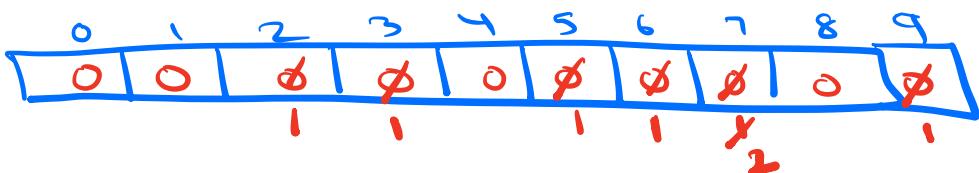
int freq[10]

digit & idx



$A[] = \langle 7, 2, 3, 9, 5, 6, 7 \rangle$

freq



$A[] = \langle 2, 3, 5, 6, 7, 7, 9 \rangle$

Count sort → method of sort based
on frequency counting

// code

void smallestno (int a[], int N){

int freq[10] = {0};

for (i=0 ; i<N ; i++) { → N

| freq[a[i]] ++

// iterate the freq arr

int idx = 0

→ N

```
for (dig = 0 ; dig <= 9 ; dig++) {
```

```
    for (cnt = 1 ; cnt <= freq[dig] ; cnt++) {
```

```
        A[idx] = dig  
        idx++
```

7

TC: O(N)

SC: O(1)

↓
10

dig

0

1

2

⋮

9

freq[dig]

cnt

f₀

f₁

f₂

f₃

⋮

f₉

N

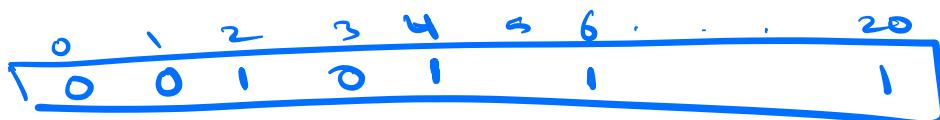
$$f_0 + f_1 + f_2 + f_3 + \dots + f_9 \\ = N$$

Count Sort

A = [4, 6, 2, 11, 13, 20, 14, 15]

freq[21]

idx → 0 to 20



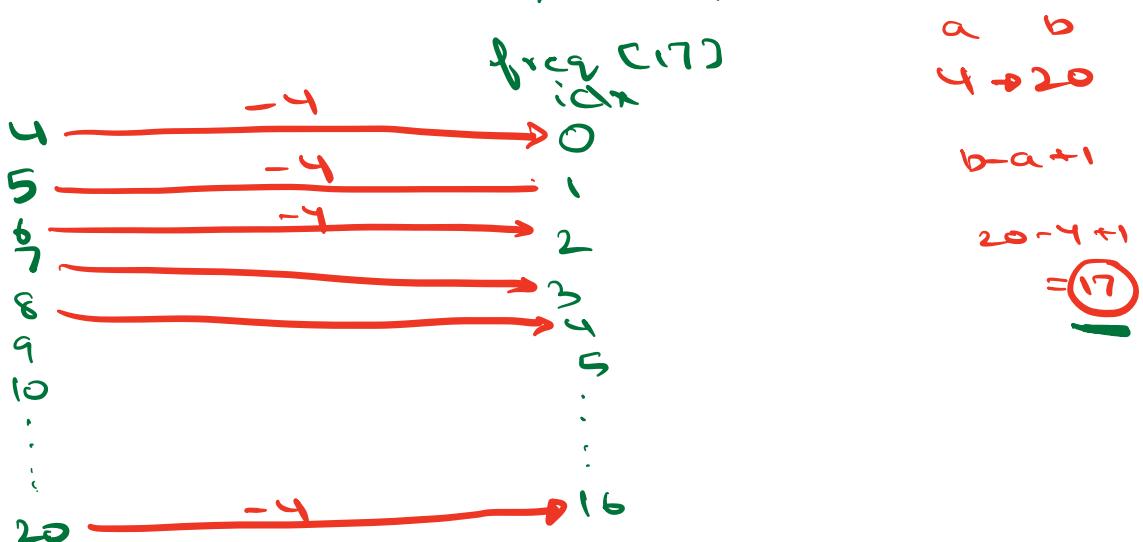
$\text{freq}[\max(\text{ele}) + 1]$ (if 0 or
the ele in
 $A[3]$)

$$A = [100, 300, 600, 10000]$$

$\text{freq}[100001]$



$$A = [-4, 6, 8, 20, 15]$$



$\text{freq}[17]$

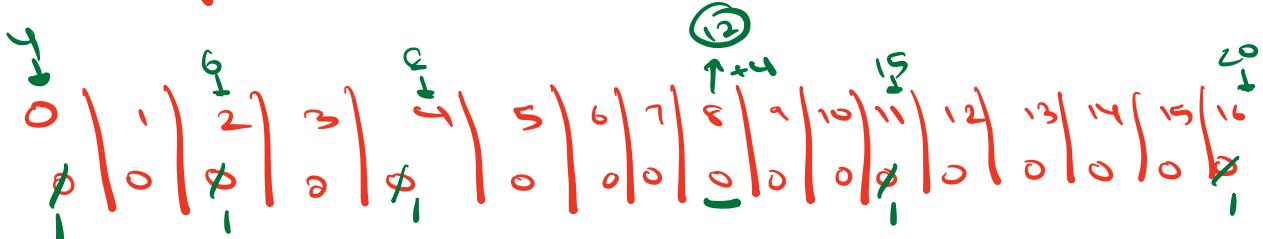
$\text{dig} - \text{min} \rightarrow \text{idx}$

① int freq[max-min+1]

② ele info \rightarrow ele-min = idx

$A = [4, 6, 8, 20, 15]$

$\text{freq}[20-4+1] \rightarrow \text{freq}[17]$



$de - \min \rightarrow \text{idt}$

$de \leftarrow \text{idt} + \min$

If range of A[i] is $0 - 10^5$

int freq[10^5] ✓

int freq[10^6] ✓

$$10^6 \times 4 \text{ B} = 4 \text{ MB}$$

If range of data is $0 - 10^9$

int freq[10^9] X

↓ ✓

1B 10^9 integers

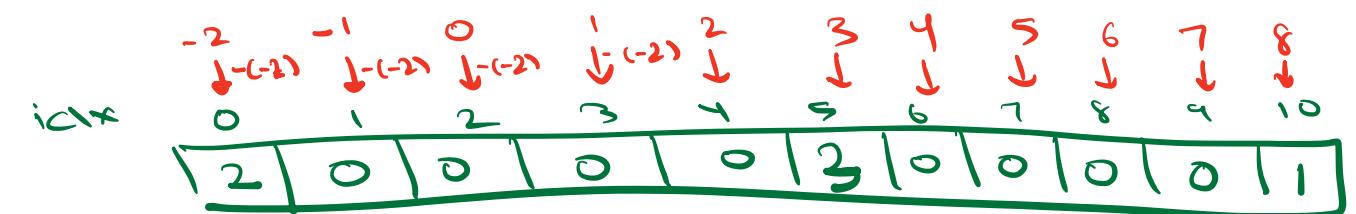
$$10^9 \times 1 \text{ B} = 1 \text{ GB} X$$

MLE occurs

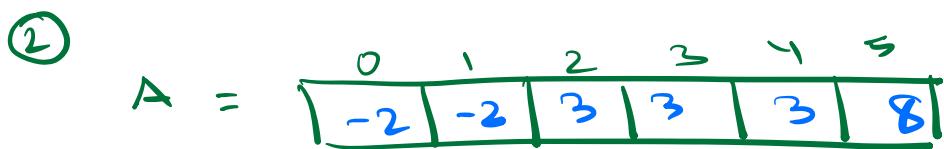
Count sort on -ve nos.

$$A = [-2, 3, 8, 3, -2, 3] \quad \begin{array}{l} \text{Min} \\ \text{Max} \end{array}$$

$-2 \rightarrow 8$
 $\max - \min + 1$
 $8 - (-2) + 1$



$\text{ele} - \min \rightarrow \text{idx}$
 $\text{ele} \leftarrow \text{idx} + \min$



```

void smallestno (int arr[], int N):
    // get max & min → N
    int freq[max-min+1] = {0}
    for (i=0 ; i<N ; i++) < → N
        freq [arr[i]-min] ++
    // iterate the freq arr

    int idx = 0
    for (i = 0 ; i < max-min+1 ; i++) < → N
        for (cnt=1 ; cnt ≤ freq[i] ; cnt++) <
            A [idx] = i + min
            idx++
    }

    TC : O(N)
    SC : O(R)
    ↓
    freq [] size → range of
    original arr []

```

Stable sort

If 2 data points have same parameter value, we maintain original order.

Name	Marks	stable sort
Amrit	40	Harshal
Chitra	25	Kalu
Tushar	20	Tushar
Veena	25	Chitra
Kal	16	Veena
Harshal	12	Saswata
Saswata	30	Amrit

Harshal	12	
Kalu	16	
Tushar	20	
Veena	25	unstable sort
Chitra	25	
Saswata	30	
Amrit	40	

Ex 2	7, 2, 3, 4, 2, 8, 1	Sorting based on value
Stable sort	1, 2, 2, 3, 4, 7, 8	
Unstable sort	1, 2, 2, 3, 4, 7, 8	

Inplace (No extra space / SC: O(1))

Q. Given 2 sorted arrays of size N & M , merge and return new sorted array.

⑦ $A = [2, 5, 7, 12, 20, 24, 29]$

⑥ $B = [6, 9, 10, 14, 18, 19]$

⑬ $C = [2, 5, 6, 7, 9, 10, 12, 14, 18, 19, 20, 24, 29]$

$N \ A \rightarrow [\quad]$

$M \ B \rightarrow [\quad]$

$N+M \ C \rightarrow [\quad \quad \quad \quad]$
↓
sort

Approach 1 : copy all ele of A and B into C and then sort C .

① copy $\rightarrow N+M$

② sort $C \rightarrow N+M \log(N+M)$

TC: $O(N+M \log(N+M))$

SC: $O(\text{sorting algo})$

Approach 2:

$$A = [\boxed{2}, \boxed{5}, \boxed{7}, \boxed{12}, \boxed{20}, \boxed{24}, \boxed{29}] \quad \textcircled{7}$$

$$B = [\boxed{6}, \boxed{9}, \boxed{10}, \boxed{14}, \boxed{18}, \boxed{19}] \quad \textcircled{6}$$

$$C = [\begin{matrix} & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ & 2 & 5 & 6 & 7 & 9 & 10 & 12 & 14 & 18 & 19 & 20 & 24 & 29 \end{matrix}]$$

$\nwarrow_{N+M} \qquad T_C: O(M+N) \qquad SC: O(1)$

`int C[] merge (int A[], int B[], int N, int M) {`

```
    int C[N+M]
    int i=0, j=0, k=0
```

```
    while (i < N && j < M) {
```

```
        if (A[i] ≤ B[j]) {
```

```
            C[k] = A[i]
```

```
            k++
            i++
```

```
        } else { // A[i] > B[j]
```

```
            C[k] = B[j]
```

```
            k++
            j++
```

// loop will end if i reaches
end of A or j reaches end of B

while ($i < N$) & // check if A is
remaining

$C[k] = A[i]$

$k++ \quad i++$

while ($j < M$) & // check if B is
remaining

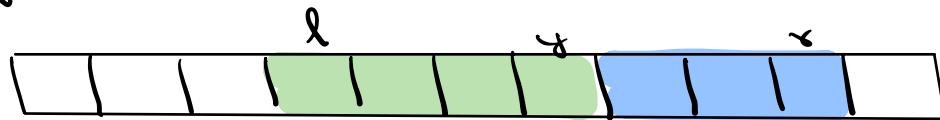
$C[k] = B[j]$

$k++ \quad j++$

7

10:46

Q. Given an array of size N and 3 indices l, y, r . In the array, subarray from $l-y$ and $y+1-r$ are sorted. Sort subarray from $l-r$



$A =$	0	1	2	3	4	5	6	7	8	9
	8	1	3	6	11	2	4	9	7	6

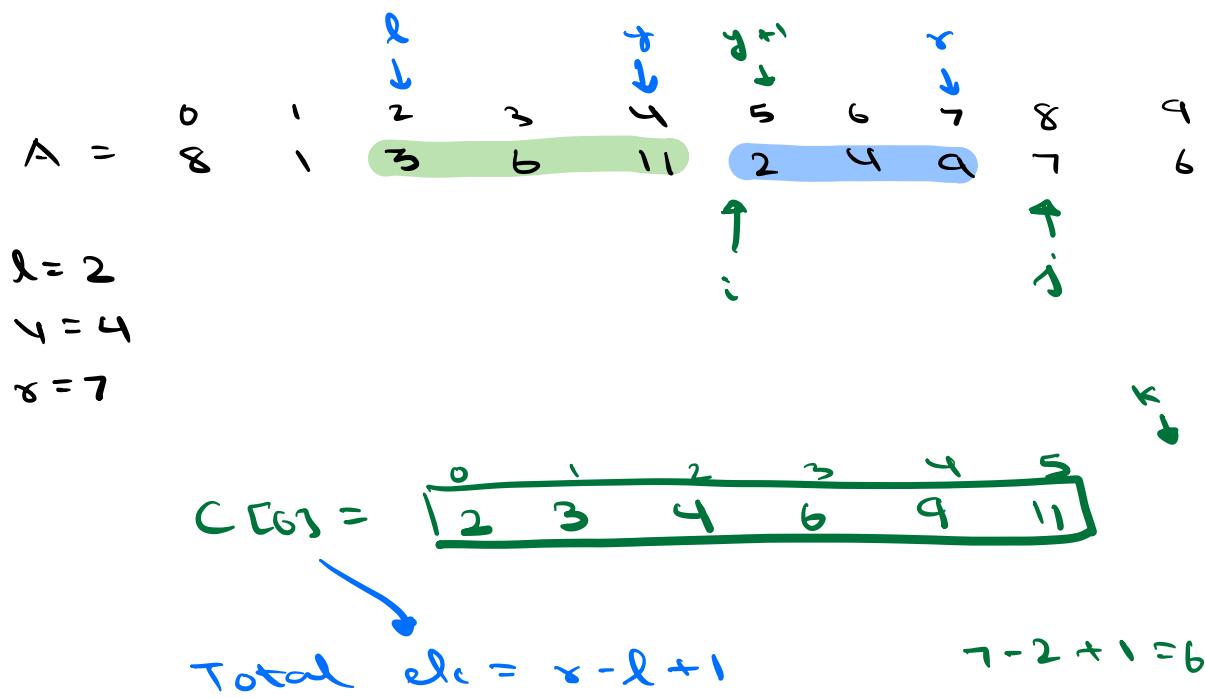
$l=2$
 $y=4$
 $r=7$

$A =$	0	1	2	3	4	5	6	7	8	9
	8	1	7	2	4	3	6	9	7	6

$l=2$
 $y=4$
 $r=7$

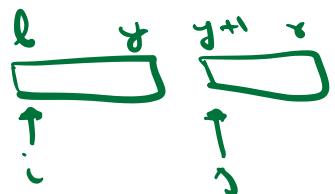


Swapping data \rightarrow breaking
sorted property of
subarrays



```
int [] merge (int A[], int l, int y, int s) {
```

```
    int C[s-l+1]
    int i=l, j=y+1, k=0
```



```

        while (i <= y || j <= s) {
            if (A[i] <= A[j]) {
                C[k] = A[i]
                k++
                i++
            }
            else // A[i] > B[j]
            {
                C[k] = A[j]
                k++
                j++
            }
        }
    }
```

// loop will end if i reaches
end of A or j reaches end of B

while ($i \leq y$) & // check if A is
| remaining
| $C[k] = A[i]$
| $k++$ $i++$
|
|

while ($i \leq x$) & // check if B is
| remaining
| $C[k] = B[j]$
| $k++$ $j++$
|
|

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// copy C arr

int i = l , k = 0

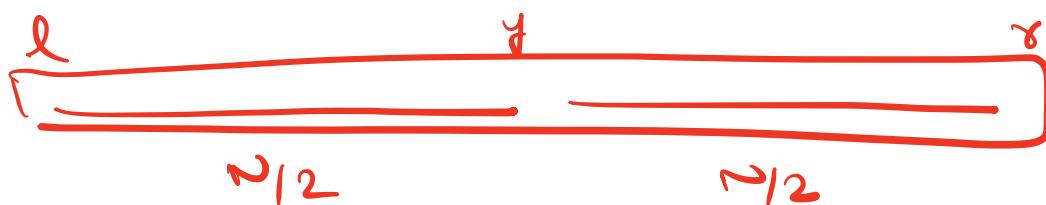
while ($i \leq x$) & $A[i] \leftarrow C[0]$

| $A[i] = C[0]$
| $i++$ $k++$

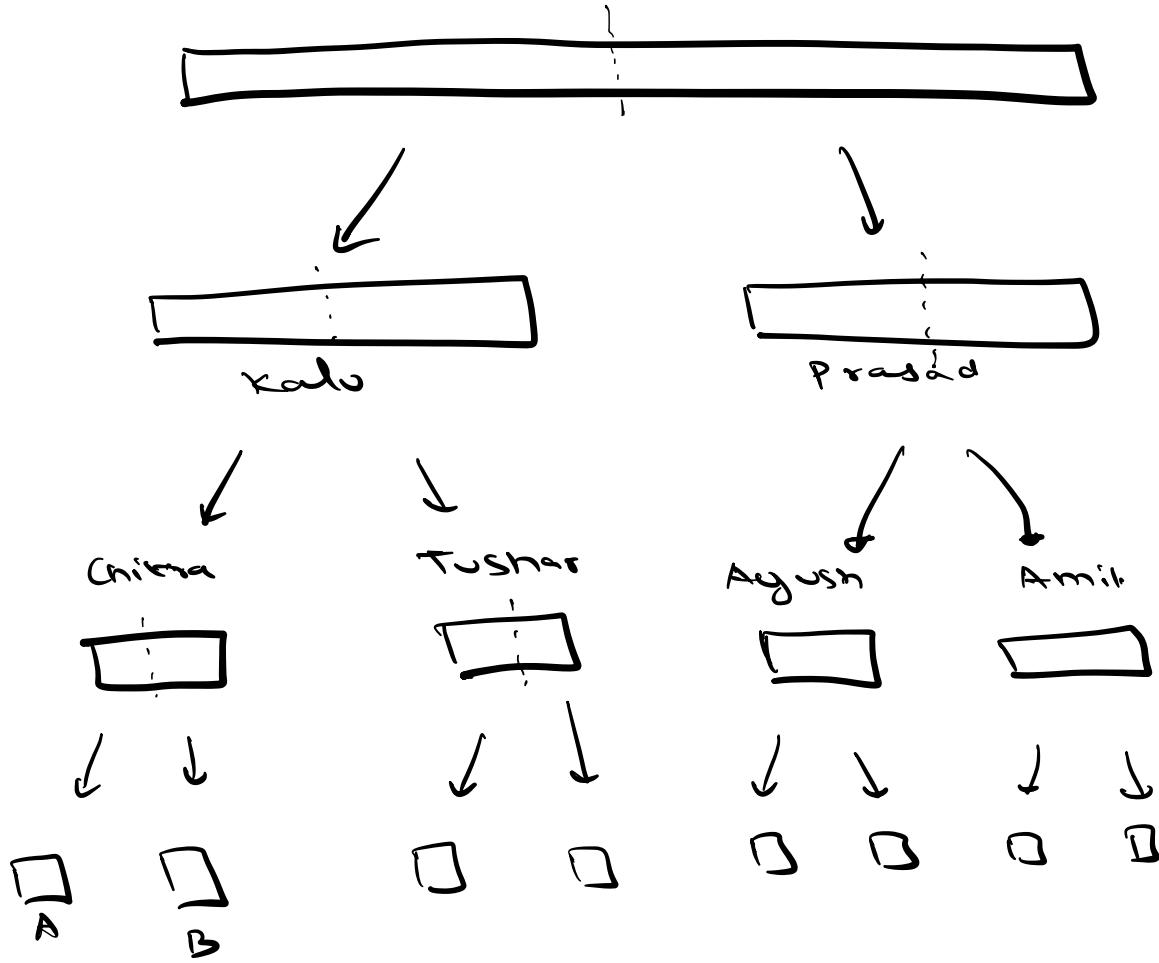
$O(N+m)$

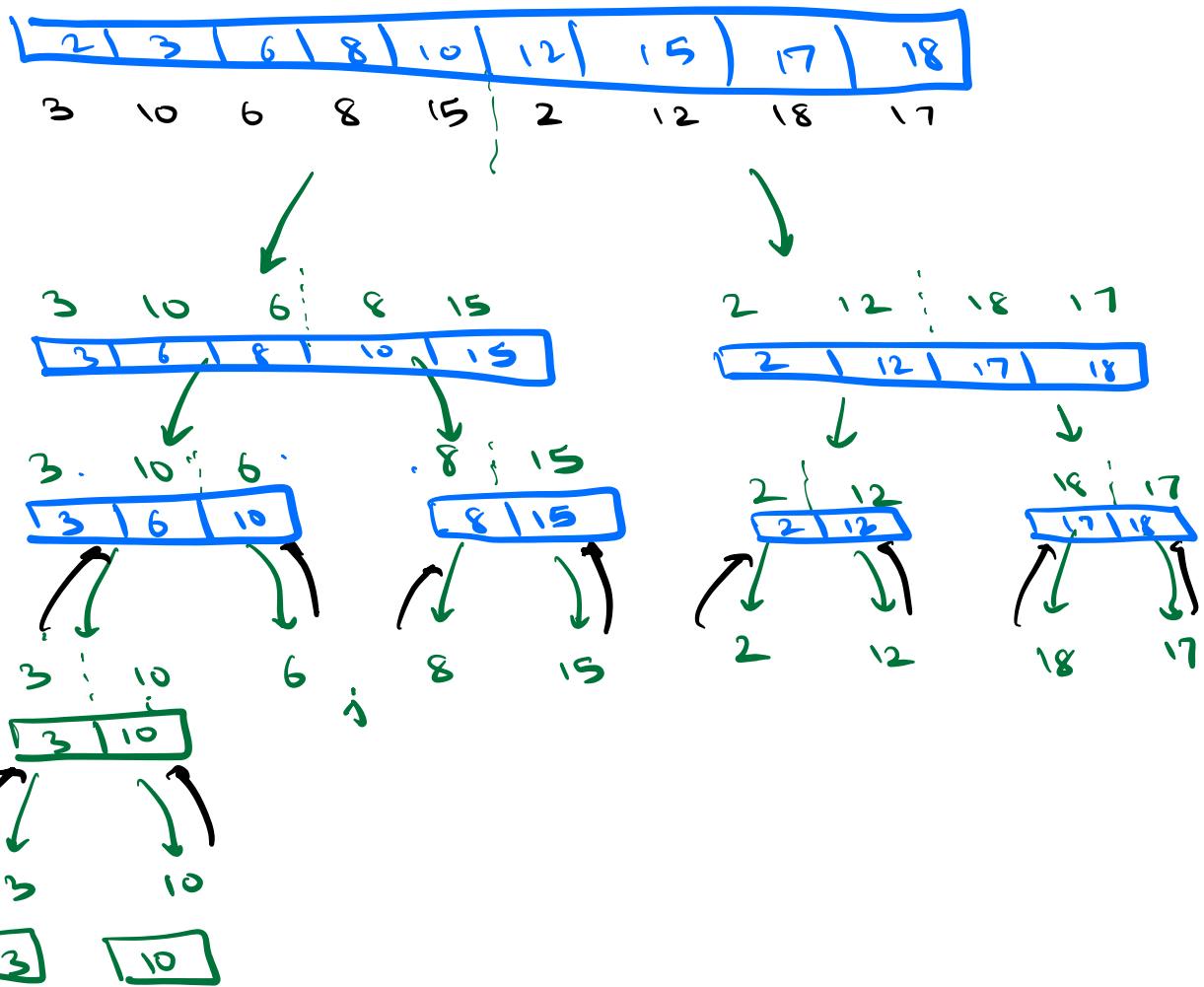
$T_C: O(N)$

$S_C: O(N)$



Merge sort



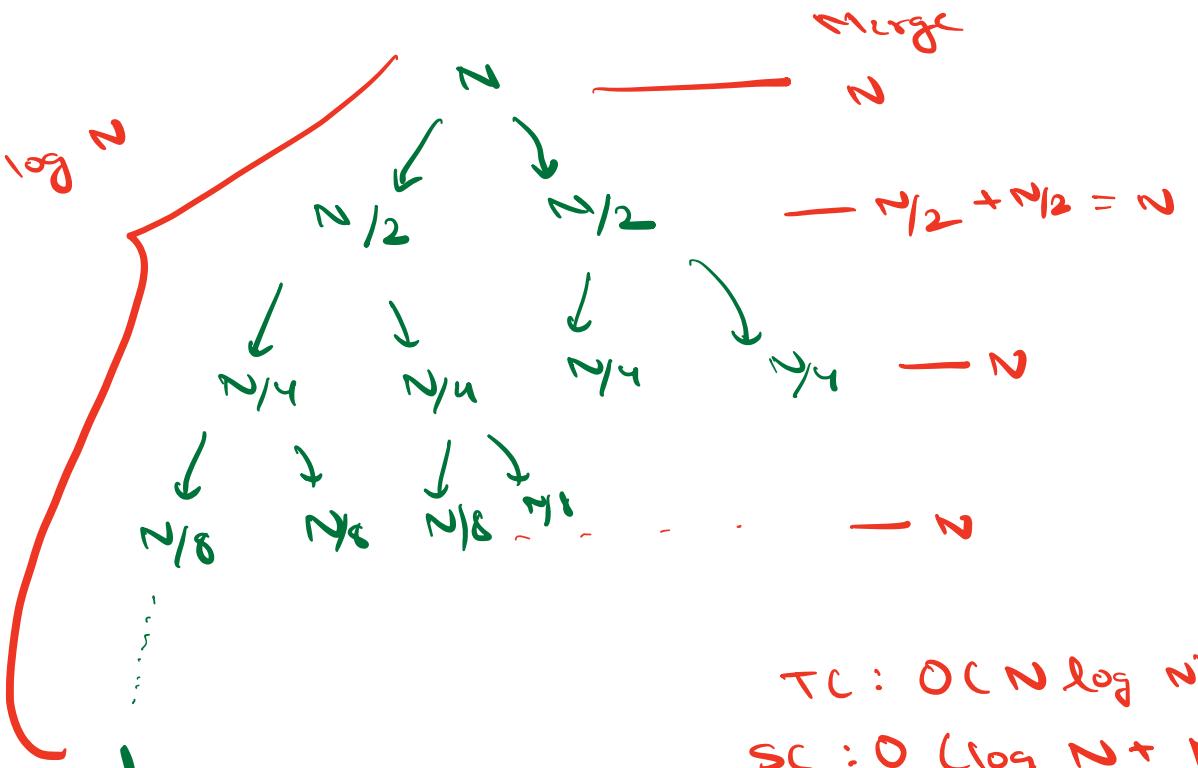


// Given l and r, sort A[] from l → r

```
void mergesort (int A[ ], l, r) {
    if (l == r) return
    int mid = (l+r)/2
    mergesort (A, l, mid)
    mergesort (A, mid+1, r)
    merge (A, l, mid, r) → C[ ]
}
```

Dry
our
code





$T_C : O(N \log N)$

$S_C : O(\log N + N)$

Call stack + fn

- Q. Given two array $A[n]$ and $B[m]$. calculate no. of pairs such that $A[i] > B[j]$

$$A[3] = \langle 7 \quad 3 \quad 5 \rangle$$

$$B[3] = \langle 2 \quad 0 \quad 6 \rangle$$

$T_C : O(N \times M)$

$cnt =$
7 pairs

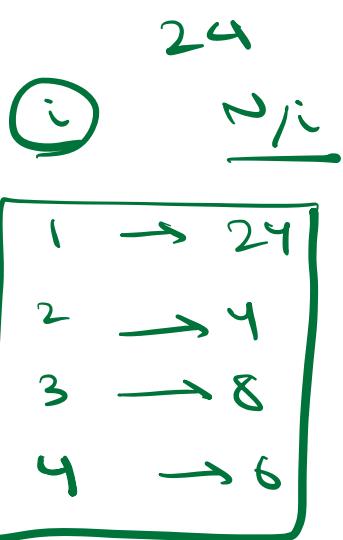
$7,2 \quad 7,0 \quad 7,6$

$3,2 \quad 3,0 \quad 5,2 \quad 5,0$

Doubles

1 2 3 4 5 6 7 8 9 10

Nos. with odd factors \rightarrow



$$24 \quad \frac{N}{i}$$

$$1 \rightarrow \sqrt{24}$$

$$100$$

$$1 \rightarrow \sqrt{100}$$

$$\begin{matrix} i \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{matrix} \quad \frac{N}{i}$$

$$10 = 100/10 = 10$$

$$\begin{matrix} i = N/i \\ i^2 = N \end{matrix}$$

$$1 \rightarrow A$$

perfect square

O(A)

$$1 \rightarrow 10 \rightarrow 1, 4, 9 \quad (3)$$

$$1 \rightarrow 20 \rightarrow 1, 4, 9, 16 \quad (4)$$

Factorial Array

$B[2] \rightarrow A[1]$

$$[4]A = [2^0, 3^1, 2^2, 3^3]$$



$$[4]B = [2^0, 6^1, 2^2, 6^3] \quad 4]$$

$$\begin{matrix} & \swarrow \\ \langle 2, 2 \rangle & \end{matrix} \quad \begin{matrix} \nearrow \\ \langle 2, 2, 7 \rangle \end{matrix}$$

$$\langle 2, 2, 6 \rangle \quad \times$$

subset
↓
subsequence

Subset \rightarrow every elem
should have
same prime
factors

$$N = 10^5$$

$$\begin{matrix} & \downarrow \\ 2^N & \\ 2^{10^5} & \end{matrix}$$