

Single Number I

Single Number II

Single Number III

Max AND Pair

Max AND Pairs

1. We are given an integer array where every no. occurs twice except for one number which occurs just once. Find that number.

$A[] = 4, 5, 5, 4, 1, 6, 6$  ans = 1

$A[ ] = 7, 5, 5, 1, 7, 6, 1, 6, 4$  ans = 4

BF  $\rightarrow$  Go to every element, calculate freq of element (by traversing array)

TC:  $O(N^2)$       SC:  $O(1)$

Outer loop  $\rightarrow$  to find an element  
Inner loop  $\rightarrow$  to count freq

Approach 2 :  $TC : O(N)$   $SC : O(N)$

$$A[] = 4, 5, 5, 4, 1, 6, 6$$

Build hashmap  $\langle \text{elem}, \text{freq} \rangle$



① Travers array and store freq

② Iterate on hashmap and return key where value = 1  
                     ↓                         ↓  
                 item                         freq

etc : freq

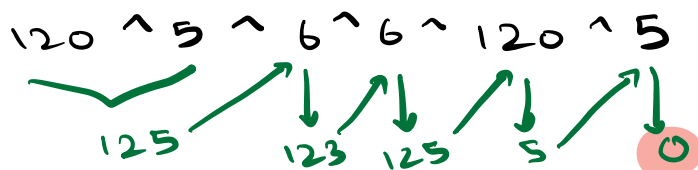
4	:	2
1	:	1
5	:	2
6	:	2

Approach 3:

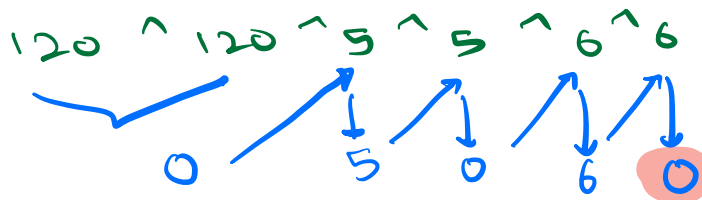
TC:  $O(N)$

SC:  $O(1)$

Idea:  $A \wedge A = 0$



XOR  
↓  
pairs  
cancel  
out  
each  
other



$A \wedge A = 0$   
 $0 \wedge A = A$

$$(5 + 10) + 7$$

$$15 + 7 = 22$$

$$(7 + 5) + 10$$

$$12 + 10 = 22$$

~~5~~, ~~5~~, ~~5~~, ~~5~~, ~~5~~, ~~5~~

int xor = 0

for (i = 0; i < n; i++)

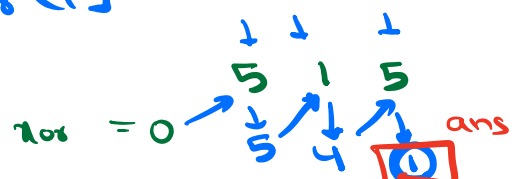
TC:  $O(N)$

SC:  $O(1)$

xor = xor ^ arr[i]

return xor

xor = 0  
5, 5, 0  
xor = 0



every ele  $\rightarrow$  4 times  
 unique ele  $\rightarrow$  1 time

Delete  
 $\downarrow$   
 Even

ans  
 $\downarrow$   
 odd

4 4 3 2 2 4 4 2 2

Approach 4:

ans = 5

$x \rightarrow 2$   
 $y \rightarrow 0$   
 $z \rightarrow 2$

	pos 2	pos 1	pos 0
x	0	1	1
x	0	1	1
y	0	0	1
y	0	0	1
z	0	1	1
z	0	1	1
	0	4	6

total cnt of set bits = even

every ele  
 $\uparrow$   
 twice

$A[] = 2, 3, 5, 6, 3, 6, 2$

	2	1	0	
2	0	1	0	
3	0	1	1	
5	1	0	1	
6	1	1	0	
3	0	1	1	
6	1	1	0	
2	0	1	0	
	3	6	3	→ 11
11 ①	1	0	1	
	2 <sup>11</sup>	3 <sup>11</sup>	6 <sup>11</sup>	

Cnt of  
set bits at  
every bit  
pos

10 1 → ans

	i
x	1
x	1
y	0
y	0
2	1
	2 → 3

0  
2 → 2

Cnt of  
set bits at  
(at pos x)

→ even  
→ odd

unique no.

unset at pos x  
bit  
set bit at pos x



set a bit in bit pos

5 4 3 2 1 0

int

odd even

0 0 0 0 1 0

ans =

2. Given integer array, all elements occur thrice except one. Find the unique element.

$A[] = 4, 5, 5, 4, 1, 6, 6, 4, 5, 6$

ans = 1

occurs only once

BF  $\rightarrow$  1 loop  $\rightarrow$  to find de

2 loop  $\rightarrow$  traverse arr and count freq

TC:  $O(N^2)$

SC:  $O(1)$

Approach 2 - Store freq of every elem in HM

TC:  $O(N)$

SC:  $O(N)$

Approach 3

ans = 9

A[] = 5, 7, 5, 9, 7, 11, 11, 7, 5, 11

	3	2	1	0
5	0	1	0	1
7	0	1	1	1
5	0	1	0	1
9	1	0	0	1
7	0	1	1	1
11	1	0	1	1
11	1	0	1	1
7	0	1	1	1
5	0	1	0	1
11	1	0	1	1
	4	6	6	10
	1	0	0	1

3 1  
111 1

3 3 3 1  
111 111 111 1

ans = 1 0 0 1

x → 3

y → 0

z → 3

	x	y	z
x	→ 1	0	
x	→ 1	0	
x	→ 1	0	
y	→ 0	1	
y	→ 0	1	
y	→ 0	1	
z	→ 1	0	
z	→ 1	0	
z	→ 1	0	
	6	3	

x → 0

y → 3

z → 0

cnt of set bits will be multiple of 3



cnt set bits for every bit pos  
 cnt set bits at pos  $x$   $\xrightarrow{\text{cnt} \% 3 = 1}$  set bit at pos  $x$  in unique elem  
 $\xrightarrow{\text{cnt} \% 3 = 0}$  unset bit at pos  $x$  in unique elem

int ans = 0

for (i=0 ; i < 32 ; i++) <  
   int cnt = 0  
   for (j=0 ; j < n ; j++) <  
     if (arr[j] & (1 << i) != 0) cnt++  
     if (cnt % 3 == 1)  
       ans = ans | (1 << i)  
   return ans  
 Tc: O(32 \* n)  
 Sc: O(1)

what if? Every elem comes thrice  
 except unique elem comes twice

3, 3, 1, 1, 1, 2, 2, 3    ans = 2

$2 \rightarrow 3$   
 $2 \rightarrow 0$

		i	k	l
2	→	1	1	0
2	→	1	1	0
2	→	1	1	0
2	→	0	1	1
2	→	0	1	1
2	→	0	1	1
d	→	1	1	0
d	→	1	1	0
<hr/>				
3	→	5	8	3
<hr/>				
		1	1	0

cnt % 3 == 2

cnt of set bits % 3

what if?      every elem comes 5 times  
                     unique elem comes 3 times

cnt of set bits → groups of 5  
 (ignoring unique)

bit pos  
 2

cnt % 5 == 3

unique elem  
 T → 1  
 F → 0

10 : 45

3. Given integer array, all elements occur twice except two. Find these two elements

$$A[] = 4, 5, 4, 1, 6, 6, 5, 2$$

ans = 1, 2

$$\begin{array}{cccc} 4 & 4 & 5 & 5 \\ \hline 0 & 1 & 0 & 0 \end{array} \quad \begin{array}{cccc} 6 & 6 & 1 & 2 \\ \hline 1 & 0 & 1 & 2 \end{array}$$

$$\text{xor} \rightarrow 1 \wedge 2$$

$$\text{xor} \rightarrow 7$$

$$\begin{array}{l} 2, 5 \rightarrow \\ 1, 6 \rightarrow 7 \\ 3, 4 \rightarrow \end{array}$$

$$A[] = 3, 7, 6, 7, 3, 8, 9, 9$$

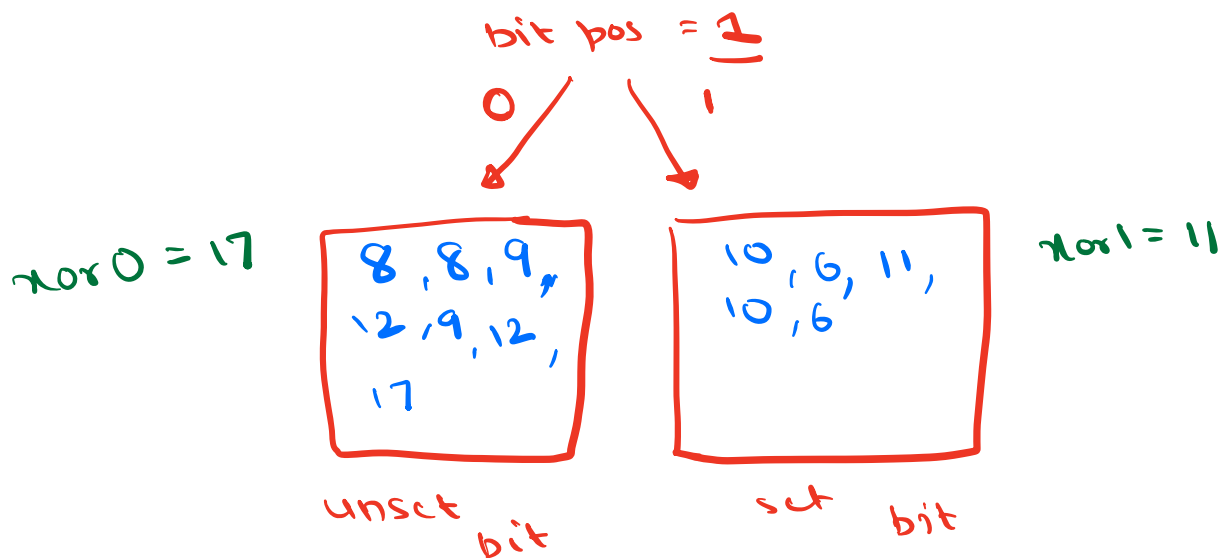
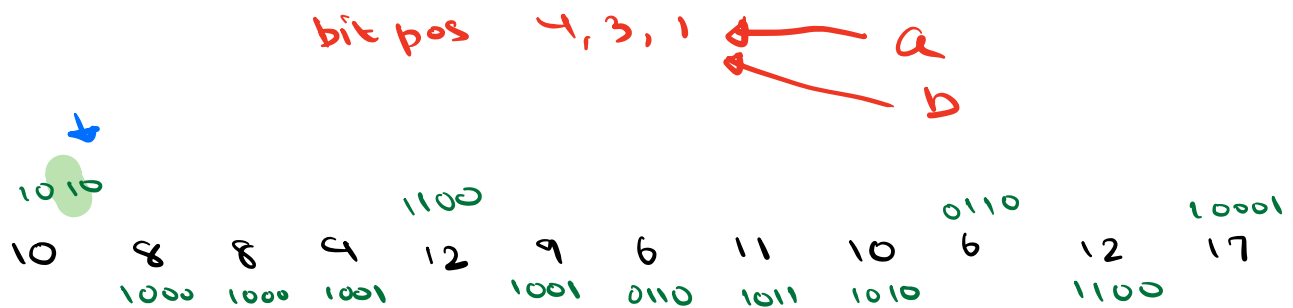
ans = 6, 8

ans = 11, 17

$$\begin{array}{cccccccccccc} 1010 & & & & 1100 & & & & 0110 & & & 10001 \\ 10 & 8 & 8 & 9 & 12 & 9 & 6 & 11 & 10 & 6 & 12 & 17 \\ 1000 & 1000 & 1001 & & 1001 & 0110 & 1011 & 1010 & & & 1100 & \end{array}$$

$$\begin{array}{cccccccc} 10 & 8 & 8 & 9 & 12 & 9 & 6 & 11 & 10 & 6 & 12 & 17 \\ \hline 10 & 8 & 8 & 9 & 12 & 9 & 6 & 11 & 10 & 6 & 12 & 17 \\ \hline & 4 & 3 & 2 & 10 & & & & & & & \\ \text{xor} & 11 & 0 & 10 & 11 & & & & & & & \\ & 17 & 1 & 0 & 0 & 0 & 1 & & & & & \\ \hline & & 1 & 1 & 0 & 1 & 0 & & & & & \end{array}$$

xor of arr[]  
xor of unique elem



```

int xorall = 0
① for (i = 0 ; i < n ; i++) {
    xorall = xorall ^ arr[i]
}

② // Look for a set bit in xorall
for (pos = 0 ; pos < 32 ; pos++) {
    if (xorall & 1 << pos != 0)
        break
}

```

```

// pos
int num1 = 0, num2 = 0

for (i = 0 ; i < n ; i++) {
    if ((arr[i] & (1 << pos)) != 0)
        num1 = num1 ^ arr[i]
    else
        num2 = num2 ^ arr[i]
}

return num1, num2

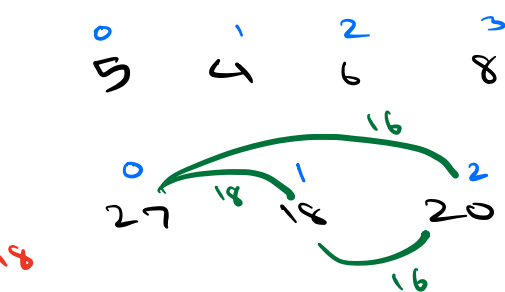
```

TC:  $O(N)$   
SC:  $O(1)$

4. Given  $N$  array elements, choose 2 indices  $(i, j)$  such that  $i \neq j$  and  $(arr[i] \oplus arr[j])$  is maximum

max &  
val = 5

18  
↓  
pair 27, 18  
ans → (0, 1)



6	0	1	1	0
8	1	0	0	0
	<u>0</u>	<u>0</u>	<u>0</u>	<u>0</u>
6	8	8	→ 0	

ans (0, 4)

{ 5, 4  
 5, 6  
 5, 8  
 5, 5 → 5  
 4, 6  
 4, 8  
 4, 5

	A	-	-	-	-
&	B	-	-	-	-
	ans	<u>          </u>			

Magnitude  
↓  
set bit

num →

4	3	2	1	0
<del>0</del>	1	<u>0</u>	<del>0</del>	<u>0</u>
	↓		↓	
		$2^3$	$+ 2^1$	
↓				
$2^4 +$				

- ① & value → good no. of set bits
- ② & value → set bit → A and B have set bit

1 & 1 → 1

2 nos. have more and more set bits at common pos

1 pair

4	3	2	1	0
1	0	1	1	1
1	0	0	0	0
<hr/>				
1	0	0	0	0
<hr/>				
↓				
$2^4$				

>

3 pairs

4	3	2	1	0
1	0	1	1	1
0	0	1	1	1
<hr/>				
0	0	1	1	1
<hr/>				
		↓		
		$2^2 + 2^1 + 2^0$		

A	1
B	-
&	<u>1</u>

Left side

MSB → LSB

26, 13, 23, 28, 7, 25

	4	3	2	1	0
26	1	1	0	1	0
13	0	1	1	0	1
23	1	0	1	1	1
28	1	1	1	0	0
27	1	1	0	1	1
7	0	0	1	1	1
25	1	1	0	0	1

mark &  
value → 26

26, 27

ans  
↓  
& value

1 1 0 1 0



2 nos.  
set bit



4 3 2 1 0  
1 1 0 1 0

$2^4 + 2^3 + 2^1$   
 $16 + 10$

26

	4	3	2	1	0	
26	1	1	0	1	0	
0 13	0	1 0	1 0	0	1 0	X
23	1	0	1	1	1	X
28	1	1	1	0	0	
27	1	1	0	1	1	
0 7	0	0	1 0	1 0	1 0	X
25	1	1	0	0	1	

ans  
↓

1 1

TC:  $O(N^2)$   
SC:  $O(1)$

```
int ans = 0
for (i = 31 ; i >= 0 ; i--) {
    int cnt = 0
    for (j = 0 ; j < n ; j++) {
        if ((arr[j] & (1 << i)) != 0)
            cnt++
        if (cnt >= 2) {
            ans = ans | (1 << i)
            for (j = 0 ; j < n ; j++) {
                if ((arr[j] & (1 << i)) == 0)
                    arr[j] = 0
            }
        }
    }
}
```

// ans  $\rightarrow$  max & value

// no. with non 0 value  $\rightarrow$   
can be pair

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5. Count all pairs with MAX & value

$N$  elements  $\rightarrow$  non zero

Pairs  $\frac{N(N-1)}{2}$

TC:  $O(N)$

SC:  $O(1)$

4 ex



$$\frac{3 \times 4}{2} \rightarrow$$

= 6 pairs

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