Addition and Multiplication Rule

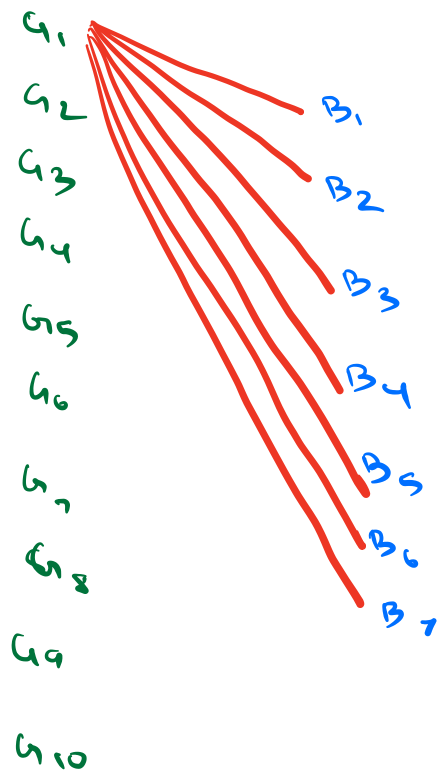
Permutation Basics

Combination Basics & Properties

Pascal Triangle

Find Nth column title

Ex 1: Given 10 girls and 7 boys, in how many ways we can form a different pair?

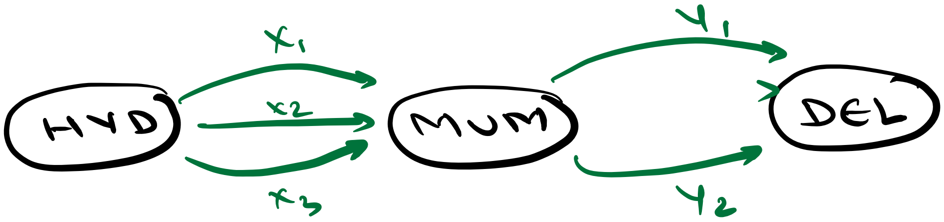


1 pair
↓
1 girl AND 1 boy

G_1 B_1
G_2 B_2
G_1 B_3
B_4
B_5
B_6
B_7

Girls * boys
10 * 7 = 70 ways

Ex 2 Total no of paths from HYD → DEL via MUM



$x_1 y_1$
$x_1 y_2$
$x_2 y_1$
$x_2 y_2$
$x_3 y_1$
$x_3 y_2$

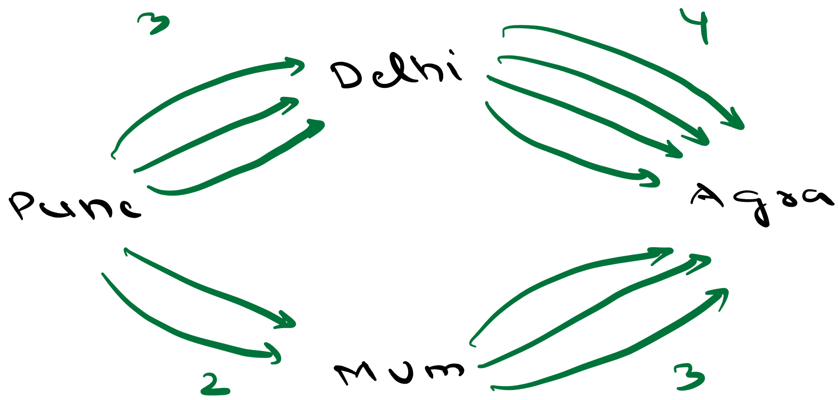
HYD → MUM AND MUM → DEL

3 2

$= 3 * 2$

$= 6$ unique paths

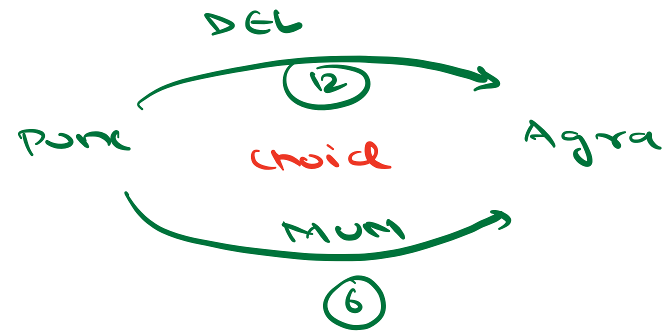
Ex 3



No. of ways to reach Pune → Agra!

→ via DEL OR via MUM

PUN→DEL AND DEL→AGR	PUN→MUM AND MUM→AGR
$3 * 4$	$2 * 3$
$= 12$	$= 6$



Total no. of ways = via MUM or via DEL

= 6 + 12

= 18

Rule of ADDITION AND MULTIPLICATION

No. of ways/ counting possibilities

Occur together in sequence → AND (×)

Occurs a choice / in separate ways → OR (+)

Permutation → arrangement of objects
↓
order matters $(i,j) \neq (j,i)$

(ph, IPen) ≠ (IPen, ph)

Given 3 distinct characters, in how many ways can we arrange them?

a, b, c

a	b	c
a	c	b
b	c	a
b	a	c
c	a	b
c	b	a

6

$$\underset{3}{\_\_} \times \underset{2}{\_\_} \times \underset{1}{\_\_}$$

a → b → c
a → c → b

b → c → a
b → a → c

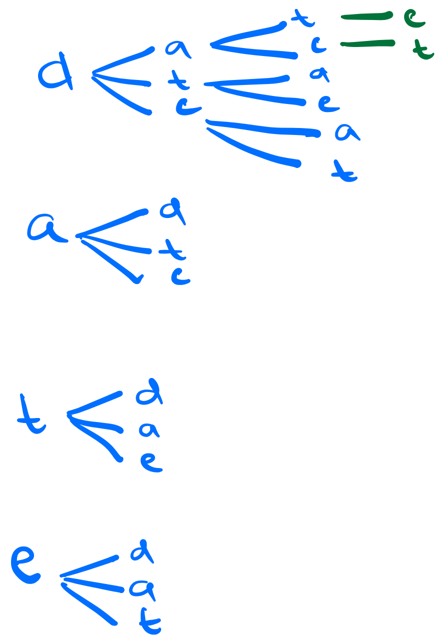
c → a → b
c → b → a

Total arrangements $= 3 \times 2 \times 1 = 6$

Given 4 distinct characters, in how many ways can we arrange them?

d, a, t, e

$\underline{4} * \underline{3} * \underline{2} * \underline{1} = 24 = 4!$



In how many ways, n distinct characters can be arranged?

$\underline{n}$ $\underline{n-1}$ $\underline{n-2}$ .... $\underline{1}$

1st slot, 2nd slot, 3rd slot, Nth slot

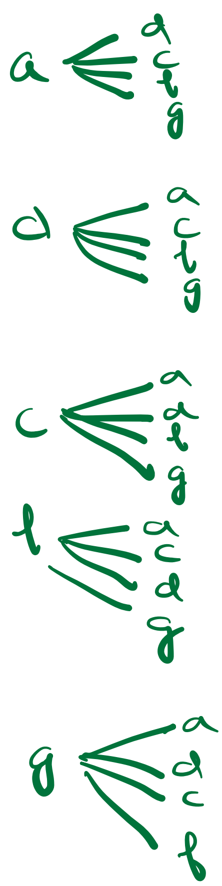
Total no. of arrangements $= N * (N-1) * (N-2) * \ldots * 1$

$= N!$

Given 5 distinct characters, no. of ways to arrange 2 out of 5 char?

a d c f g

$\underline{5} * \underline{4}$



a	d	c	g
d	a	g	c
c	f		
f	c		

Total no. of ways = 20

Given N distinct chars, no. of ways to arrange R out of N chars?

$\underline{N}$	$\underline{N-1}$	$\underline{N-2}$	. . . . .	$\underline{N-R+1}$
1st slot	2nd slot	3rd slot		Rth slot

N-0   N-(2-1)   N-(3-1)   N-(R-1)

Total no. of ways to arrange R out of N chars

=

ans = $N * (N-1) * (N-2) * \ldots * (N-R+1)$

$$ans = \frac{N * (N-1) * (N-2) * \ldots * (N-R+1) * N-R * N-R-1 * N-R-2 \ldots * 1}{(N-R) * (N-R-1) * (N-R-2) * \ldots * 1}$$

$$^{N}P_{R} = \frac{N!}{(N-R)!}$$

Total no. of ways to arrange R obj out of N objects = $\frac{N!}{(N-R)!}$

Total no. of ways to arrange 2 char out of 5 chars

N = 5, R = 2

$$ans = \frac{5!}{(5-2)!} = \frac{5!}{3!}$$

$$= \frac{5 \times 4 \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{3} \times \cancel{2} \times \cancel{1}} = 5 \times 4 = 20$$

Total no. of ways to arrange N obj out of N objects = $\frac{N!}{(N-R)!}$

$= \frac{N!}{(N-N)!} = \frac{N!}{0!}$

$= N!$

Combination → no. of ways to select something

Team → 2 Players

Total → 15 players

Same team
V R
R V

V R
R V

Combination
Order does not matter → select

Permutation → select and arrange

Given 4 players, count no. of ways of selecting 3 players

$\langle P_1 \quad P_2 \quad P_3 \quad P_4 \rangle$

$P_1 \quad P_2 \quad P_3$
$P_1 \quad P_2 \quad P_4$
$P_1 \quad P_3 \quad P_4$
$P_2 \quad P_3 \quad P_4$

4 ways

Given 4 players, count no. of ways of arranging 3 players

$\langle P_1 \quad P_2 \quad P_3 \quad P_4 \rangle$

$${}^4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!} = 24$$

__ __ __

6

$P_1\ P_2\ P_3$	$P_1\ P_2\ P_4$	$P_1\ P_3\ P_4$
$P_1\ P_3\ P_2$	$P_1\ P_4\ P_2$	$P_1\ P_4\ P_3$
$P_2\ P_3\ P_1$	$P_2\ P_4\ P_1$	$P_4\ P_3\ P_1$
$P_2\ P_1\ P_3$	$P_2\ P_1\ P_4$	$P_4\ P_1\ P_3$
$P_3\ P_1\ P_2$	$P_4\ P_1\ P_2$	$P_3\ P_1\ P_4$
$P_3\ P_2\ P_1$	$P_4\ P_2\ P_1$	$P_3\ P_4\ P_1$

P_2	P_3	P_4
P_2	P_4	P_3
P_3	P_4	P_2
P_3	P_2	P_4
P_4	P_2	P_3
P_4	P_3	P_2

3 players out of 4

No. of arrangements $= 24$

Every 6 arrangement $\rightarrow$ 1 unique selection

No. of selections $= \dfrac{24}{6} = 4$ unique selections

No. of ways to arrange r objects out of n objects

$$^nP_r = \frac{n!}{(n-r)!}$$

No. of ways to select r objects out of n objects

$$\text{Total no. of select} = \frac{^nP_r}{r!} = \frac{n!}{(n-r)!\,r!}$$

No. of ways to arrange r objects

$$= r!$$

$r!$ arrangements $\rightarrow$ 1 unique selection

No. of ways to select r objects out of N $= {}^NC_R = \frac{N!}{(N-R)!\,R!}$

10:46

Properties of Combination

① $\boxed{{}^{N}C_{N} = 1} = \dfrac{N!}{(N-N)!\ N!}$

pick everything

${}^{N}C_{R} = \dfrac{N!}{(N-R)!\ R!}$

② $\boxed{{}^{N}C_{0} = 1} = \dfrac{N!}{(N-0)!\ 0!} = \dfrac{N!}{N!}$

don't pick anything

③ ${}^{N}C_{N-R} = \dfrac{N!}{(N-(N-R))!\ (N-R)!}$ $\boxed{{}^{N}C_{R} = \dfrac{N!}{(N-R)!\ R!}}$

$= \dfrac{N!}{R!\ (N-R)!}$

$\boxed{{}^{N}C_{N-R} = {}^{N}C_{R}}$

${}^{N}C_{R} = {}^{5}C_{3} = {}^{5}C_{2}$ ${}^{N}C_{N-R}$

$N = 5$

$R = 3$

(3) select r items out of N

___ ___ ___ . . . . ___ ___

1st 2nd 3rd (N-1)th Nth

select OR not select

$^{N-1}C_{R-1}$ $^{N-1}C_{R}$

$$\boxed{{}^{N}C_{R} = \underbrace{{}^{N-1}C_{R-1}}_{\text{pick } N^{th} \text{ item}} + \underbrace{{}^{N-1}C_{R}}_{\text{leave } N^{th} \text{ item}}}$$

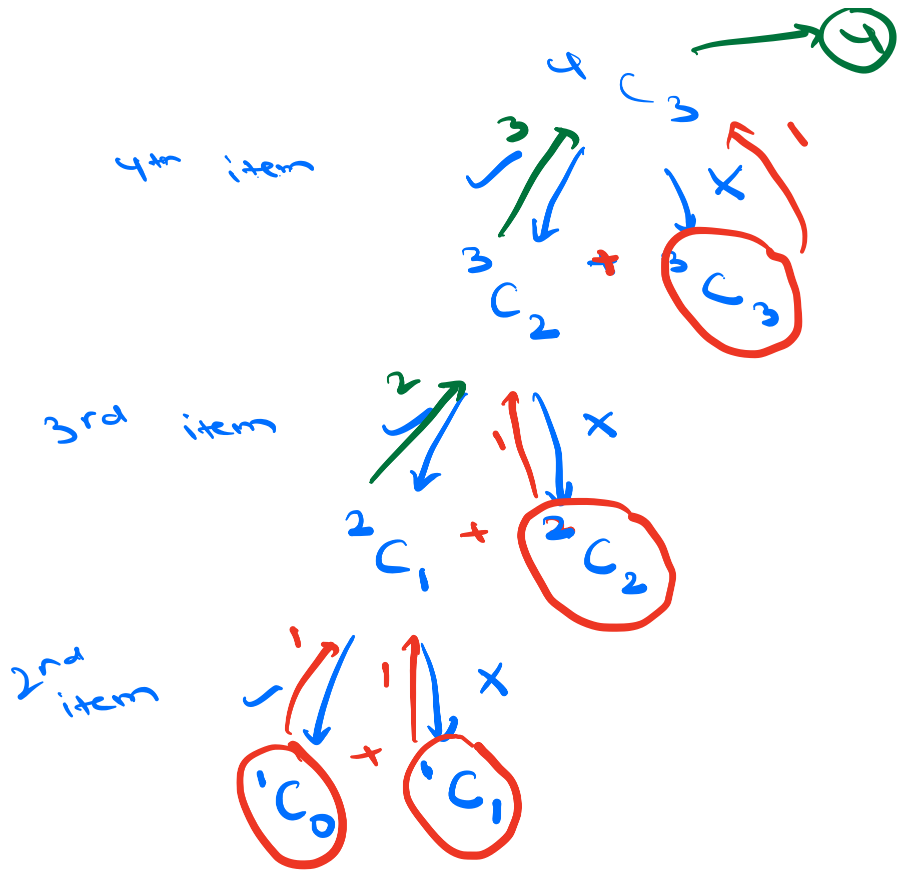
$N = 10 \quad R = 3 \qquad {}^{10}C_3 = \frac{10!}{(10-3)!\,3!}$

$${}^{N}C_{R} = \frac{N!}{(N-R)!\,R!}$$

$^{10000}C_{9999}$

$$\frac{10000!}{1!\;9999!}$$

$${}^{N}C_{R} = {}^{N-1}C_{R-1} + {}^{N-1}C_{R}$$



if (n==r)
  return 1
if (r==0)
  return 1

$^NC_N = 1$

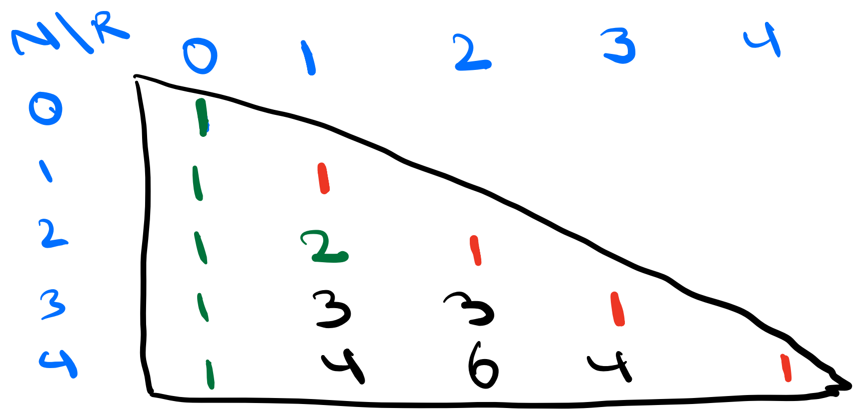
$^NC_0 = 1$

Q1. Pascal's Triangle

Given value of n, generate pascal's Δ

$n=4$

0	$^0C_0$				
1	$^1C_0$	$^1C_1$			
2	$^2C_0$	$^2C_1$	$^2C_2$		
3	$^3C_0$	$^3C_1$	$^3C_2$	$^3C_3$	
4	$^4C_0$	$^4C_1$	$^4C_2$	$^4C_3$	$^4C_4$



$^{N}C_0 = 1$

$^{N}C_N = 1$

$^{N}C_1 = N$

Pascal [N+1][N+1]

1st loop → to fin row no. / N
2nd loop → to fin col no. / R

N=6   i = 0 to N+1
j = 0 to i

$$mat[i][j] = \frac{i!}{(i-j)!\, j!}$$

↓
overflow

$$^{N}C_R = {}^{N-1}C_{R-1} + {}^{N-1}C_R$$

$$mat[i][j] = {}^{i}C_j = \underbrace{{}^{i-1}C_{j-1} + {}^{i-1}C_j}$$

mat[i][j] = mat[i-1][j-1] + mat[i-1][j]

N = 4 mat[5][5]

N\R	0	1	2	3	4
0	1	0	0	0	0
1	1	1	0	0	0
2	1	$\boxed{2}$	1	0	0
3	1	3	3	1	0
4	1	4	6	4	1

${}^0C_0 = 1$

${}^NC_0 = 1$

${}^NC_N = 1$

mat[2][1] = ${}^2C_1 = {}^1C_1 + {}^1C_0$

```
void pascal (int N) {
    int mat[N+1][N+1] = {0}
    for (int i=0; i <= N ; i++) {
        for ( j= 0 ; j <= i ; j++ ) {
            if ( j==0 || j==i)
                mat[i][j]=1
            else
                mat[i][j] = mat[i-1][j]+
                            mat[i-1][j-1]
        }
    }
}
```

TC : $O(N^2)$

SC : $O(N^2)$

Prob 2 Find Nth col Title

1	2	3		26	27	28		52	53		
A	B	C	......	Z	AA	AB	...	AZ	BA	BB	... BZ

CA ... CZ ... ZZ AAA AAB ....

	col
N = 3	C
N = 30	AD
N = 50	AX

Dec → Binary 12

A → Z (26) rem → 0 — 25

50

$$\begin{array}{r|l}
26 & 50-1 = 49 \quad - 23 \\ \hline
26 & 1-1 = 0 \quad - 0 \\ \hline
 & 0
\end{array}$$

0	1	2	....	25
↓	↓	↓		↓
A	B	C		Z

	0	23
	↓	↓
50 →	A	X

```
string colTitle (int n) {
    string ans = " "
    while (n > 0) {
        rem = (n-1) % 26
        char ch = rem + 65 / 'A'
        ans += ch
        n = (n-1)/26
    }
    reverse (ans)
    return ans
}
```

$$\begin{array}{ccc} 0 & 1 & 2 \\ \downarrow & \downarrow & \downarrow \\ A & B & C \end{array}$$

TC: $O(\log_{26} N)$

SC: $O(1)$

$$N \longrightarrow N/26 \longrightarrow N/26^2 \rightarrow \dots \rightarrow 0$$

$$N \rightarrow N/2 \rightarrow N/4 \rightarrow \dots \rightarrow 0$$

$\log_2 N$

Doubts

Total select

N items → R items

1 2 3 . . . . . . N

select OR Not select

$^{N-1}C_{R-1}$ $^{N-1}C_R$

$$^{N}C_R = {}^{N-1}C_{R-1} + {}^{N-1}C_R$$

Every item → select R items

→ Not selected

	A	B	C	D	E
ABC	✓	✓	✓	X	X
CDE	X	X	✓	✓	✓

$^{5}C_3$

3 → select

2 → not select