

## AGENDA

Bitwise operators and Properties  
Left and Right shift operator  
Concepts

Prob {  
    Check  $i^{\text{th}}$  bit  
    Count set bits  
    Unset  $i^{\text{th}}$  bit  
    Set bits in range

(9-10:30 PM)

Adv Contest 1 → 13 Oct Friday

Syllabus → Arrays 1, 2, 3

Bit Manipulations

10:30

Discussion

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Friday 6 Oct → Normal lecture

Bits  $\rightarrow$  0 unset      1 set

Bitwise Operators (&, |, ^, ~, <<, >>)

at least 1 set bit

A	B	A & B	A   B	A ^ B	~A
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

Same  
Same  
puppy  
shame

## Basic AND Properties

int A = 10  
int B = 4  
int C = A & B  
    ↓  
    0

10  $\rightarrow$  1010  
4  $\rightarrow$  0100  
    0 ← 0000

1) A & 1

↓      ↓

1      0

0th bit is set      0th bit is unset

&

A  $\rightarrow$  - - - - - 1

1  $\rightarrow$  0 0 0 0 0 0 0 0 1

---

A  $\rightarrow$  - - - - - 0

1  $\rightarrow$  0 0 0 0 0 0 0 0 1

↓  
odd

A → even  
6 5 4 3 2 1 0  
A → - - - - - 1  
↓  
 $2^0$   
odd no.

A → 10

0 0 0 0 1 0 1 0 even

A → 3

0 0 0 0 0 0 1 1 odd

A → 7

0 0 0 0 0 1 1 1 odd

0<sup>th</sup> bit is 1 → odd

0<sup>th</sup> bit is 0 → even

A & 1

↓ 1

0<sup>th</sup> bit is  
set

↓  
odd

↓ 0

0<sup>th</sup> bit is  
unset

↓  
even

2)  $A \& 0 = 0$

A = - - - - -  
& 0 = 0 0 0 0 0 0  
-----  
0 0 0 0 0 0

3)  $A \& A = A$   
1 1 = 1

A → 0 1 0 1 1  
× A → 0 1 0 1 1  
-----  
0 1 0 1 1

## Basic OR properties

1.  $A \vee 0 = A$

$$\begin{array}{r} A = 1 \ 0 \ 1 \ 0 \ 1 \\ 0 = 0 \ 0 \ 0 \ 0 \ 0 \\ \hline (A) \rightarrow 1 \ 0 \ 1 \ 0 \ 1 \end{array}$$

$$\begin{array}{l} 0 \vee 0 = 0 \\ \text{otherwise} \\ 1 \vee 1 = 1 \end{array}$$

$$\begin{array}{l} 1 \vee 1 = 1 \\ \text{otherwise} \\ 0 \vee 0 = 0 \end{array}$$

2.  $A \vee A = A$

$$\begin{array}{r} A = 1 \ 0 \ 1 \ 0 \ 1 \\ A = 1 \ 0 \ 1 \ 0 \ 1 \\ \hline 1 \ 0 \ 1 \ 0 \ 1 \end{array}$$

## Basic XOR Properties

1.  $A \wedge 0 = A$

bit  
 $x \wedge 0 =$

$$\begin{array}{l|l} x & \\ \hline 1 & 1 \wedge 0 = 1 \\ 0 & 0 \wedge 0 = 0 \end{array}$$

$$\begin{array}{r} A \rightarrow 1 \ 0 \ 1 \ 0 \ 1 \\ \wedge \ 0 \rightarrow 0 \ 0 \ 0 \ 0 \ 0 \\ \hline 1 \ 0 \ 1 \ 0 \ 1 \end{array}$$

$$\begin{array}{r} A \rightarrow 0 \ 1 \ 0 \ 1 \ 1 \ 0 \\ \wedge \ 0 \rightarrow 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ \hline 0 \ 1 \ 0 \ 1 \ 1 \ 0 \end{array}$$

2.  $A \wedge A = 0$

$$\begin{array}{r} A \rightarrow 1 \ 0 \ 1 \ 0 \ 1 \\ \wedge \ A \rightarrow 1 \ 0 \ 1 \ 0 \ 1 \\ \hline 0 \ 0 \ 0 \ 0 \ 0 \end{array}$$

Commutative Property  
AND, OR, XOR

$$5+3=3+5$$

↓  
Order of operands does not change  
answer

$$A \& B = B \& A$$

$$A \mid B = B \mid A$$

$$A \wedge B = B \wedge A$$

$$2-3=3-2$$

Associative Property

↓  
grouping of operands does not  
change answer

$$\begin{array}{c} (2+5)+7 \\ \downarrow \quad \downarrow \\ 7+7 \\ =14 \end{array}$$

$$A \& B \& C$$

$$\begin{array}{c} 2+(5+7) \\ \downarrow \\ 2+12 \\ 14 \end{array}$$

$$(A \& B) \& C = A \& (B \& C)$$

$$A \mid B \mid C$$

$$(A \mid B) \mid C = A \mid (B \mid C)$$

$$A \wedge B \wedge C$$

$$(A \wedge B) \wedge C = A \wedge (B \wedge C)$$

$$\begin{array}{c|c}
 \begin{array}{c} 2+4 \times 5 \\ \hline 2+20 \\ \hline 22 \end{array} & 
 \begin{array}{c} 2+4 \times 5 \\ \hline 6 \times 5 \\ \hline \downarrow \\ 30 \end{array}
 \end{array}
 \neq$$

Evaluate expression  $a^b a^d b$

$$a^b a^d b$$

↓

$$a^b a^b b^d b$$

↓

$$\begin{array}{cccc}
 a^b & a^b & b^d & b^d \\
 \hline
 \downarrow & \nearrow & \downarrow & \nearrow \\
 0 & b & 0 & d
 \end{array}$$

ans → d

Evaluate expression  $1^3 5^3 2^1 5$

$$\begin{array}{cccc}
 1^3 & 1^3 & 5^3 & 5^3 & 2^1 & 2^1 \\
 \hline
 \downarrow & \wedge & \downarrow & \wedge & \downarrow & \wedge \\
 0 & & 0 & & 0 & 2
 \end{array}$$

ans = 2

# Left shift (<<)

8 bit no

a = 10  
 7 6 5 4 3 2 1 0  
 0 0 0 0 1 0 1 0

a → 10

a << 1  
 discarded  
 0 0 0 1 0 1 0 0

1 shift = 20 = 10 × 2<sup>1</sup>

a << 2  
 discarded  
 0 0 1 0 1 0 0 0

2 shift = 40  
 ↓  
 10 × 2<sup>2</sup>

a << 3  
 7 6 5 4 3 2 1 0  
 0 0 0 0 1 0 1 0  
 0 1 0 1 0 0 0 0

3 shift = 80  
 ↓  
 10 × 2<sup>3</sup>

a << 4  
 1 0 1 0 0 0 0 0

10 << 4 = 160

a << 5  
 0 1 0 0 0 0 0 0

64 expected 320

Overflow → bigger value cannot be stored in smaller data type / memory space

10 << 5 → 320  
 10 << 6 → 640  
 } big no. won't fit in 8 bits

$$10 \ll 1 = 10 \times 2^1$$

$$a \ll n = a \times 2^n$$

$$\rightarrow 1 \ll n = 2^n$$

$$2^4 \rightarrow 1 \ll 4$$

Right shift ( $\gg$ )

a	<div> <div>7 6 5 4 3 2 1 0</div> <div>0 0 0 1 0 1 0 0</div> </div> <div> </div>	20
a >> 1	<div> <div>0 0 0 0 1 0 1 0</div> </div> <div> </div>	1 shift 10 = $20/2$
a >> 2	<div> <div>0 0 0 0 0 1 0 1</div> </div> <div> </div>	2 shift 5 = $20/2^2$
a >> 3	<div> <div>0 0 0 0 0 0 1 0</div> </div> <div> </div>	2
a >> 4	<div> <div>0 0 0 0 0 0 0 1</div> </div> <div> </div>	1
a >> 5	<div> <div>0 0 0 0 0 0 0 0</div> </div> <div> </div>	0

Overflow does not occur



$$A \gg N = \frac{A}{2^N}$$

$$1 \gg N = \frac{1}{2^N}$$

$$1 \ll 3$$

$$1 \ll n = 2^n$$

$$1 \ll 3 = 2^3 = 8$$

(10 : 29)

$$1$$

	7	6	5	4	3	2	1	0
	0	0	0	0	0	0	0	1

$$1 \ll 3$$

	7	6	5	4	3	2	1	0
	0	0	0	0	1	0	0	0

set bit  $\rightarrow$  0<sup>th</sup> pos

set bit  $\rightarrow$  3<sup>rd</sup> pos

$$1$$

	7	6	5	4	3	2	1	0
	0	0	0	0	0	0	0	1

$$1 \ll i$$

	7	...	i	...	2	1	0
	0	0	0	1	0	0	0

Set  $i^{\text{th}}$  bit

$$N = N | (1 \ll i)$$

$$i = 3$$

1

$$N = \begin{array}{r} \text{5 4 3 2 1 0} \\ 010110 \\ 001000 \\ \hline 011110 \end{array}$$

bit

$$x | 0 = \_$$

↓

ans

$$1 \quad 1 | 0 = 1$$

$$0 \quad 0 | 0 = 0$$

Ⓐ bit  $| 0 = \text{bit}$

OR

$$i$$

$$N \quad \begin{array}{r} \text{5 4 3 2 1 0} \\ 001000 \\ \hline xy1abc \end{array}$$

Toggle  $i^{\text{th}}$  bit in  $N$

2

$$\begin{array}{r} \text{5 4 3 2 1 0} \\ 101101 \\ \hline \text{①} \downarrow \text{1101} \end{array}$$

$$\begin{array}{r} \text{5 4 3 2 1 0} \\ 101101 \\ \hline \text{①} \downarrow \text{10101} \end{array}$$

$$\begin{array}{r}
 \text{N} \\
 \begin{array}{cccccc}
 5 & 4 & 3 & 2 & 1 & 0 \\
 1 & 0 & 1 & 1 & 0 & 1 \\
 0 & 1 & 0 & 0 & 0 & 0 \\
 \hline
 1 & 1 & 0 & 1 & 1 & 0 & 1
 \end{array}
 \end{array}$$

(a) bit 10 = bit

$$i = 4$$

$$\begin{array}{rcl}
 \text{bit} & & \\
 N & \& & 10 \\
 N & & \rightarrow \text{ans} \\
 1 & & 1 \\
 0 & & 0
 \end{array}$$

$$N = N \wedge (1 \ll i)$$

Toggle  $i^{\text{th}}$  bit  
in  $N$

Check whether  $i^{\text{th}}$  bit is set or not in  $N$

$$\begin{array}{r}
 \text{N} \\
 \begin{array}{cccccc}
 5 & 4 & 3 & 2 & 1 & 0 \\
 1 & 0 & 1 & 1 & 0 & 1 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 \hline
 0 & 0 & 0 & 1 & 0 & 0 \rightarrow \text{non zero}
 \end{array}
 \end{array}$$

$$\begin{array}{rcl}
 N & = & 1010 \\
 \& 1 & = & 0001
 \end{array}$$

$$\begin{array}{rcl}
 i = 2 & \rightarrow & \text{True} \\
 i = 4 & \rightarrow & \text{False}
 \end{array}$$

$$\begin{array}{r}
 \text{N} \\
 \begin{array}{cccccc}
 5 & 4 & 3 & 2 & 1 & 0 \\
 1 & 0 & 1 & 0 & 0 & 1 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 \hline
 0 & 0 & 0 & 0 & 0 & 0 \rightarrow \text{zero}
 \end{array}
 \end{array}$$

if ans is 0, then  $i^{\text{th}}$  bit is unset  
ans is non 0, then  $i^{\text{th}}$  bit is set

$$\text{ans} = N \& (1 \ll i)$$

set its bit

Toggle in bit  
XOR

Check if  
i<sup>th</sup> bit is set **AND**

$$Z = Z \mid (1 \leq i)$$

$$Z = Z^{\sim} \quad (1 \leq i)$$

ans = N & (1<i)

0 ↓ bit unset  
 ≠ 0 ↓ bit set

$$N = 3$$
 $i = 4$  $i = 1$ 

false

4-200

7 1 5 4 3 2 1 0  
0 0 0 0 0 1 1 1

if  $(N \geq (1 - \epsilon) = 0)$

return false

else

return true

TC:  $O(n)$

SC: O(1)

$\begin{array}{r} \phantom{0}N \\ 8 \quad 12 < i \end{array}$

$\begin{array}{ccccccc} & & & & i & & \\ & & & & \textcircled{0} & & \\ - & - & - & - & - & - & - \\ 0 & 0 & 0 & 1 & 0 & 0 & \\ \hline 0 & 0 & 0 & & 0 & 0 & \end{array}$

2. Given an integer  $N$ , count no. of set bits in  $N$ .

$N = 12$       00001100      cnt 2

31      2      1      0

N      —      —      —      —      —      —

int N  
 ↓  
 4 B  
 ↓  
 32 bits

BF: Go to every bit pos, check whether bit is 1. If it is 1, count it

```

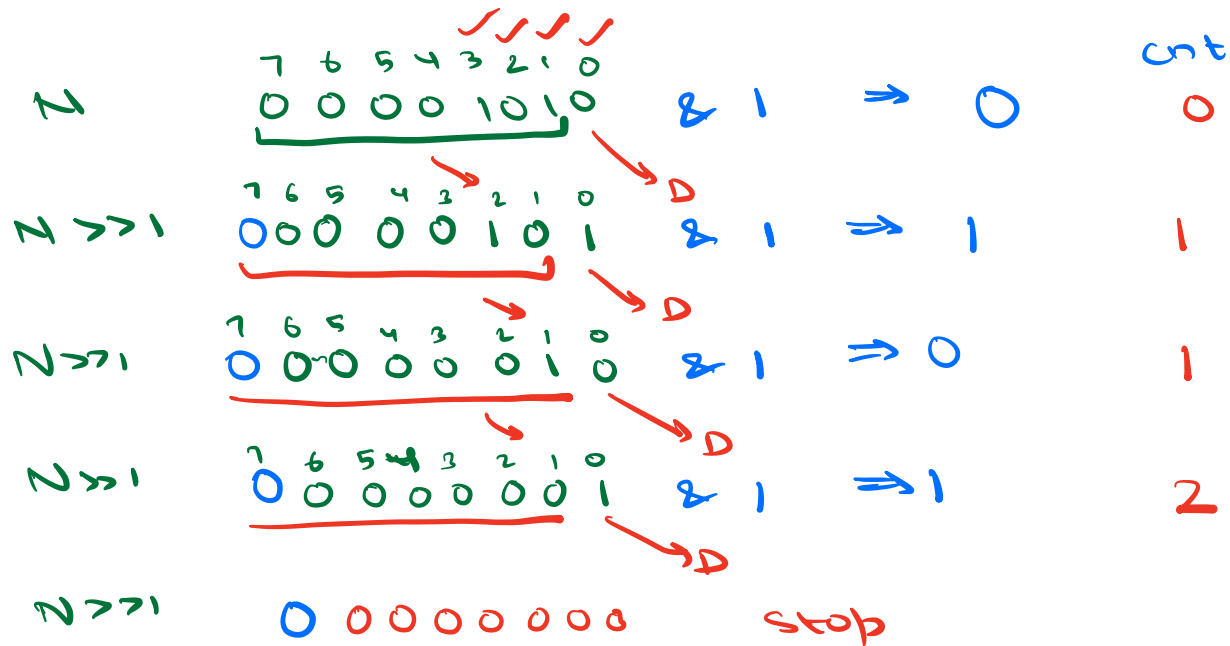
int countbit (int N) {
    int cnt = 0
    for (i = 0 ; i < 32 ; i++) {
        // ith bit pos → check if set
        if (checkbit (N, i))
            cnt++
    }
    return cnt
}

```

32 itr  
↓  
TC : O(1)  
SC : O(1)

## Approach 2

$N = 10$



Time Complexity:  $O(\log N)$

```

int countbit (int N) {
    int cnt = 0;
    while (N > 0) {
        if (N & 1 == 1)
            cnt++;
        N = N >> 1;
        N = N / 2;
    }
}
    
```

Sequence of values for  $N$ :  $10 \rightarrow 5 \rightarrow 2 \rightarrow 1 \rightarrow 0$

Sequence of values for  $N/2$ :  $N \rightarrow N/2 \rightarrow N/4 \rightarrow \dots \rightarrow 0$

$\log N$  steps

31 . . . . . 1 0  
 X X X X X X X

3. Unset  $i^{\text{th}}$  bit

$N = 12$   
 $i = 2$   
 $\text{ans} = 8$

32 10  
 11 00  
 ↓  
10 00

$N = 12$   
 $i = 1$   
 $\text{ans} = 12$

32 10  
 11 00  
11 00

Check  $i^{\text{th}}$  bit is set or not  
 ↓  
 Toggle  $i^{\text{th}}$  bit

↓  
 X

$$N = N \wedge (1 \ll i)$$

ans =  
 $N \& (1 \ll i)$

fn unsetbit ( $N, i$ ) <  
 if (checkbit ( $N, i$ )) < true  
 $N = N \wedge (1 \ll i)$   
 >

↓  
 >

TC :  $O(1)$   
 SC :  $O(1)$

4. make a binary no. which has B 1s followed by C 0s

$$B = 3 \quad C = 2$$

1100  $\rightarrow$  ans 28

7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0

$$B = 3 \quad C = 2$$

7	6	5	4	3	2	1	0
0	0	0	0	0	0	0	0
	1	1	1	1			

$$B = 4 \quad C = 3$$

First bit  $\rightarrow$  c bit pos

Last bit  $\rightarrow c + B - 1$  bit pos

cnt = B

1. — 2.

$$j - i + 1 = B$$

$$E \rightarrow \textcircled{A}$$

$$j - C + 1 = B$$

$$\Rightarrow j = B + C - 1$$



```

int num = 0
for (i = C ; i ≤ C + B - 1 ; i++) {
    num = num | (1 << i)
}

```

8 iter  
 Tc:  $O(B)$   
 Sc:  $O(1)$

---

Doubts

A = 1 2 3 → 132

1, 2, 3  
 132 ✓  
 321 ✓  
 213 ✓  
 231 ✓  
 312 ✓  
 123

Vector <pair <int, int>> V

v.push\_back(<1, 3>)  
 v.push\_back(<4, 6>)

Interval  
 1, 3  
 4, 6



v[0].first → 1  
 v[0].second → 3

---



[illegible]

d  
c

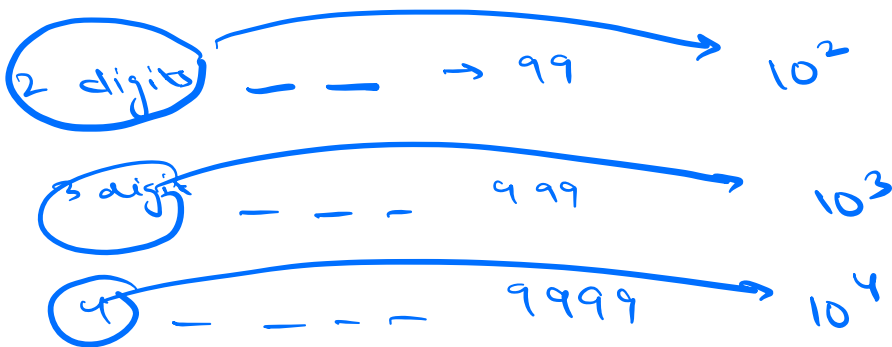
$$C = \text{sum} / 10$$

$$\text{sum} = \text{dig of } a + \text{dig of } b + c$$

Num = 122  
    ↑ 1

Num   
int 

A 123


$$A_{100} = \text{---}$$

$10^{100}$

1 1 1 3 2 1 1 2 5 9 6 5  
1 0 0 0 0 0 0 0 0 0

int  
 $10^9$