- What is Dynamic Programming?
- Conditions to use DP
- Why DP? → Fibonacci Series
- No. of Stairs
- Min. Perfect Squares

Contest 5 → Next Fri

Contest 3 and 4 → Reattempt

DP

Graphs

Jan End

↓

DSA Mock Interview

Fibonacci Series

N	0	1	2	3	4	5	6	.....
	0	1	1	2	3	5	8	......

$$fib(n) = fib(n-1) + fib(n-2)$$

Base Case

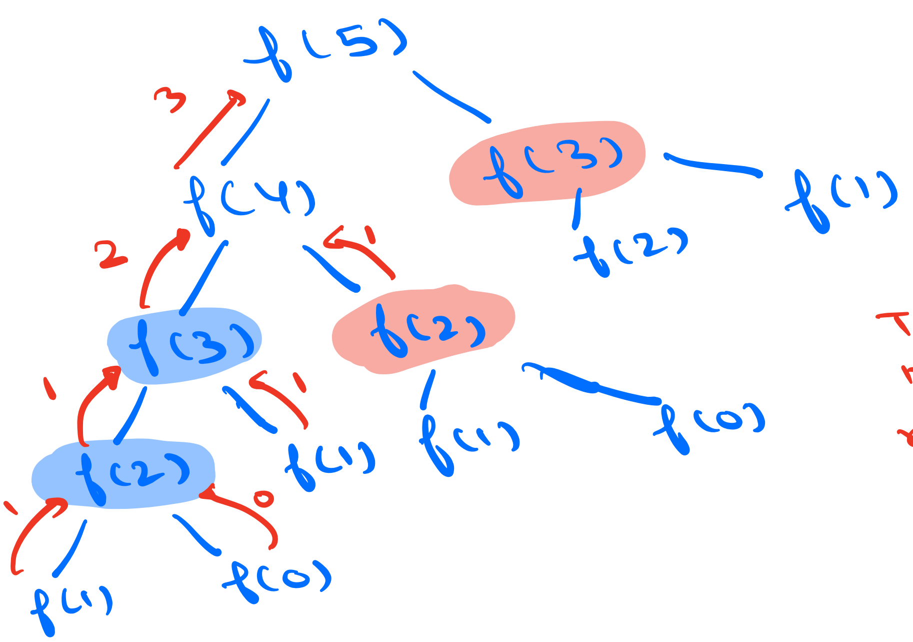
Recursive relation

```
int fib(n) {
    if (n <= 1)
        return n
    return fib(n-1) + fib(n-2)
}
```

→ TC : $O(2^N)$
SC : $O(N)$

$N = 10$ Iterations → $2^{10} = 1024 = 10^3$

$N = 20$ Iterations → $2^{20} = 10^6$



Too many repetitive calls

DP → when some problems repeat again, store their ans

Conditions for DP

1. Optimal substructure : solving a problem by breaking into similar subproblems
2. Overlapping subproblems

```
int dp[N+1] = {-1}

int fib(n) {
    if (n <= 1)
            return n
    if (dp[N] != -1)
            return dp[N]

    dp[N] = fib(n-1) + fib(n-2)
    return dp[N]
}
```

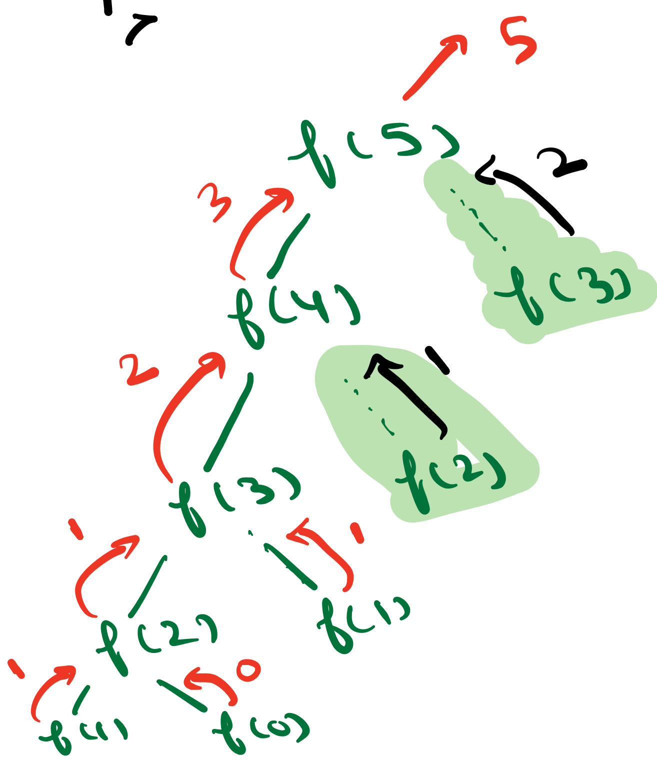
N

N-1

N-2

.
.
.

0



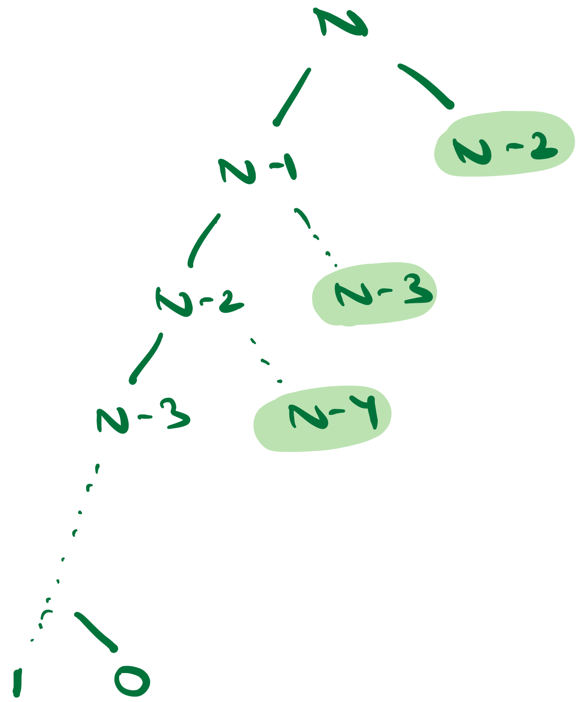
0	1	2	3	4	5
-1	-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~
		1	2	3	5

TC : O(N)

SC : O(N + N)

↓

O(N)



fn (N) {

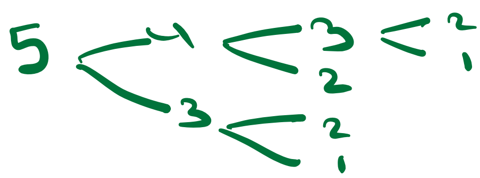
Base Case

if (ans for N already stored) return from memory

memory = Recursive call

return memory

}



Types of DP

Recursion
Top Down
(Memoization)

Iterative
Bottom Up
(Tabulation)

5 → 4 → 3 → 2 → 1 0

```
dp[N+1] = <-1>

dp[0] = 0
dp[1] = 1

for (i=2 ; i ≤ N ; i++) <
    dp[i] = dp[i-1] + dp[i-2]
>
return dp[N]
```

TC : O(N)
SC : O(N)

N=5

0	1	2	3	4	5
~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~	~~-1~~
0	1	1	2	3	5

f(3)
f(2)

* Bottom up DP with SC: O(1) // 0th → Nth term

a = 0
b = 1

for (i = 2 ; i ≤ N ; i++) {
  c = a + b
  a = b
  b = c
}

return c

i=2   a b c
        0 1 1

i=3   a b c
        1 1 2

i=4   a b c
        1 2 3

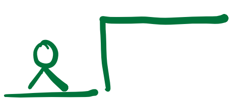
a b c
0 1 1
   a b

TC : O(N)
SC : O(1)

---

2. Given N stairs, in how many ways we can go from 0th to Nth stair if we take a jump of 1 stair or 2 stairs at a time?

N = 1



<1>

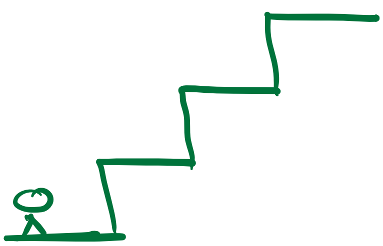
ans = 1

N = 2

<1, 1>
<2>

ans = 2

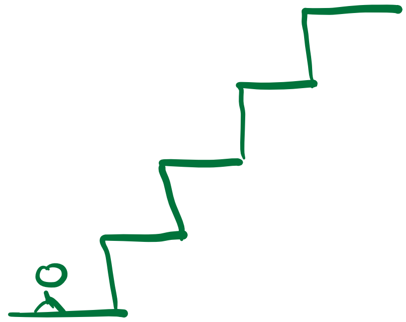
N = 3



<1, 1, 1>
<1, 2>
<2, 1>

ans = 3

$N=4$



<1, 1, 1, 1>

<1, 1, 2>

<1, 2, 1>

<2, 1, 1>

<2, 2>

ans = 5

4th step ⇒ 5

1 stair

2 stairs

3rd step

<1, 1, 1> 1

<1, 2> 1

<2, 1> 1

2nd step

<1, 1>, 2

<2>, 2

ways (N^{th} step)

1 stair

2 stairs

ways($N-1^{th}$)

1

1

1

ways($N-2^{th}$)

2

2

2

2

ways(N) = ways(N-1) + ways(N-2)

5th step → 8

1 2

5 4th step 3rd step 3

<1, 1, 1, 1> 1	<1, 1, 1> 2
<1, 1, 2> 1	<1, 2> 2
<1, 2, 1> 1	<2, 1> 2
<2, 1, 1> 1	
<2, 2> 1	

① DP relation

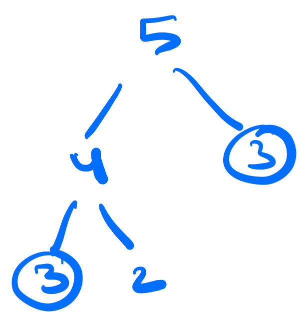
ways(N) = ways(N-1) + ways(N-2)

② Base case

N=1 ways(1) = 1

N=0 ways(0) = 1

Do nothing



ways(2) = ways(1) + ways(0)

2 1 1

ways(-5) = 0

No ways

N = 2 ways(2) = 2

3. Find minimum number of perfect squares required to get sum = N

$\downarrow$ 1, 4, 9, 16, 25, ...

N = 2 $\quad 1^2 + 1^2 \quad$ cnt 2

N = 3 $\quad 1^2 + 1^2 + 1^2 \quad$ cnt 3

N = 4 $\quad 1^2 + 1^2 + 1^2 + 1^2 \quad$ cnt 1

2^2

N = 5 $\quad 1^2 + 1^2 + 1^2 + 1^2 + 1^2 \quad$ cnt 2

$2^2 + 1^2$

✗ Greedy Approach → To make no. n, use biggest perfect square possible

$$12 \xrightarrow{-3^2} 3 \xrightarrow{-1^2} 2 \xrightarrow{-1^2} 1 \xrightarrow{-1^2} 0$$

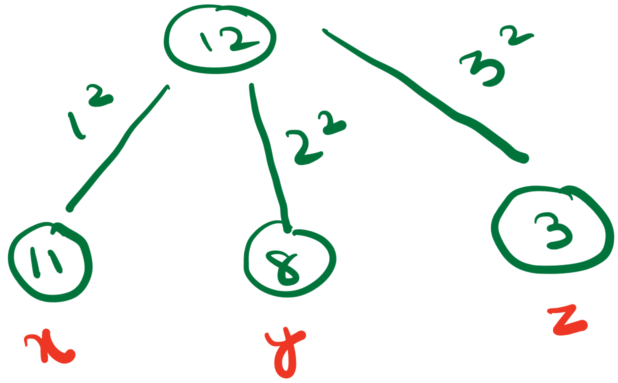
greedy → 4 terms

$\downarrow$

$2^2 + 2^2 + 2^2$

actual ans = 3

Brute Force : Try every possible way to form sum



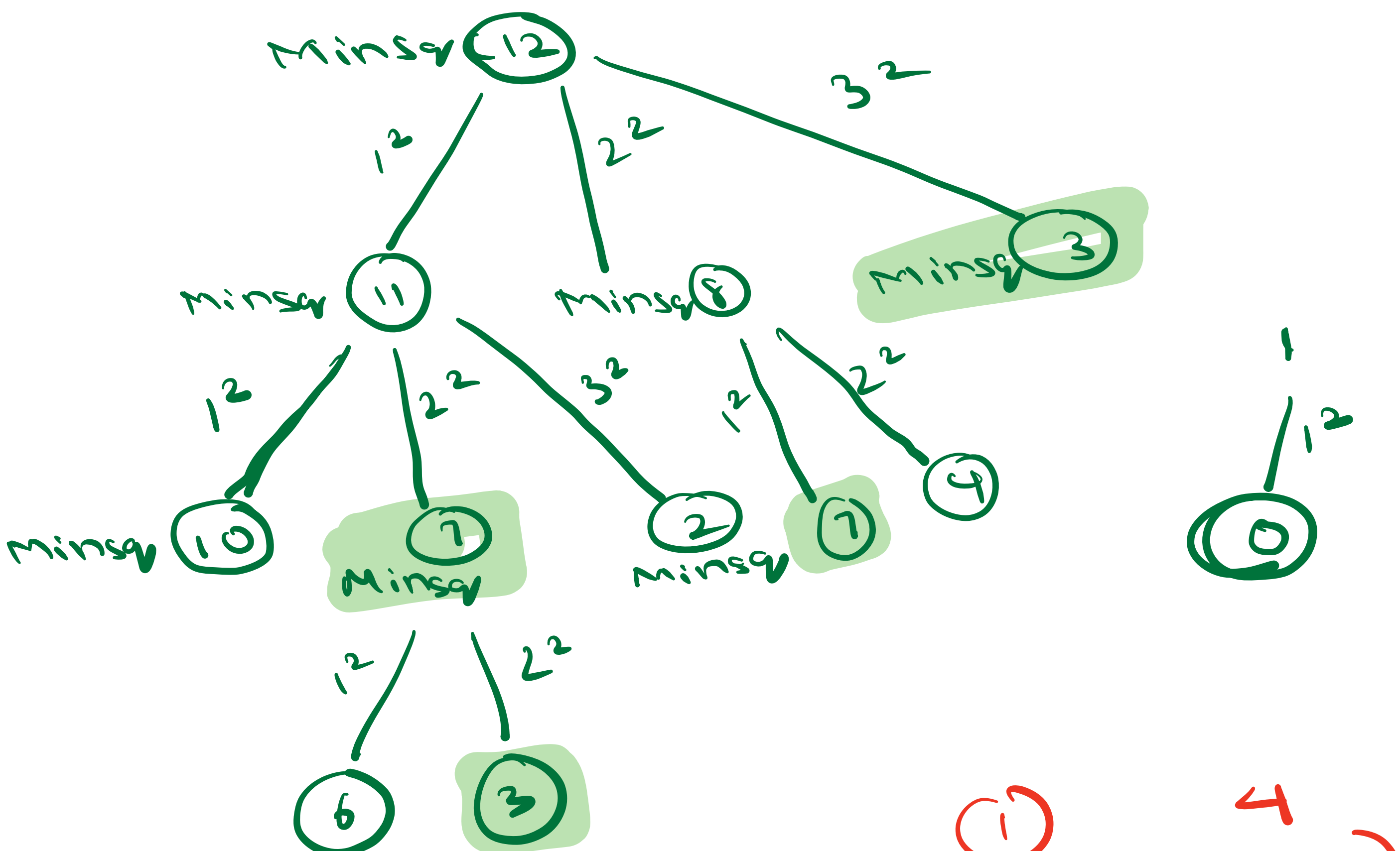
Minsq(12)

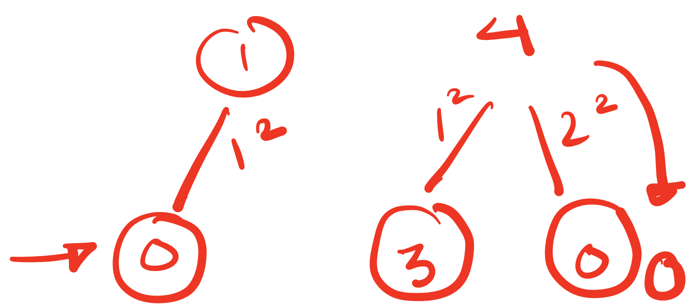
↓

min(x,y,z) +1

Minsq(12) = Min (Minsq(11), Minsq(8), Minsq(3))

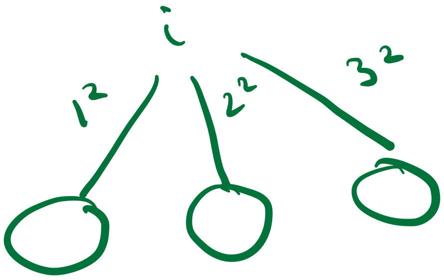
+ 1



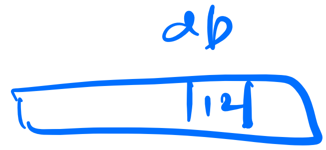


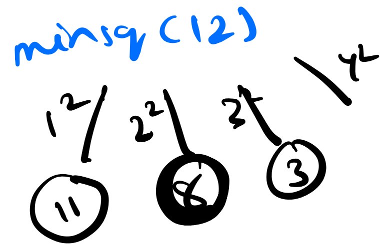
$Minsq(i) = \min\{Minsq(i - x^2)\} + 1$

for all $x \Rightarrow x^2 \le i$



$MinSq(0) = 0$






```
int dp[N+1] = {-1}

int minsq(int N) {
    if (N == 0)
        return 0

    if (dp[N] != -1)
        return dp[N]

    int minval = INT_MAX
    for( x = 1 ; x * x <= N ; x++) {
        minval = min(minval, minsq( N - x^2))
    }

    dp[N] = minval + 1
    return dp[N]
}
```

TC: $O(N\sqrt{N})$

SC: $O(N)$

Iterative

$0 \to 1 \to 2 \to 3$

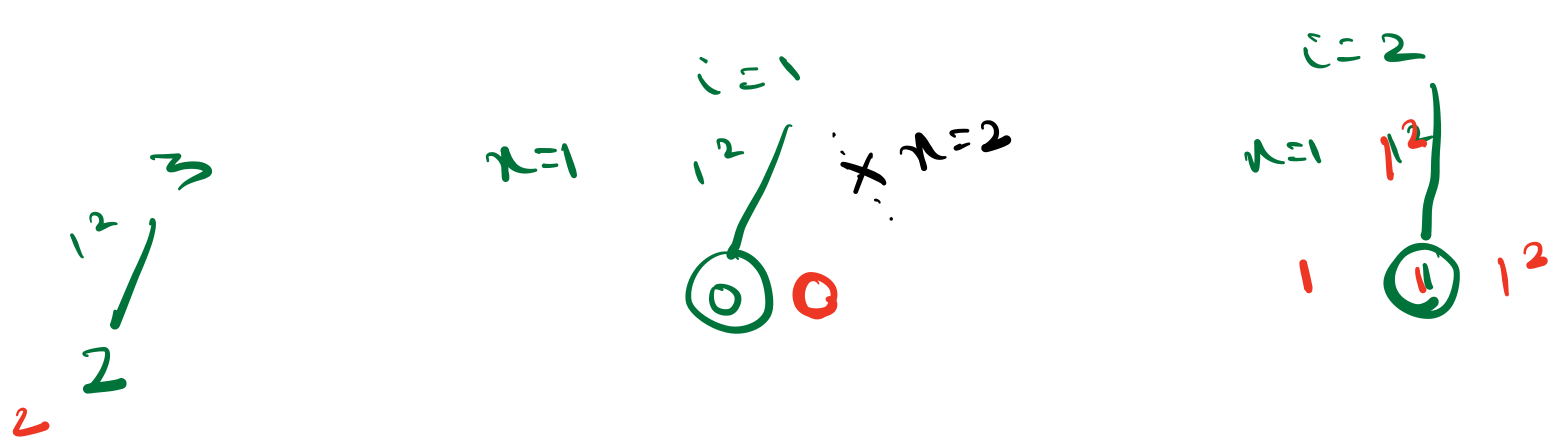
```
int dp[N+1] = {-1}
 dp[0] = 0

for (i=1 ; i <= N ; i++) {
        int   minval = INT_MAX
        for (x=1 ; x*x <= i ; x++) {
            minval = min(minval, dp[i - x^2])
        }
        dp[i] = minval + 1
}
```

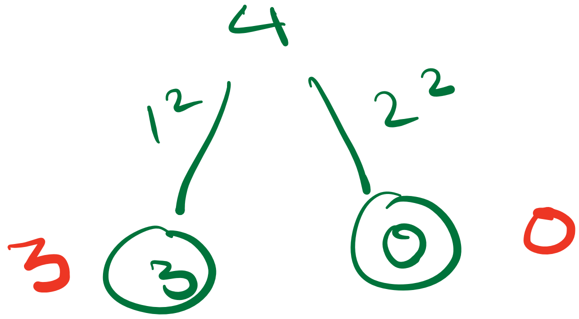
TC : $O(N\sqrt{N})$
SC : $O(N)$

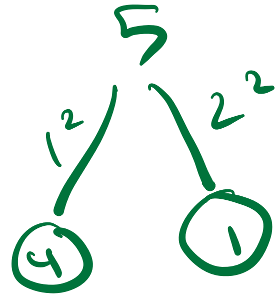
dp[i] = Minsq to make i

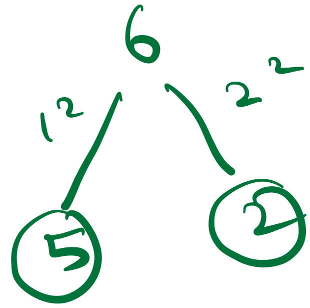
N = 9

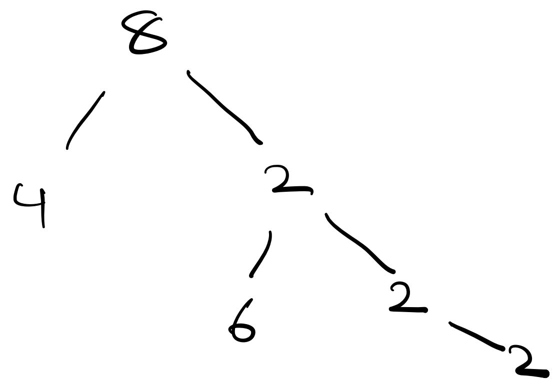
0	1	2	3	4	5	6	7	8	9
0	1	2	3	1					




4
1²
2²
3
3
0
0


5
1²
2²
4
1


6
1²
2²
5
2


8
4
2
6
2
2