Projective 3 Points PnP

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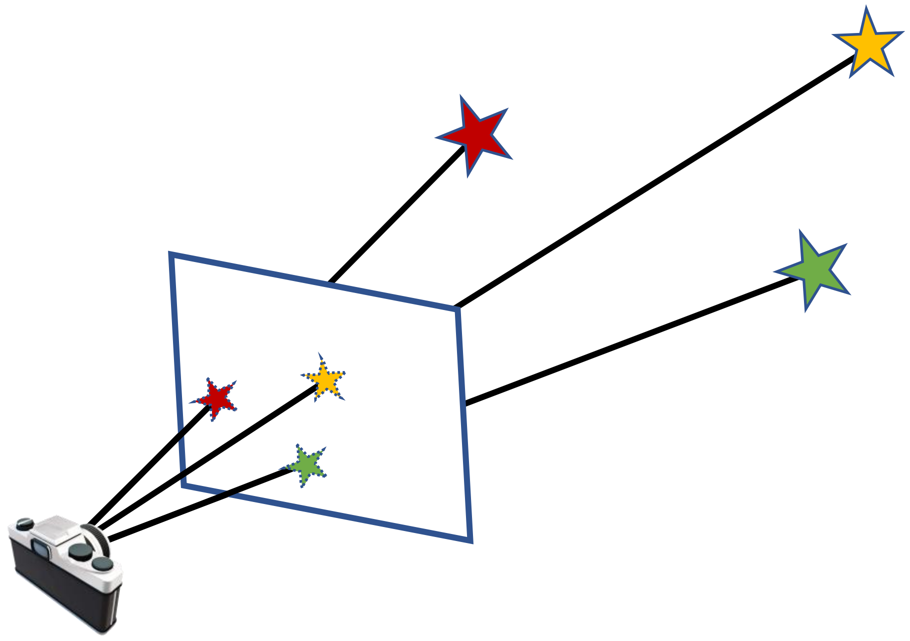
Projective 3 Points

- Estimate the pose of a calibrated camera given control points

- Uses ≥ 3 points
- Often used with RANSAC
- P3P, PnP, Spatial Resection
- We present Grunert's Method

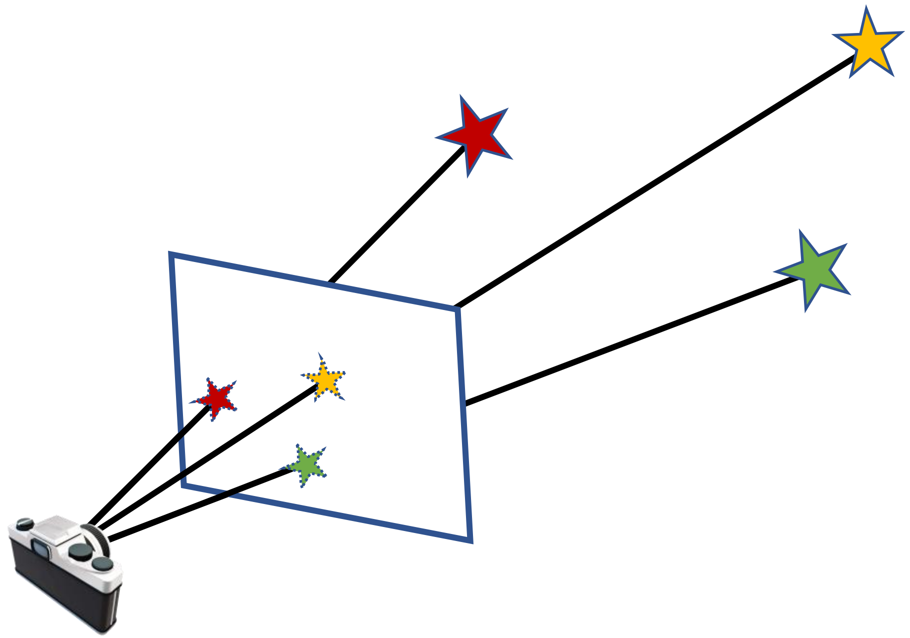


P3P

?

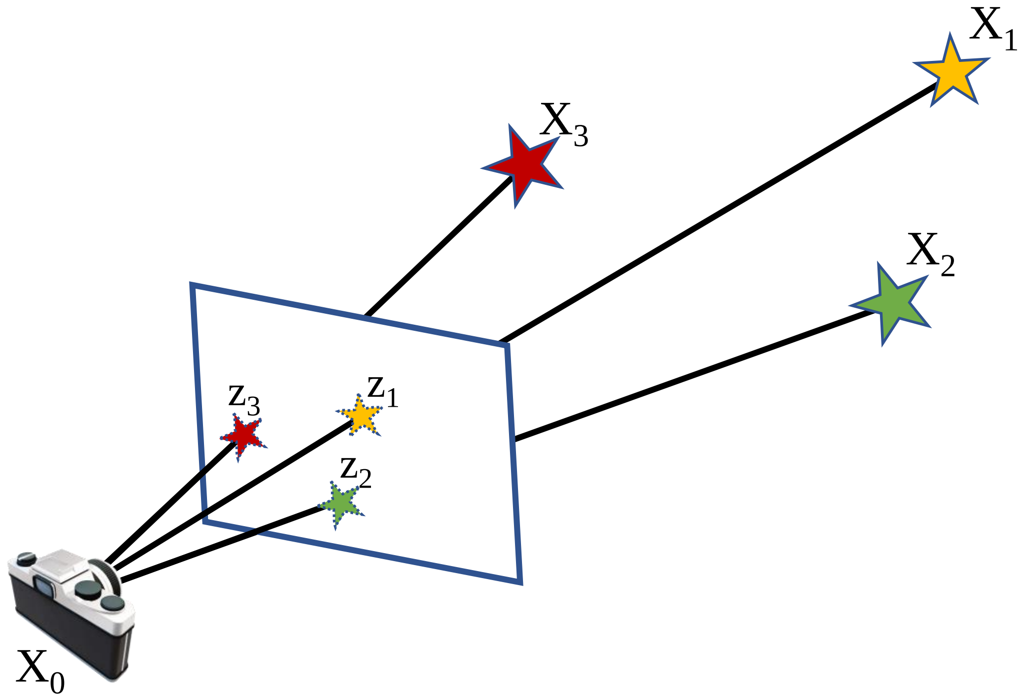
P3P

- $\lambda p = K[R|t]X$
- Total of 6dof to estimate
  - We need at least 3 points



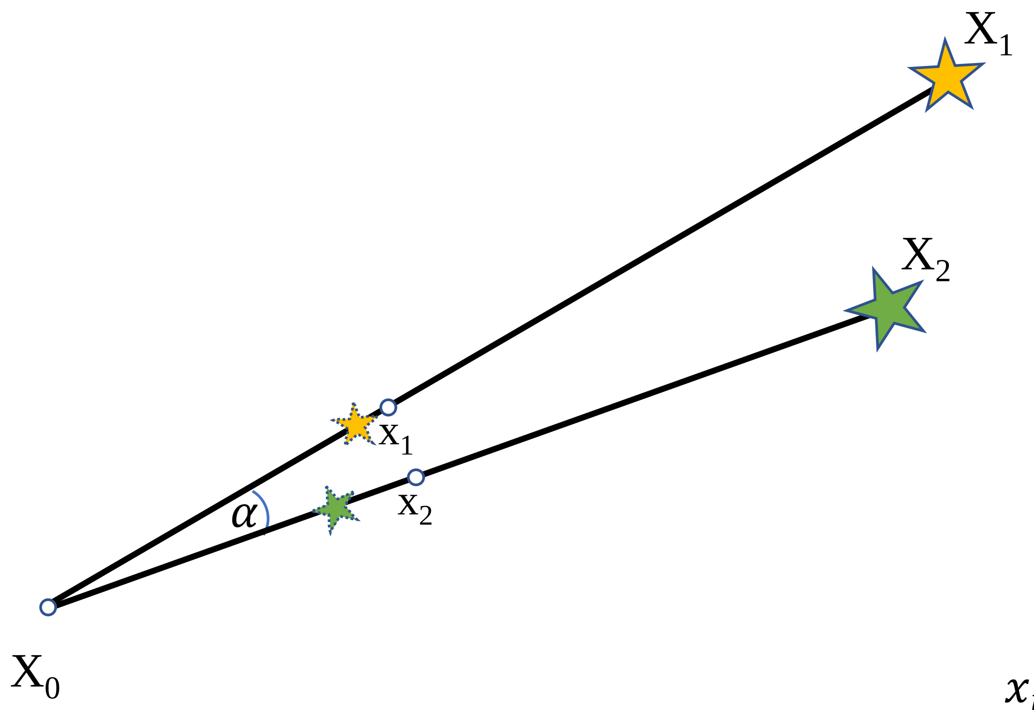
P3P

- Given: $X_1, X_2, X_3, z_1, z_2, z_3$



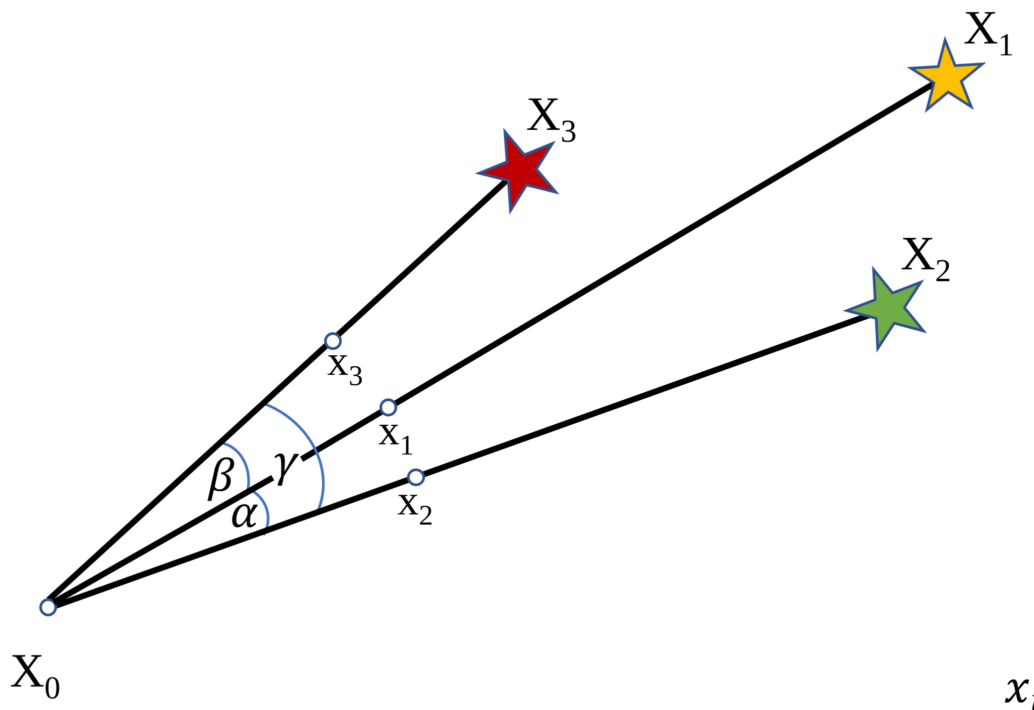
$$\lambda z_i = KX_i \quad \Rightarrow \quad X_i = \lambda K^{-1} z_i$$

P3P
angles



$$x_i = \frac{K^{-1} z_i}{\|K^{-1} z_i\|}$$

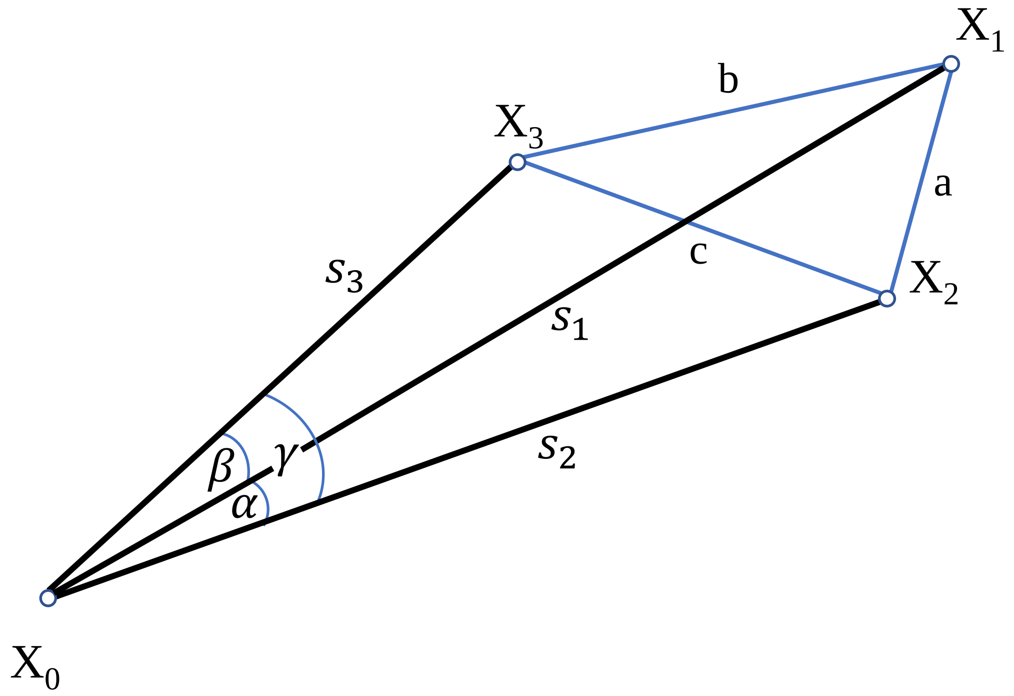
P3P
angles



$$\alpha = \cos^{-1}(x_1^T x_2)$$
$$\beta = \cos^{-1}(x_1^T x_3)$$
$$\gamma = \cos^{-1}(x_2^T x_3)$$

$$x_i = \frac{K^{-1} z_i}{\|K^{-1} z_i\|}$$

P3P
distances

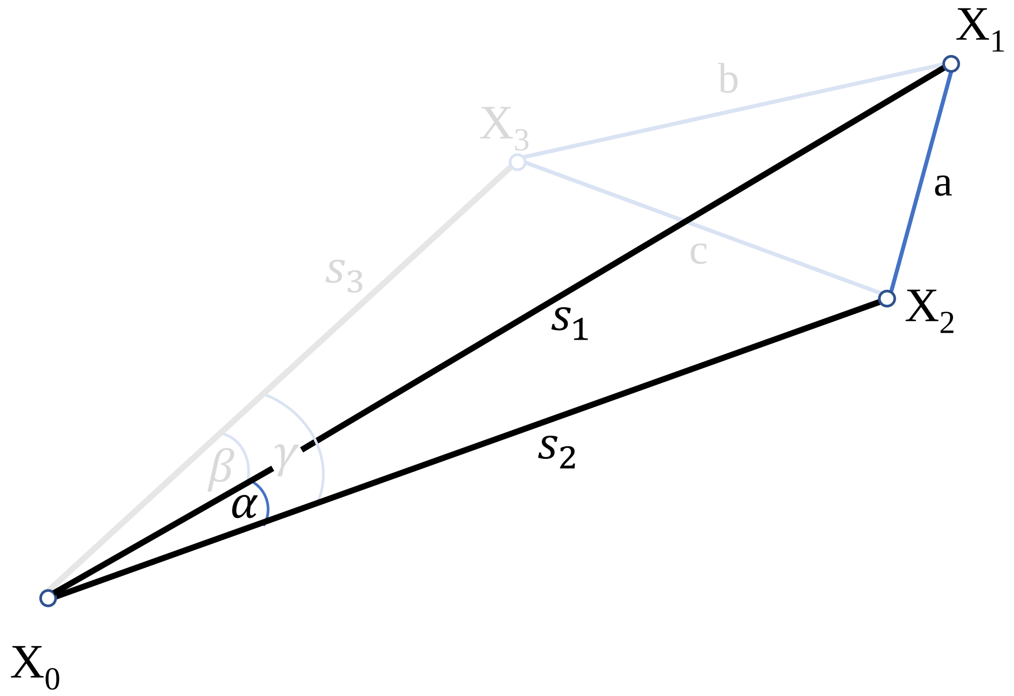


$$a = \|X_1 - X_2\|$$

$$b = \|X_1 - X_3\|$$

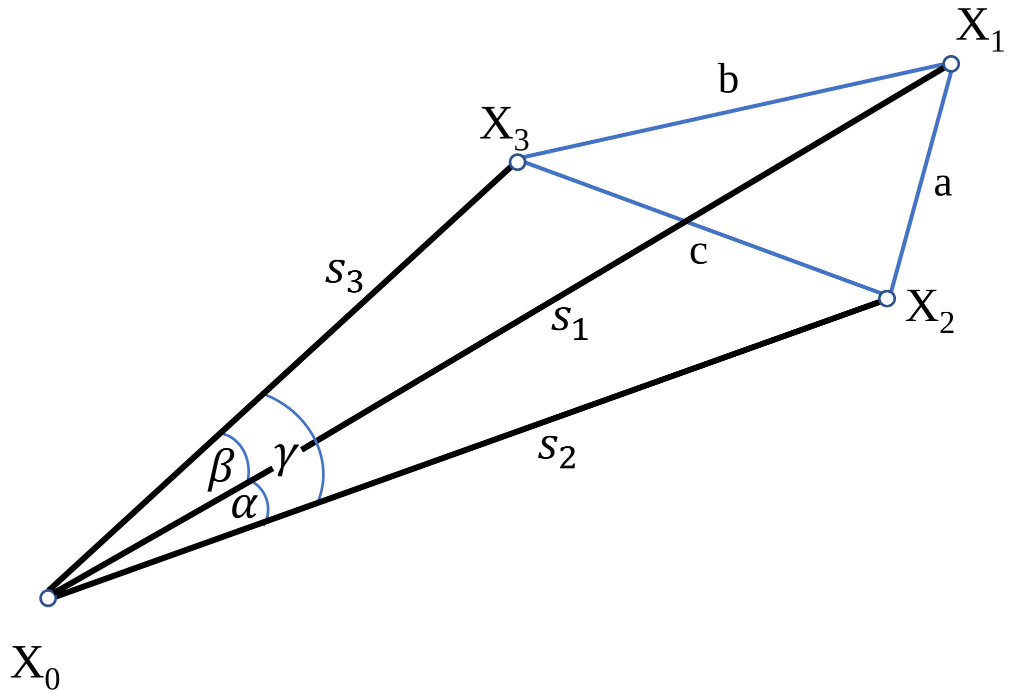
$$c = \|X_2 - X_3\|$$

P3P
distances



$$a^2 = s_1^2 + s_2^2 - 2 s_1 s_2 \cos\alpha$$

P3P
distances



$$a^2 = s_1^2 + s_2^2 - 2s_1 s_2 \cos\alpha$$
$$b^2 = s_1^2 + s_3^2 - 2s_1 s_3 \cos\beta$$
$$c^2 = s_2^2 + s_3^2 - 2s_2 s_3 \cos\gamma$$

P3P

- 4^{th} degree polynomial in $\dfrac{s_3}{s_1}$

$$\boxed{\boldsymbol{A\left(\frac{s_3}{s_1}\right)^4 + B\left(\frac{s_3}{s_1}\right)^3 + C\left(\frac{s_3}{s_1}\right)^2 + D\left(\frac{s_3}{s_1}\right) + E = 0}}$$

$$\delta_{1,2} = \frac{a^2 \pm c^2}{b^2}$$

$$A = (\delta_2 - 1)^2 - \frac{4c^2}{b^2}\cos^2\alpha$$

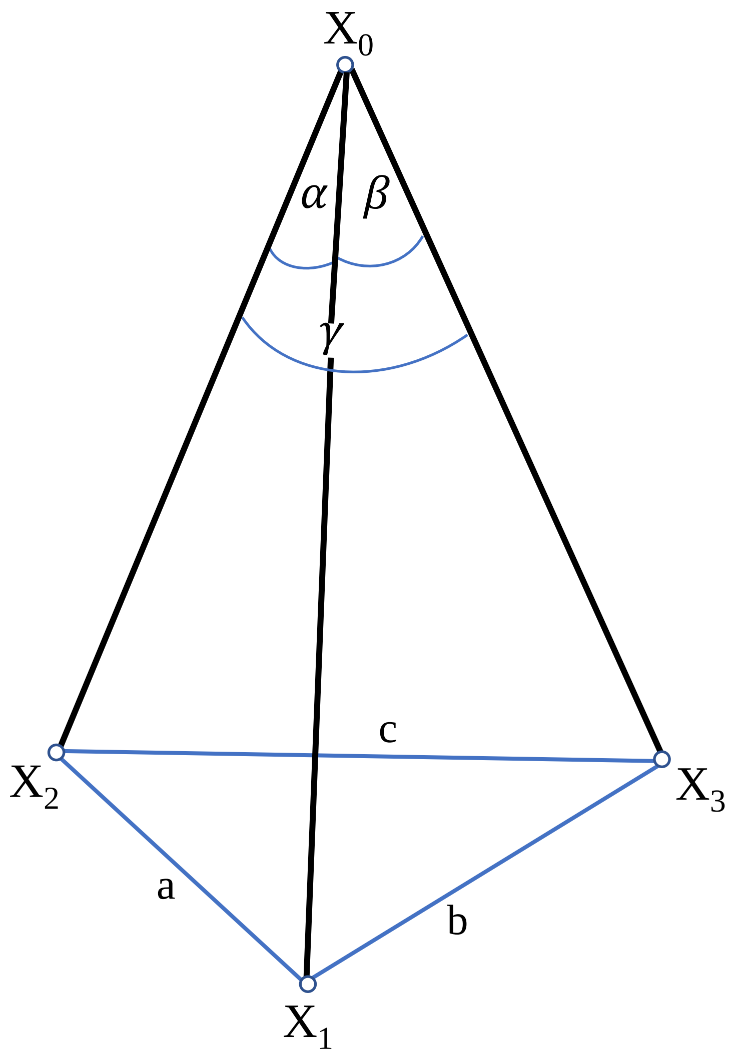
$$B = 4\left(\delta_2(1-\delta_2)\cos\beta - (1-\delta_1)\cos\alpha\cos\gamma + 2\frac{c^2}{b^2}\cos^2\alpha\cos\beta\right)$$

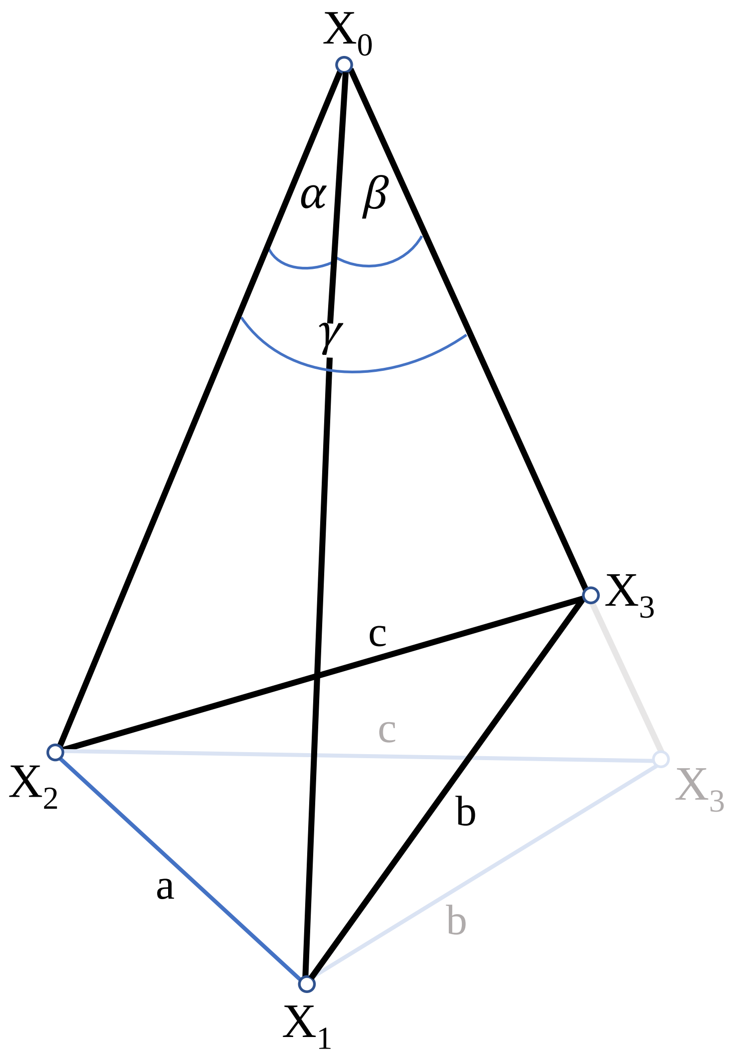
$$C = 2\left(\delta_2^2(1+2\cos^2\beta) - 1 + 2\delta_2\cos^2\alpha - 4\delta_1\cos\alpha\cos\beta\cos\gamma + 2\left(\frac{b^2-a^2}{b^2}\right)\cos^2\gamma\right)$$

$$D = 4\left(\frac{2a^2}{b^2}\cos^2\gamma\cos\beta - \delta_2(1+\delta_2)\cos\beta - (1-\delta_1)\cos\alpha\cos\gamma\right)$$

$$E = (1+\delta_2)^2 - \frac{4a^2}{b^2}\cos^2\gamma$$

P3P
ambiguity



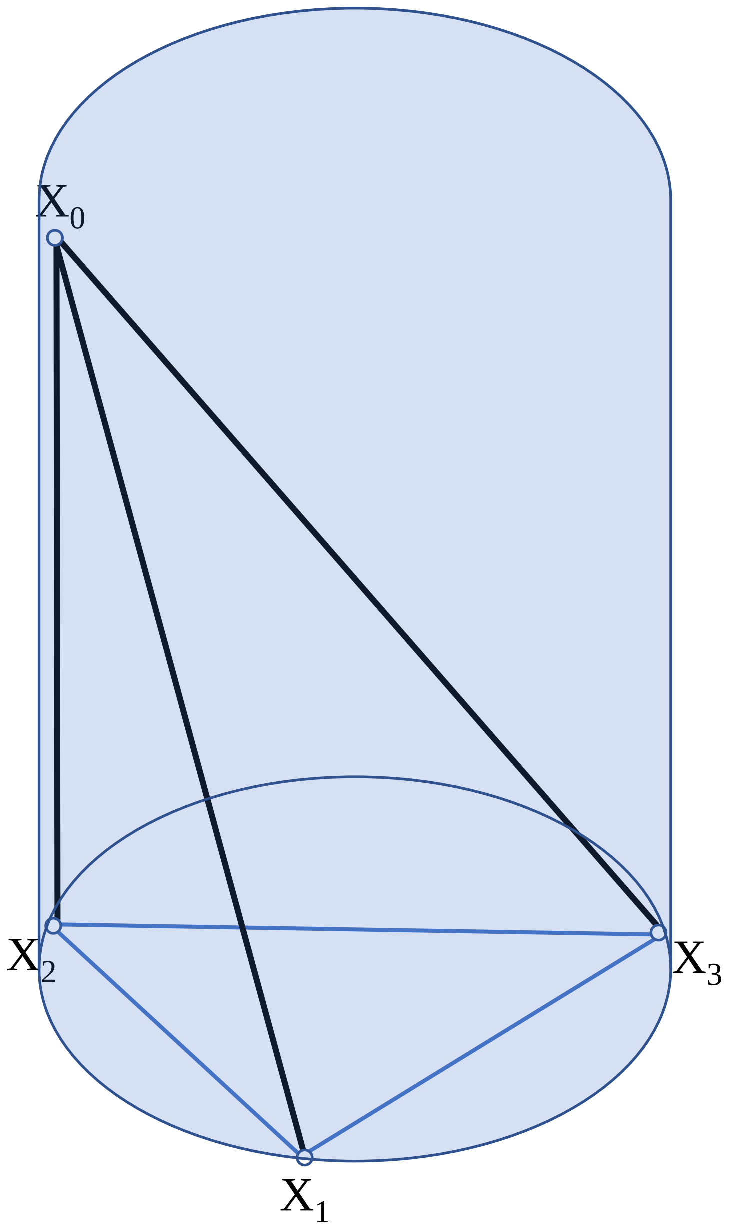


P3P ambiguity

- 4 possible positions
- Use initial guess
- Use an additional point - P4P

P3P

- Critical cylinder



P3P
world pose

- Find transformation to world coordinates

$$C = \begin{bmatrix} | & | & | \\ s_1 x_1 & s_2 x_2 & s_3 x_3 \\ | & | & | \end{bmatrix} - \bar{C}$$

$$W = \begin{bmatrix} | & | & | \\ X_1 & X_2 & X_3 \\ | & | & | \end{bmatrix} - \bar{W}$$

$$\bar{C} = mean(s_i x_i)$$

$$\bar{W} = mean(X_i)$$

P3P
world pose

- Find transformation to world coordinates

$$C = \begin{bmatrix} | & | & | \\ s_1 x_1 & s_2 x_2 & s_3 x_3 \\ | & | & | \end{bmatrix} - \bar{C}$$

$$W = \begin{bmatrix} | & | & | \\ X_1 & X_2 & X_3 \\ | & | & | \end{bmatrix} - \bar{W}$$

$$C = RW$$

$$CW^T = RWW^T$$

$$CW^T = RVDV^T = UDV^T$$

$$RV = U \quad \Longrightarrow \quad R = UV^T$$

$$R(X - \bar{W}) + \bar{C} = RX + (\bar{C} - R\bar{W})$$

$$\boxed{[R|\bar{C} - R\bar{W}]}$$

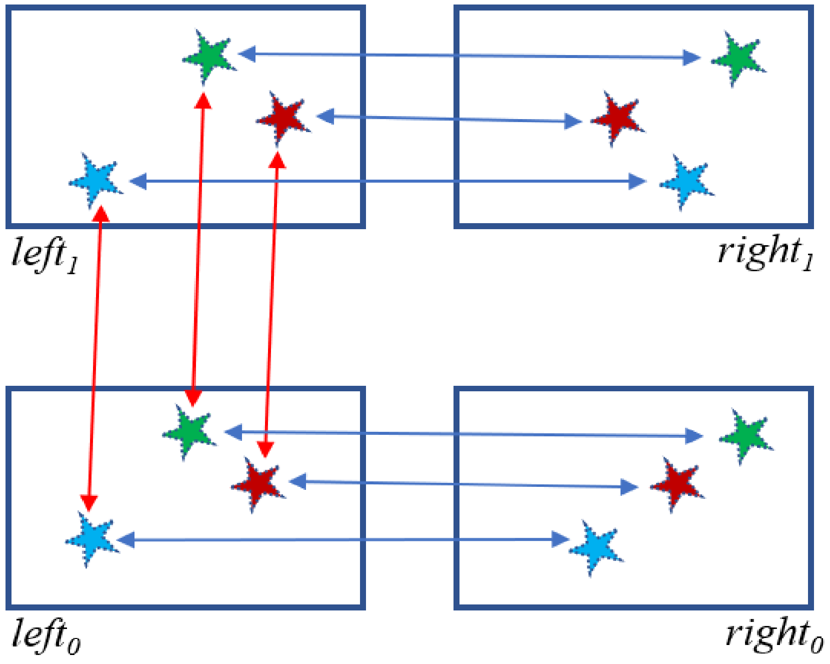
P3P

- Use RANSAC
- PnP using least squares approach

X_3

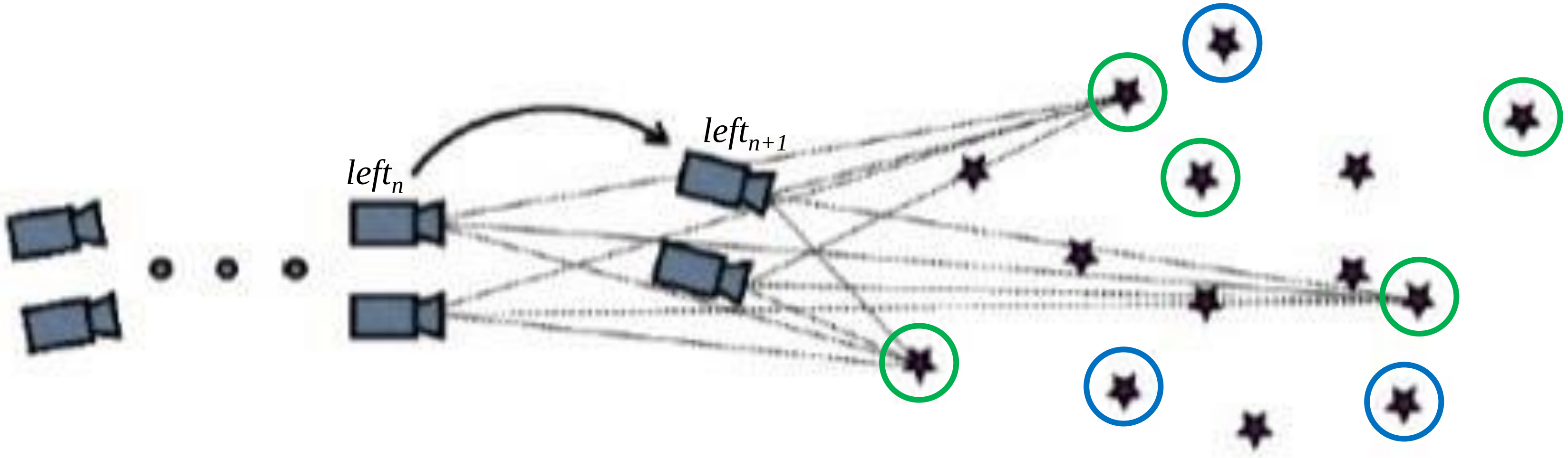
Project part II

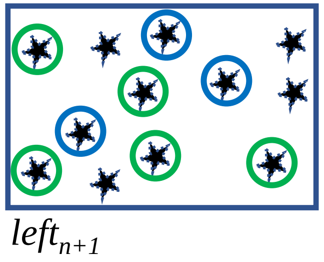
- Tracking



Consensus Matching

- Relative motion estimation
  - cv2.solvePnPRansac

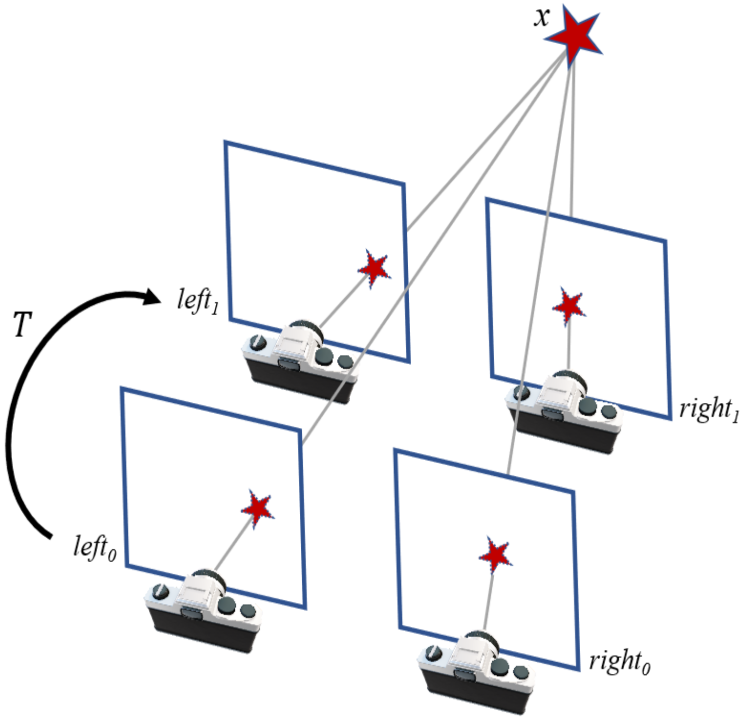


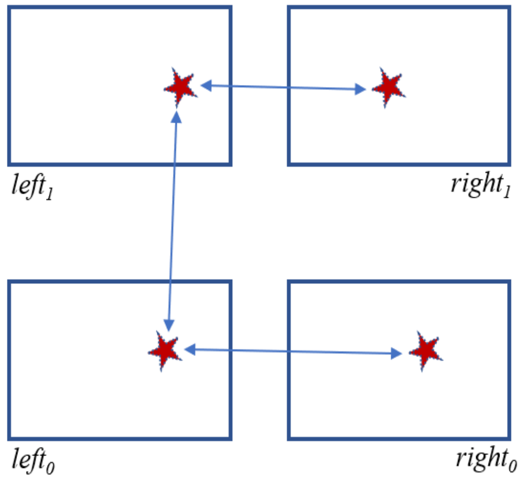


Courtesy of FrameSlam Konolige, Agrawal

Consensus Matching

- PnP with improved RANSAC
- What is an inlier?





Detection

- Non-maximal suppression
- GridAdapterFeatureDetector