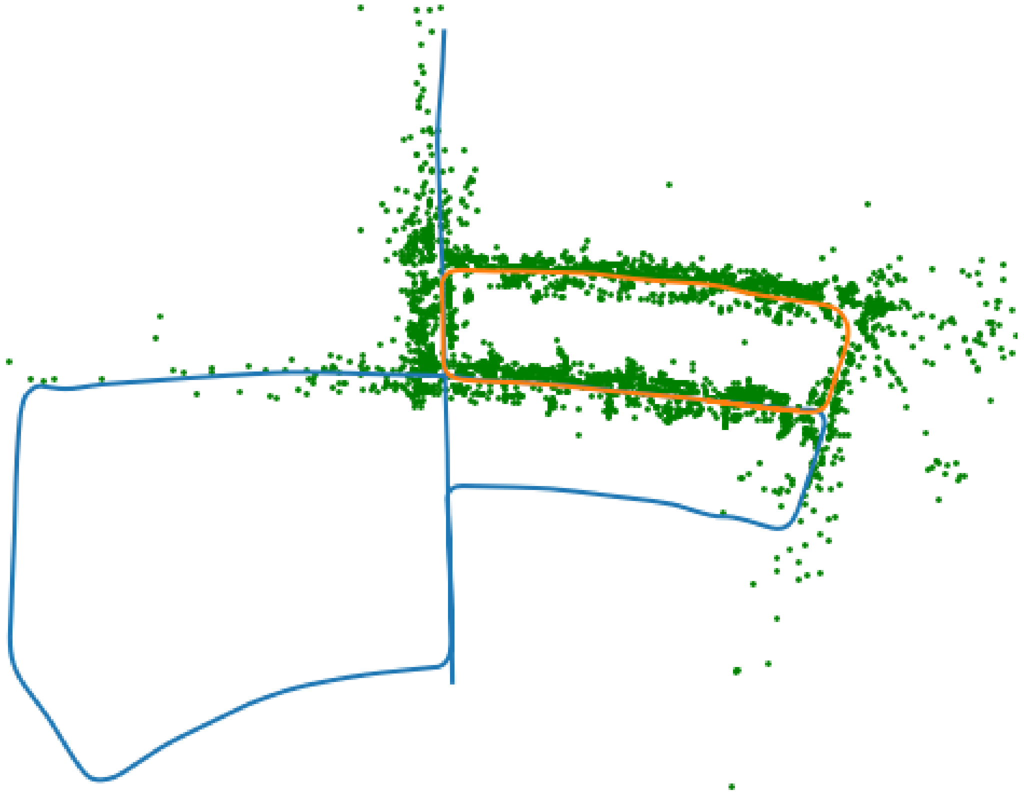
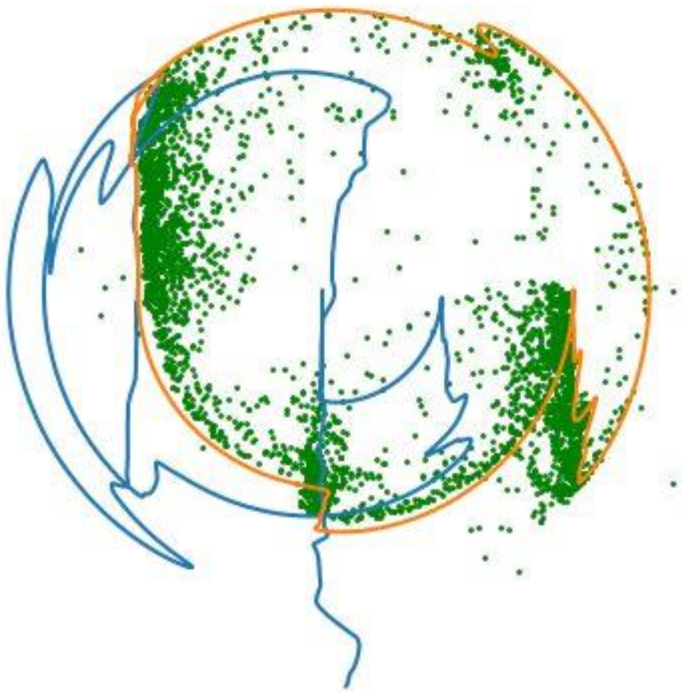
Bundle Adjustment

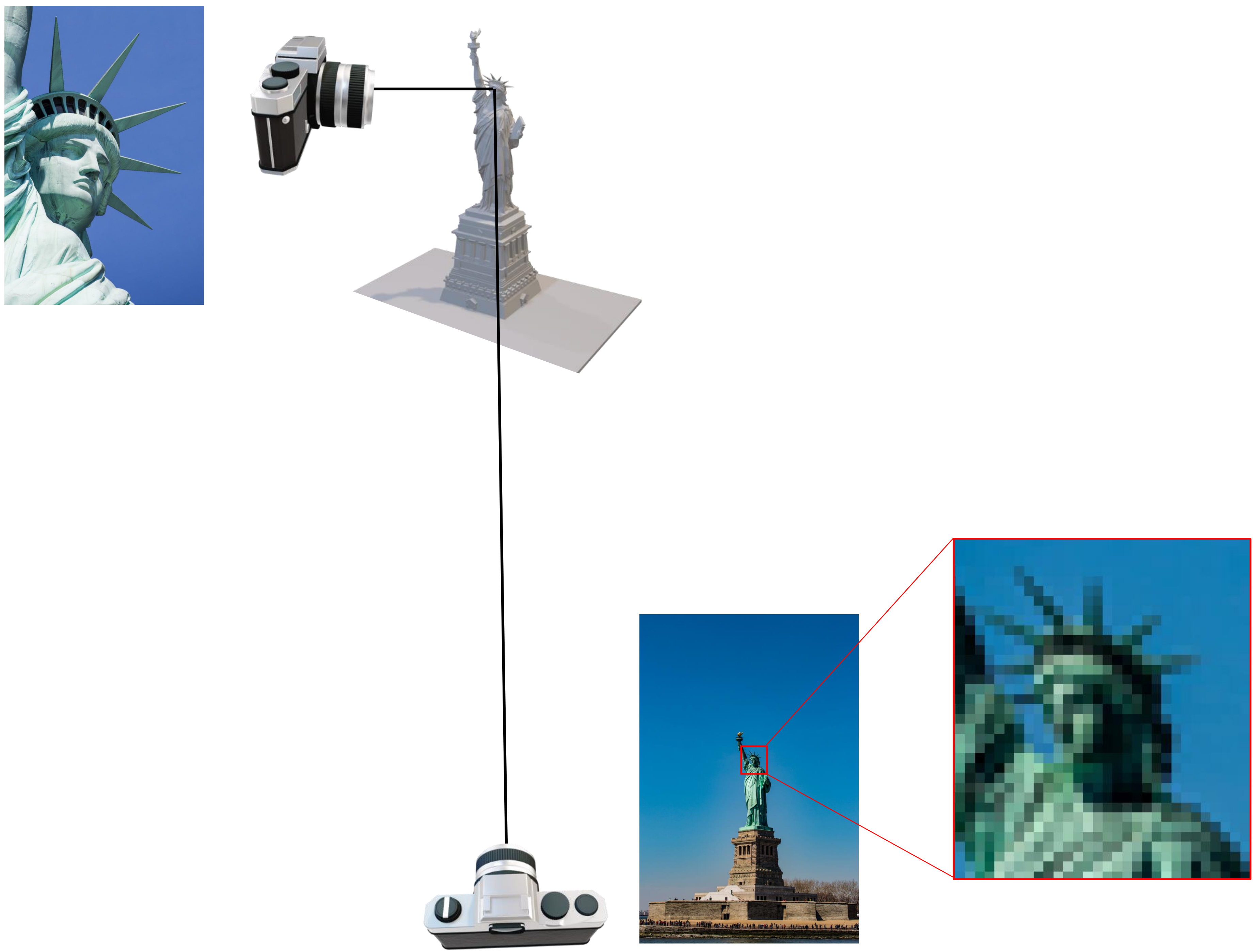
David Arnon

KITTI





Triangulation

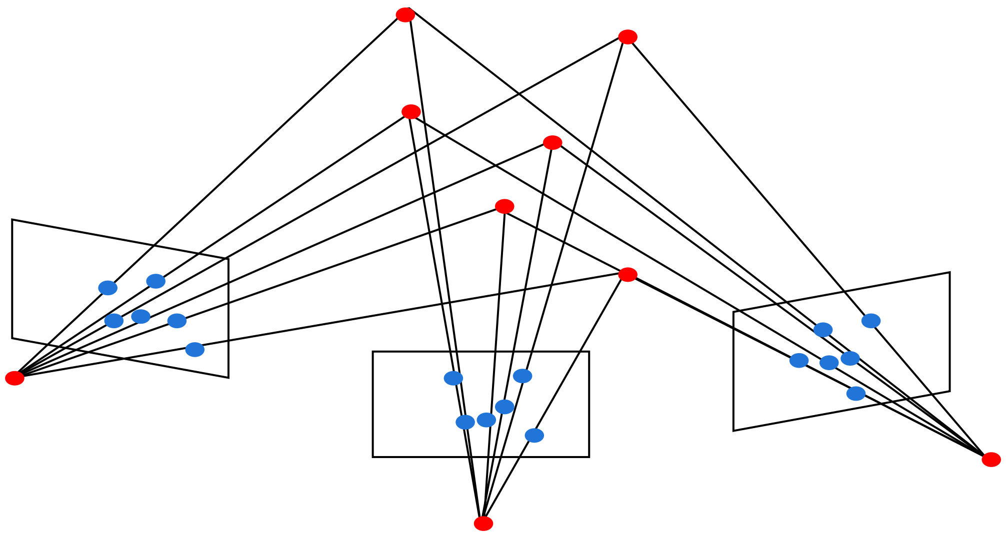


Bundle Adjustment

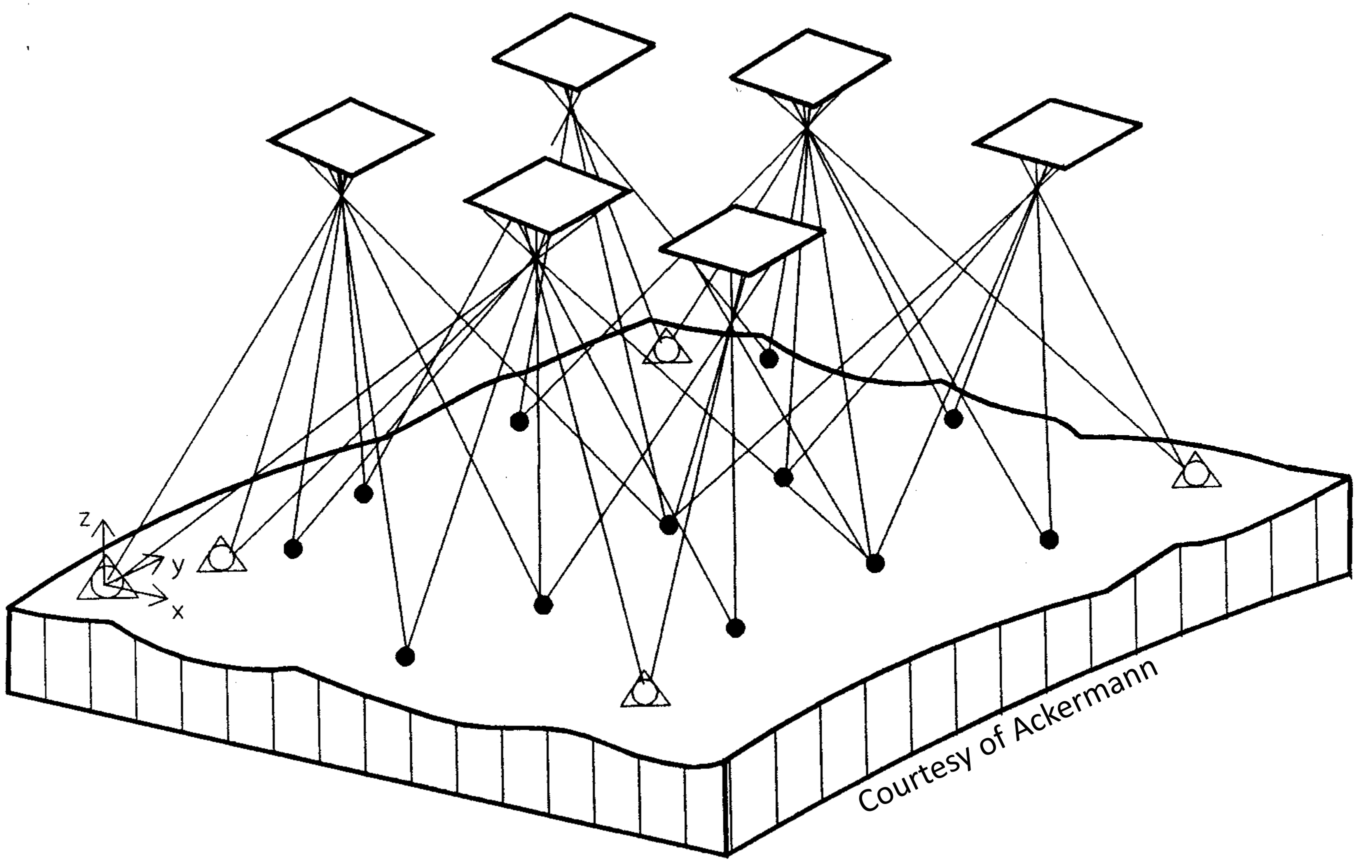
- Refines a visual reconstruction to produce jointly optimal 3D structure (world) and viewing parameters (cameras)

- *'bundle'* refers to the bundle of light rays leaving each 3D feature and converging on each camera center.

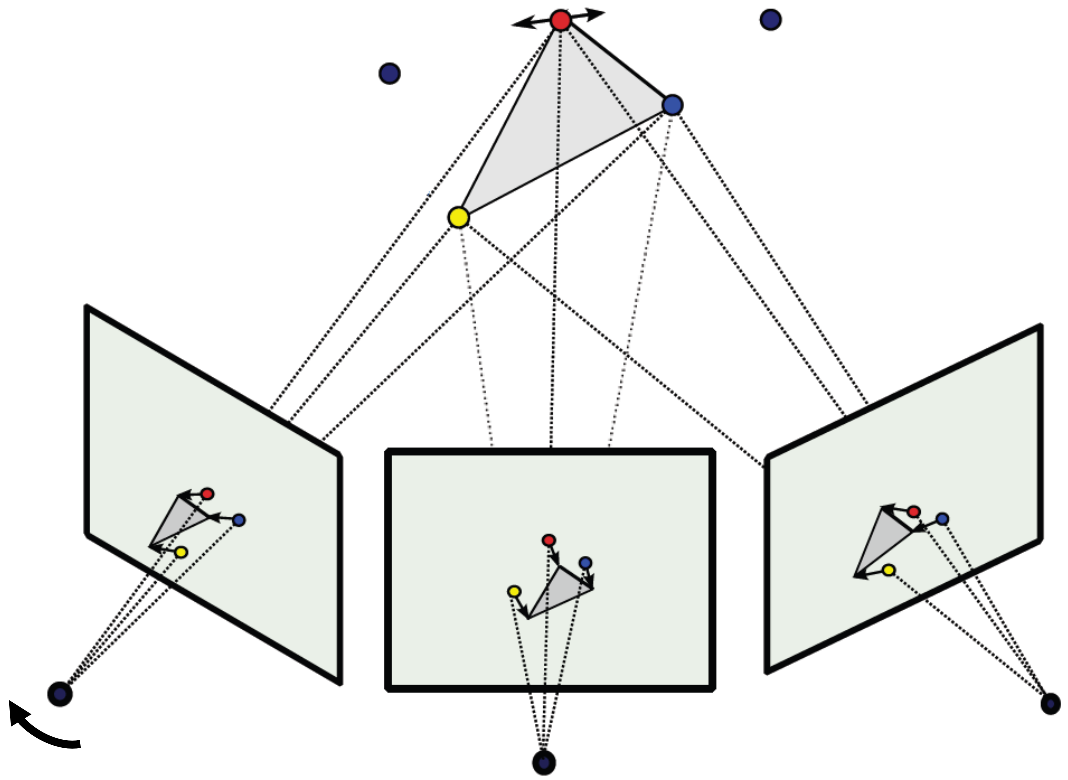
- Developed in the field of photogrammetry in the 1950´s



Bundle Adjustment



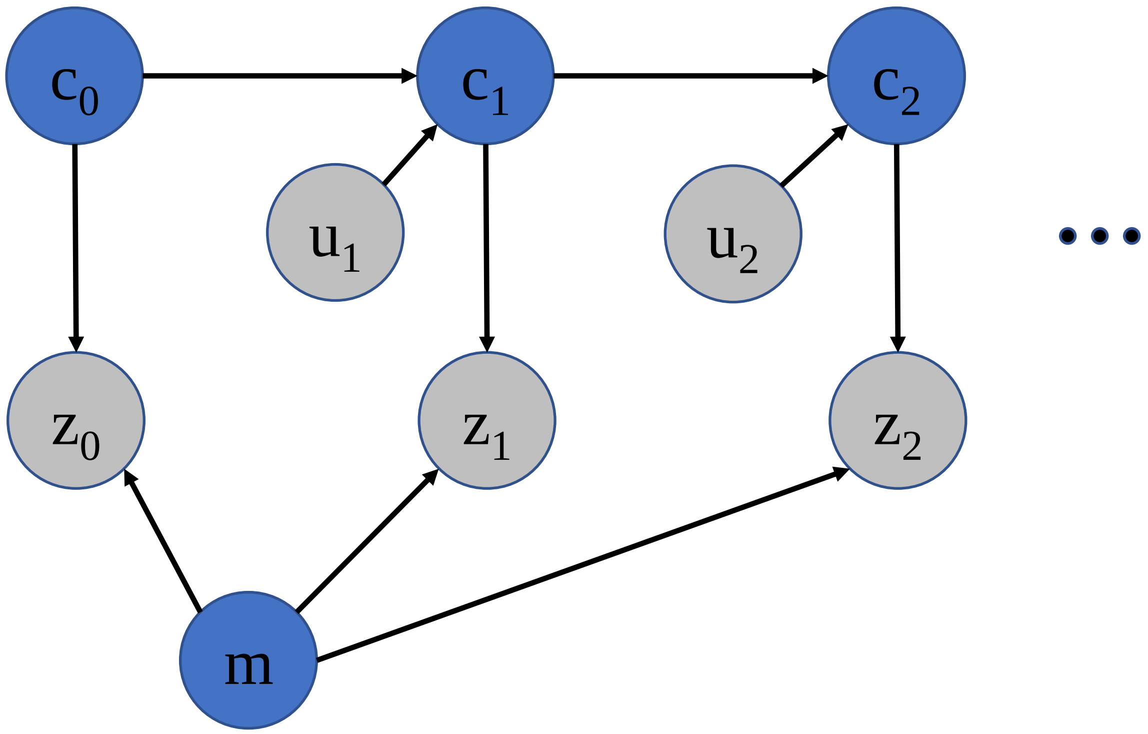
Bundle Adjustment



Courtesy of Vadim Indelman

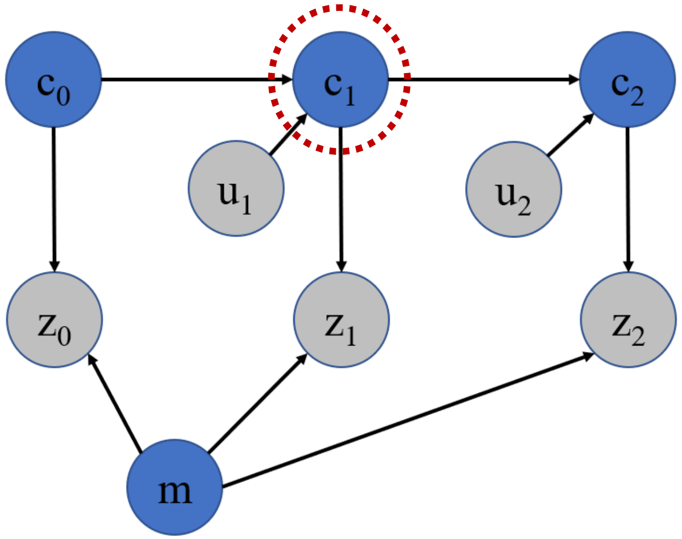
Slam

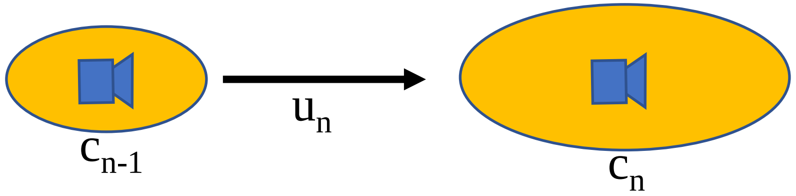
- Graphical Model



$$p(c_{0:2}, m \mid u_{1:2}, z_{0:2})$$

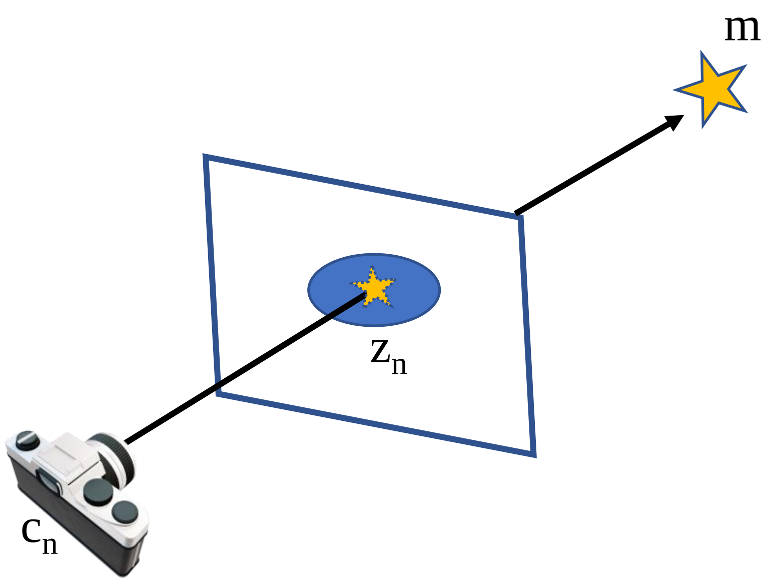
Motion Model



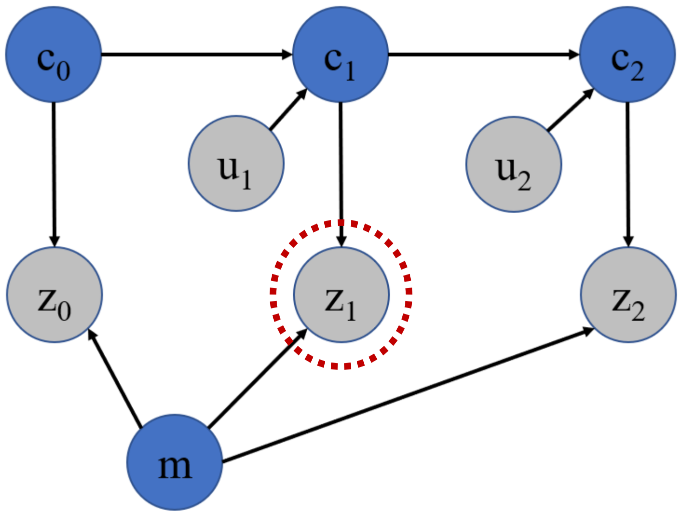


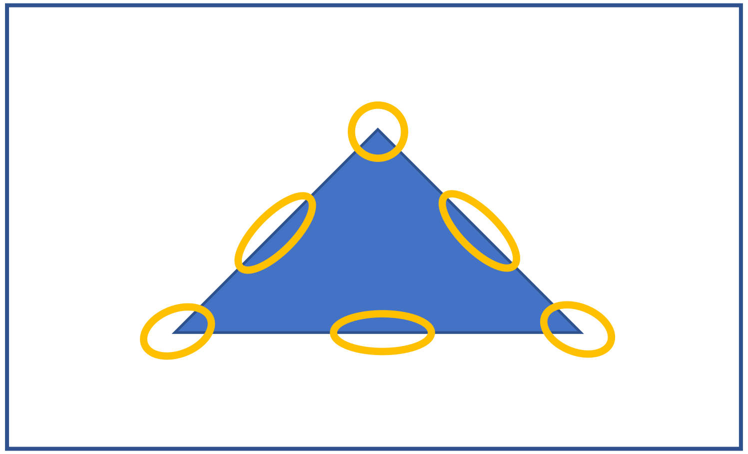
$$p(c_n|c_{n-1}, u_n)$$

Measurement Model



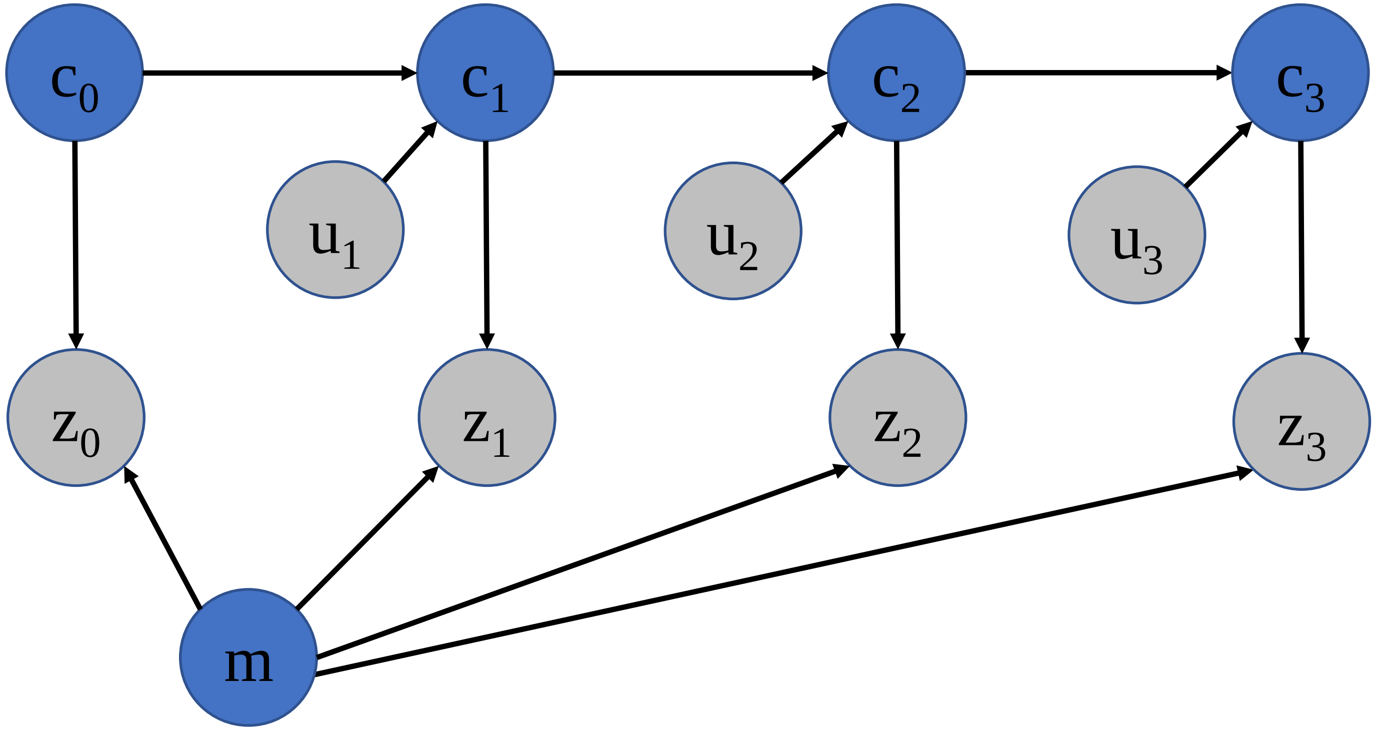
$$p(z_n | c_n, m)$$





Graphical Model

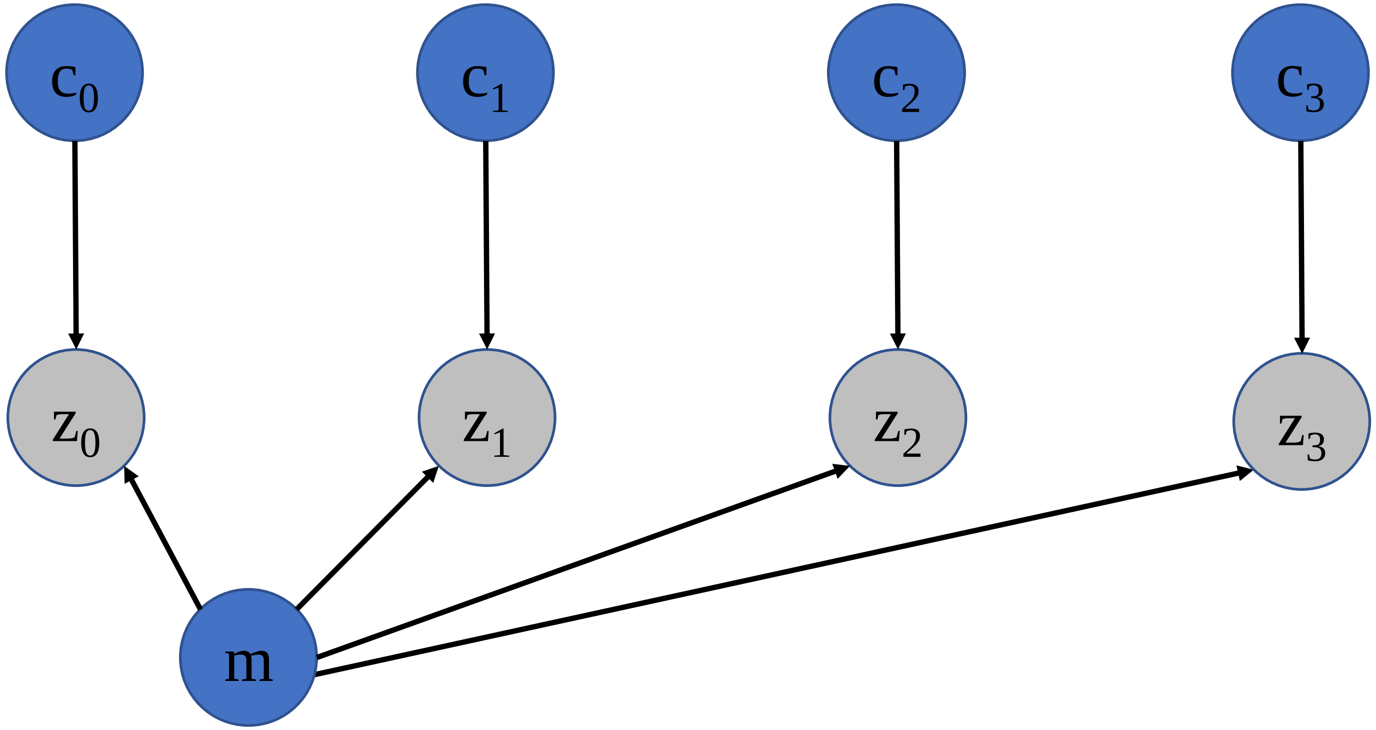
- Bundle Adjustment



$$p(c_{0:3}, m \mid u_{1:3}, z_{0:3})$$

Graphical Model

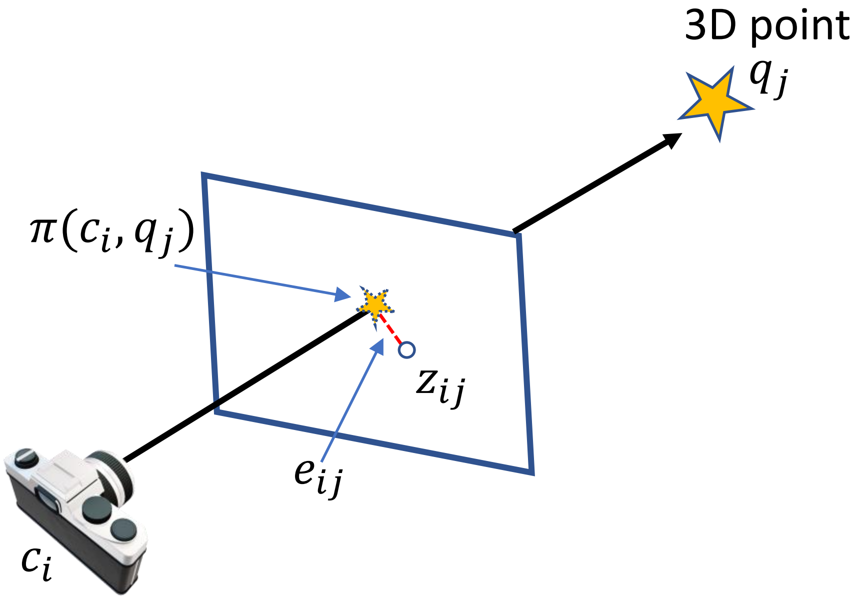
- Bundle Adjustment



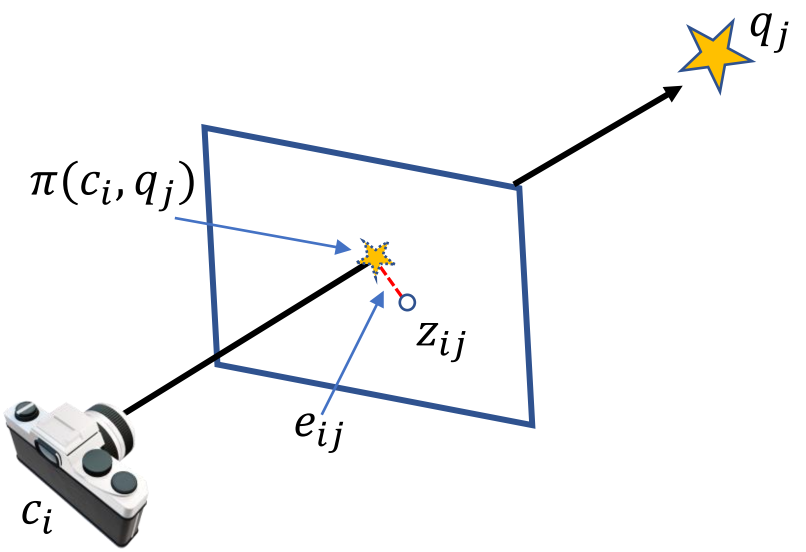
$$p(c_{0:3}, m \mid \cancel{u_{1:3}}, z_{0:3})$$

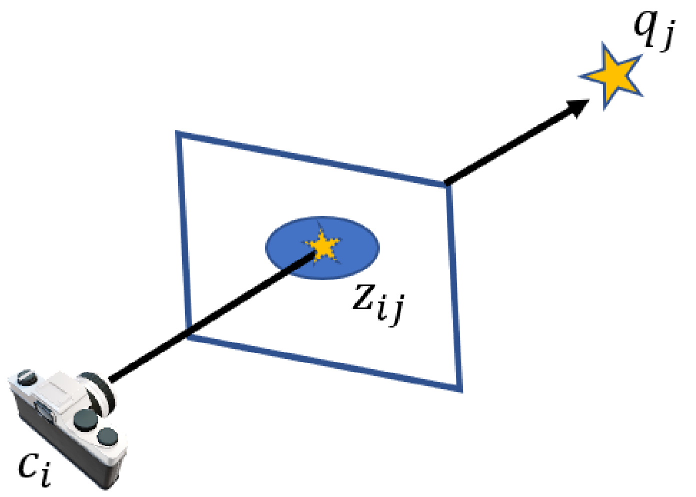
Probabilistic Formulation

- Reprojection error: $\boxed{e_{ij} \doteq z_{ij} - \pi(c_i, q_j)}$



Measurement Model





$$p\left(z_{ij} \middle| c_i, q_j\right) \sim N\left(\pi\left(c_i, q_j\right), \Sigma\right)$$

$$z_{ij} = \pi\left(c_i, q_j\right) + w, \qquad w \sim N(0, \Sigma)$$

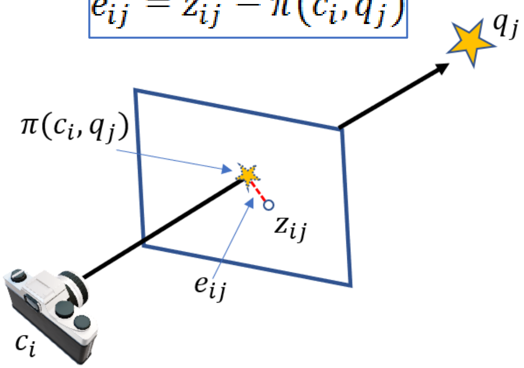
Bayes

- $p(z_{ij}|c_i, q_j) \sim N(\pi(c_i, q_j), \Sigma)$
- $p(c_i, q_j|z_{ij}) = \frac{1}{p(z_{ij})} p(z_{ij}|c_i, q_j) p(c_i, q_j)$
- $p(c_i, q_j|z_{ij}) \propto p(z_{ij}|c_i, q_j) p(c_i, q_j)$
- $p(c_i, q_j|z_{ij}) \propto p(z_{ij}|c_i, q_j)$
- $p(c_i, q_j|z_{ij}) \propto \exp\left(-\frac{1}{2}\|z_{ij} - \pi(c_i, q_j)\|_\Sigma^2\right)$
- $p(c_i, q_j|z_{ij}) \propto \exp\left(-\frac{1}{2}\|e_{ij}\|_\Sigma^2\right)$

$$\boxed{N_{\mu,\Sigma}(z) \propto exp\left(-\frac{1}{2}\|z - \mu\|_\Sigma^2\right)}$$

Reprojection error:

$$\boxed{e_{ij} \doteq z_{ij} - \pi(c_i, q_j)}$$



Bundle Adjustment

- $argmax_{C,Q}[p(C,Q|Z)]$
- $argmax_{C,Q}[p(Z|C,Q)]$
- $argmax_{C,Q}\left[\prod_{c_i}\prod_{j\in M_i} p\left(z_{ij}|c_i,q_j\right)\right]$
- $argmax_{C,Q}\left[\prod_{c_i}\prod_{j\in M_i} \exp\left(-\frac{1}{2}\|e_{ij}\|_{\Sigma}^{2}\right)\right]$
- $argmax_{C,Q}\left[\sum_{c_i}\sum_{j\in M_i} -\frac{1}{2}\|e_{ij}\|_{\Sigma}^{2}\right]$
- $argmin_{C,Q}\left[\sum_{c_i}\sum_{j\in M_i}\|e_{ij}\|_{\Sigma}^{2}\right]$
- $argmin_{C,Q}\left[\sum_{c_i}\sum_{j\in M_i}\left\|\Sigma^{-½}e_{ij}\right\|^{2}\right]$

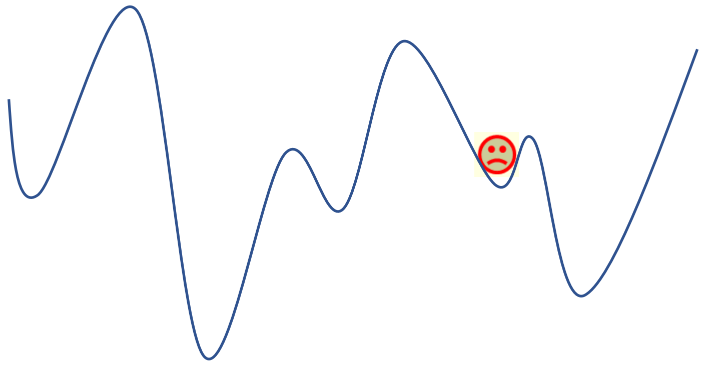
$\boxed{e_{ij} \doteq z_{ij} - \pi(c_i, q_j)}$

$\boxed{\Sigma^{½} = \mathrm{chol}(\Sigma)}$

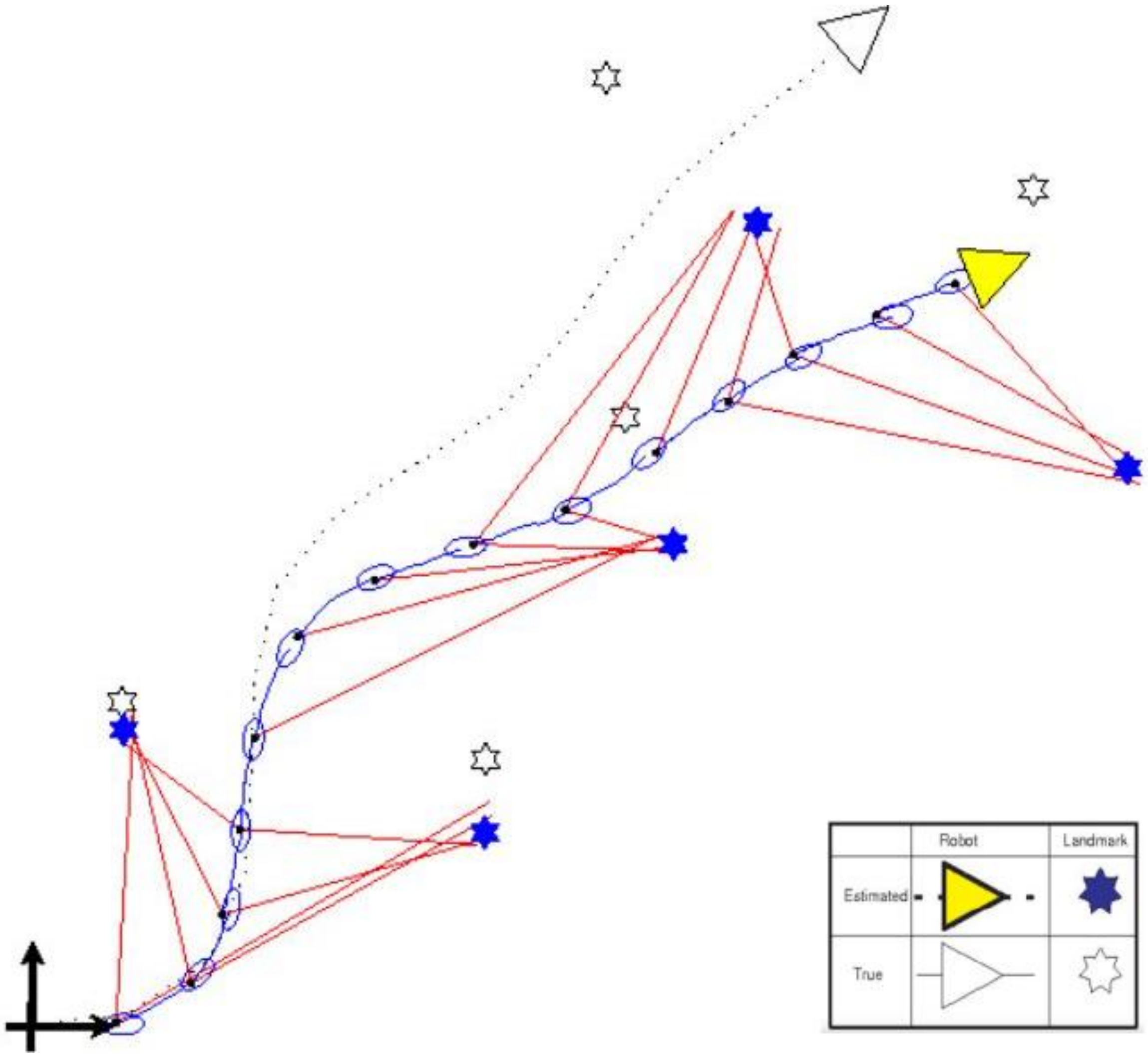
$$\boxed{\begin{array}{c}\Sigma = (\Sigma^{½})(\Sigma^{½})^{T} \\ \Sigma^{-1} = \Sigma^{-½\mathrm{T}}\Sigma^{-½}\end{array}}$$

Bundle Adjustment

- Maximum likelihood for normally distributed measurements
- Sensitive to outliers
  - The Gaussian has extremely small tail compared to most real measurement error distribution
- Non-linear least squares problem
- Solved using an iterative process
- General problem is non-convex,
  can settle in a local minima
- Requires a reasonable starting point



Bundle Adjustment



Courtesy of Durrant-Whyte, Baily; Slam: The essential algorithm

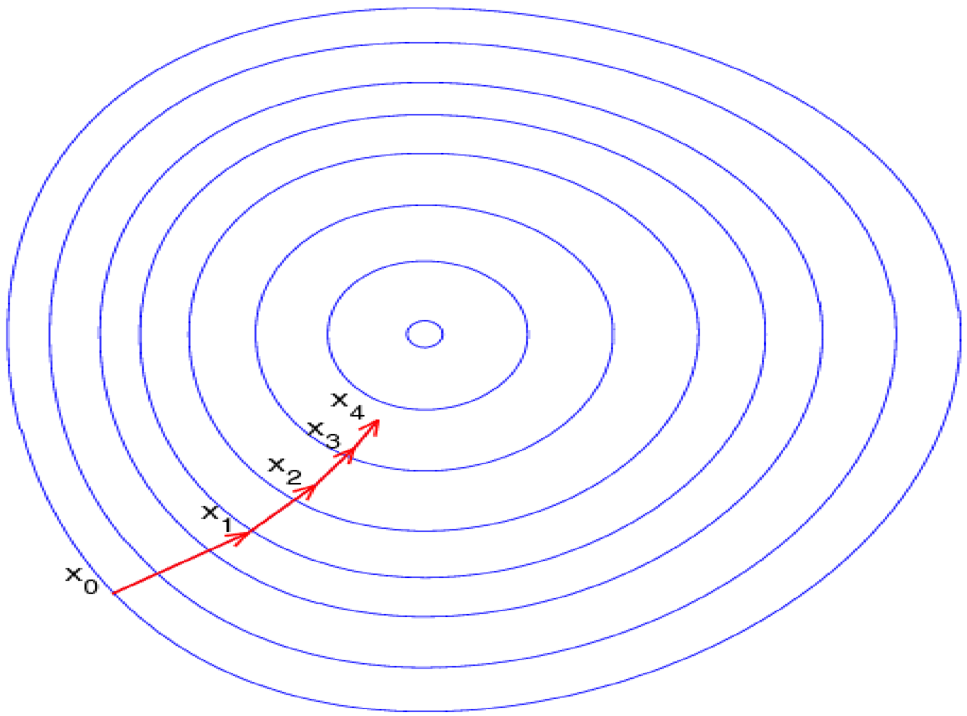
Bundle Adjustment

- Define a measurement function f that given the problem parameters calculates the expected measurements

- In optimal conditions $f(x) = z$

- Minimize $||f(x) - z||_{\Sigma}^{2}$

- Iterative solution



Bundle Adjustment
Representation

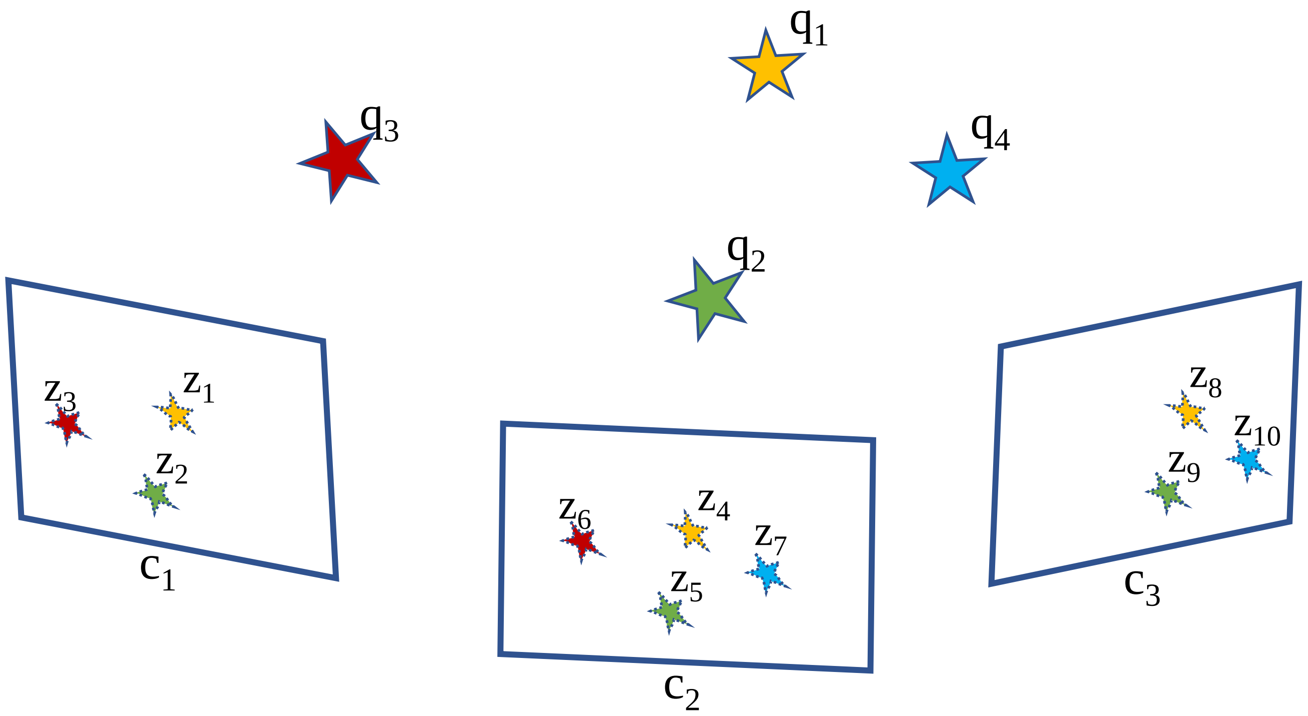
- Two system variables
  - Camera $c_i = [\psi \quad \theta \quad \phi \quad x \quad y \quad z]^T$
    - 3D position and Euler angles
    - Can produce camera matrix $K[R_i|t_i]$
    - $R_i = R(\psi, \theta, \phi) = R_z(\psi)R_y(\theta)R_x(\phi)$
    - $t_i = [x \quad y \quad z]^T$
  - Landmark $q_i = [x \quad y \quad z]^T$
    - 3D position
- Projection: $\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K[R_i|t_i] \begin{bmatrix} q_j \\ 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} u \\ v \end{bmatrix} = \pi(c_i, q_j)$

Bundle Adjustment

Representation

$$z = [z_1 \quad z_2 \quad z_3 \quad z_4 \quad z_5 \quad z_6 \quad z_7 \quad z_8 \quad z_9 \quad z_{10}]^T$$

$$x = [c_1 \quad c_2 \quad c_3 \quad q_1 \quad q_2 \quad q_3 \quad q_4]^T$$



$$f(x) \doteq \begin{bmatrix} \pi(c_1, q_1) \\ \pi(c_1, q_2) \\ \pi(c_1, q_3) \\ \pi(c_2, q_1) \\ \pi(c_2, q_2) \\ \pi(c_2, q_3) \\ \pi(c_2, q_4) \\ \pi(c_3, q_2) \\ \pi(c_3, q_3) \\ \pi(c_3, q_4) \end{bmatrix}$$

Jacobian

- $f(x + \Delta x) \cong f(x) + J(x)\Delta x$

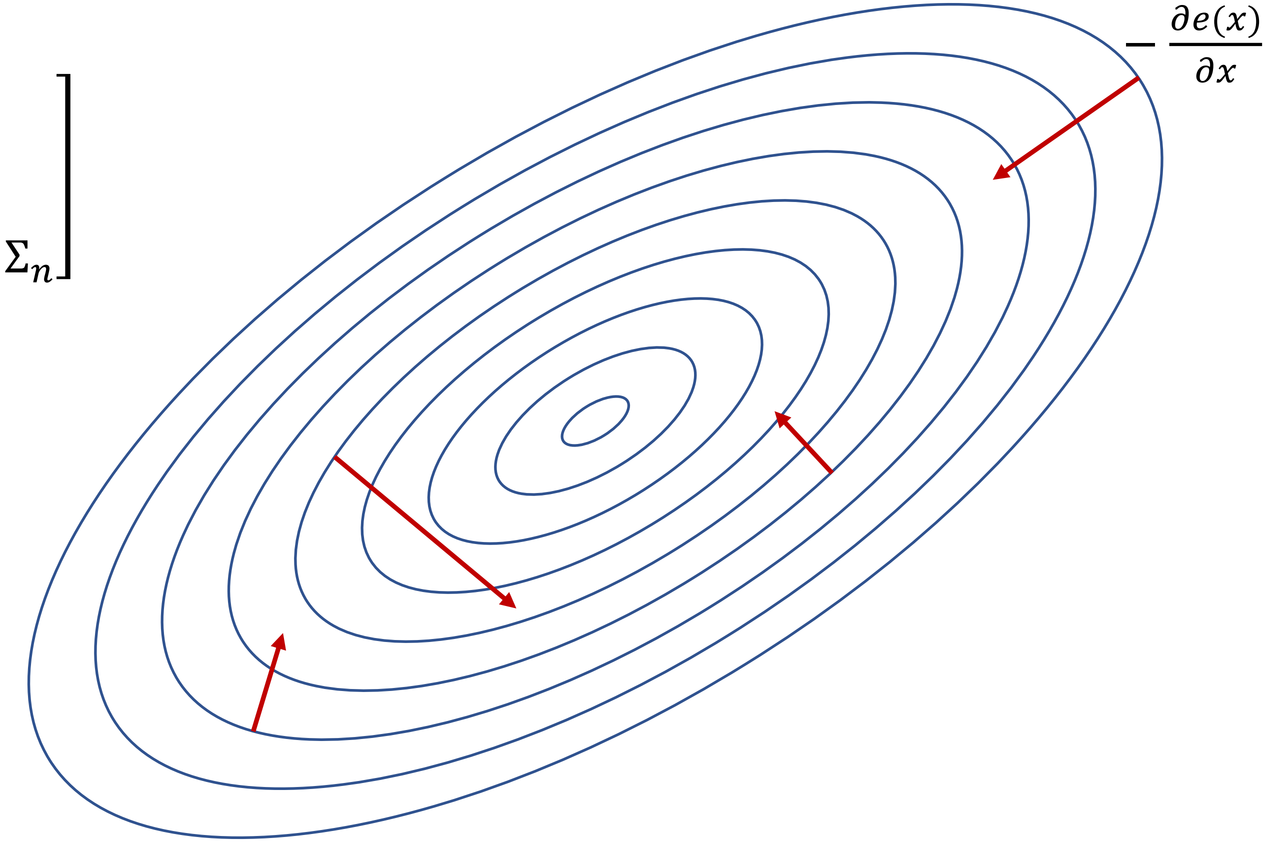
$$
\underbrace{\begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_m(x) \end{bmatrix}}_{f(x)} + \underbrace{\begin{bmatrix} \frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \cdots & \frac{\partial f_1(x)}{\partial x_p} \\ \frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \cdots & \frac{\partial f_2(x)}{\partial x_p} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_m(x)}{\partial x_1} & \frac{\partial f_m(x)}{\partial x_2} & \cdots & \frac{\partial f_m(x)}{\partial x_p} \end{bmatrix}}_{J_f(x)} \underbrace{\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_p \end{bmatrix}}_{\Delta x}
$$

Bundle Adjustment

Gradient Decent

- Error function $e(x) = \|f(x) - z\|_{\Sigma}^{2}$:

$$\Sigma = \begin{bmatrix} \Sigma_1 & & & \\ & \Sigma_2 & & \\ & & \ddots & \\ & & & \Sigma_n \end{bmatrix}$$



Bundle Adjustment
Linear Approximation

- $e(x) = \frac{1}{2}(f(x) - z)^T \Sigma^{-1} (f(x) - z)$
- $\left(\frac{\partial e(x)}{\partial x}\right)^T = J(x)^T \Sigma^{-1} (f(x) - z) = J(x)^T \Sigma^{-1} \Delta z$
- $e(x + \Delta x) \cong e(x) + \frac{\partial e(x)}{\partial x} \Delta x$
- $e(x + \Delta x) \cong e(x) - \frac{1}{\lambda} \left\| \frac{\partial e(x)}{\partial x} \right\|_2^2 < e(x)$
- $\Delta x = -\frac{1}{\lambda} J(x)^T \Sigma^{-1} \Delta z$

$$\boxed{\Delta z \doteq f(x) - z}$$

$$\boxed{\Delta x = -\frac{1}{\lambda}\left(\frac{\partial e(x)}{\partial x}\right)^T}$$

$$\boxed{g \doteq J(x)^T \Sigma^{-1} \Delta z}$$

$$\boxed{\Delta x = -\frac{1}{\lambda} g}$$

Bundle Adjustment

Gradient Decent