Projective 3 Points PnP

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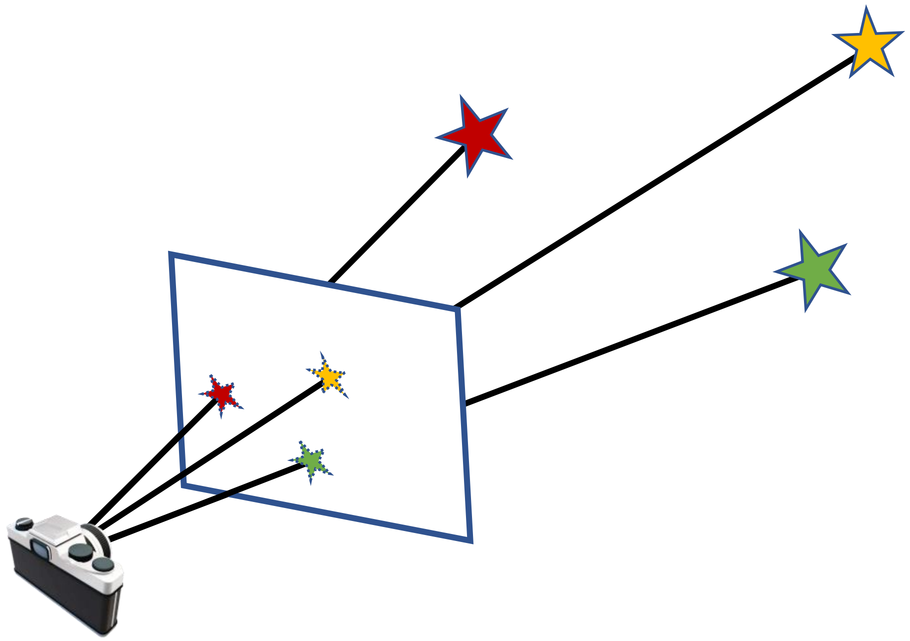
P3P

?

Projective 3 Points

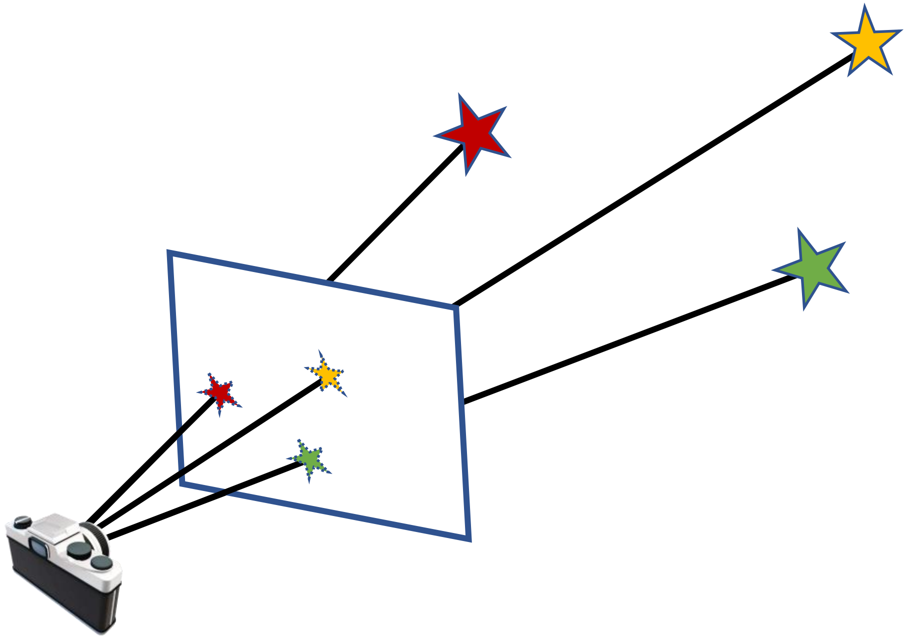
- Estimate the pose of a calibrated camera given control points
- Uses ≥ 3 points
- Often used with RANSAC
- P3P, PnP, Spatial Resection
- We present Grunert's Method



P3P

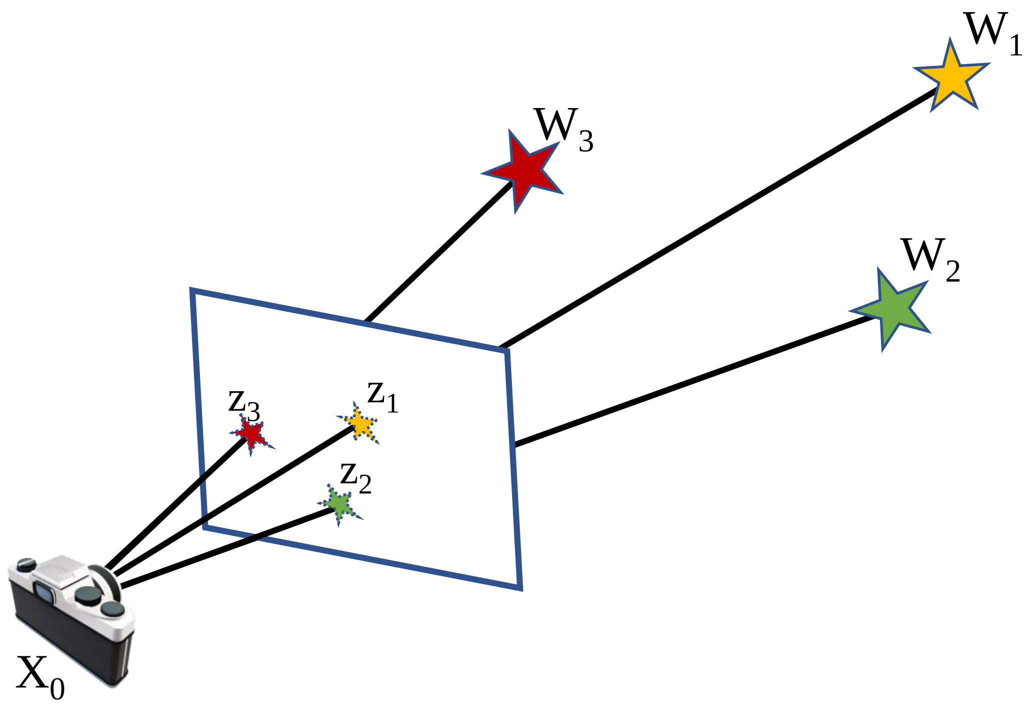
- $\lambda p = K[R|t]X$

- Total of 6dof to estimate
  - We need at least 3 points



P3P

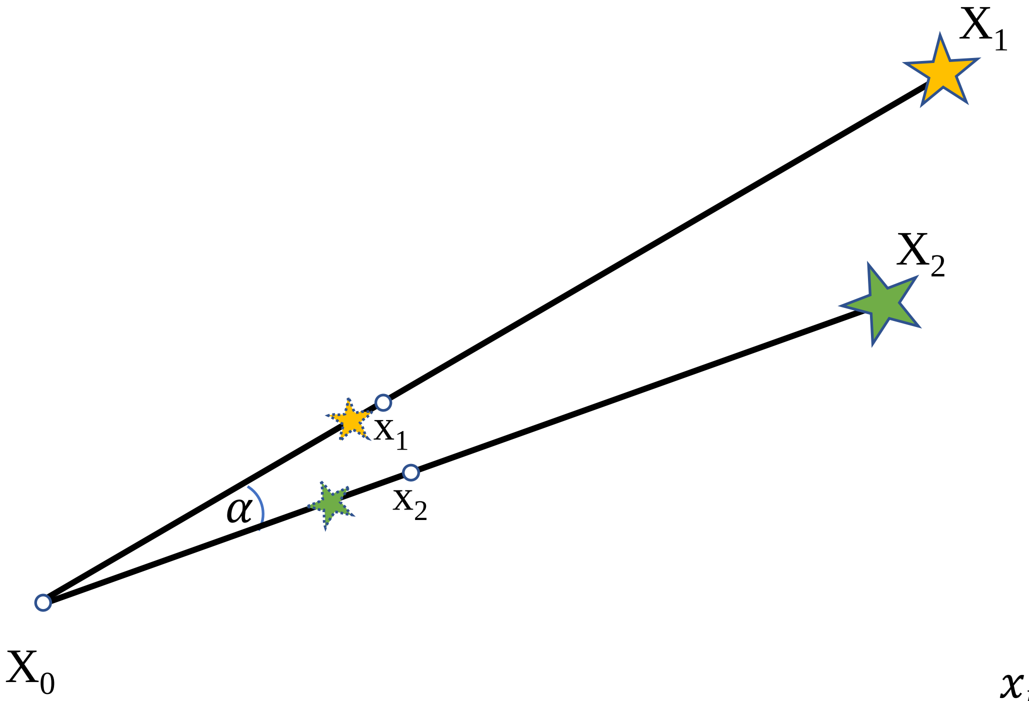
- Given: $W_1, W_2, W_3, z_1, z_2, z_3$



$$X_i = [R|T]W_i$$

$$\lambda z_i = KX_i \quad \Rightarrow \quad X_i = \lambda K^{-1} z_i$$

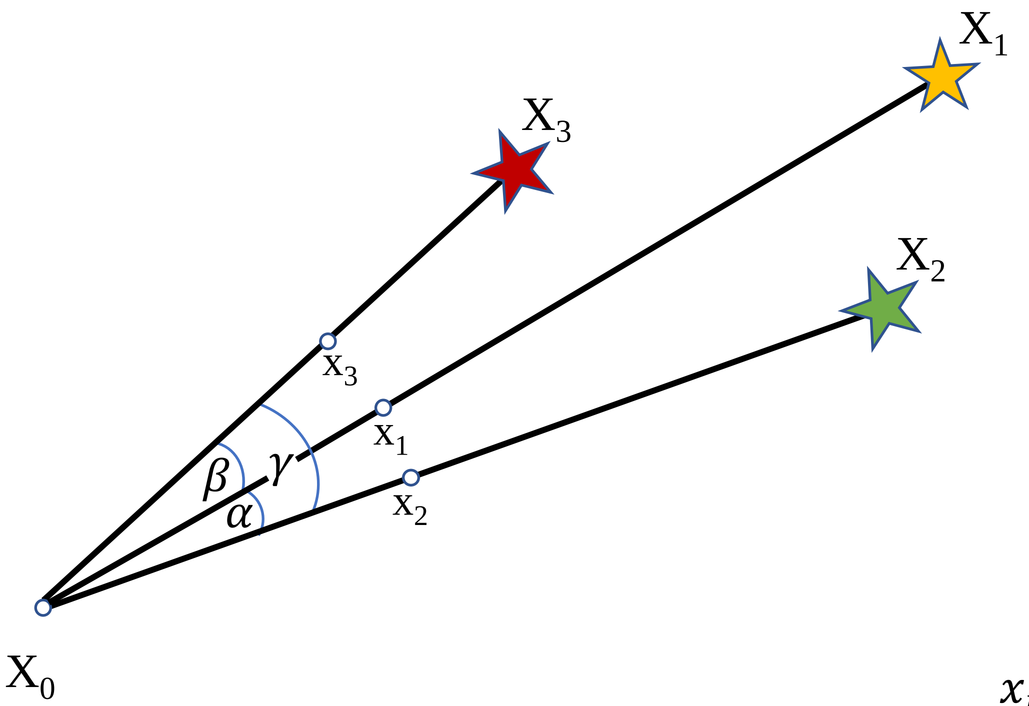
P3P
angles



$$x_i = \frac{K^{-1} z_i}{\|K^{-1} z_i\|}$$

P3P

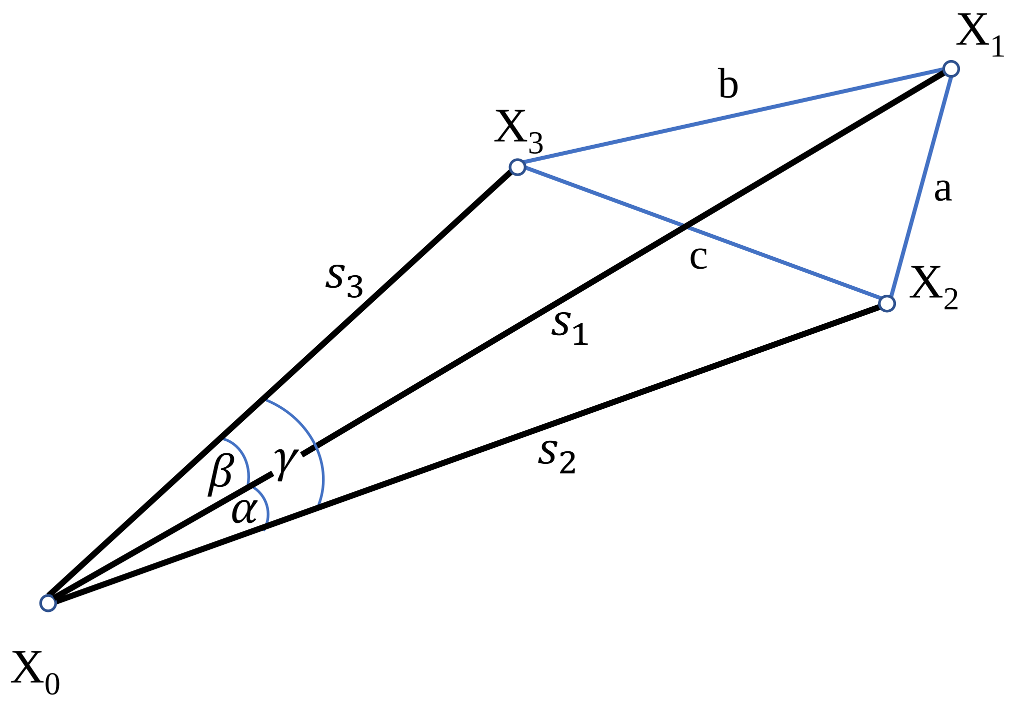
angles



$$\alpha = \cos^{-1}(x_1^T x_2)$$
$$\beta = \cos^{-1}(x_1^T x_3)$$
$$\gamma = \cos^{-1}(x_2^T x_3)$$

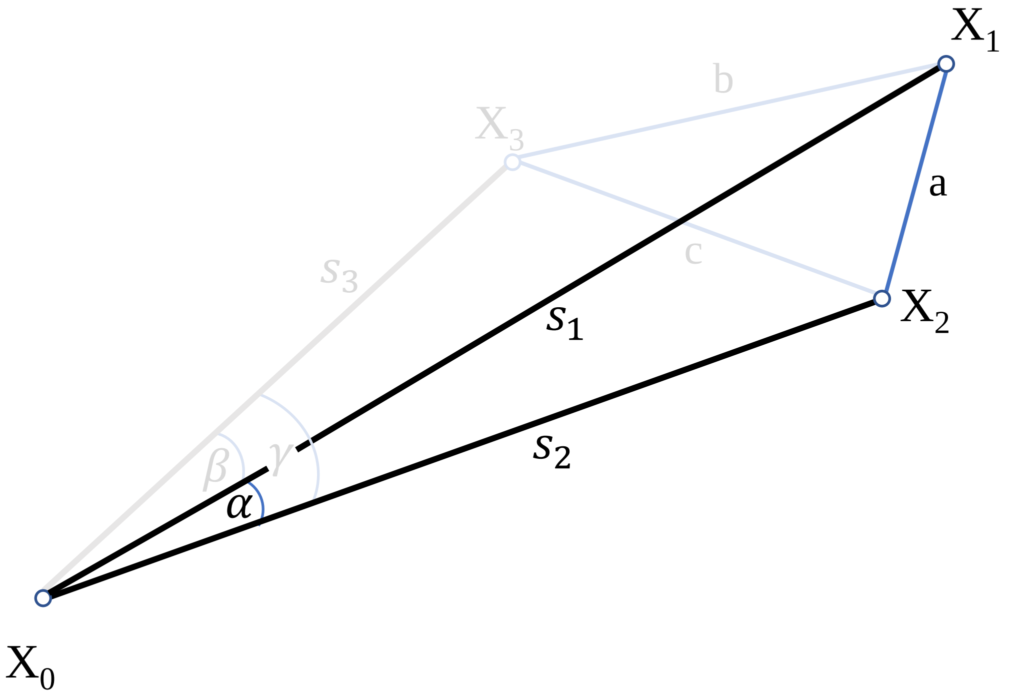
$$x_i = \frac{K^{-1} z_i}{\|K^{-1} z_i\|}$$

P3P
distances



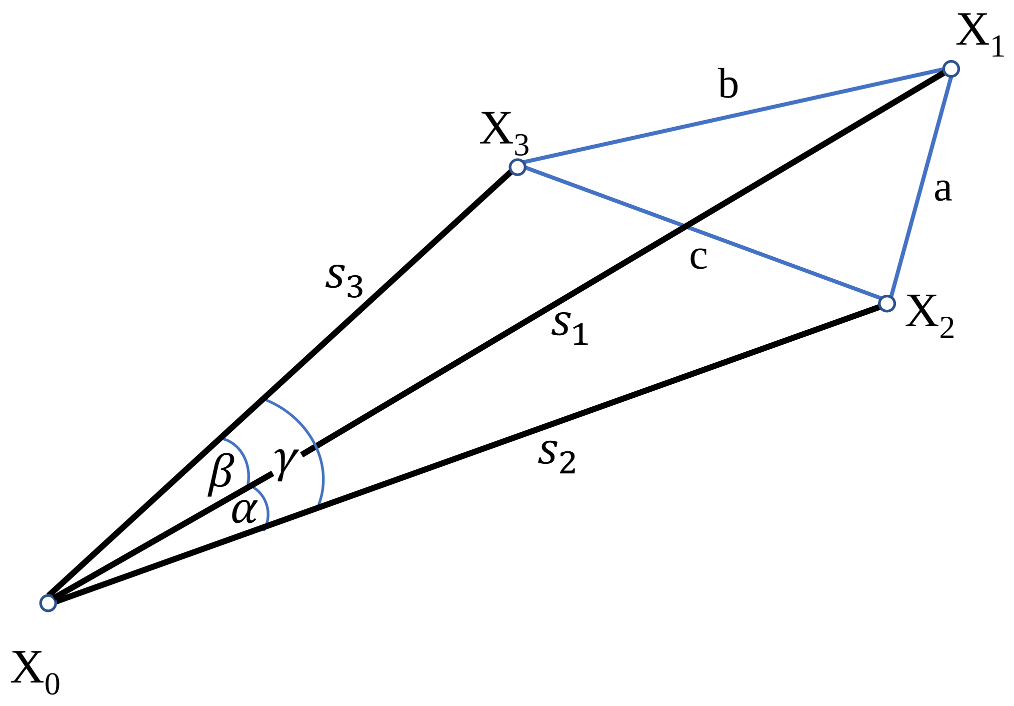
$$a = \|X_1 - X_2\|$$
$$b = \|X_1 - X_3\|$$
$$c = \|X_2 - X_3\|$$

P3P
distances



$$a^2 = s_1^2 + s_2^2 - 2s_1s_2\cos\alpha$$

P3P
distances



$$a^2 = s_1^2 + s_2^2 - 2 s_1 s_2 \cos \alpha$$
$$b^2 = s_1^2 + s_3^2 - 2 s_1 s_3 \cos \beta$$
$$c^2 = s_2^2 + s_3^2 - 2 s_2 s_3 \cos \gamma$$

P3P

- 4^{th} degree polynomial in $u = \frac{s_3}{s_1}$

$$\boxed{\boldsymbol{A}u^4 + \boldsymbol{B}u^3 + \boldsymbol{C}u^2 + \boldsymbol{D}u + \boldsymbol{E} = \boldsymbol{0}}$$

$$A = (\delta_2 - 1)^2 - \frac{4c^2}{b^2}\cos^2\alpha$$

$$\delta_{1,2} = \frac{a^2 \pm c^2}{b^2}$$

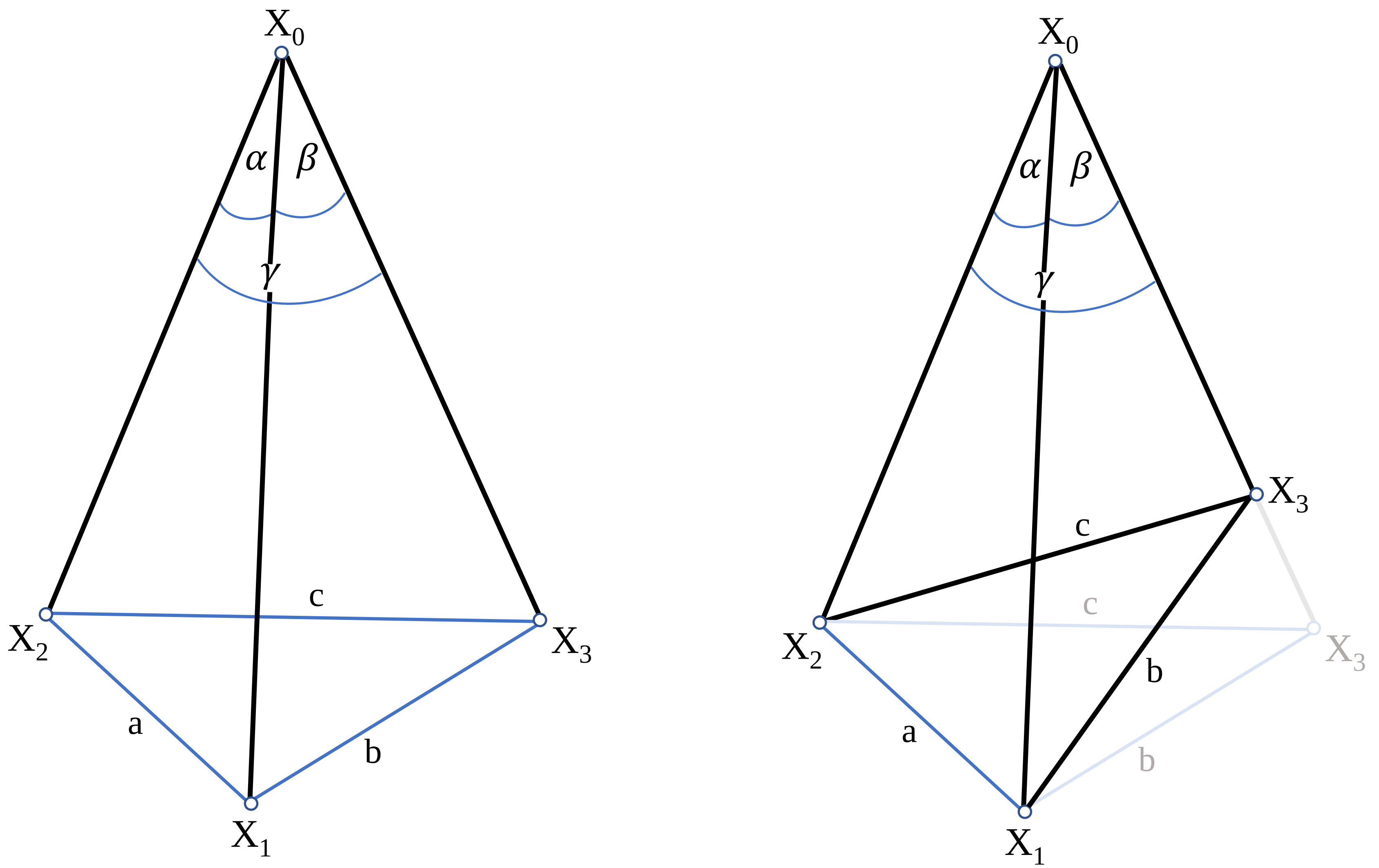
$$B = 4\left(\delta_2(1-\delta_2)\cos\beta - (1-\delta_1)\cos\alpha\cos\gamma + 2\frac{c^2}{b^2}\cos^2\alpha\cos\beta\right)$$

$$C = 2\left(\delta_2^2(1+2\cos^2\beta) - 1 + 2\delta_2\cos^2\alpha - 4\delta_1\cos\alpha\cos\beta\cos\gamma + 2\left(\frac{b^2-a^2}{b^2}\right)\cos^2\gamma\right)$$

$$D = 4\left(\frac{2a^2}{b^2}\cos^2\gamma\cos\beta - \delta_2(1+\delta_2)\cos\beta - (1-\delta_1)\cos\alpha\cos\gamma\right)$$

$$E = (1+\delta_2)^2 - \frac{4a^2}{b^2}\cos^2\gamma$$

P3P
ambiguity

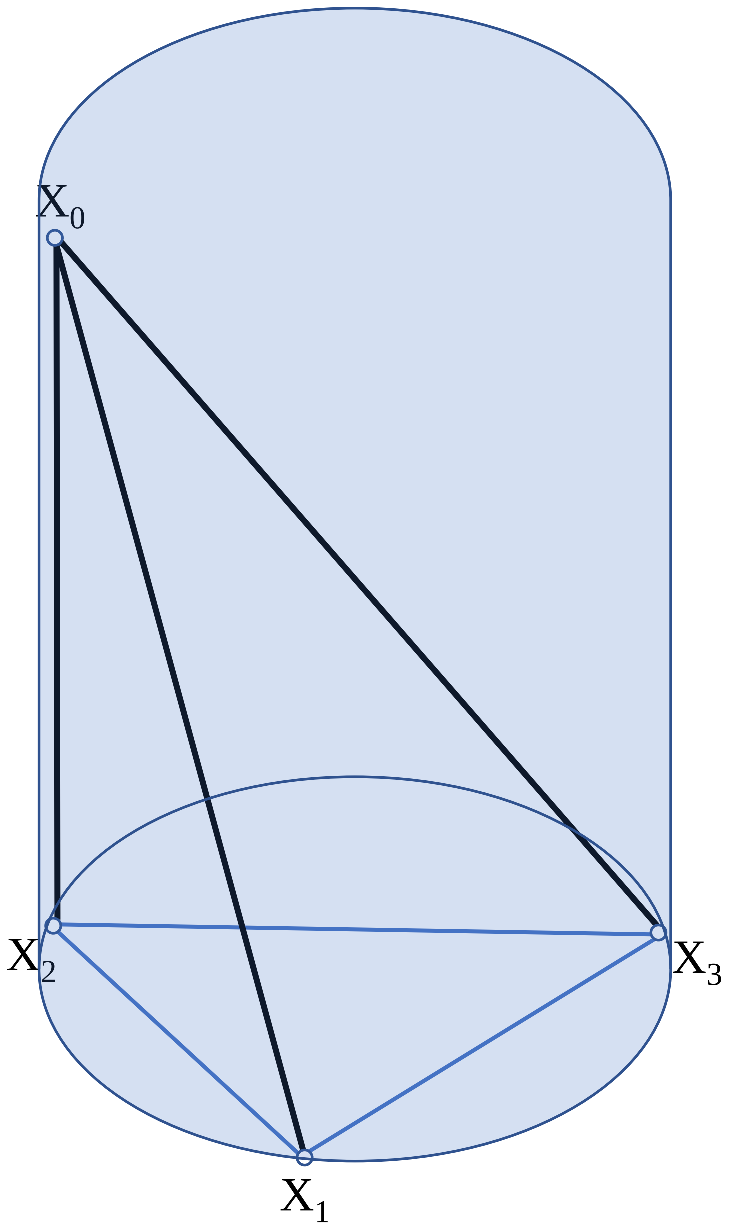


P3P
ambiguity

- 4 possible positions
- Use initial guess
- Use an additional point - P4P

P3P

- Critical cylinder



P3P
world pose

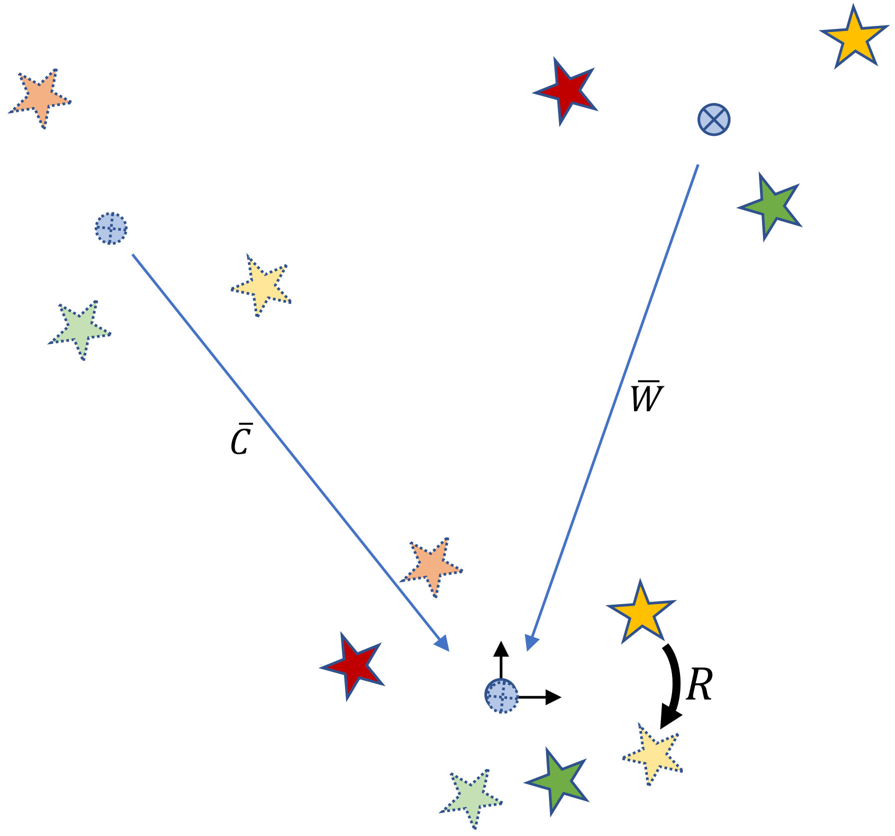
- Find transformation to world coordinates

$$C = \begin{bmatrix} | & | & | \\ s_1 x_1 & s_2 x_2 & s_3 x_3 \\ | & | & | \end{bmatrix} - \bar{C}$$

$$W = \begin{bmatrix} | & | & | \\ W_1 & W_2 & W_3 \\ | & | & | \end{bmatrix} - \bar{W}$$

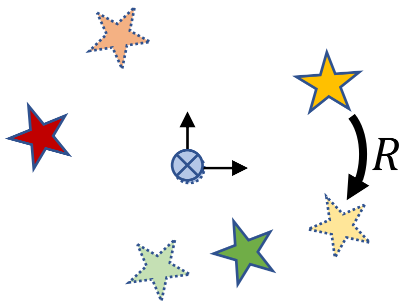
$$\bar{C} = mean(s_i x_i)$$

$$\bar{W} = mean(W_i)$$



P3P
world pose

- Find transformation from world coordinates



$$C = RW$$

$$CW^T = RWW^T$$

$$CW^T = \underbrace{RV}_{U} DV^T = UDV^T$$

$$RV = U \quad \Longrightarrow \quad R = UV^T$$

$$C = \begin{bmatrix} | & | & | \\ s_1x_1 & s_2x_2 & s_3x_3 \\ | & | & | \end{bmatrix} - \bar{C}$$

$$W = \begin{bmatrix} | & | & | \\ W_1 & W_2 & W_3 \\ | & | & | \end{bmatrix} - \bar{W}$$

$$R(p_w - \bar{W}) + \bar{C} = Rp_w + (\bar{C} - R\bar{W})$$

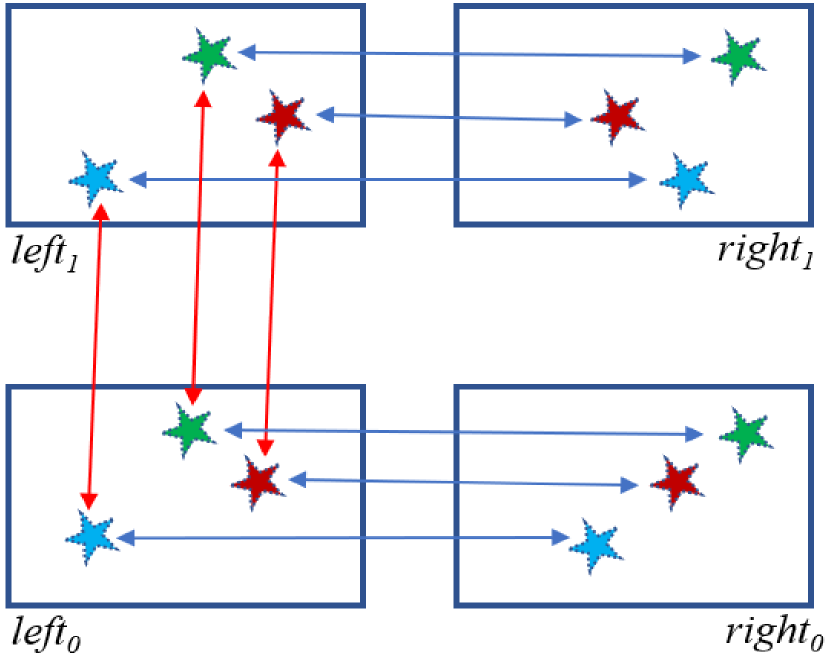
$$\boxed{[R|\bar{C} - R\bar{W}]}$$

P3P

- Use RANSAC
- PnP using least squares approach

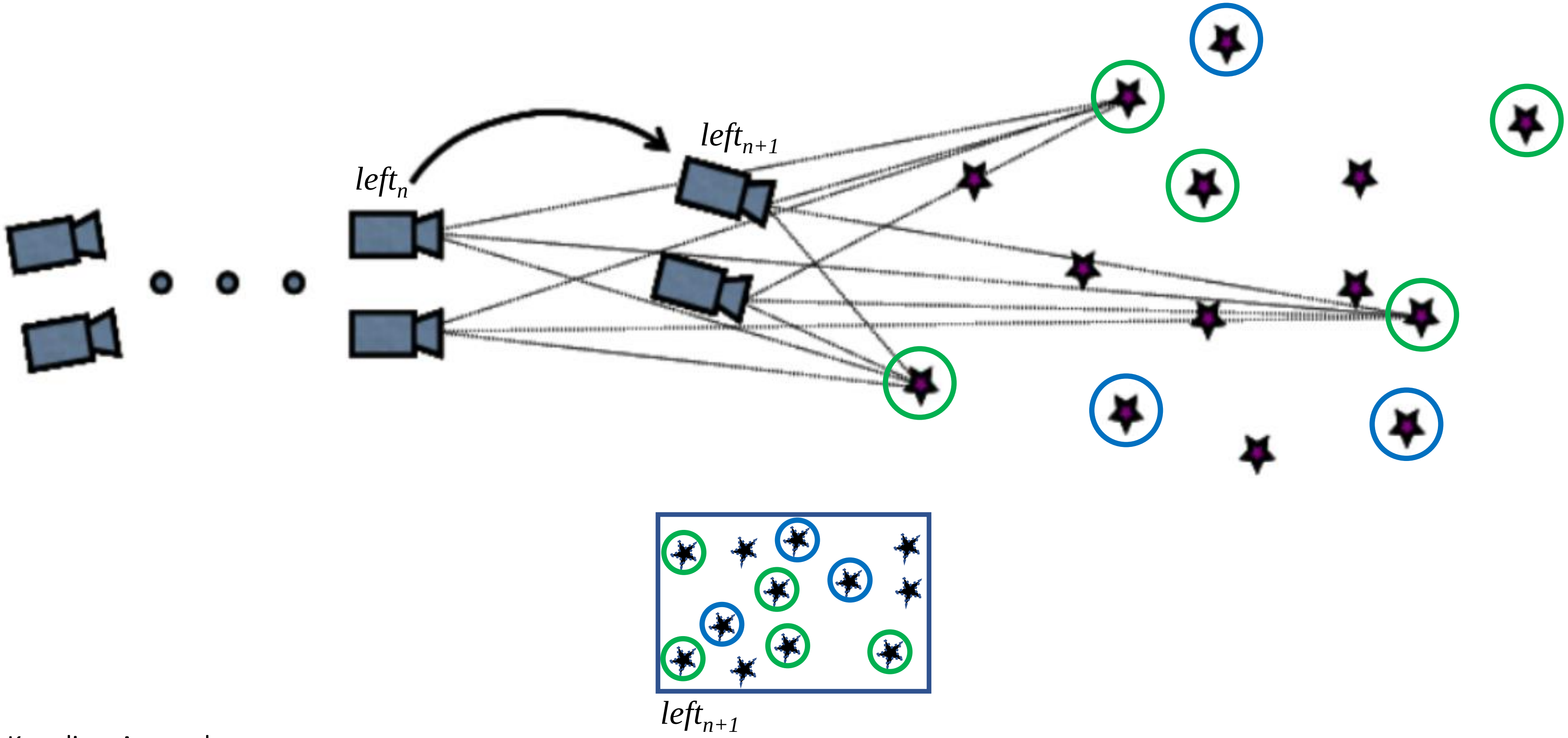
Project part III

- Tracking



Consensus Match

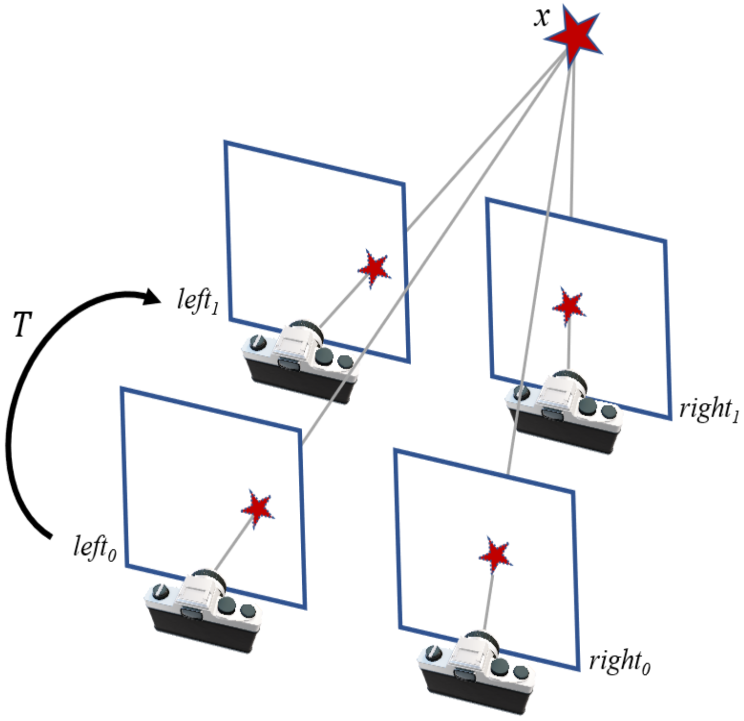
- Relative motion estimation
  - cv2.solvePnPRansac

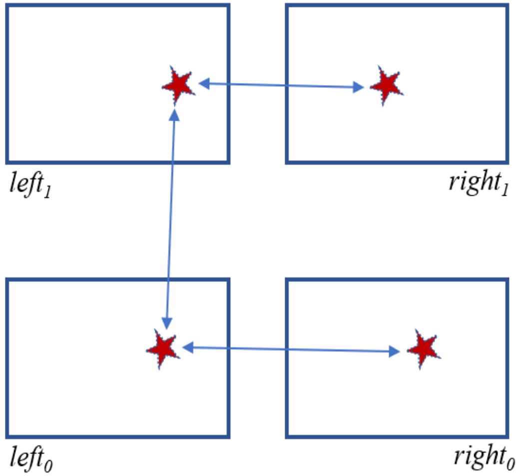


Courtesy of FrameSlam Konolige, Agrawal

Consensus Match

- PnP with improved RANSAC
- What is an inlier?





Detection

- Non-maximal suppression
- GridAdapterFeatureDetector