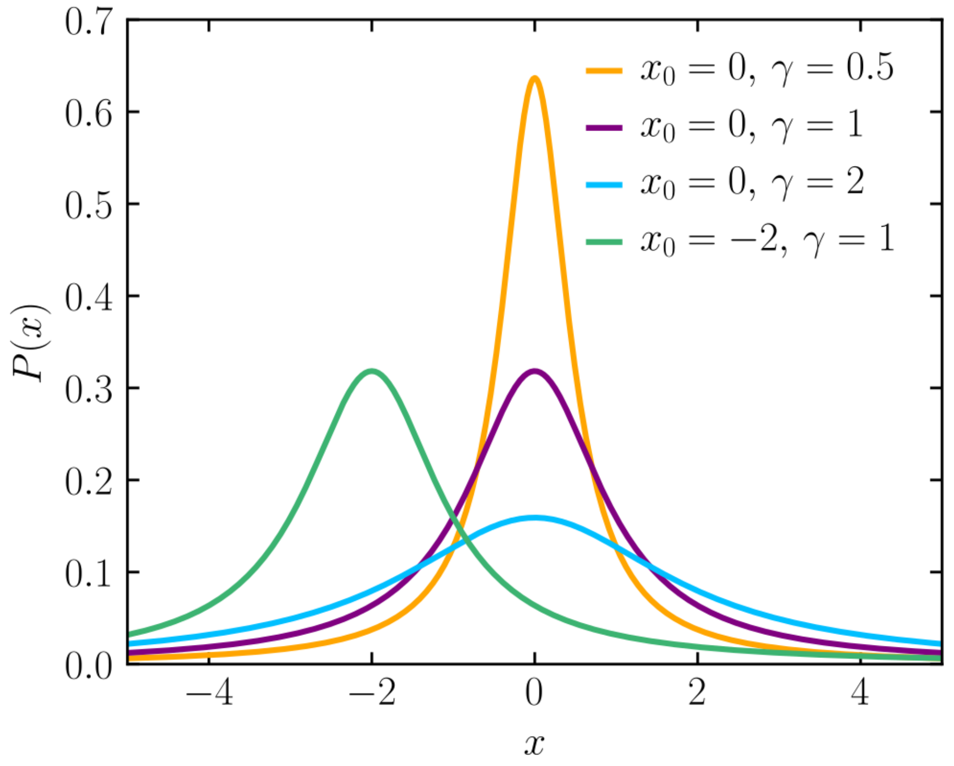
Robust Estimation

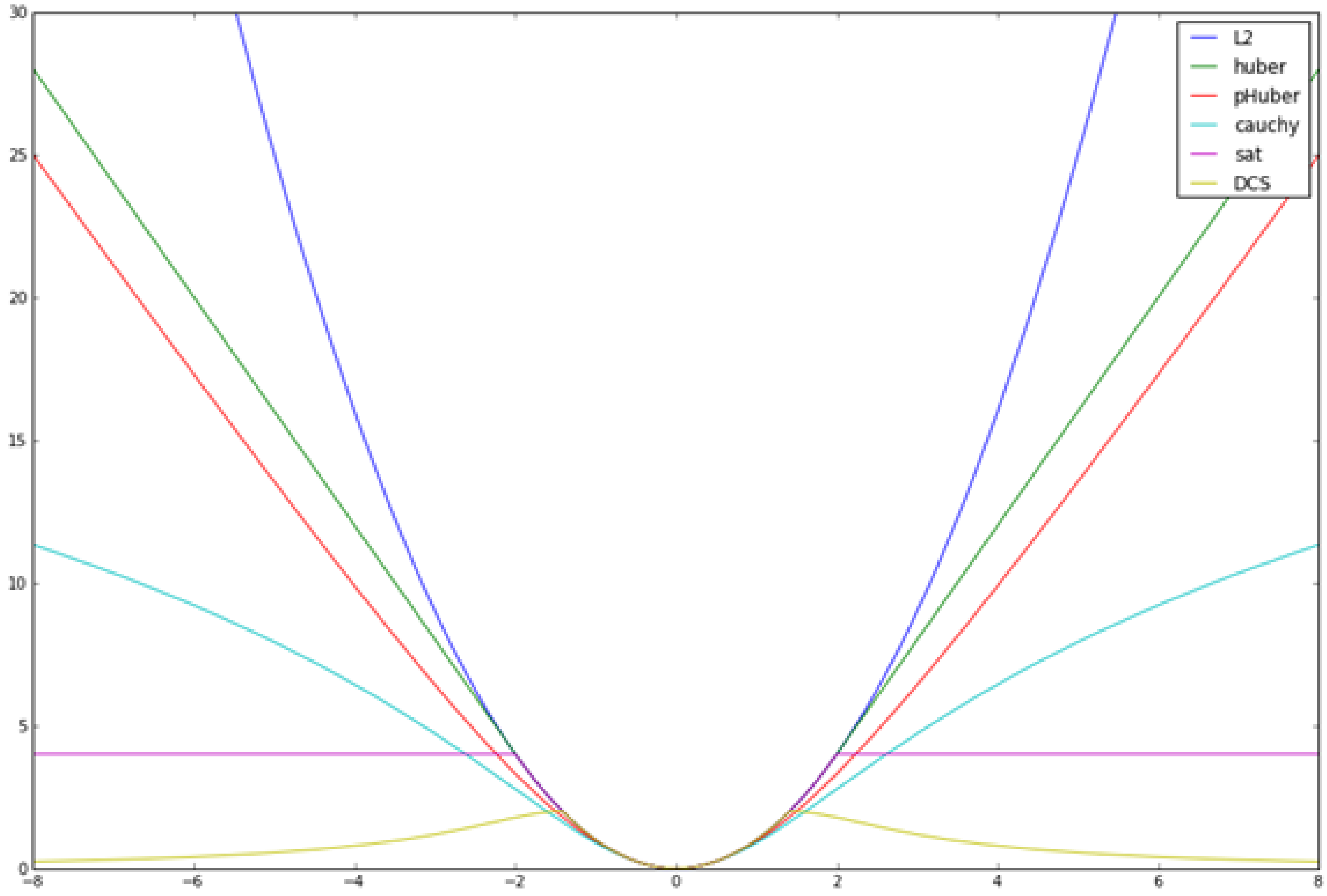
David Arnon

Robust Estimation
Kernels

- L_2: $L_\delta(x) = x^2$
- Huber: $L_\delta(x) = \begin{cases} x^2, & |x| < \delta \\ \delta(2|x| - \delta), & |x| \geq \delta \end{cases}$
- Saturated: $L_\delta(x) = \begin{cases} x^2, & |x| < \delta \\ \delta^2, & |x| \geq \delta \end{cases}$
- Cauchy: $L_\delta(x) = \delta^2 \log(1 + (x/\delta)^2)$
- Cauchy distribution: $Cauchy_{0,\gamma}(x) = \dfrac{1}{\pi\gamma\left(1 + \left(\frac{x}{\gamma}\right)^2\right)}$



Robust Estimation
Kernels



Robust Estimation

GTSAM Kernels

- **Look Ma, No RANSAC**
  https://gtsam.org/2019/09/20/robust-noise-model.html

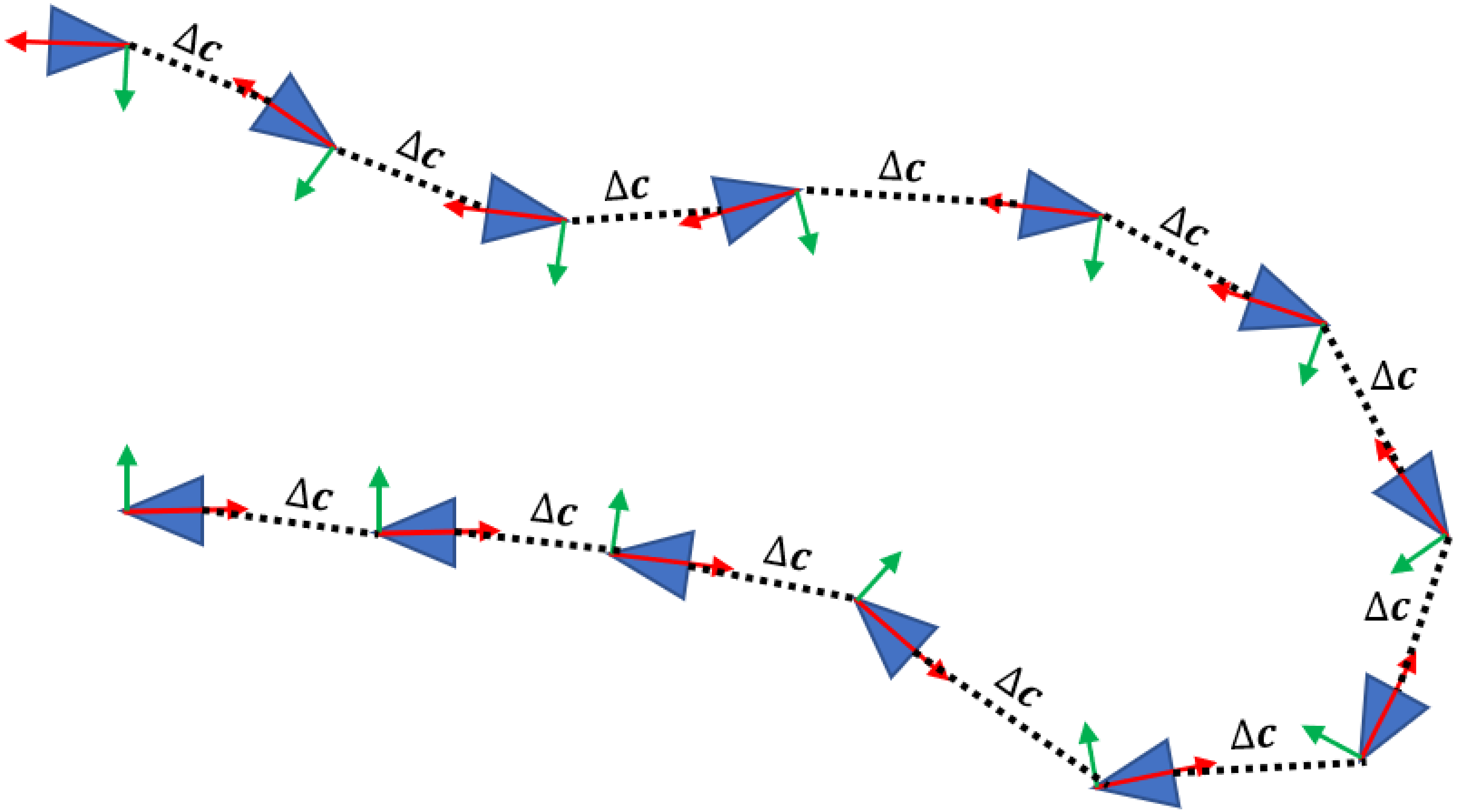
- `gaussian_model = gtsam.noiseModel.Diagonal.Sigmas(np.array([1.0, 1.0, 1.0]))`
- `cauchy = gtsam.noiseModel.mEstimator.Cauchy.Create(2)`
- `cauchy_model = gtsam.noiseModel.Robust.Create(cauchy, gaussian_model)`

$$\boxed{f(\mathbf{x};\mu,\mathbf{\Sigma},k) = \frac{\Gamma\left(\frac{1+k}{2}\right)}{\Gamma(\frac{1}{2})\pi^{\frac{k}{2}}\left|\mathbf{\Sigma}\right|^{\frac{1}{2}}\left[1+(\mathbf{x}-\mu)^T\mathbf{\Sigma}^{-1}(\mathbf{x}-\mu)\right]^{\frac{1+k}{2}}}}$$

Loop Closure

David Arnon

Pose Graph



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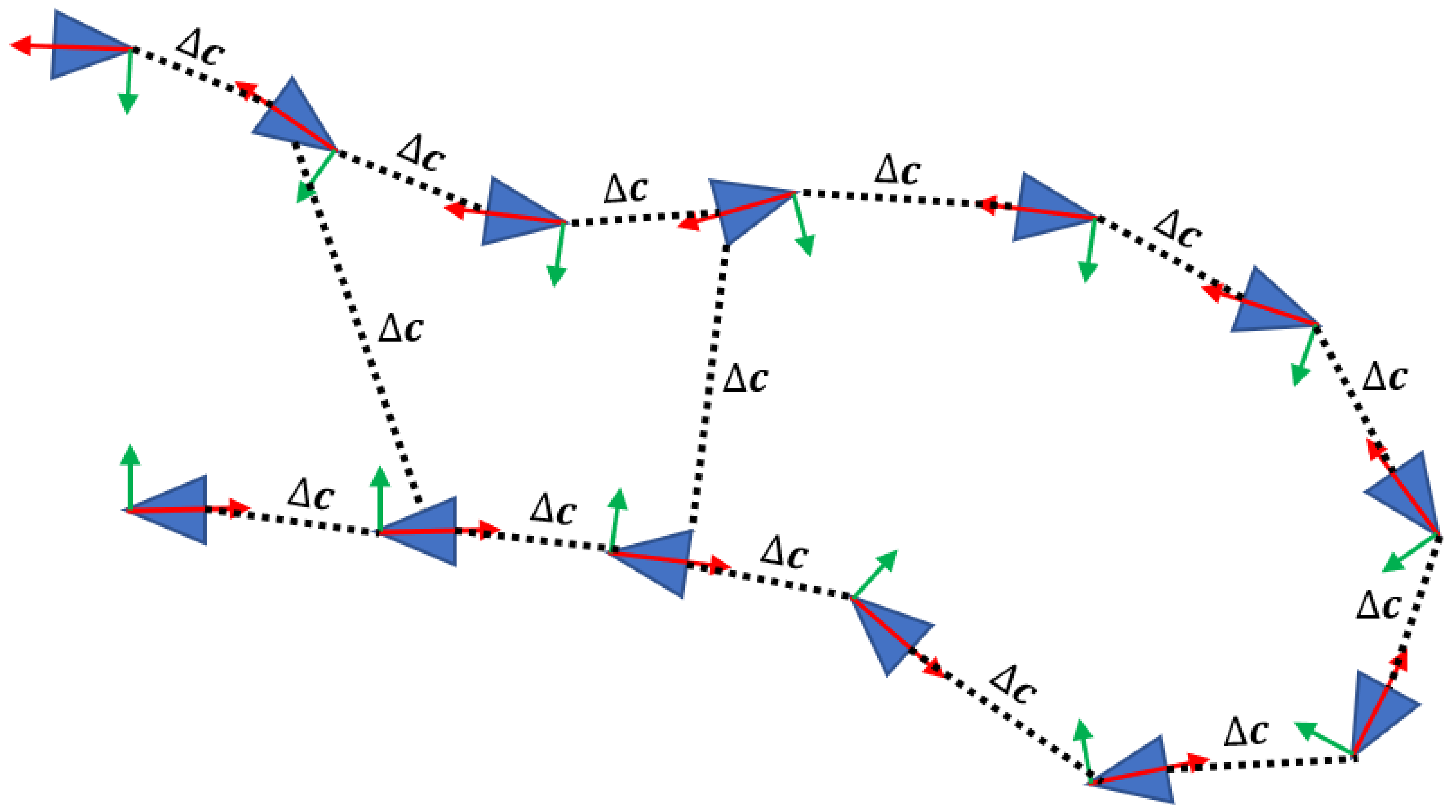
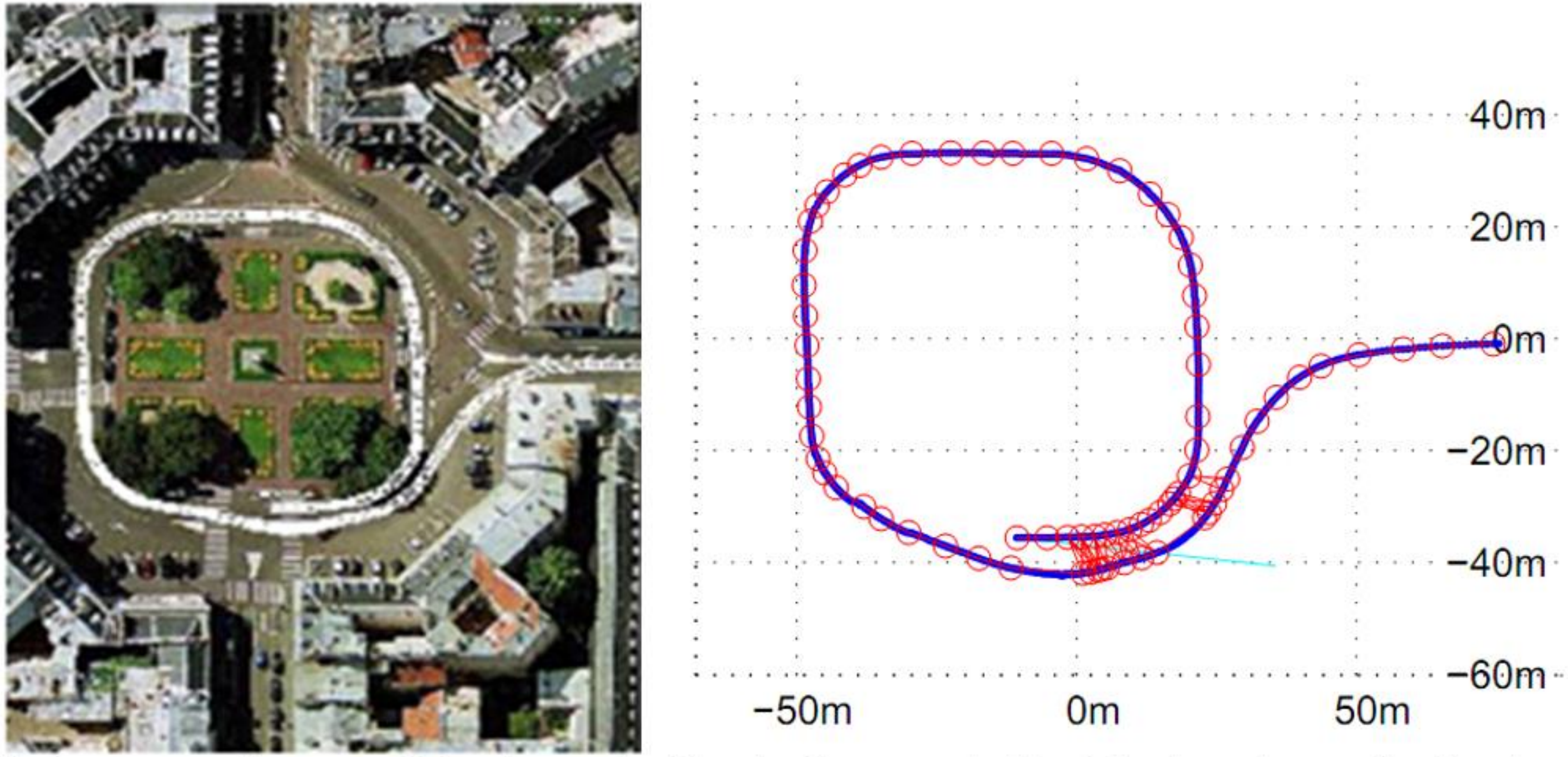
Loop Closure

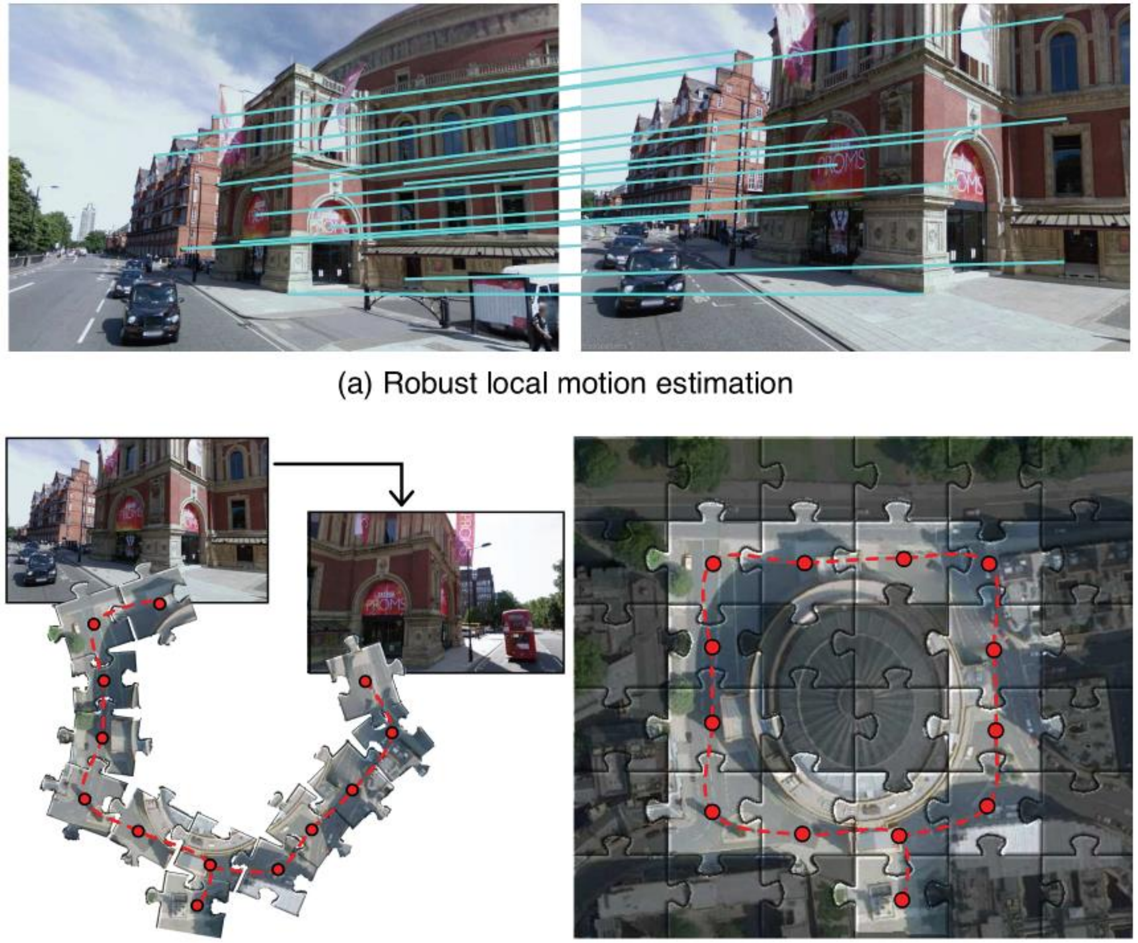


Loop Closure



Trajectory around a square. Estimation on the right – keyframes marked in circles, loop closure edges in red

Loop Closure
Detection



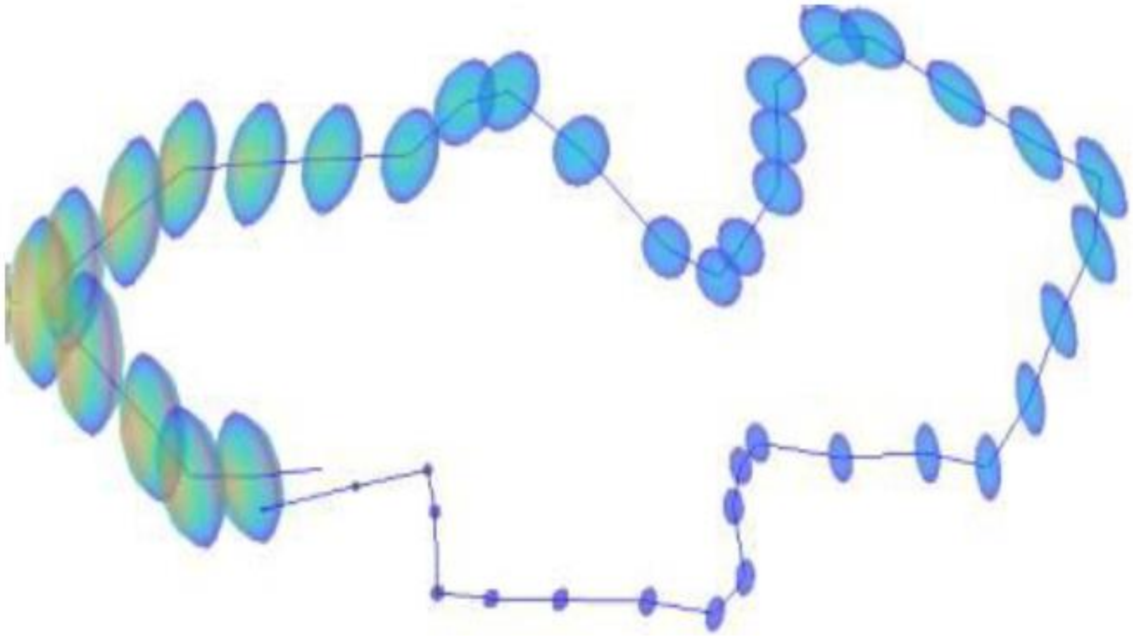
(a) Robust local motion estimation

(b) Mapping and loop-closure detection

(c) Global optimisation

Applying Information Theory to Efficient SLAM – M.Chli

Loop Closure
Detection



Gaussians

- https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf

The Matrix Cookbook

[http://matrixcookbook.com]

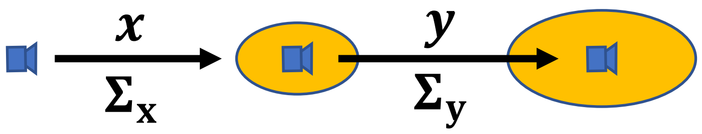
Kaare Brandt Petersen
Michael Syskind Pedersen

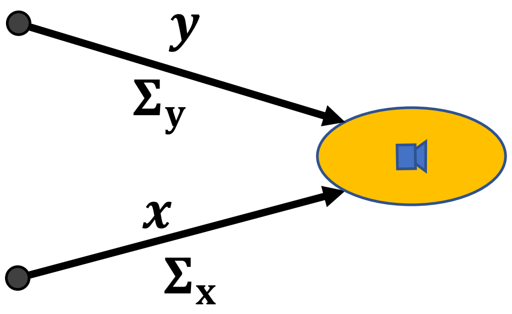
8.1.4 Linear combination

Assume $\mathbf{x} \sim \mathcal{N}(\mathbf{m}_x, \mathbf{\Sigma}_x)$ and $\mathbf{y} \sim \mathcal{N}(\mathbf{m}_y, \mathbf{\Sigma}_y)$ then

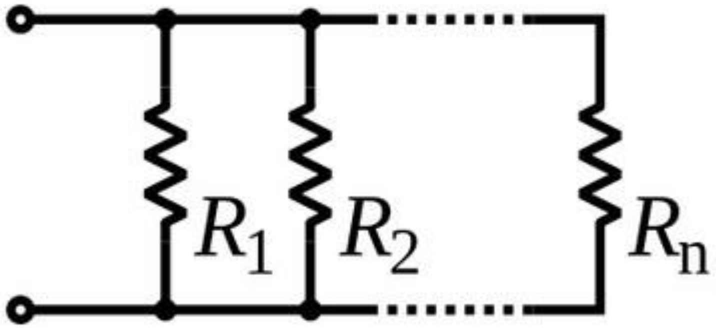
$$\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} + \mathbf{c} \sim \mathcal{N}(\mathbf{A}\mathbf{m}_x + \mathbf{B}\mathbf{m}_y + \mathbf{c}, \mathbf{A}\mathbf{\Sigma}_x\mathbf{A}^T + \mathbf{B}\mathbf{\Sigma}_y\mathbf{B}^T) \qquad (355)$$

Loop Closure
Measurements





$$R_{\text{total}} = R_s = R_1 + R_2 + \cdots + R_n$$



$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}.$$

$$I = \frac{V}{R}$$

Gaussians

8.1.8 Product of gaussian densities

Let $\mathcal{N}_{\mathbf{x}}(\mathbf{m}, \mathbf{\Sigma})$ denote a density of $\mathbf{x}$, then

$$\mathcal{N}_{\mathbf{x}}(\mathbf{m}_1, \mathbf{\Sigma}_1) \cdot \mathcal{N}_{\mathbf{x}}(\mathbf{m}_2, \mathbf{\Sigma}_2) = c_c \mathcal{N}_{\mathbf{x}}(\mathbf{m}_c, \mathbf{\Sigma}_c) \tag{371}$$

$$\begin{aligned} c_c &= \mathcal{N}_{\mathbf{m}_1}(\mathbf{m}_2, (\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2)) \\ &= \frac{1}{\sqrt{\det(2\pi(\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2))}} \exp\left[-\frac{1}{2}(\mathbf{m}_1 - \mathbf{m}_2)^T (\mathbf{\Sigma}_1 + \mathbf{\Sigma}_2)^{-1} (\mathbf{m}_1 - \mathbf{m}_2) \right] \\ \mathbf{m}_c &= (\mathbf{\Sigma}_1^{-1} + \mathbf{\Sigma}_2^{-1})^{-1} (\mathbf{\Sigma}_1^{-1}\mathbf{m}_1 + \mathbf{\Sigma}_2^{-1}\mathbf{m}_2) \\ \boxed{\mathbf{\Sigma}_c} &\boxed{= (\mathbf{\Sigma}_1^{-1} + \mathbf{\Sigma}_2^{-1})^{-1}} \end{aligned}$$

but note that the product is not normalized as a density of $\mathbf{x}$.

Loop Closure
Covariance Approximation