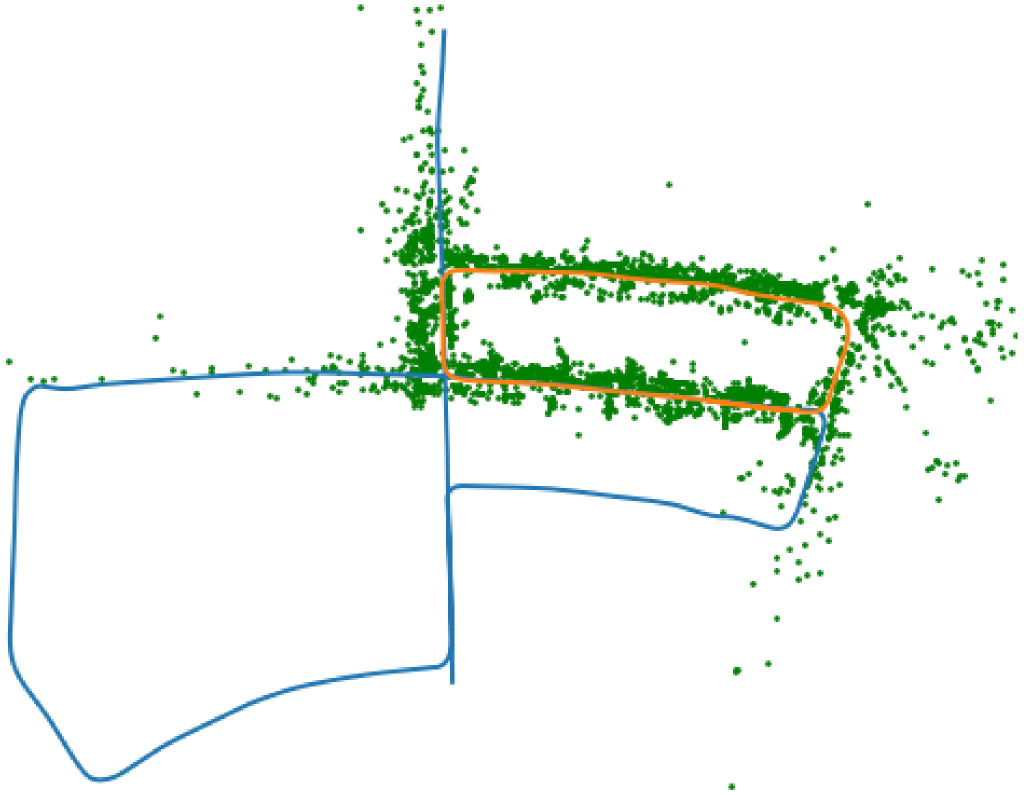
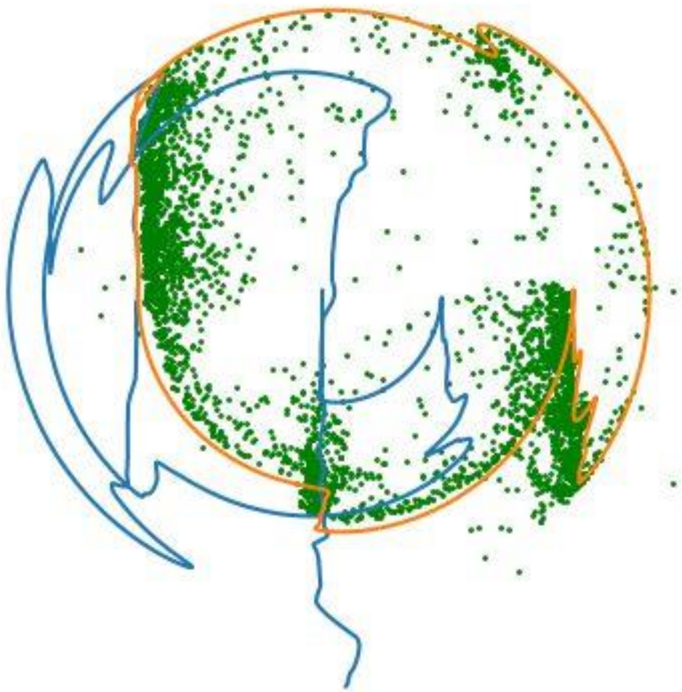
Bundle Adjustment

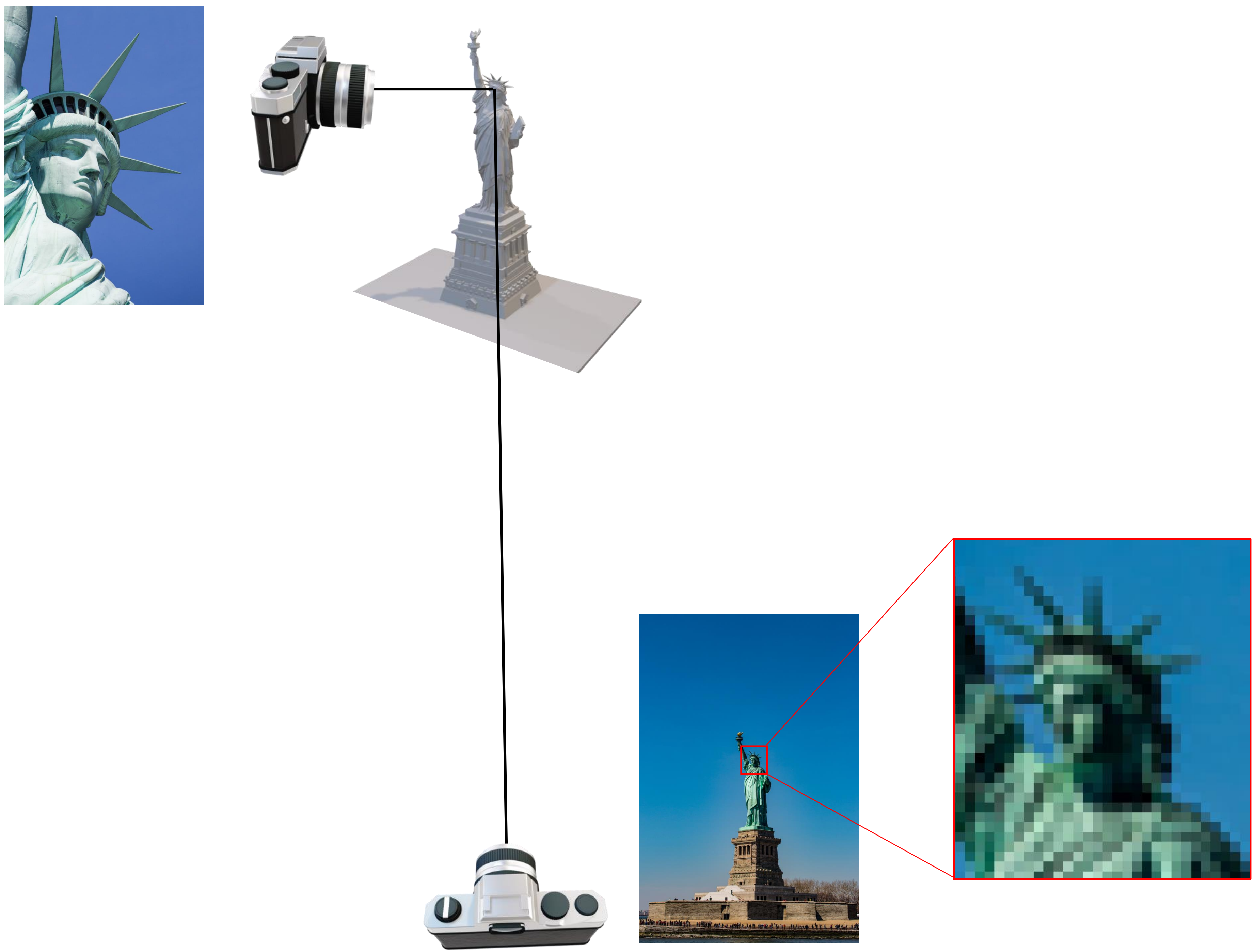
David Arnon

KITTI



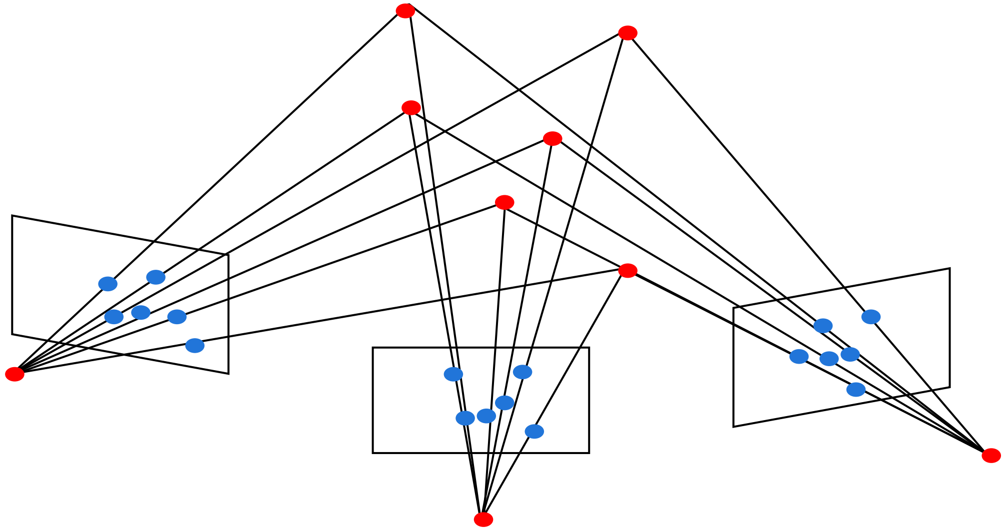


Triangulation

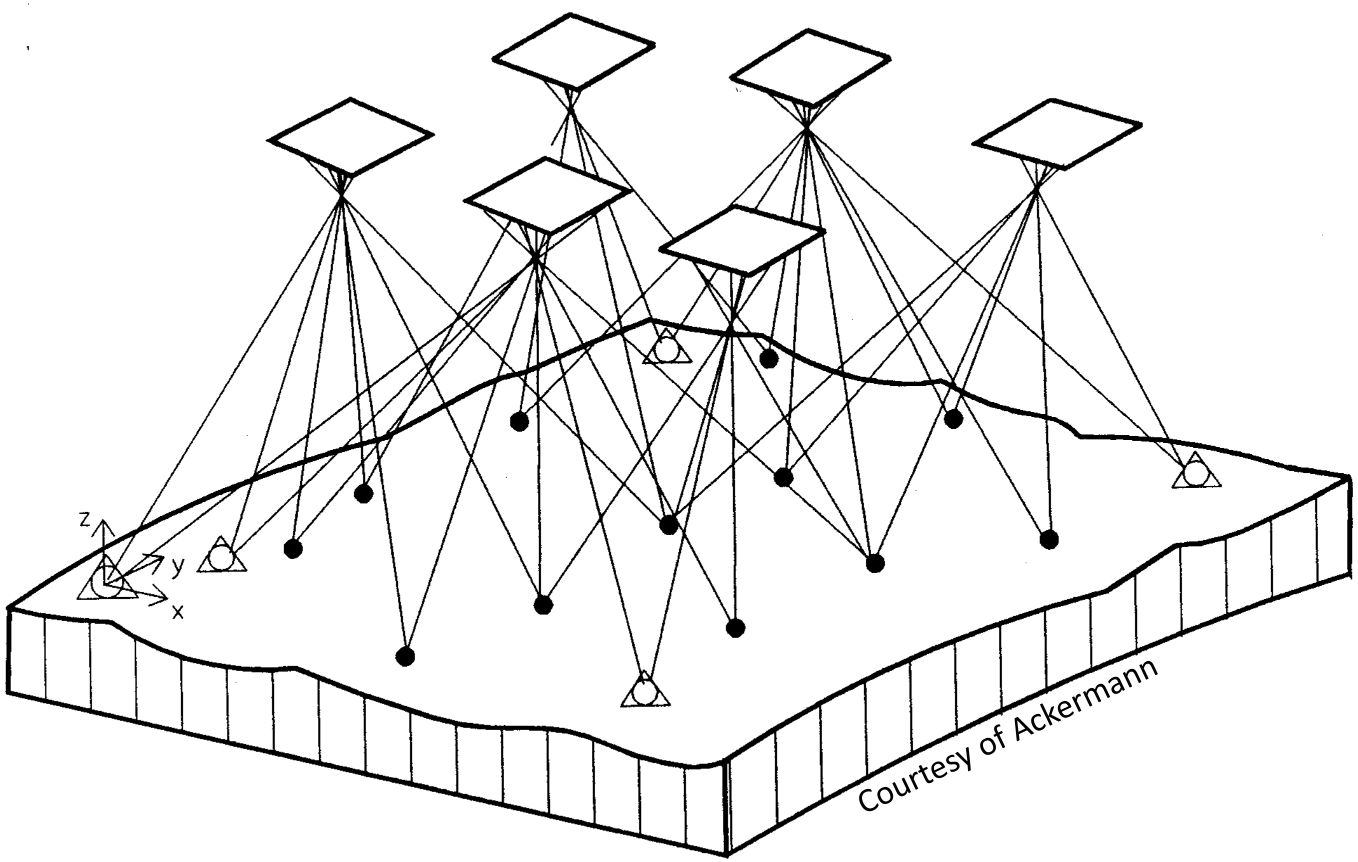


Bundle Adjustment

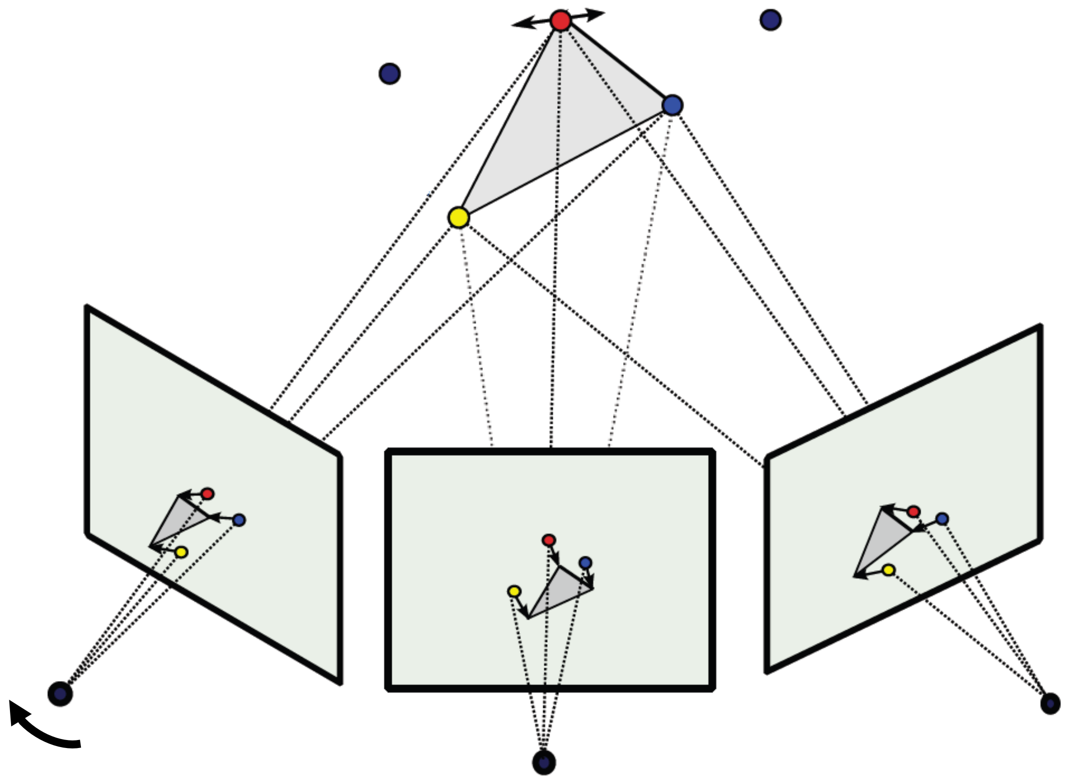
- Refines a visual reconstruction to produce jointly optimal 3D structure (world) and viewing parameters (cameras)
- *'bundle'* refers to the bundle of light rays leaving each 3D feature and converging on each camera center.
- Developed in the field of photogrammetry in the 1950´s



Bundle Adjustment



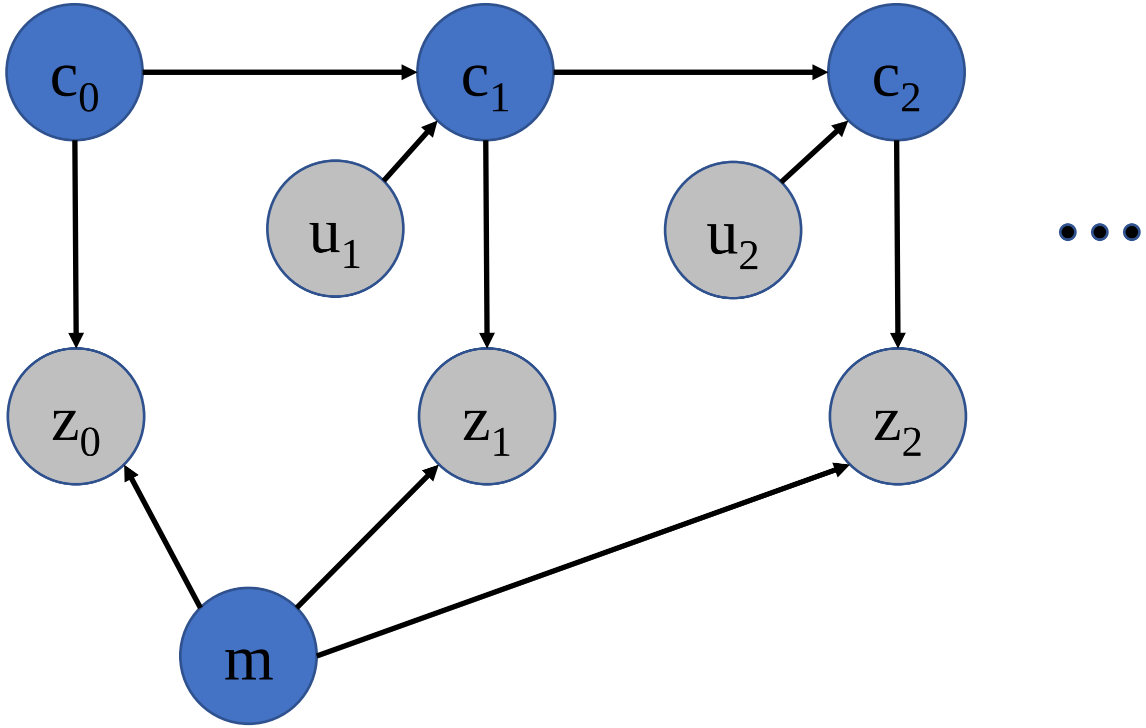
Bundle Adjustment



Courtesy of Vadim Indelman

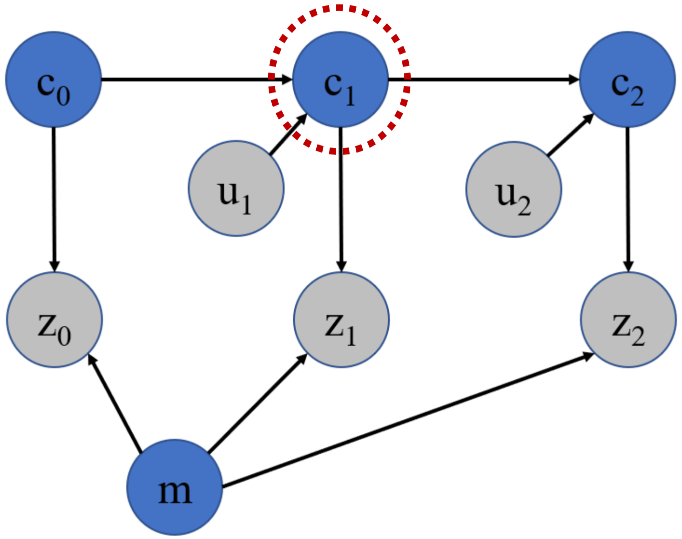
Slam

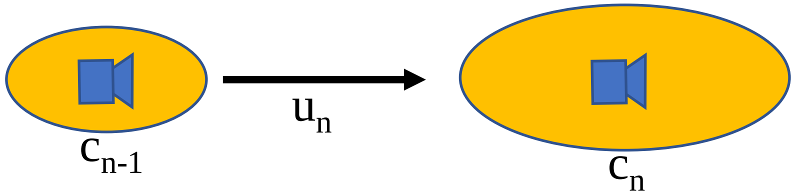
- Graphical Model



$$p(c_{0:2}, m \mid u_{1:2}, z_{0:2})$$

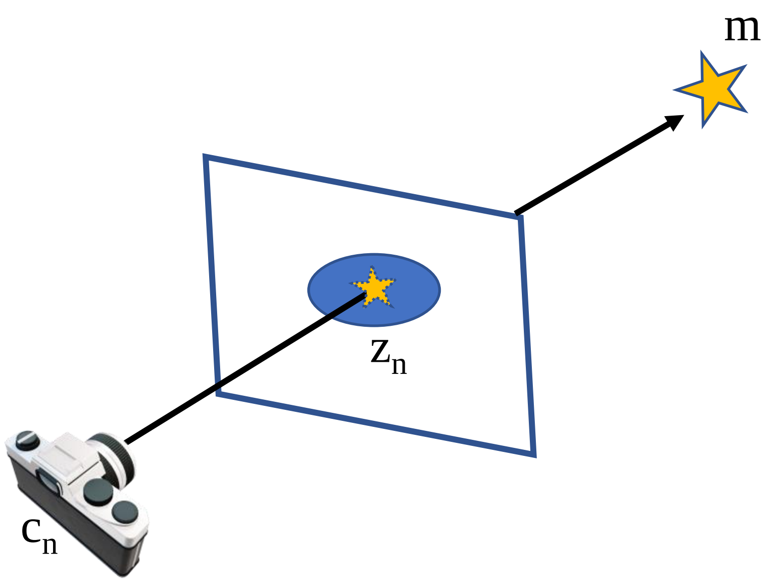
Motion Model



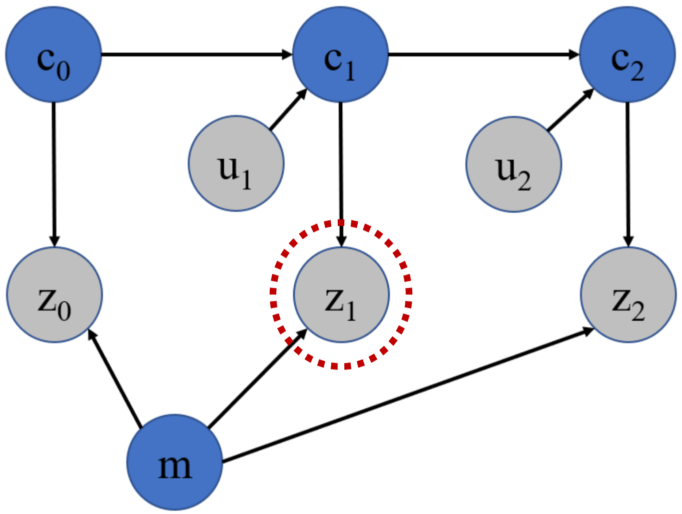


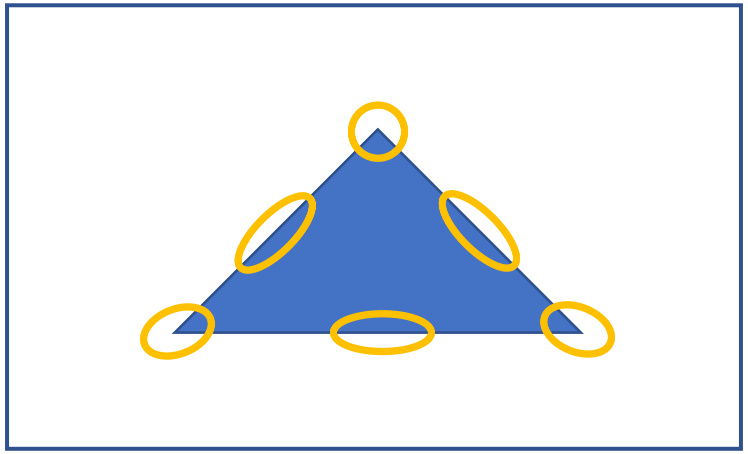
$$p(c_n | c_{n-1}, u_n)$$

Measurement Model

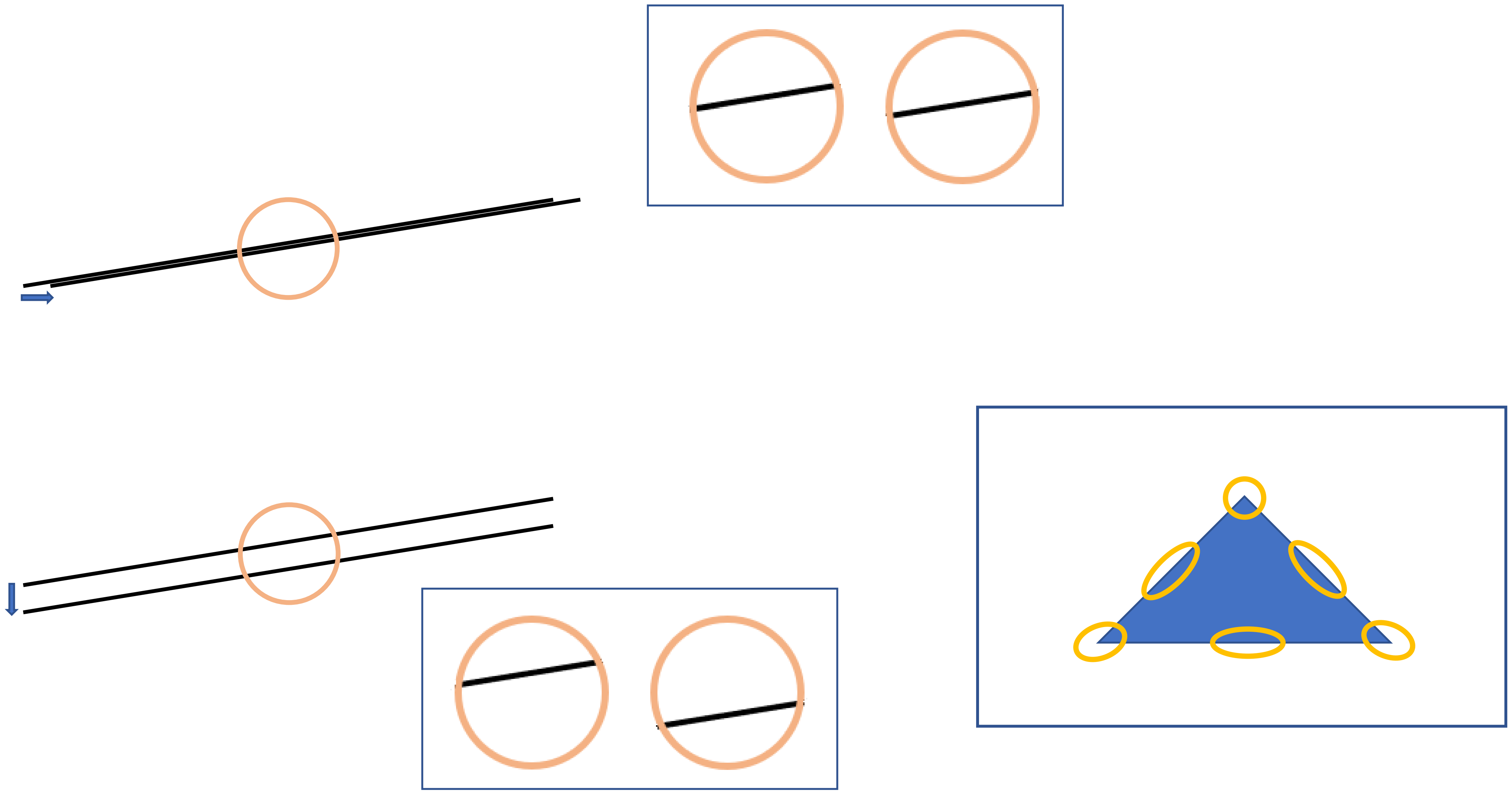


$$p(z_n|c_n, m)$$



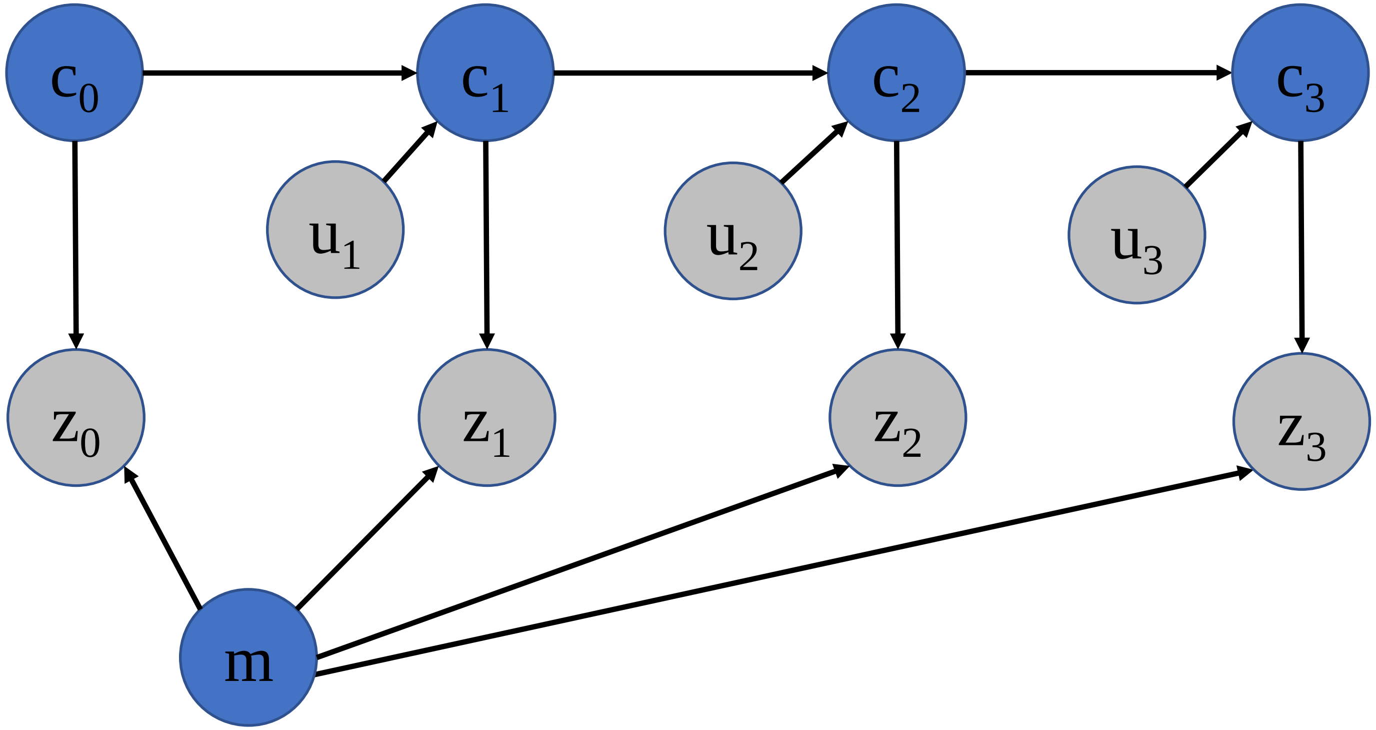


Measurement Model



Graphical Model

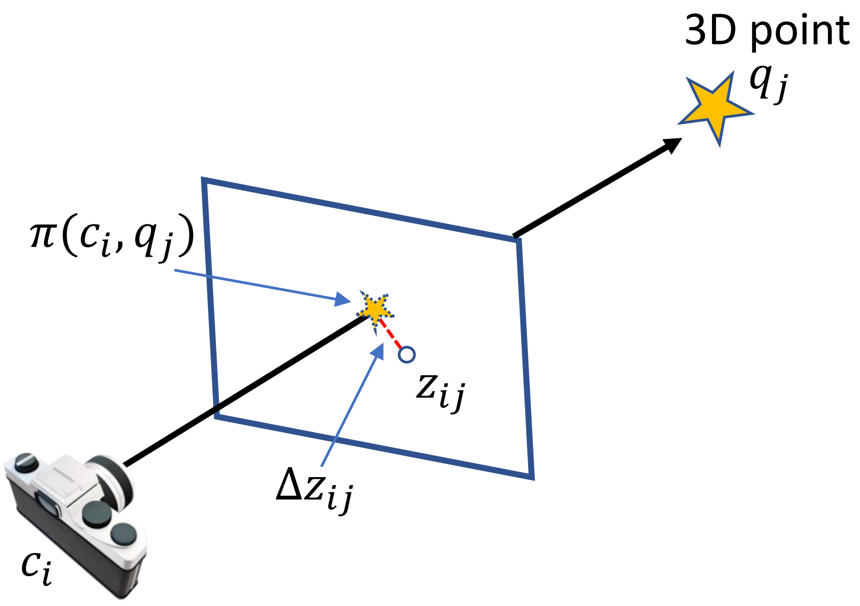
- Bundle Adjustment



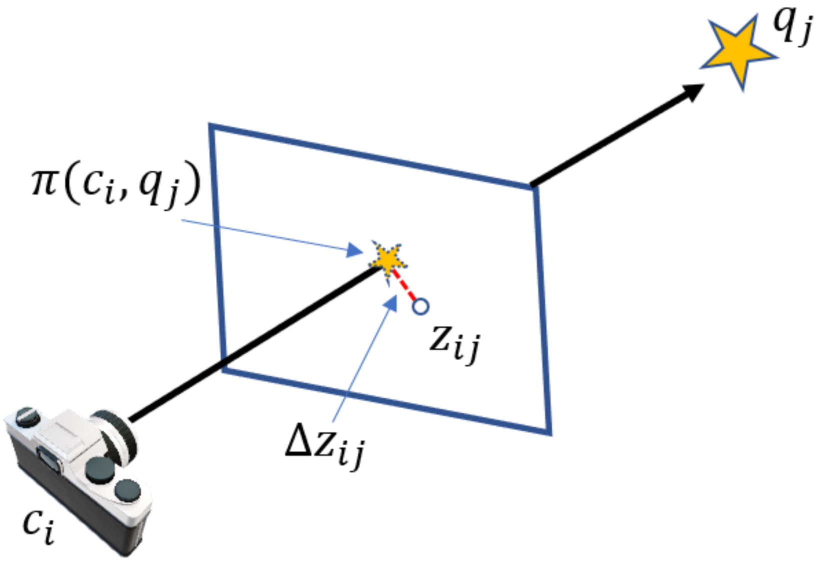
$$p(c_{0:3}, m \mid \cancel{u_{1:3}}, z_{0:3})$$

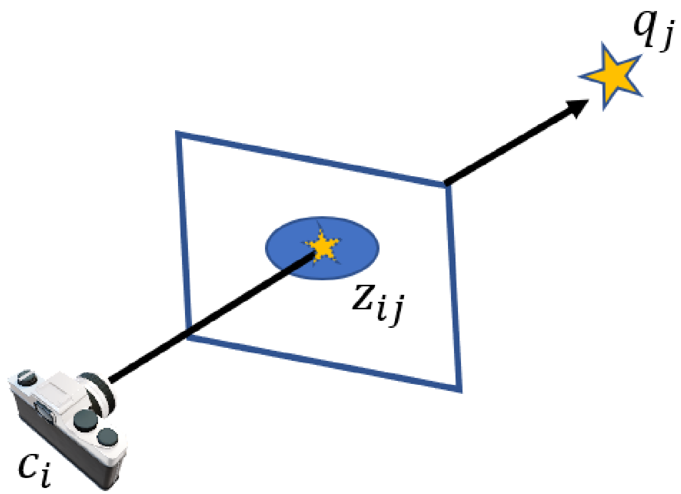
Probabilistic Formulation

- Reprojection error: $\boxed{\Delta z_{ij} \doteq \pi(c_i, q_j) - z_{ij}}$



Measurement Model





$$p\left(z_{ij} \middle| c_i, q_j\right) \sim N\left(\pi\left(c_i, q_j\right), \Sigma\right)$$

$$z_{ij} = \pi\left(c_i, q_j\right) + w, \qquad w \sim N(0, \Sigma)$$

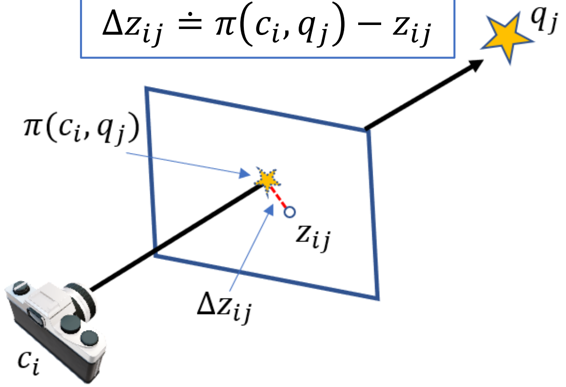
Bayes

- $p(z_{ij}|c_i, q_j) \sim N(\pi(c_i, q_j), \Sigma)$
- $p(c_i, q_j|z_{ij}) = \frac{1}{p(z_{ij})} p(z_{ij}|c_i, q_j) p(c_i, q_j)$
- $p(c_i, q_j|z_{ij}) \propto p(z_{ij}|c_i, q_j) p(c_i, q_j)$
- $p(c_i, q_j|z_{ij}) \propto p(z_{ij}|c_i, q_j)$
- $p(c_i, q_j|z_{ij}) \propto \exp\left(-\frac{1}{2}\left\|z_{ij} - \pi(c_i, q_j)\right\|_{\Sigma}^{2}\right)$
- $p(c_i, q_j|z_{ij}) \propto \exp\left(-\frac{1}{2}\left\|\Delta z_{ij}\right\|_{\Sigma}^{2}\right)$

$$\boxed{N_{\mu,\Sigma}(z) \propto exp\left(-\frac{1}{2}\|z-\mu\|_{\Sigma}^{2}\right)}$$

Reprojection error:

$$\boxed{\Delta z_{ij} \doteq \pi(c_i, q_j) - z_{ij}}$$



Bundle Adjustment

- $argmax_{C,Q}[p(C,Q|Z)]$
- $argmax_{C,Q}[p(Z|C,Q)]$
- $argmax_{C,Q}\left[\prod_{c_i}\prod_{j\in M_i} p\left(z_{ij}\middle|c_i,q_j\right)\right]$
- $argmax_{C,Q}\left[\prod_{c_i}\prod_{j\in M_i}\exp\left(-\frac{1}{2}\left\|\Delta z_{ij}\right\|_\Sigma^2\right)\right]$
- $argmax_{C,Q}\left[\sum_{c_i}\sum_{j\in M_i} -\frac{1}{2}\left\|\Delta z_{ij}\right\|_\Sigma^2\right]$
- $argmin_{C,Q}\left[\sum_{c_i}\sum_{j\in M_i}\left\|\Delta z_{ij}\right\|_\Sigma^2\right]$
- $argmin_{C,Q}\left[\sum_{c_i}\sum_{j\in M_i}\left\|\Sigma^{-½}\Delta z_{ij}\right\|^2\right]$

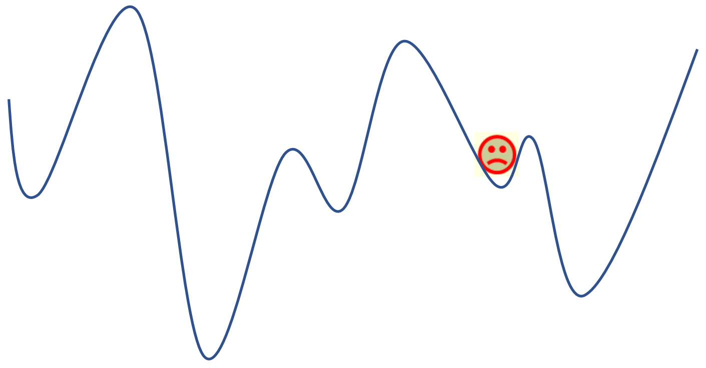
$\boxed{\Delta z_{ij} \doteq \pi\left(c_i,q_j\right) - z_{ij}}$

$\boxed{\Sigma^{½} = \mathrm{chol}(\Sigma)}$

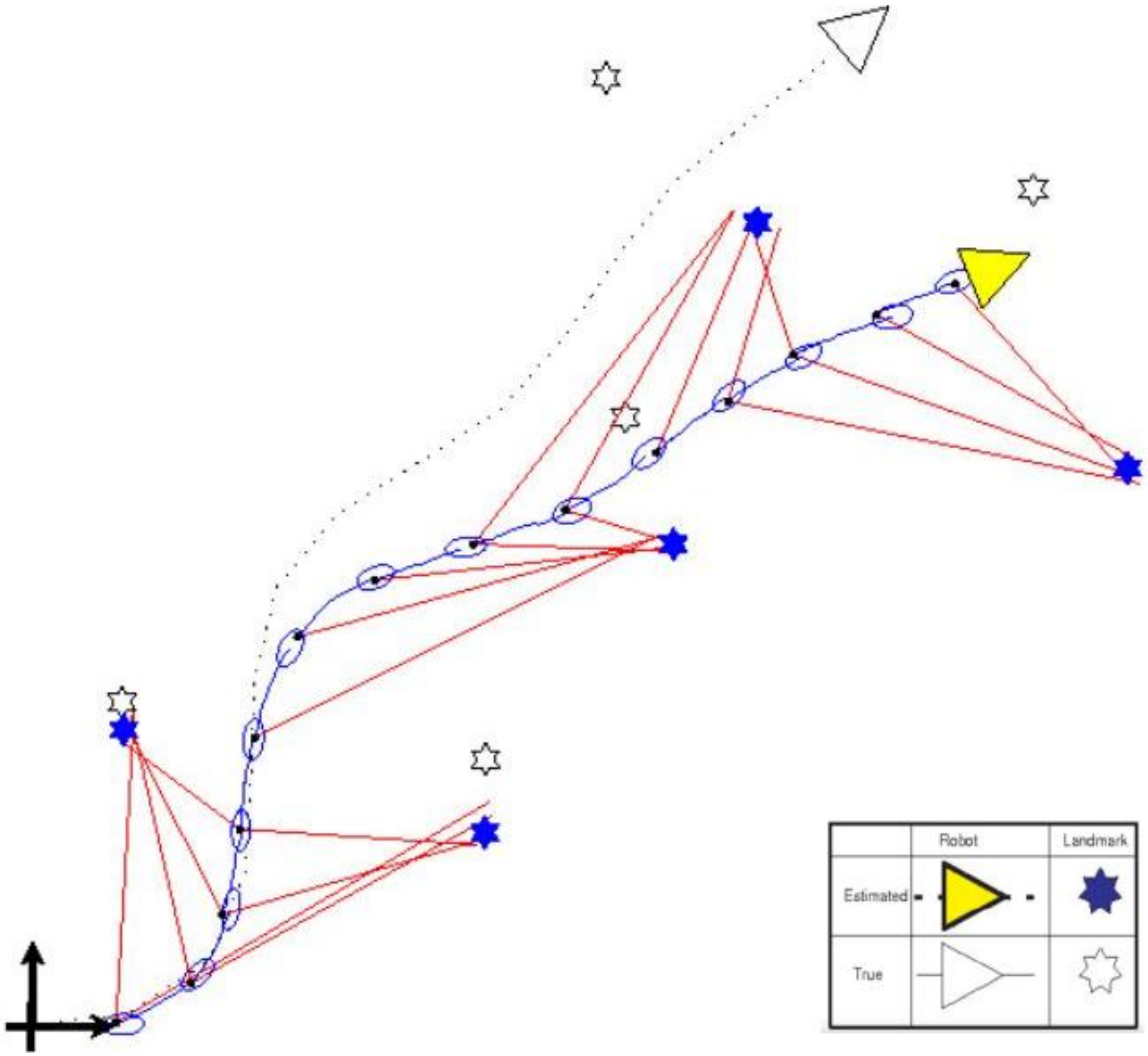
$$\boxed{\begin{array}{c}\Sigma = (\Sigma^{½})(\Sigma^{½})^T \\ \Sigma^{-1} = \Sigma^{-½\mathrm{T}}\Sigma^{-½}\end{array}}$$

Bundle Adjustment

- Maximum likelihood for normally distributed measurements
- Sensitive to outliers
  - The Gaussian has extremely small tail compared to most real measurement error distribution
- Non-linear least squares problem
- Solved using an iterative process
- General problem is non-convex,
  can settle in a local minima
- Requires a reasonable starting point



Bundle Adjustment



Courtesy of Durrant-Whyte, Baily; Slam: The essential algorithm

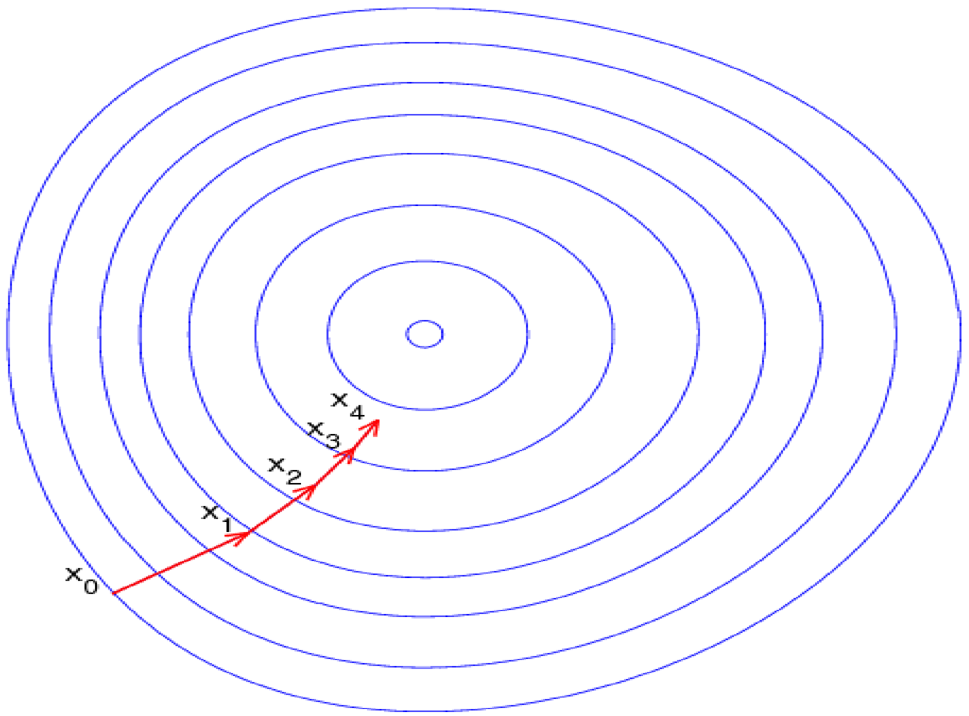
Bundle Adjustment

- Define a measurement function f that given the problem parameters calculates the expected measurements

- In optimal conditions $f(x) = z$

- Minimize $||f(x) - z||_{\Sigma}^{2}$

- Iterative solution

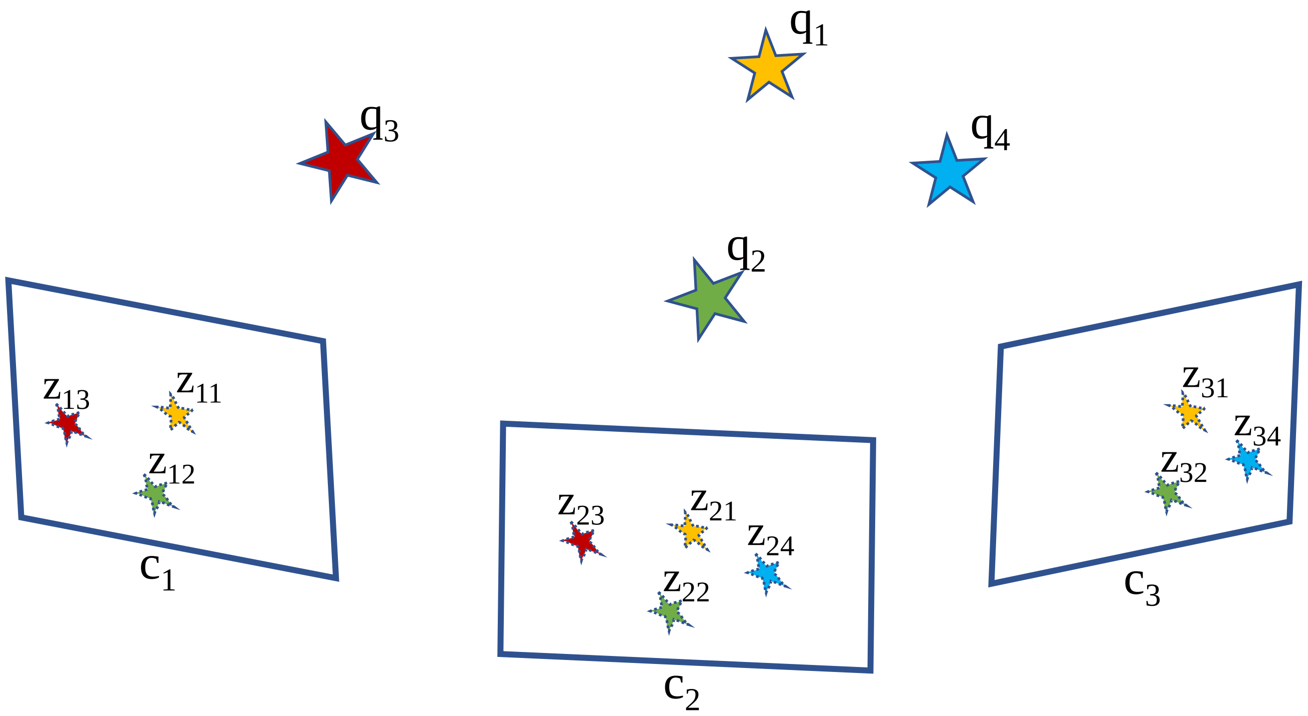


Bundle Adjustment

Representation

$$z^T = [z_{11}^T \quad z_{12}^T \quad z_{13}^T \quad z_{21}^T \quad z_{22}^T \quad z_{23}^T \quad z_{24}^T \quad z_{31}^T \quad z_{32}^T \quad z_{34}^T]$$

$$x^T = [c_1^T \quad c_2^T \quad c_3^T \quad q_1^T \quad q_2^T \quad q_3^T \quad q_4^T]$$



$$f(x) \doteq \begin{bmatrix} \pi(c_1, q_1) \\ \pi(c_1, q_2) \\ \pi(c_1, q_3) \\ \pi(c_2, q_1) \\ \pi(c_2, q_2) \\ \pi(c_2, q_3) \\ \pi(c_2, q_4) \\ \pi(c_3, q_1) \\ \pi(c_3, q_2) \\ \pi(c_3, q_4) \end{bmatrix}$$

Bundle Adjustment

Representation

- Two system variables
  - Camera $c_i = [\psi \quad \theta \quad \phi \quad x \quad y \quad z]^T$
    - 3D position and Euler angles
    - Can produce camera matrix $K[R_i|t_i]$
    - $R_i = R(\psi, \theta, \phi) = R_z(\psi)R_y(\theta)R_x(\phi)$
    - $t_i = [x \quad y \quad z]^T$
  - Landmark $q_i = [x \quad y \quad z]^T$
    - 3D position
- Projection: $\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = K[R_i|t_i] \begin{bmatrix} q_j \\ 1 \end{bmatrix} \longrightarrow z_{ij} = \begin{bmatrix} u \\ v \end{bmatrix} = \pi(c_i, q_j)$

Jacobian

- $f(x + \Delta x) \cong f(x) + J(x)\Delta x$

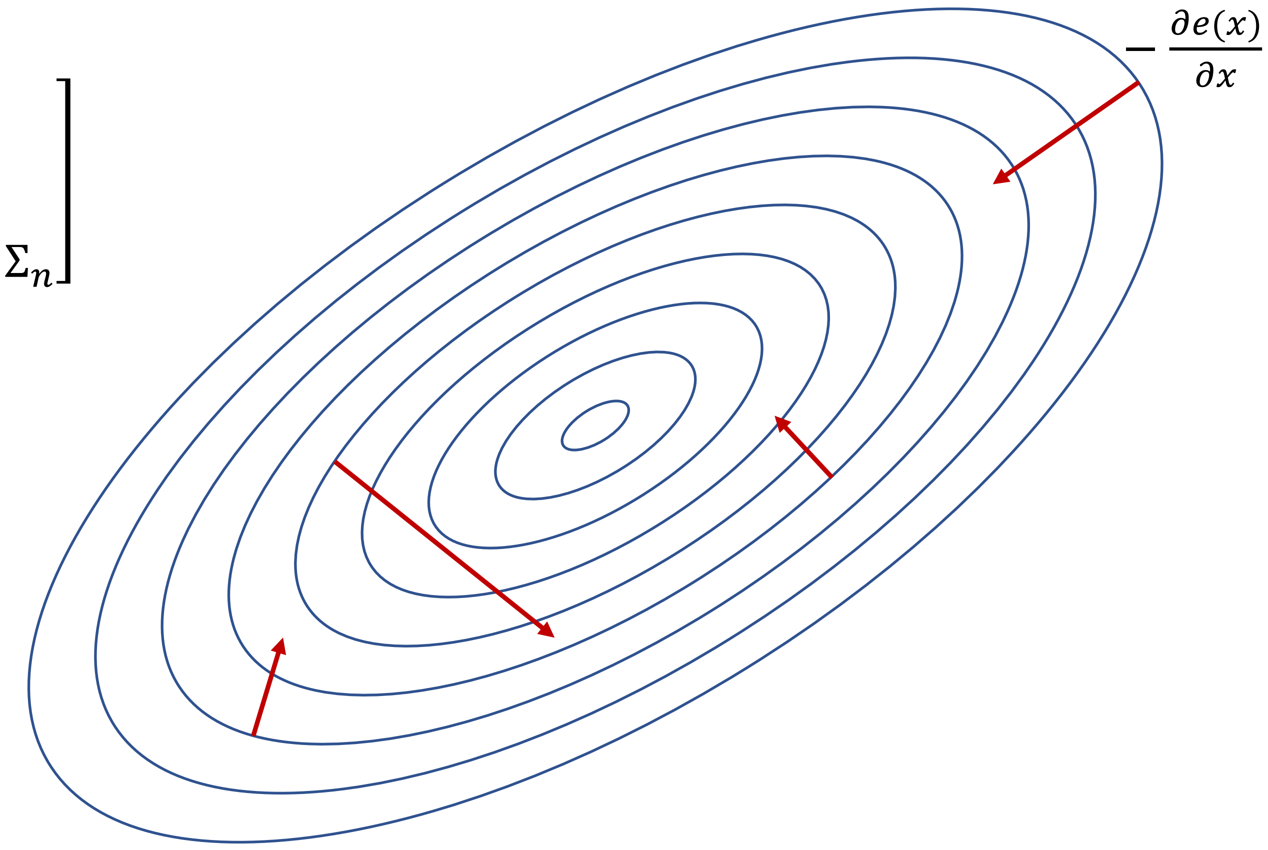
$$
\begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_m(x) \end{bmatrix}
+
\begin{bmatrix}
\frac{\partial f_1(x)}{\partial x_1} & \frac{\partial f_1(x)}{\partial x_2} & \cdots & \frac{\partial f_1(x)}{\partial x_p} \\
\frac{\partial f_2(x)}{\partial x_1} & \frac{\partial f_2(x)}{\partial x_2} & \cdots & \frac{\partial f_2(x)}{\partial x_p} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_m(x)}{\partial x_1} & \frac{\partial f_m(x)}{\partial x_2} & \cdots & \frac{\partial f_m(x)}{\partial x_p}
\end{bmatrix}
\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_p \end{bmatrix}
$$

$f(x)$ $\qquad\qquad$ $J_f(x)$ $\qquad\qquad$ Δx

Bundle Adjustment
Gradient Decent

- Error function $e(x) = \|f(x) - z\|_\Sigma^2$:

$$\Sigma = \begin{bmatrix} \Sigma_1 & & & \\ & \Sigma_2 & & \\ & & \ddots & \\ & & & \Sigma_n \end{bmatrix}$$



Bundle Adjustment
Linear Approximation

- $e(x) = \frac{1}{2}(f(x) - z)^T \Sigma^{-1} (f(x) - z)$
- $\left(\frac{\partial e(x)}{\partial x}\right)^T = J(x)^T \Sigma^{-1} (f(x) - z) = J(x)^T \Sigma^{-1} \Delta z$

$$\boxed{\Delta z \doteq f(x) - z}$$

- $e(x + \Delta x) \cong e(x) + \frac{\partial e(x)}{\partial x} \Delta x$
- $e(x + \Delta x) \cong e(x) - \frac{1}{\lambda} \left\| \frac{\partial e(x)}{\partial x} \right\|_2^2 < e(x)$

$$\boxed{\Delta x = -\frac{1}{\lambda} \left(\frac{\partial e(x)}{\partial x}\right)^T}$$

- $\Delta x = -\frac{1}{\lambda} J(x)^T \Sigma^{-1} \Delta z$

$$\boxed{g \doteq J(x)^T \Sigma^{-1} \Delta z}$$

$$\boxed{\Delta x = -\frac{1}{\lambda} g}$$

Bundle Adjustment

Gradient Decent