VAN course
Lesson 3

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Reminder: Homogeneous Coordinates

- A point (x, y) can be re-written in homogeneous coordinates as (x_h, y_h, h)
- The homogeneous parameter h is a non-zero value such that:

$$x = \frac{x_h}{h} \qquad y = \frac{y_h}{h}$$

- We can then write any point (x, y) as (hx, hy, h)
- We can conveniently choose $h = 1$ so that (x, y) becomes $(x, y, 1)$
- $(x, y, 1) = (5x, 5y, 5) = (hx, hy, h) \neq (0x, 0y, 0)$
  - This removes one DOF, hence it is still a 2D representation

Why Homogeneous Coordinates?

- Common transformations are affine but not linear
  - But are linear in Homogeneous Coordinates
- Allows us use matrix multiplication to calculate transformations – extremely efficient!

Homography

- Combine the geometric transformation into a single matrix with 3x3 matrices
- Two-Dimensional translation matrix:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = 1x + 0y + 1t_x = x + t_x$$

$$y' = 0x + 1y + 1t_y = y + t_y$$

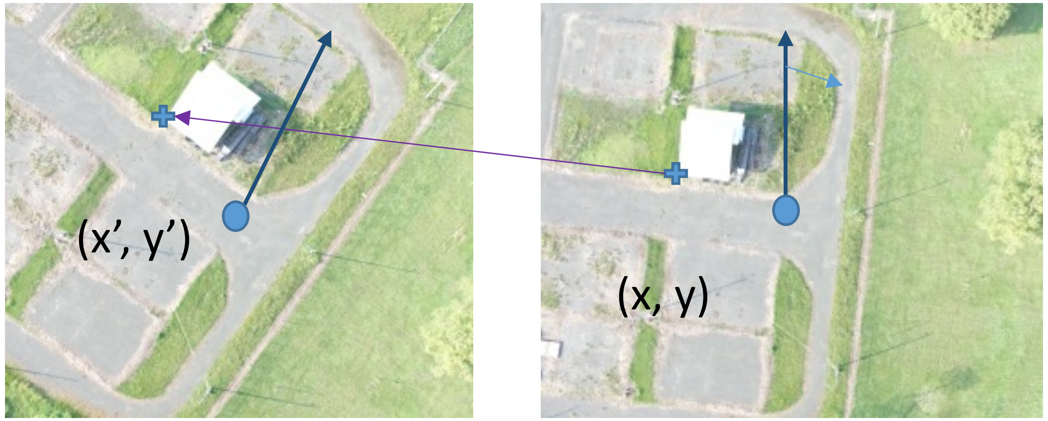
$$h = 0x + 0y + 1 = 1$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Homography

- Two-Dimensional rotation matrix

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$x' = x\cos\theta - y\sin\theta + 1\cdot 0 = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta + 1\cdot 0 = x\sin\theta + y\cos\theta$$
$$h = 0x + 0y + 1 = 1$$

Example: $\theta = 90^0$

$$x' = x\cos 90^0 - y\sin 90^0 + 1\cdot 0 = -y$$
$$y' = x\sin 90^0 + y\cos 90^0 + 1\cdot 0 = x$$
$$h = 0x + 0y + 1 = 1$$

Homography

- Two-Dimensional **scaling** matrix

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$x' = s_x x + 0y + 1 \cdot 0 = s_x x$$

$$y' = 0x + s_y y + 1 \cdot 0 = s_y y$$

$$h = 0x + 0y + 1 = 1$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \\ 1 \end{bmatrix}$$

Homography

- Linear transformation - a combination of:

Scale, Rotation and Translation transformations

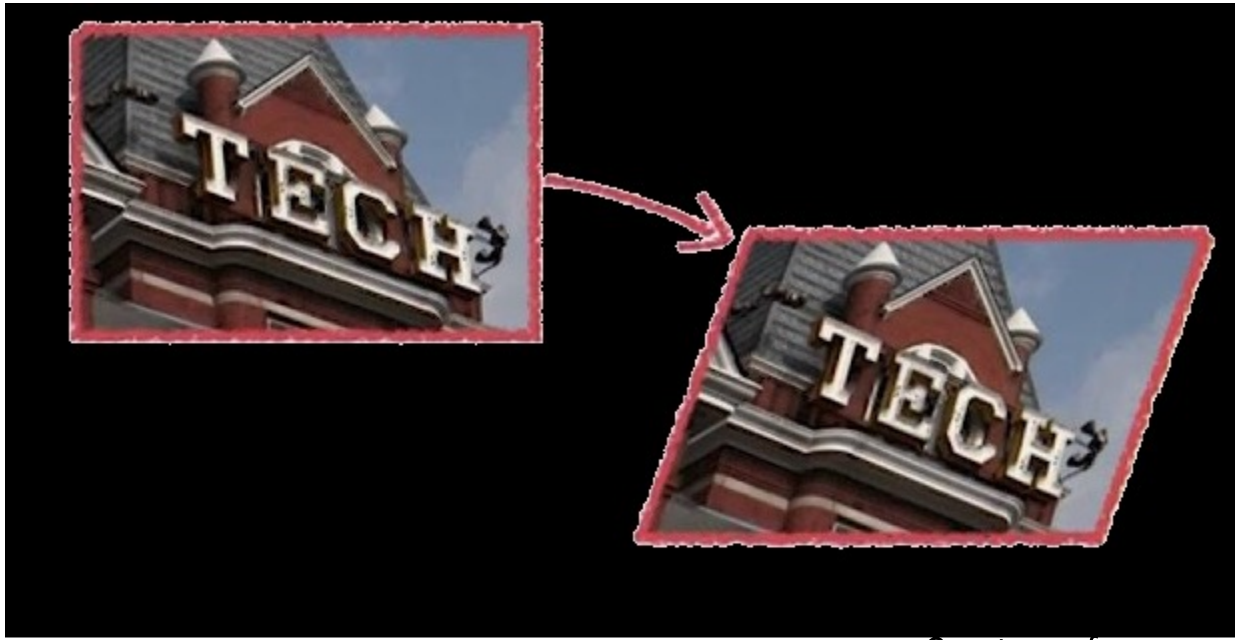
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Also called “similarity”
- 4 DOF: S , Ɵ, t_x , t_y

Homography

- Affine Transformation:
- 6 DOF: a, b, c, d, e, f

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



Courtesy of coursera

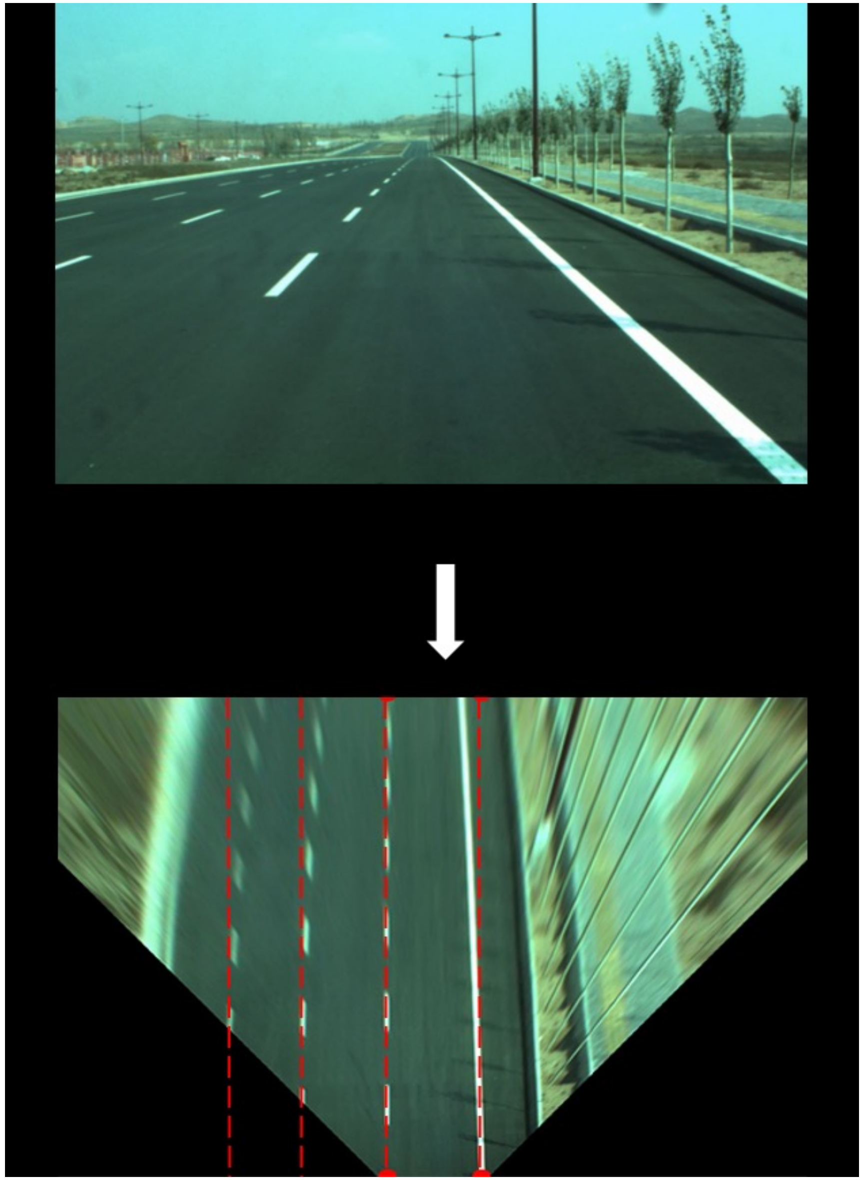
- First transformation to change angles!
- Turns squares into parallelogram
- Any affine trans is equal to:
    - Rotation -> uneven scale -> another rotation -> translation

Homography

- Perspective Transformation:
- 8 DOF: a, b, c, d, e, f, h, g

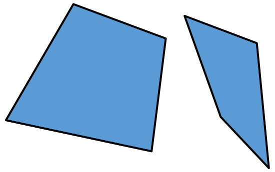
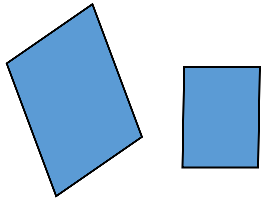
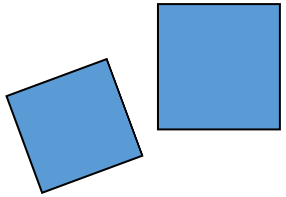
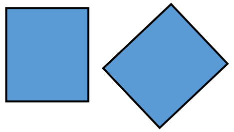
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ h & g & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Note that h, g are typically very small (~0.00001)
- Turns squares into a Quadrilateral
  - Usually quazi-trapezoids
- Can describe change of perspective on planes



Courtesy of line.17qq.com/

Hierarchy of 2D transformations

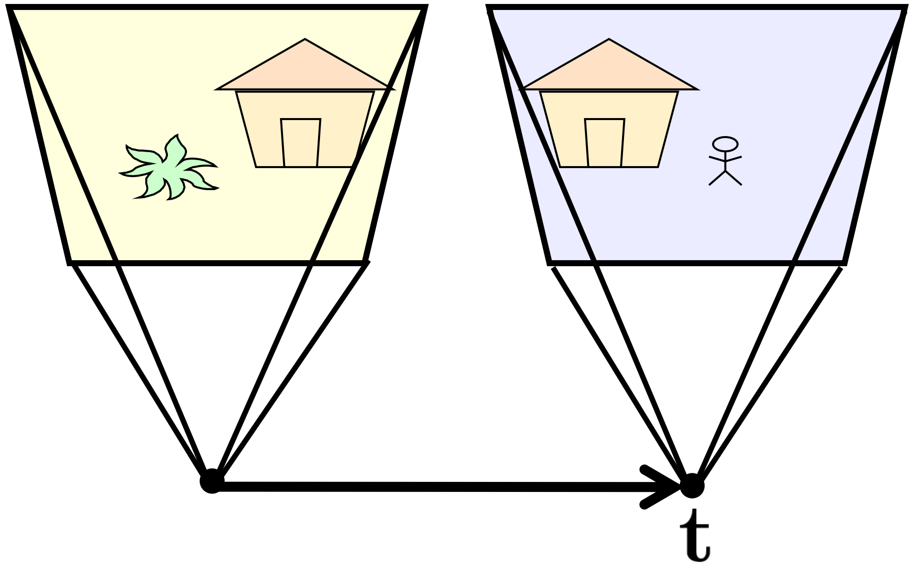
		transformed squares	invariants
Projective 8dof	$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$		Concurrency, collinearity, order of contact (intersection, tangency, inflection, etc.), cross ratio
Affine 6dof	$\begin{bmatrix} a_{11} & a_{12} & t_x \\ a_{21} & a_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Parallellism, ratio of areas, ratio of lengths on parallel lines (e.g midpoints), linear combinations of vectors (centroids). **The line at infinity l_∞**
Similarity 4dof	$\begin{bmatrix} sr_{11} & sr_{12} & t_x \\ sr_{21} & sr_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		Ratios of lengths, angles. **The circular points I,J**
Euclidean 3dof	$\begin{bmatrix} r_{11} & r_{12} & t_x \\ r_{21} & r_{22} & t_y \\ 0 & 0 & 1 \end{bmatrix}$		lengths, areas.

Courtesy of Marc Pollefeys

Stereo

- In stereo cameras-couple there is only translation along the 3D X axes, with no 3D rotation.

$$\mathbf{R} = \mathbf{I}_{3\times 3}$$
$$\mathbf{t} = [\ 1 \quad 0 \quad 0\ ]^T$$

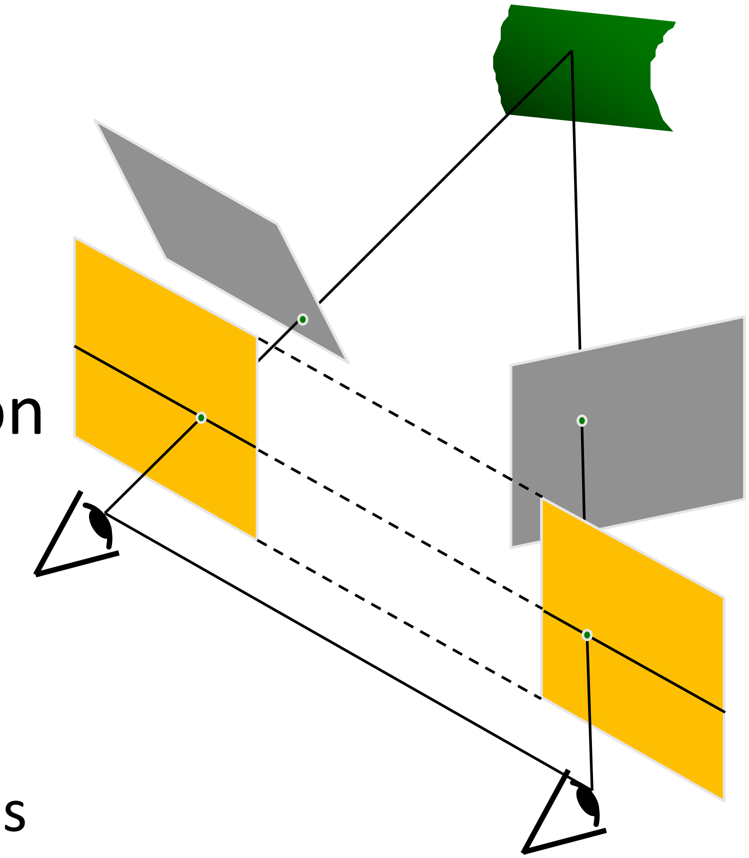


- Therefore, all pixel-matches has same Y coordinate
- And the depth is a function of the disparity (x_1-x_2)
- This makes matching process much easier:
  - Faster – small search area, good initial guess
  - Robust – less possible false matches
- But is highly unlikely to get!
  - Nano-movements break the stereo-assumption
  - Homography to the rescue: stereo rectification

Stereo image rectification

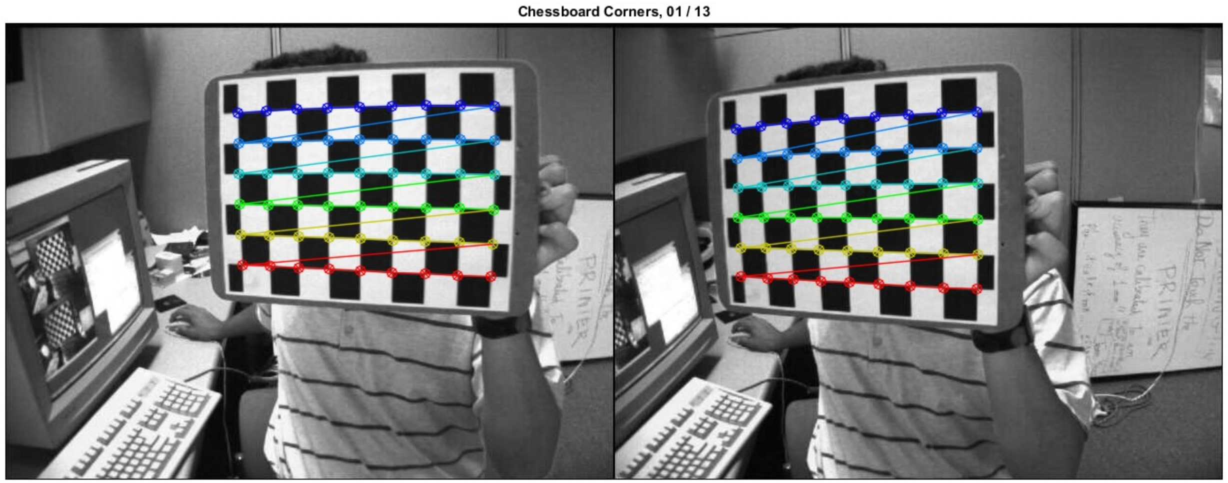
- Re-project image planes:
  - onto a common plane
  - parallel to the line between optical centers
- Pixel motion is horizontal after this transformation
- The rectification is two homographies:
  - Two 3x3 homographic transformations
  - One for each input image re-projection

    C. Loop and Z. Zhang. Computing Rectifying Homographies for Stereo Vision CVPR 1999
  - We usually warp the images using the transformations



Stereo image rectification

- Why it works?
  - True rectification can be achieved using two 3D rotations
  - Perspective transformation model camera rotation
- Stereo Calibration:
  - finding the transformations
  - Usually also includes lens-distortion calibration
  - Kitti already did this for us



Courtesy of amroamroamro.github.io/