CS-725: Assignment 2 (Question-2)

Question 2 Part (a) – Bands of Blue

Proposed Solution – To classify the given regions, we'll employ a neural network with a specific architecture.

Our activation function is given as:

$$g(x;T) = \begin{cases} 1 & \text{If } x \geq T \\ 0 & \text{Otherwise} \end{cases}$$

Network Architecture:

I. **Input Layer:**
Two input neurons representing the x_1 and x_2 coordinates of a data point.

II. **Hidden Layer:**
Four neurons, each representing a specific region boundary:

1. Neuron 1: Detects the region below the line $x_1 + x_2 = 1$.
2. Neuron 2: Detects the region above the line $x_1 + x_2 = 4$.
3. Neuron 3: Detects the region below the line $x_1 + x_2 = 6$.
4. Neuron 4: Detects the region above the line $x_1 + x_2 = 9$.

III. **Output Layer:**

Neuron	**Weights** *(w1, w2)*	**Threshold**
1	**(-1, -1)**	**-1**
2	**(1, 1)**	**4**
3	**(-1, -1)**	**-6**
4	**(1, 1)**	**9**

The output neuron will have weights of 1 from each hidden neuron and a threshold of 2.

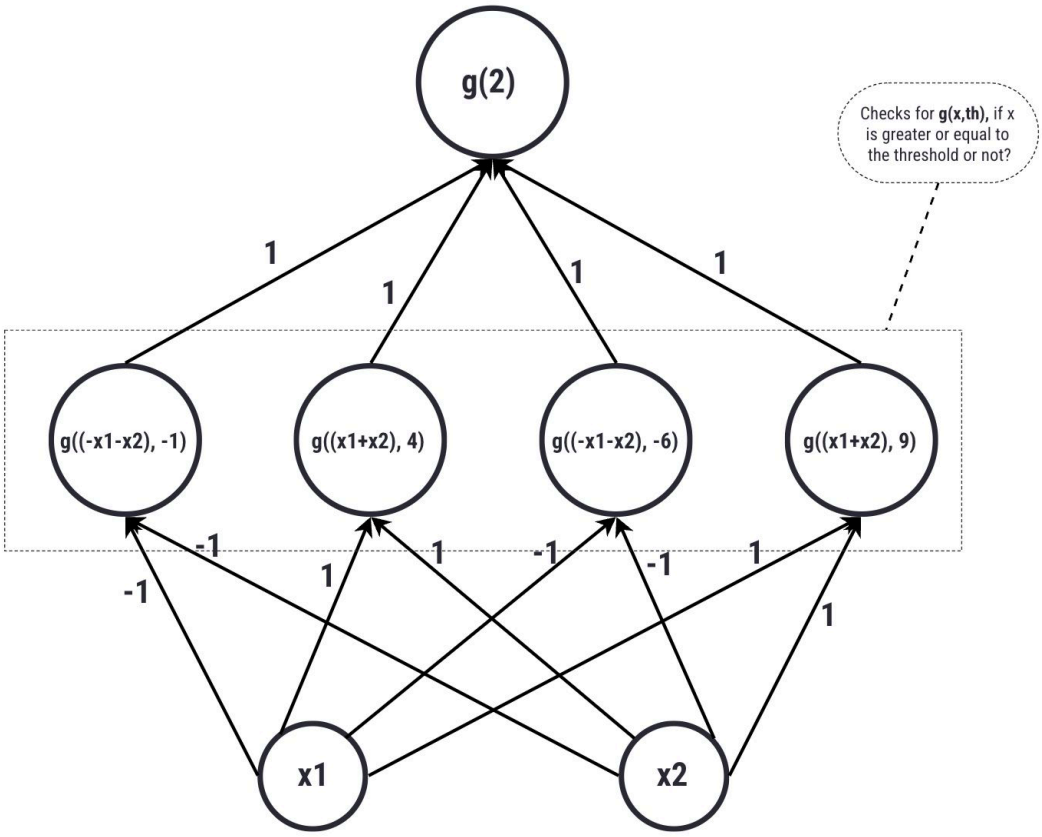
Classification Process:

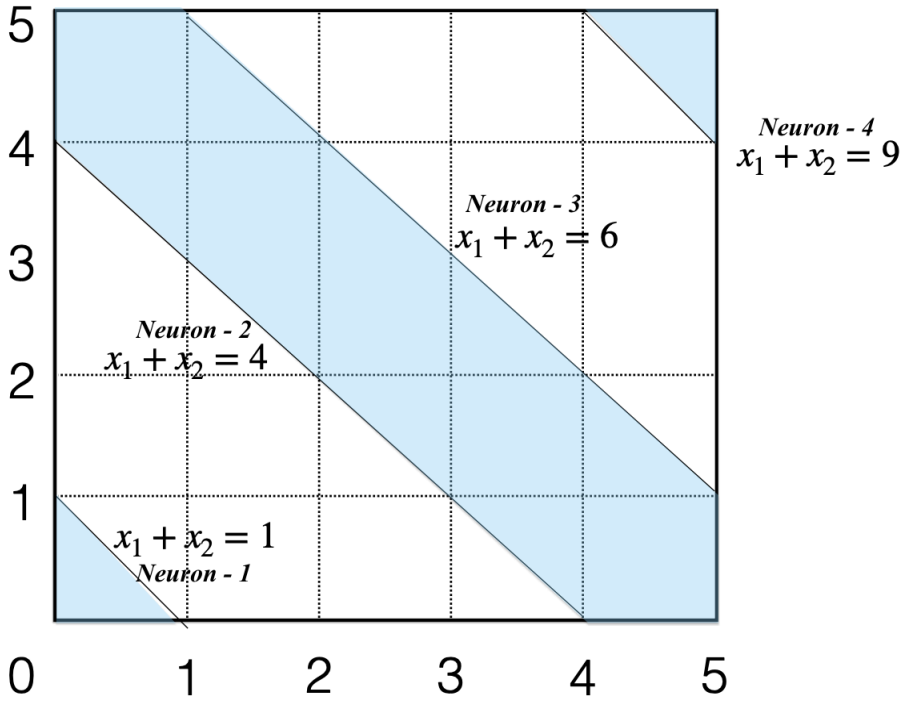
1. **Input:** A data point (x_1, x_2) is fed into the input layer.
2. **Hidden Layer:** Each hidden neuron calculates its weighted sum and applies the step function.
3. **Output Layer:** The output neuron sums the outputs of the hidden neurons and applies the step function.
    - If the sum is greater than or equal to 2, the output is 1 (class 1).
    - Otherwise, the output is 0 (class 0).

Rationale:

- **Region Detection:** The hidden neurons are designed to identify specific regions based on their weight and threshold combinations.
- **Intersection Logic:** The output neuron's threshold of 2 ensures that only points belonging to the intersection of two or more regions are classified as class 1.
- **Class Assignment:** Points that satisfy the intersection criteria are assigned to class 1; others are assigned to class 0.

By following this approach, the neural network can accurately classify the given regions into two classes.


g(2)
Checks for g(x,th), if x is greater or equal to the threshold or not?
1
1
1
1
g((-x1-x2), -1)
g((x1+x2), 4)
g((-x1-x2), -6)
g((x1+x2), 9)
-1
1
1
-1
-1
1
-1
1
x1
x2


5
4
3
2
1
0
1
2
3
4
5
Neuron - 4
$x_1 + x_2 = 9$
Neuron - 3
$x_1 + x_2 = 6$
Neuron - 2
$x_1 + x_2 = 4$
$x_1 + x_2 = 1$
Neuron - 1

Question 2 Part (b) – Catch the Star

Proposed Solution – To classify the given regions, we'll employ a neural network with a specific architecture.

Our activation function is the same as above.

Network Architecture:

I. **Input Layer:**
Two input neurons representing the x1 and x2 coordinates of a data point.

II. **Hidden Layer:**
Four neurons, each representing a specific region boundary:

1. Neuron 1: Detects the region below the line $10x_1 + 4x_2 = 80$.
2. Neuron 2: Detects the region above the line $10x_1 - 4x_2 = 20$.
3. Neuron 3: Detects the region below the line $x_2 = 6$.
4. Neuron 4: Detects the region above the line $7x_1 + 10x_2 = 80$.
5. Neuron 5: Detects the region above the line $-7x_1 + 10x_2 = 10$.

III. **Output Layer:**

Neuron	**Weights** *(w1, w2)*	**Threshold**
1	**(-10, -4)**	**-80**
2	**(10, -4)**	**20**
3	**(0, -1)**	**-6**
4	**(7, 10)**	**80**
5	**(-7, 10)**	**10**

The output neuron will have weights of 1 from each hidden neuron and a threshold of 4.

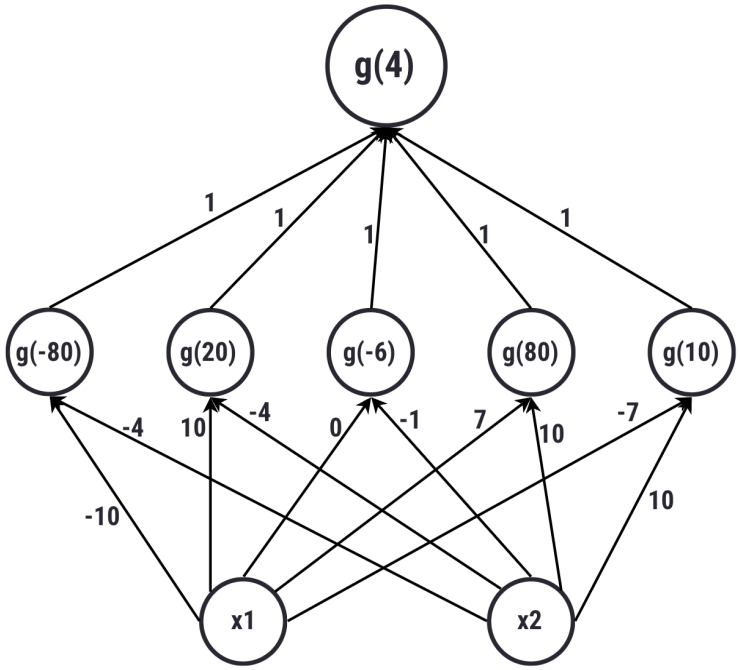
Classification Process:

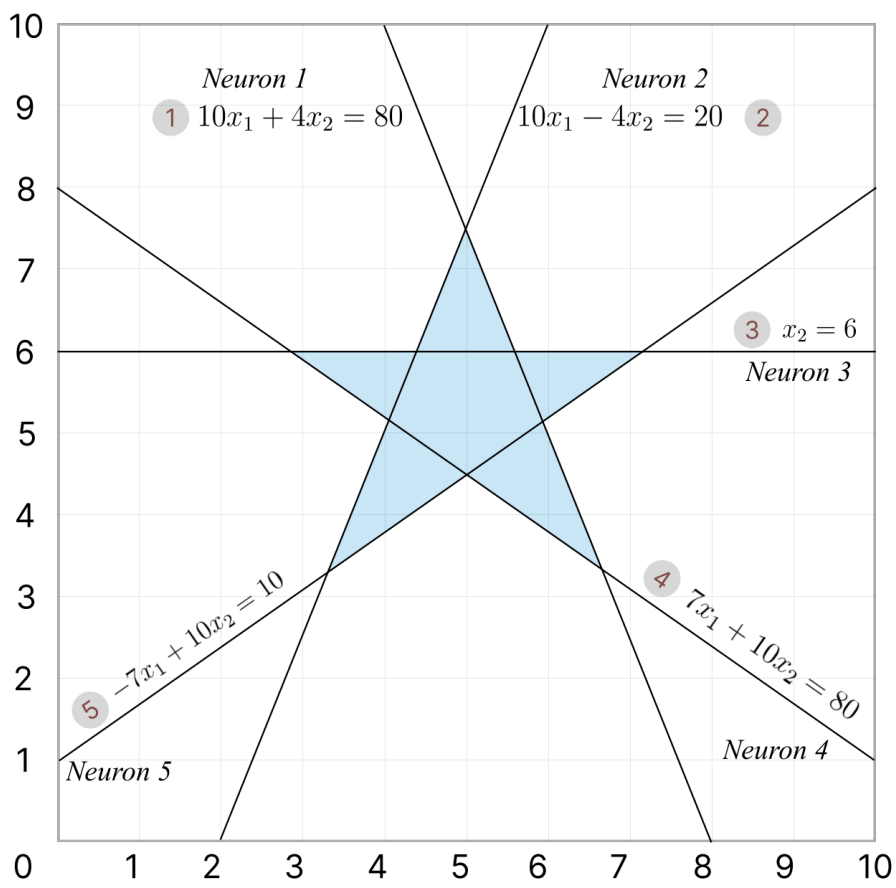
1. **Input:** A data point (x_1, x_2) is fed into the input layer.
2. **Hidden Layer:** Each hidden neuron calculates its weighted sum and applies the step function.
3. **Output Layer:** The output neuron sums the outputs of the hidden neurons and applies the step function.
    - If the sum is greater than or equal to 4, the output is 1 (class 1).
    - Otherwise, the output is 0 (class 0).

Rationale:

- **Region Detection:** The hidden neurons are designed to identify specific regions based on their weight and threshold combinations.
- **Intersection Logic:** The output neuron's threshold of 4 ensures that only points belonging to the intersection of at least four out of the five regions are classified as class 1.
- **Class Assignment:** Points that satisfy the intersection criteria are assigned to class 1; others are assigned to class 0.

By following this approach, the neural network can accurately classify the given regions into two classes.




Neuron 1
1 $10x_1 + 4x_2 = 80$
Neuron 2
$10x_1 - 4x_2 = 20$ 2
3 $x_2 = 6$
Neuron 3
4 $7x_1 + 10x_2 = 80$
Neuron 4
5 $-7x_1 + 10x_2 = 10$
Neuron 5
0 1 2 3 4 5 6 7 8 9 10