PH3205 ODE Problem Set 3

Q 1) a) Implement the algorithm

$$q_{i+1} = q_i + p_i \delta t$$
$$p_{i+1} = p_i - q_i \delta t$$

to implement the first order Euler algorithm to solve the SHM equation

$$\ddot{q} = -q$$

subject to $q = 1$, $\dot{q} = 0$ at $t = 0$. Implement this for $\delta t = 0.01$ from $t = 0$ to $t = 100$. Plot a graph of the variation of the conserved quantity

$$E = \frac{1}{2}\left(\dot{q}^2 + q^2\right)$$

versus t.

b) Repeat this problem for the algorithm

$$q_{i+1} = q_i + p_i \delta t$$
$$p_{i+1} = p_i - q_{i+1} \delta t$$

c) Again, for the algorithm

$$p_{i+1} = p_i - q_i \delta t$$
$$q_{i+1} = q_i + p_{i+1} \delta t$$

Q 2) Repeat the last problem for the algorithm

$$q_{i+1} = q_i + p_i \delta t - \frac{1}{2} q_i \delta t^2$$
$$p_{i+1} = p_i - \frac{1}{2}\left(q_i + q_{i+1}\right) \delta t$$

Compare the error in the exact solution $q(t)$ for the differential equation

$$\ddot{q} = -q$$

with that calculated using this algorithm and that in Q 1b)

Q 3) Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the initial condition

$$u(x, 0) = \begin{cases} x & \text{for } 0 < x < \frac{1}{2} \\ 1 - x & \text{for } \frac{1}{2} \leq x < 1 \end{cases}$$

and the boundary condition

$$u(0,t) = u(1,t) = 0$$

using the finite difference approximation

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} \approx \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2}$$

Use linspace (from numpy) or equivalent to create an array of 100 equally spaced points between 0 and 1 (both ends inclusive). Evolve the initial value of u using (i) $\delta t = 0.00005$ and (ii) $\delta t = 0.000052$

Explain the behavior in the two cases.

To animate the evolution using matplotlib's pyplot library, you need the following lines of code

```
import matplotlib.pyplot as plt
...
line, = plt.plot(xs,us)       # Note the comma!
...
while ...
    ...
    line.set_ydata(us)        # to update the curve
    plt.plot(.0001)
```

Unfortunately the animation works seamlessly only on a desktop/laptop (more difficult to get it to work on a phone).