PH3105 Problem Set 7

Q 1) a)Solve

$$\frac{dy}{dt} = -1000y + 3000 - 2000e^{-t}, \qquad y(0) = 0$$

analytically.

b) Now solve this numerically using the explicit Euler method. Use step sizes of $h = 0.0005$ and $h = 0.0015$ to solve for y between $t = 0$ and $t = 0.006$. Plot your solutions alongside the exact one. Comment on the stability of the solution.

c) Now repeat this using the implicit Euler method with a step size of 0.05 between $t = 0.05$ and 0.4

Q 2) Use RK4 with adaptive step size to solve the equation

$$\frac{dy}{dx} + 0.6y = 10\exp\left(-\frac{(x-2)^2}{2(0.075)^2}\right)$$

using an initial step size of 0.5 and an error bound $\epsilon = 0.00005$.

Q 3) Solve the following initial-value problem from $t = 1.5$ $t = 2.5$

$$\frac{dy}{dt} = -\frac{2y}{1+t}$$

Use the fourth-order Adams predictor-corrector method. Employ a step size of 0.05 and the fourth-order RK method to predict the starting values, given $y(0) = 2$. Plot the error as a function of t (you will need the exact solution for this). Repeat for h= 0.02. Comment on what this says about the error in this method.

The coefficients for the Adams-Bashforth method are

$$\beta_1 = \frac{55}{24}, \beta_2 = -\frac{59}{24}, \beta_3 = \frac{37}{24}, \beta_4 = -\frac{3}{8}.$$

Those for the Adams Moulton method are

$$\beta_0 = \frac{3}{8}, \beta_1 = \frac{19}{24}, \beta_2 = -\frac{5}{24}, \beta_3 = \frac{1}{24}$$