Computer Vision Homework 4

Q1. Invariance: A property where a transformation to the input does not change the output. The result remains the same regardless of the transformation.

Example: A circle detection algorithm is invariant to rotation because the circle remains a circle after rotation.

Equivariance: A property where the output transforms in the same way as the input under a specific transformation.

Example: In object detection, if the input is translated, the detected bounding boxes will also translate correspondingly.

Harris Corner Detector: Invariance vs. Equivariance

Translation: The Harris corner detector finds corners by computing gradients and detecting areas of high intensity changes. Translating an image will shift corners by the same amount, making the detector **equivariant** to translation.

Rotation: The Harris corner detector is **equivariant** to rotation, the detected corners will also rotate correspondingly. The algorithm detects the same corners in the sense that their spatial relationships and local features are preserved under rotation, but their coordinates change according to the rotation applied to the image.

Horizontal Flipping: Flipping does not change the nature of the gradients at corner points. Hence, corners remain detectable at their flipped positions. So Horizonal Flipping is **invariant**

Scaling: The Harris corner detector is **neither invariant nor equivariant** to scaling. It is sensitive to the size of features in the image and does not adapt inherently to scale changes.

Adding a constant to every pixel intensity (ignoring overflow): The Harris corner detector depends on intensity gradients, which are differences between neighboring pixel intensities. Adding a constant does not affect these differences, so the detector is **invariant** to this transformation.

Q2.

Benefits of Using Image Gradients, Histograms, and Cells in Feature Descriptors like SIFT

1. Image Gradients

Gradients capture edges and patterns in an image, which are robust to changes in illumination or pixel intensities.

Gradients help describe the local structure of a region, such as edges, corners, or textures, which are crucial for feature matching.
Example in SIFT:
Gradients at keypoints are used to calculate the orientation, allowing for rotation invariance.

2. Histograms

By creating a histogram of gradient directions, the descriptor becomes robust to small variations in gradient magnitude or orientation due to noise or distortions.
Aggregating gradients into histograms reduces sensitivity to small local variations, focusing on dominant orientations instead.

Example in SIFT:
Gradients within a keypoint region are grouped into orientation bins, creating a robust representation of the region's dominant structure.

3. Cells

Dividing the region around a keypoint into smaller cells preserves the spatial distribution of gradients.
This helps distinguish between similar textures with different spatial arrangements.
Cells ensure the descriptor retains a balance between localization and robustness.

Example in SIFT:
A 4x4 grid of cells around each keypoint captures the spatial layout of gradient histograms, making the descriptor robust to small shifts and distortions.

Q3.

1. Shift to the right by 100 pixels

   [1 0 100]
   [0 1 0]
   [0 0 1]

2. Clockwise rotation by 45 degrees

   [cos(-45°) -sin(-45°) 0]
   [sin(-45°) cos(-45°) 0]
   [0 0 1]

   =>

   [√2/2 √2/2 0]
   [-√2/2 √2/2 0]
   [0 0 1]

3. Rotate around the point (20, 20) in the counterclockwise direction by 90 degree
   Translation to the origin

[1 0 -20]
[0 1 -20]
[0 0   1]
Counterclockwise rotation by 90 degrees:

[0 -1   0]
[1   0   0]
[0   0   1]

Translation back:

[1 0 20]
[0 1 20]
[0 0   1]

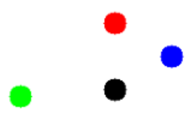
Translation back * Rotation * translation origin =
[1 0 20] [0 -1 0] [1 0 -20]
[0 1 20] * [1   0 0] * [0 1 -20]
[0 0   1] [0   0 1] [0 0   1]

=>

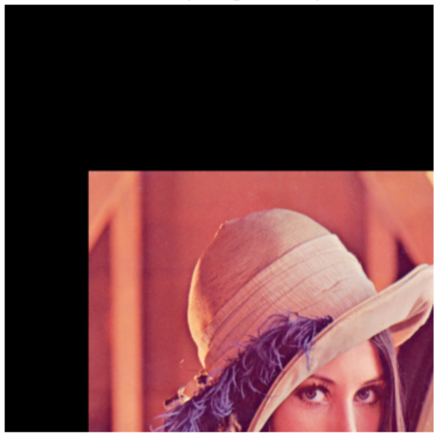
[0 -1 40]
[1   0 -20]
[0   0   1]

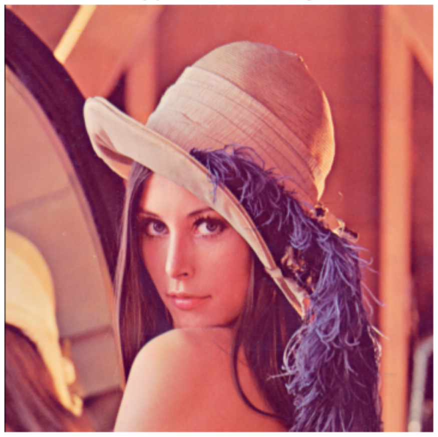
Programming Assignment Results

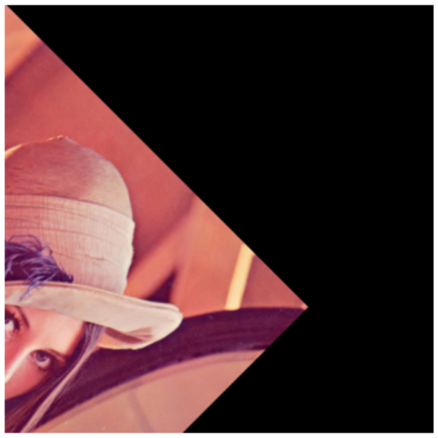
Q1.

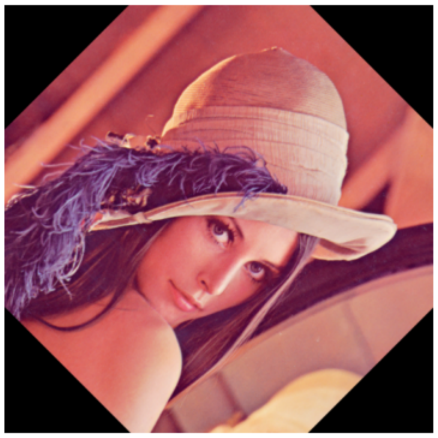

Geometric Transformations

Q2.


Translated (100px right, 200px down)


Flipped Horizontally


Rotated 45° Clockwise (Origin)


Rotated 45° Clockwise (Center)