Simple Computational Study Of Spring Pendulum Motion

Introduction

We wish to determine the trajectory of a spring pendulum bob which has been subjected to some initial physical conditions. The diagram on the left clearly illustrates the pendulum system. We treat the system using classical newtonian physics and obtain a pair of ***coupled non-linear differential equations*** as shown below.

Here,
x = Length of the spring.
θ = Angular displacement of the bob.
m = Mass of the of the bob.
k = Spring constant of the spring.

Note: All derivates are of the above quantities are taken with respective to time.

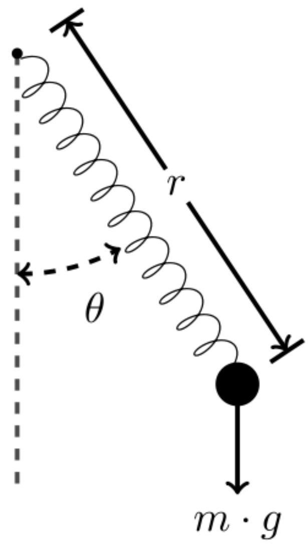


Diagram of spring pendulum system.
Credit: https://ths.rwth-aachen.de/research/projects/hypro/spring-pendulum/

$$\ddot{x} - x\dot{\theta}^2 = g\cos(\theta) - \frac{k}{m}x. \quad \text{......................... (1)}$$

$$x\ddot{\theta} + 2\dot{x}\dot{\theta} = -\,g\sin(\theta). \quad \text{............................... (2)}$$

We solve the above system of equations using Euler method in this article.

The results of the simulation is shown in the coming pages.

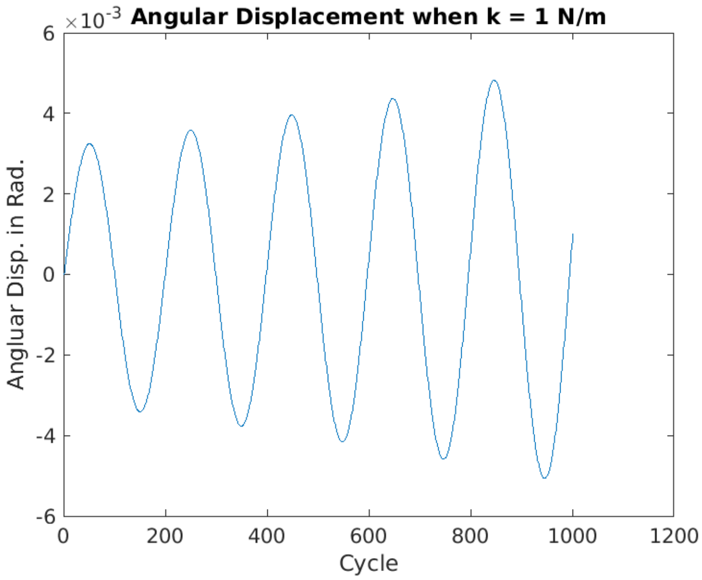
We implemented the simulation program in Java programming language. The repository for the source code is available at https://github.com/AmitabhaR/SpringPendulum.

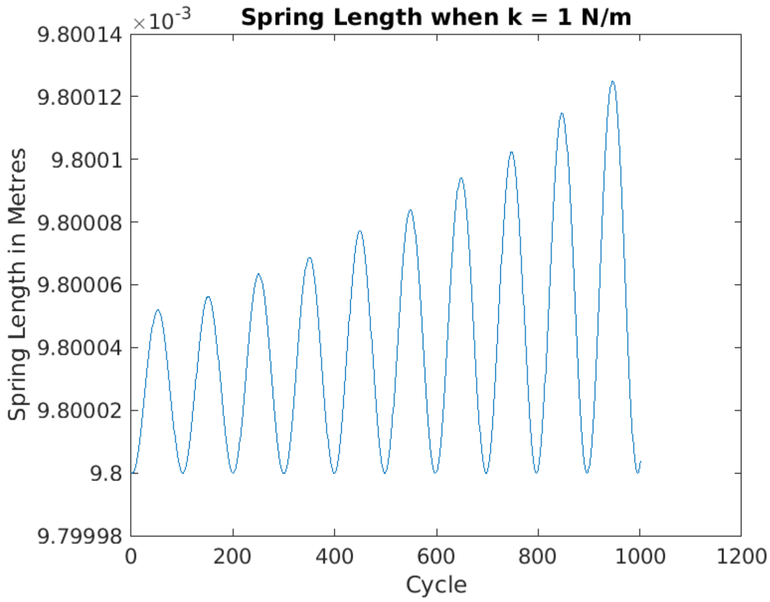
Results

We consider two cases in which in one case we vary the spring constant of the pendulum and in the other case we vary the mass of the pendulum bob. The above cases are farther subdivided into two cases for different values of the respective variables. All the simulations are done for either 1000 or 2000 steps with 0.001 sec as unit time increment.

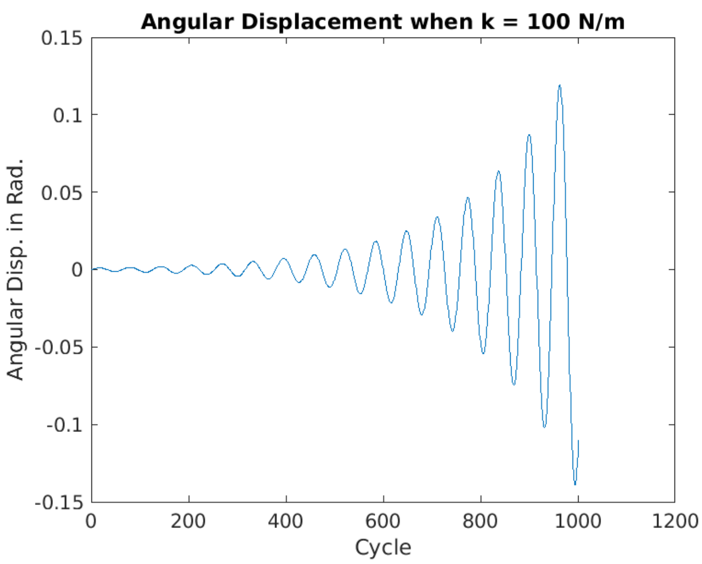
1. Variable spring constant (k)

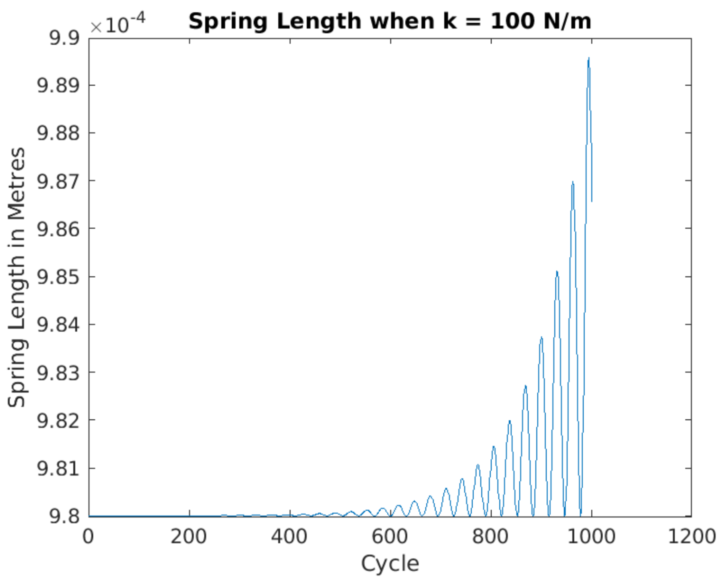
1. **Case-1:** m = 0.01 kg, k = 1 N/m, x_0 = 0.098 m, θ_0 = 0 rad, $\dot{\theta}_0$ = 0.1 rad/sec, $\ddot{\theta}_0$ = 0 rad/sec^2.





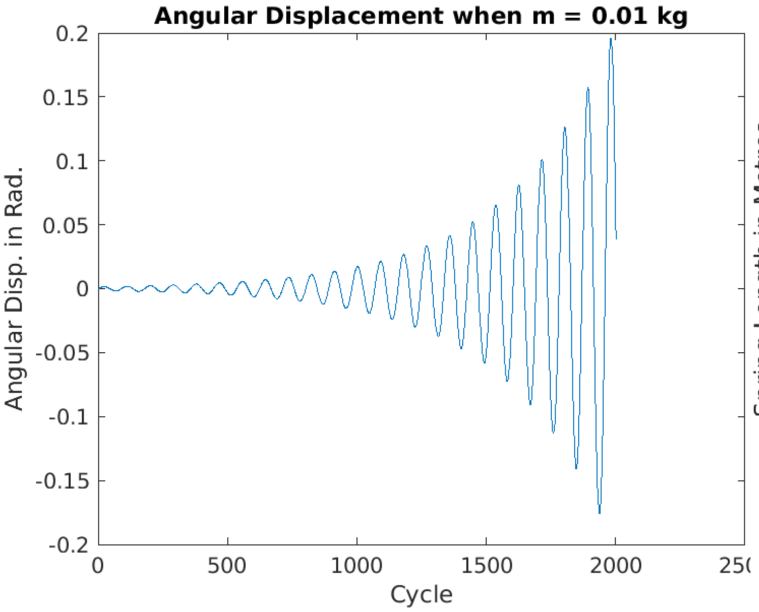
2. **Case-2**: m = 0.01 kg, k = 100 N/m, x_0 = 0.098 m, θ_0 = 0 rad, $\dot{\theta}_0$ = 0.1 rad/sec, $\ddot{\theta}_0$ = 0 rad/sec^2.

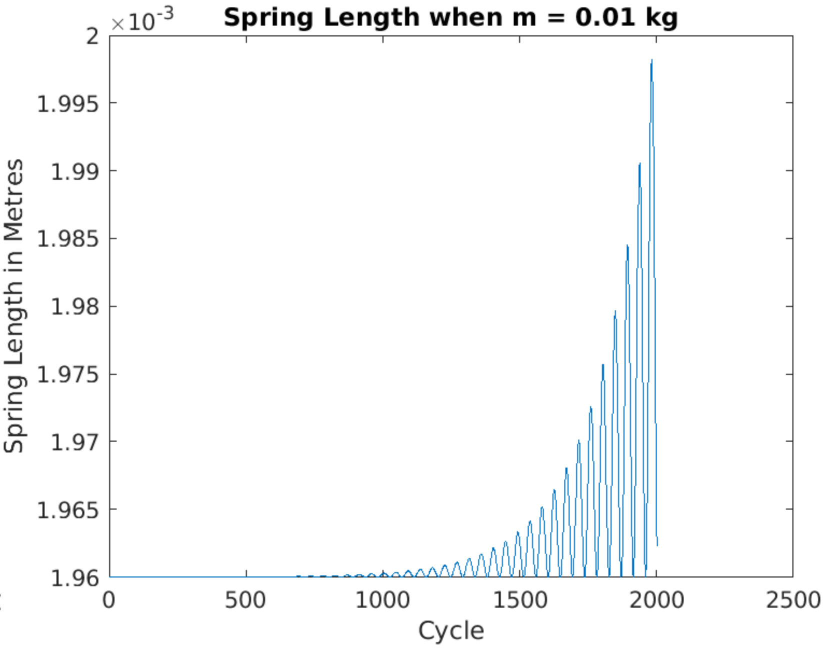




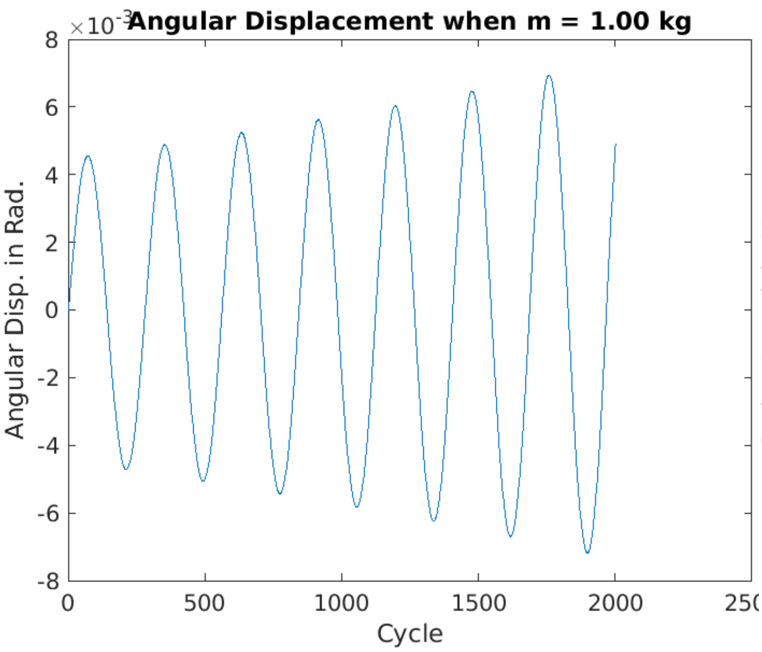
2. Variable pendulum bob mass (m)

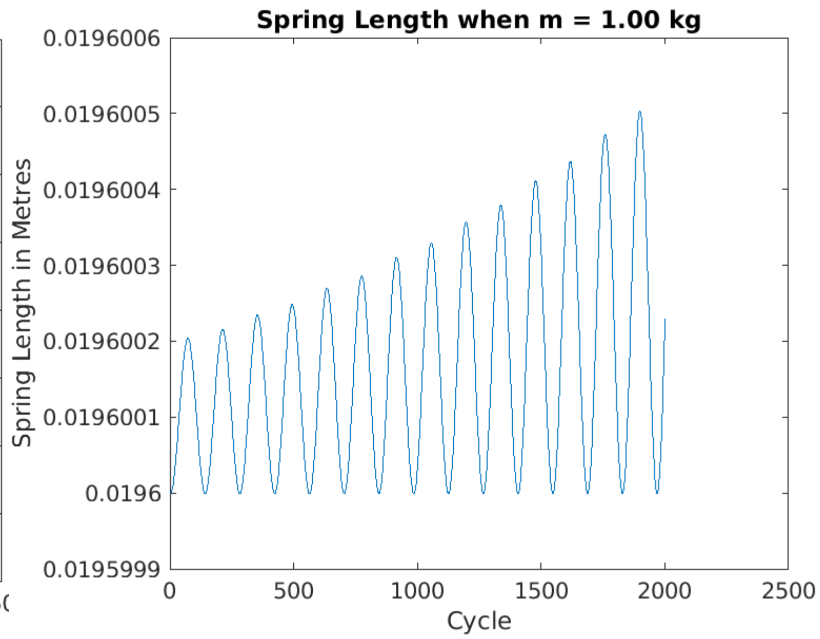
1. **Case-1:** m = 0.01 kg, k = 50 N/m, x_0 = 0.198 m, θ_0 = 0 rad, $\dot{\theta}_0$ = 0.1 rad\sec, $\ddot{\theta}_0$ = 0 rad/sec^2.





2. **Case-2:** m = 1.00 kg, k = 50 N/m, x_0 = 0.198 m, θ_0 = 0 rad, $\dot{\theta}_0$ = 0.1 rad\sec, $\ddot{\theta}_0$ = 0 rad/sec^2.





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