Vision Aided Navigation 2024 - Exercise 4

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https://github.com/AmitaiOvadia/SLAMProject/blob/main/VAN_ex/code/ex4/Ex4.py
And
https://github.com/AmitaiOvadia/SLAMProject/blob/main/VAN_ex/code/utils/utils.py

Question 4.2

Present the following tracking statistics. All stats should not include trivial (length 1) tracks:
This is how I did the SALM:

- Blured each image with gaussian filter of $\sigma = 2$ for avoiding noisy features.
- Used AKAZE feature extraction algorithem, with threshold of 0.0001 (10 times smaller the default, to compensate for the blur).
- Removed feature pairs with for the 1 pixel difference in the y axis.
- Performed the outlier removal using pnp over every 2 cosecutive stereo pairs, with reprojection error of 1.5 pixels.

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Tracking Statistics:
Total number of tracks: 270510
Number of frames: 3360
Mean track length: 5.57
Maximum track length: 154
Minimum track length: 2
Mean number of frame links: 448.68
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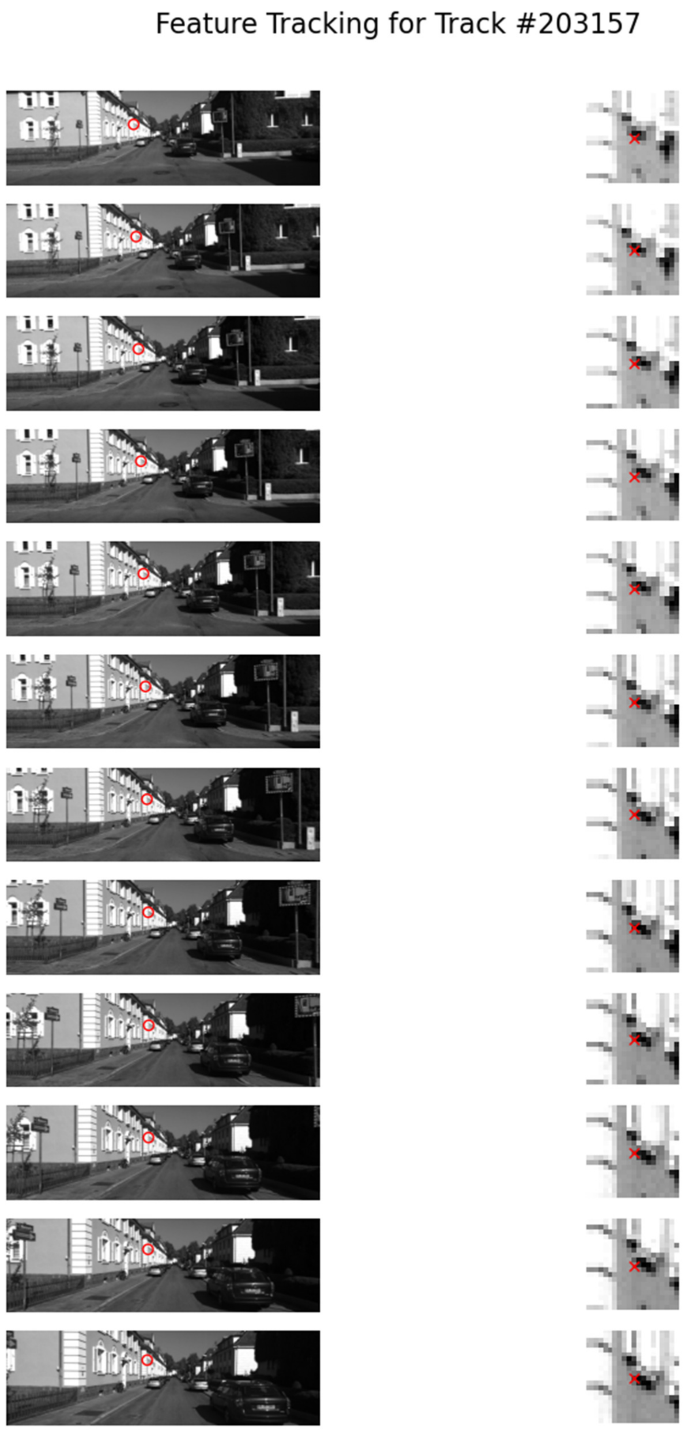
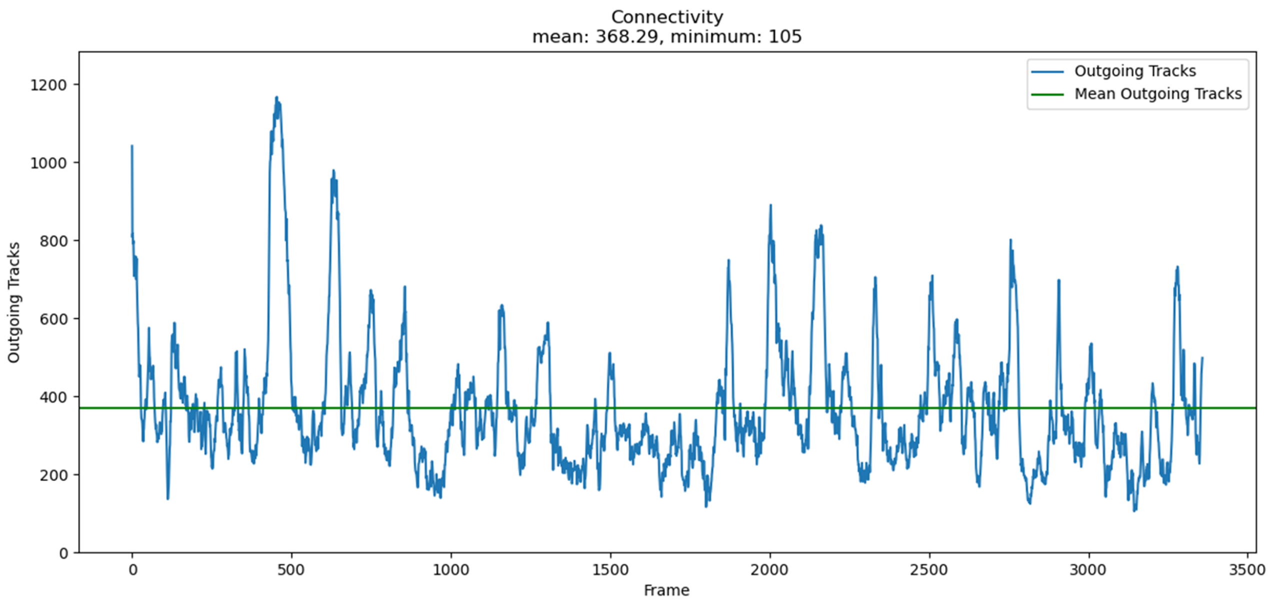
Question 4.3

Display the feature locations on all the relevant (left) images as a 20x20 square.
Cut a region of 20x20 pixels (subject to image boundaries) around the feature from the left image and mark the feature.
Present this for all the images of the track:
A track of size 12:



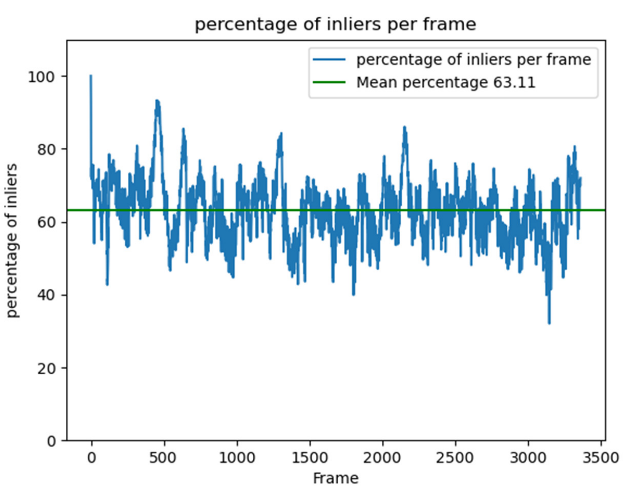
Question 4.4

Present a connectivity graph: For each frame, the number of tracks outgoing to the next frame (the number of tracks on the frame with links also in the next frame)



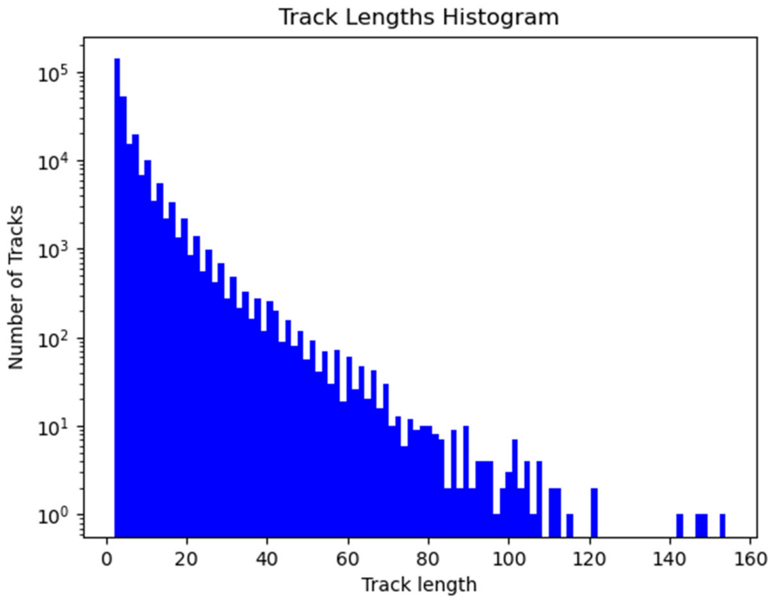
Question 4.5

Present a graph of the percentage of inliers per frame.
The inliers ratio was caluclated like this:
A = all the features that are shared across the 2 stereo pairs
B = all the features that remained after the RANSAC phase
$inliers\ ratio = \frac{B}{A}$



Question 4.6

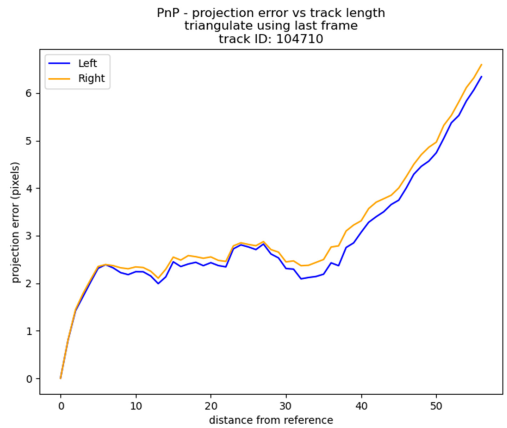
Present a track length histogram graph



Question 4.7

- Present a graph of the reprojection error size (L2 norm) over the track's images.

  Here the 3D feature triangulated in the last frame, reprojected to the other frames (given the ground truth cameras), and compared to it's correspnding pixels.

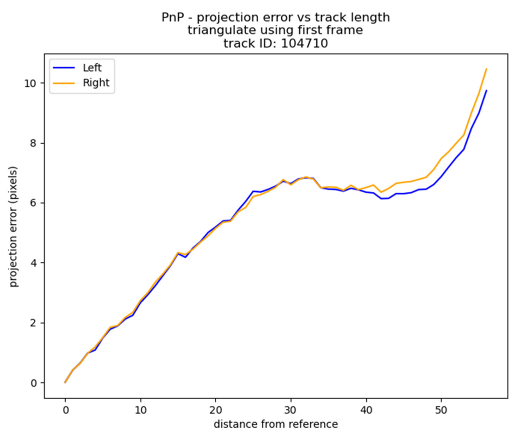
  

- What would happen if we triangulated from the first frame instead of the last?

  How would you expect the reprojection error to change? Why?

  Here is the graph of the same track reprojections,

  but here the triangulation is from the first frame stereo pair.

  You can see that the triangulation is better if done using the last frame (the reprojection error is smaller)

  The reason is probably that the closer you are to the 3D point,

  then the ratio between the b the distance between the left and right cameras

  and Z the distance from the point, $\frac{b}{Z}$, is bigger and so the triangulation is better (the rays are intersecting in a bigger angle).

  Exceptions to that can araise if the feature is just better viewed from the first frames,

  or is on the side of the vehicle (but then it could be out of the field of view)

Question 4.8

Let $x \sim N(\mu_x, \Sigma_x)$ and $y = Ax$ for a nonsingular (invertible) matrix A

- Prove that the mean and covariance of y are $\mu_y = A\mu_x + b$ and $\Sigma_y = A\Sigma_x A^T$:

  $E[y] = E[Ax + b] = AE[x] + E[b] = A\mu_x + b$

  $\Sigma_y = E\left[(y - \mu_y)(y - \mu_y)^T\right] = E\left[((Ax + b) - (A\mu_x + b))((Ax + b) - (A\mu_x + b))^T\right]$

  $E\left[(Ax - A\mu_x)(Ax - A\mu_x)^T\right] = E\left[A(x - \mu_x)(x - \mu_x)^T A^T\right] = AE\left[(x - \mu_x)(x - \mu_x)^T\right] A^T = A\Sigma_x A^T$

- Prove that y is normally distributed $y \sim N(\mu_y, \Sigma_y)$ using the random vector transformation:

  Define the transformation and its inverse:

  Let $y = g(x) = Ax + b$.

  To find x in terms of y:

  $x = g^{-1}(y) = A^{-1}(y - b)$

  Find the Jacobian determinant:

  $J = \frac{\partial g(x)}{\partial x} = A$

  Since A is a constant matrix, the determinant of the Jacobian is simply $|A|$.

  $f_y(y) = f_x(g^{-1}(y)) \left| \frac{\partial g^{-1}(y)}{\partial y} \right|$

  Since $g^{-1}(y) = A^{-1}(y - b)$, the Jacobian of $g^{-1}(y) = A^{-1}(y - b)$ with respect to y is A^{-1}.

  Therefore, the determinant of the Jacobian is $|A^{-1}|$.

  Calculate the pdf:

  $Given\ x \sim N(\mu_x, \Sigma_x), the\ pdf\ of\ x\ is:$

  $f_x(x) = \frac{1}{\sqrt{(2\pi)^n |\Sigma_x|}} \exp\left(-\frac{1}{2}(x - \mu_x)^T \Sigma_x^{-1}(x - \mu_x)\right)$

  $Substitute\ x = A^{-1}(y - b):$

  $f_x(A^{-1}(y - b)) = \frac{1}{\sqrt{(2\pi)^n |\Sigma_x|}} \exp\left(-\frac{1}{2}(A^{-1}(y - b) - \mu_x)^T \Sigma_x^{-1}(A^{-1}(y - b) - \mu_x)\right)$

  $The\ determinant\ of\ the\ Jacobian\ |A^{-1}| = \frac{1}{|A|}:$

  $f_y(y) = f_x(A^{-1}(y - b)) \left| \frac{1}{|A|} \right|$

  $Simplifying:$

  $f_y(y) = \frac{1}{|A|} \frac{1}{\sqrt{(2\pi)^n |\Sigma_x|}} \exp\left(-\frac{1}{2}(A^{-1}(y - b) - \mu_x)^T \Sigma_x^{-1}(A^{-1}(y - b) - \mu_x)\right)$

  Mahalanobis norm property:

  The Mahalanobis norm property states:

  $\|Mu\|_\Sigma^2 = \|u\|_{M^T \Sigma M}^2$

  $Apply\ this\ to\ our\ context\ where\ M = A^{-1}\ and\ u = y - b:$

  $\|A^{-1}(y - b) - \mu_x\|_{\Sigma_x}^2 = (A^{-1}(y - b) - \mu_x)^T \Sigma_x^{-1}(A^{-1}(y - b) - \mu_x)$

  $Therefore,\ we\ can\ rewrite\ f_y(y)\ as:$

  $f_y(y) = \frac{1}{\sqrt{(2\pi)^n |A\Sigma_x A^T|}} \exp\left(-\frac{1}{2}(y - (A\mu_x + b))^T (A\Sigma_x A^T)^{-1}(y - (A\mu_x + b))\right)$

Conclusion:

This shows that y follows a multivariate normal distribution with mean $A\mu_x + b$ and covariance matrix $A\Sigma_x A^T$.

$Therefore,\ y \sim N(A\mu_x + b, A\Sigma_x A^T).$