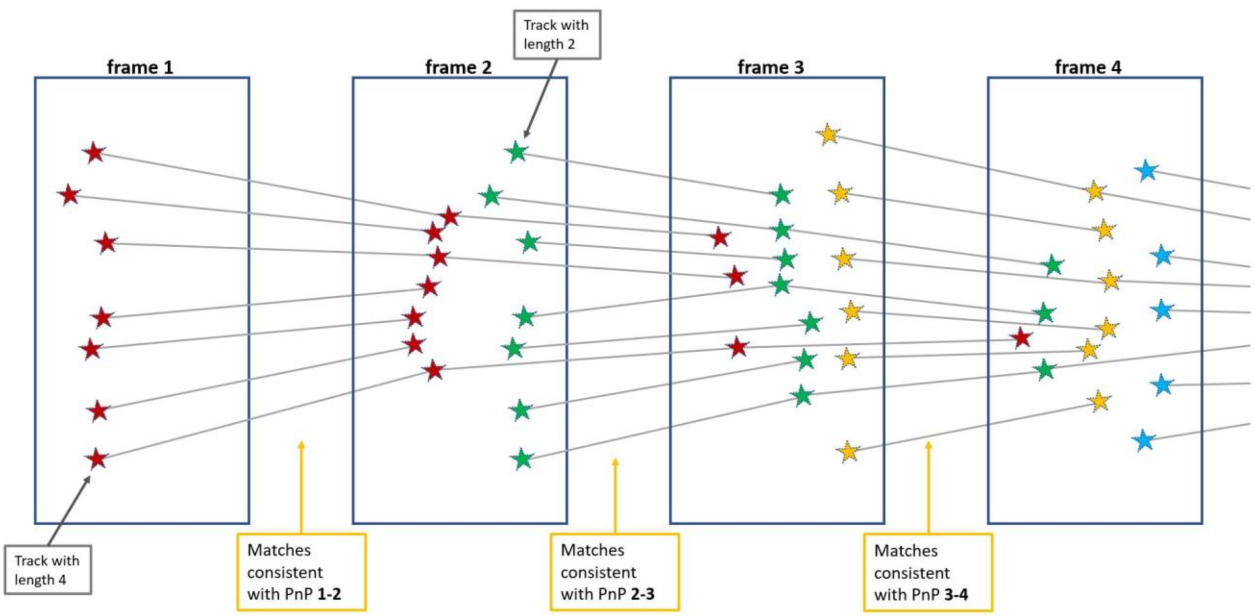
# Vision Aided Navigation 2024 - Exercise 4

## Prefix:

In **exercise 3** we tracked features between consecutive pairs of stereo images along the vehicle trajectory, removed the outliers and used PnP with Consensus Matching to estimate the relative motion and produce an initial estimate for the trajectory of the vehicle.

In this exercise we extend the feature tracking across multiple frames and implement a suitable database for the tracked features. We use the matches we got in exercise 3 with careful bookkeeping to keep track (no pun intended) of which feature was tracked across what frame.
This information will be important in future stages when we build the Bundle Adjustment / Loop Closure optimization.




**4.1** We call a 3D landmark that was matched across multiple pairs of stereo images (frames) a ***track***. In the previous exercise we recognized tracks of length 2 (matched over two pairs of images) - we hope to extend the tracks by matching features over more pairs.
Implement a suitable database:

- Every track should have a unique id, we will refer to it as *TrackId*.
- Every image stereo pair should have a unique id, we will refer to it as *FrameId*.
- Implement a function that returns all the *TrackIds* that appear on a given *FrameId*.
- Implement a function that returns all the *FrameIds* that are part of a given *TrackId*.

- Implement a function that for a given (*FrameId*, *TrackId*) pair returns:
  - Feature locations of track *TrackId* on both left and right images as a triplet $(x_l, x_r, y)$ with:
    - $(x_l, y)$ the feature location on the left image
    - $(x_r, y)$ the feature location on the right image

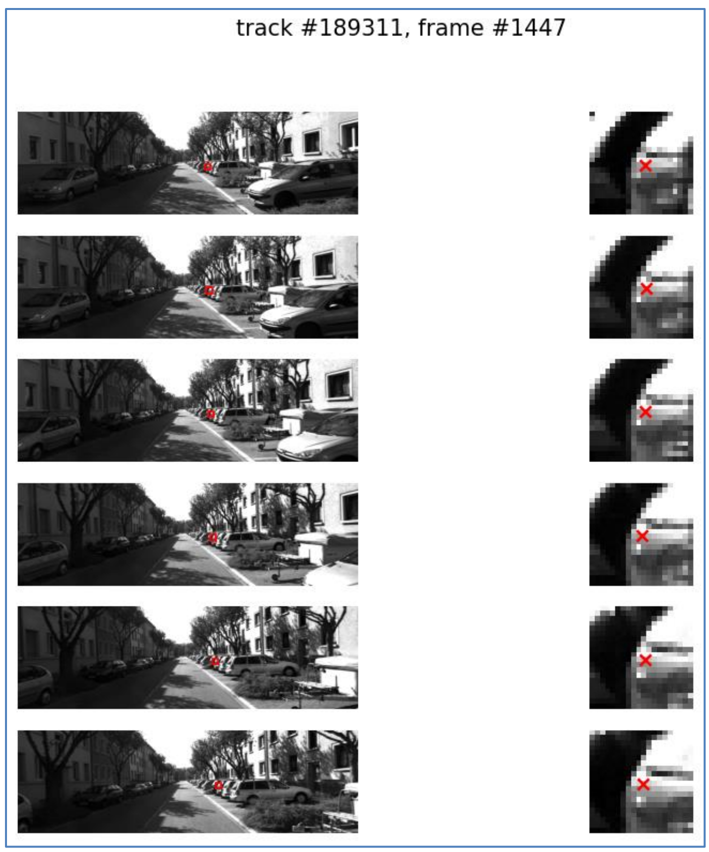
    Note that the $y$ value is shared on both images.
- Implement the ability to extend the database with new tracks on a new frame as we match new stereo pairs to the previous ones.
- For a given frame *FrameId*, implement the ability to serialize to disk the tracking information of that frame and to read this information from a file.
- Implement serialization of the entire database.

**4.2** Present the following tracking statistics. All stats should not include trivial (length 1) tracks:

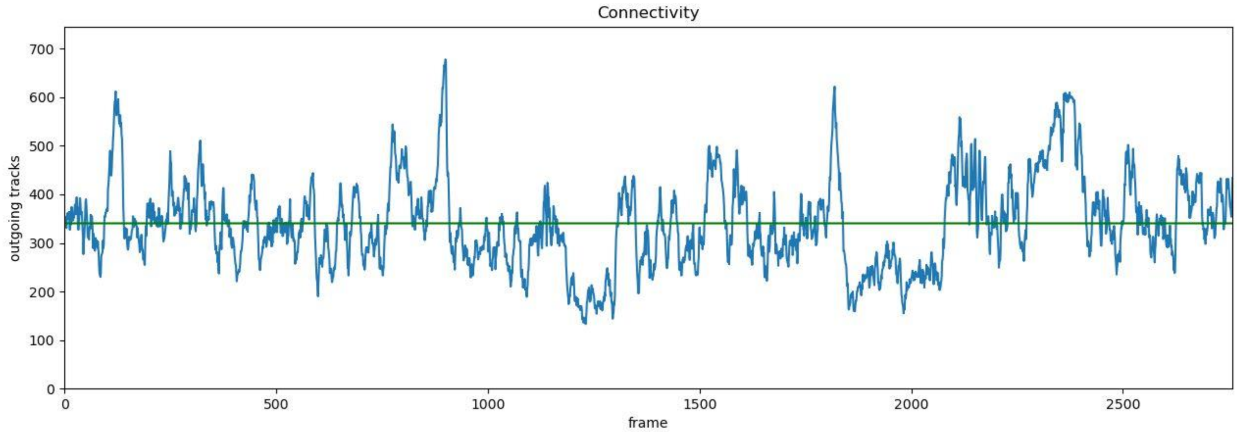
- Total number of tracks
- Number of frames
- Mean track length, maximum and minimum track lengths
- Mean number of frame links (number of tracks on an average image)

**4.3** Pick a track of length ≥ 6.

- Display the feature locations on all the relevant (left) images as a 20x20 square.
  Cut a region of 20x20 pixels (subject to image boundaries) around the feature from the left image and mark the feature.
  Present this for all the images of the track.

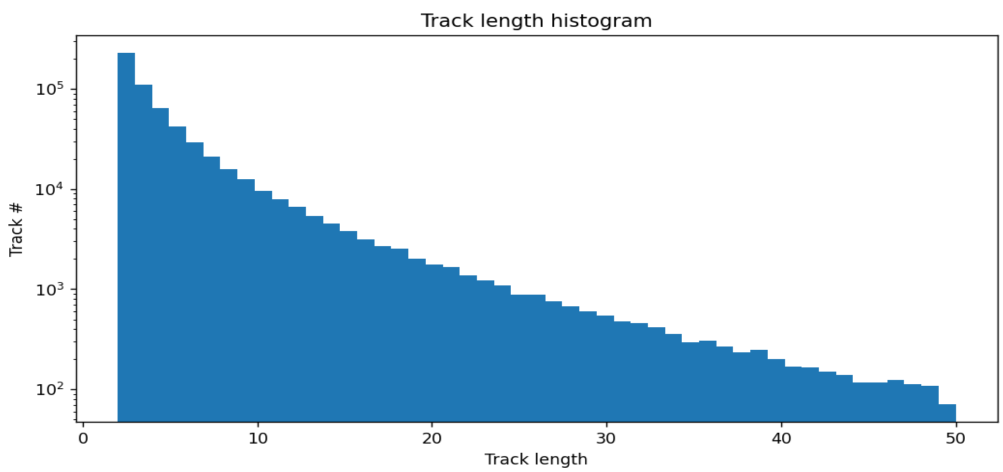



**4.4** Present a connectivity graph: For each frame, the number of tracks **outgoing** to the next frame (the number of tracks on the frame with links also in the next frame)




**4.5** Present a graph of the percentage of inliers per frame

**4.6** Present a track length histogram graph

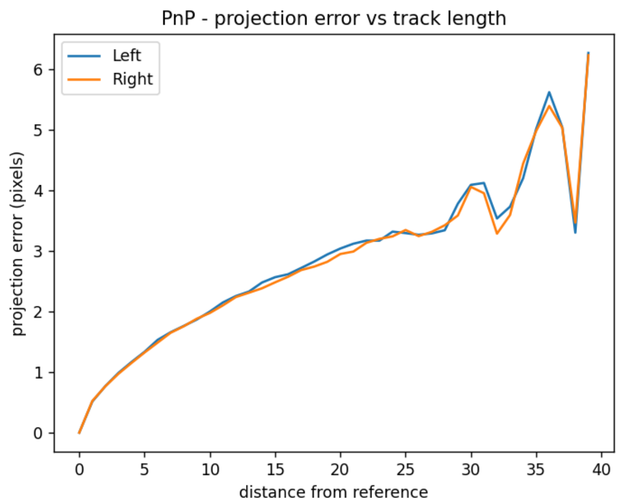



**4.7** Pick a random track of length ≥ 10.

Read the ground truth camera matrices (in *\poses\00.txt*). Using those matrices triangulate a 3d point in world coordinates from the features in the **last** frame of the track. Project this point to all the frames of the track (both left and right cameras).

We'll define the **reprojection error** for a given camera as the difference between the projection and the tracked feature location on that camera.

- Present a graph of the reprojection error size ($L_2$ norm) over the track's images.
- What would happen if we triangulated from the **first** frame instead of the last? How would you expect the reprojection error to change? Why?




**4.8** Let $x \sim N(\mu_x, \Sigma_x)$ and $y = Ax + b$ for a nonsingular matrix $A$.

- Prove that the mean and covariance of $y$ are $\mu_y = A\mu_x + b,\ \Sigma_y = A\Sigma_x A^T$.
- Prove that $y$ is normally distributed: $y \sim N(\mu_y, \Sigma_y)$.
    - Use the **random vector transformation formula**:
      For $x \sim f_x(x)$ and $y = g(x)$ we have the resulting pdf of $y$:
      $$f_y(y) = f_x\left(g^{-1}(y)\right)\left|\boldsymbol{J}_{g^{-1}}\right|$$
      With $\left|\boldsymbol{J}_{g^{-1}}\right|$ the determinant of the Jacobian of $g^{-1}$.
      In our case $x \sim N(\mu_x, \Sigma_x)$, so $f_x(x) = \frac{1}{\sqrt{(2\pi)^n|\Sigma_x|}}\exp\left(-\frac{1}{2}\|x-\mu_x\|^2_{\Sigma_x}\right)$
      and $g(x) = Ax + b$; We want to show that $f_y(y)$ is a normal distribution.
    - Hint: Proving the following Mahalanobis norm property may be useful:
      $\|Mu\|^2_\Sigma = \|u\|^2_{M^{-1}\Sigma M^{-T}}$