

Vision Aided Navigation 2024 - Exercise 3

Prefix:

In **exercise 2** we triangulated the matches between the stereo images and got a 3D point cloud. We also found that despite our best efforts to avoid mismatches, there are still erroneous 3D points in our point cloud.

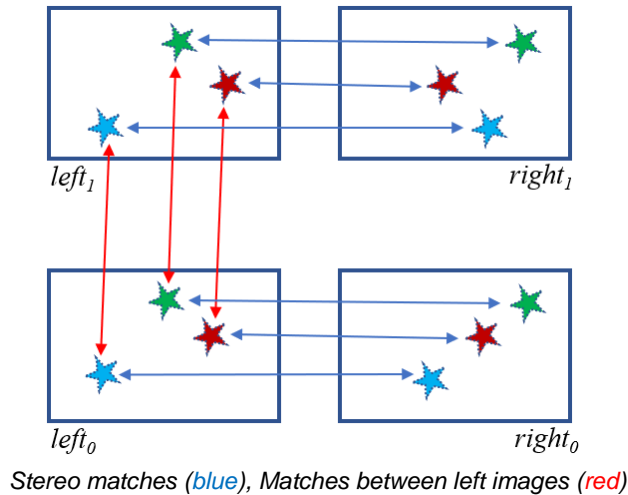
In this exercise we will move forward in time to the next stereo pair and match the left image to the previous left image and run PnP using RANSAC iterations to estimate the relative motion. We will also use the extra information - we now have two stereo pairs - to reject (almost all of) the remaining outliers.

3.1 Use the code from the previous exercises to create a point cloud for the next stereo pair (i.e. match features, remove outliers and triangulate):

left₁: VAN_ex\dataset\sequences\00\image_0\000001.png
right₁: VAN_ex\dataset\sequences\00\image_1\000001.png

We now have two 3D point clouds, one for pair 0 (from exercise 2) and one for pair 1.

3.2 Match features between the two left images (*left₀* and *left₁*)



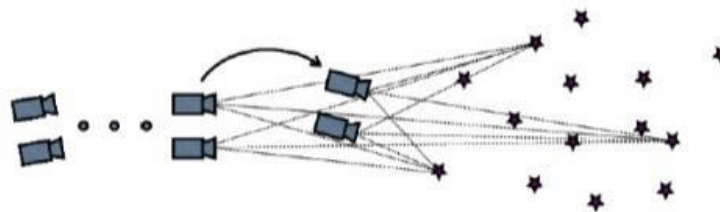
Note that this is not a stereo pair, and therefore not rectified, so we cannot use the outlier rejection pattern we used in exercise 2.

3.3 Choose 4 key-points that were matched on all four images. This means we have both 3D locations from the triangulation of pair 0 and matching pixel locations on pair 1. Apply the PnP algorithm between the point cloud and the matching pixel locations on *left₁* to calculate the extrinsic camera matrix $[R|t]$ of *left₁*, with R a 3×3 rotation matrix and t a 3×1 translation vector.

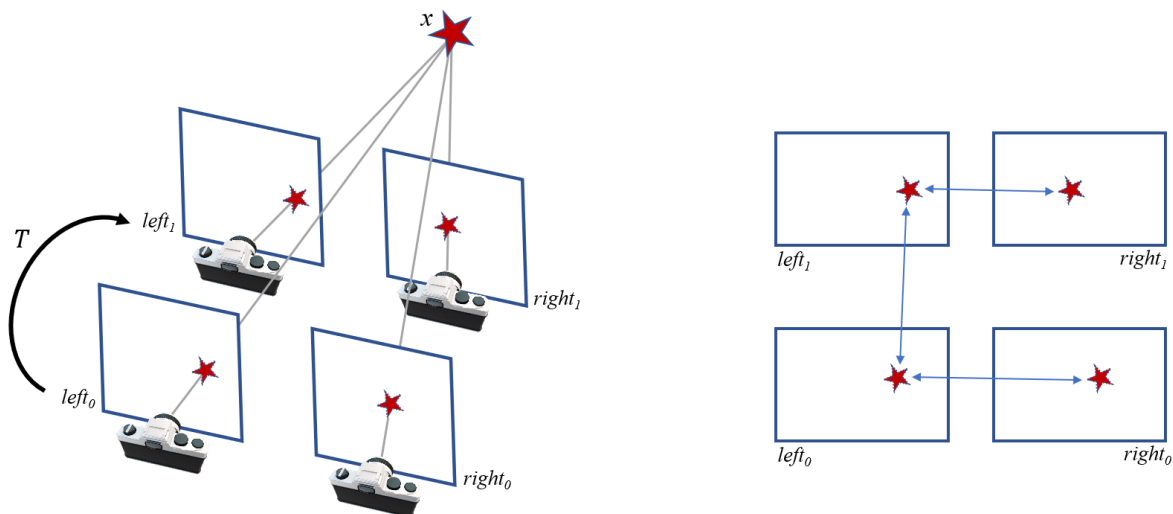
- Describe how to define from $[R|t]$ a transformation T that transforms from $left_0$ coordinates to $left_1$ coordinates.
- For three cameras A, B, C : If camera A has extrinsic matrix $[I|0]$, transformation $T_{A \rightarrow B}(x) = R_1x + t_1$ transforms from the coordinates of A to the coordinates of camera B and transformation $T_{B \rightarrow C}(x) = R_2x + t_2$ transforms from the coordinates of B to the coordinates of camera C , express transformation $T_{A \rightarrow C}$ and the extrinsic matrix of C using R_1, R_2, t_1, t_2 .



- For a camera with extrinsic matrix $[R|t]$, what is the location of the camera in the global coordinate system?
Hint: if the location is $(x \ y \ z)^T$, what would we expect the result of $[R|t](x \ y \ z \ 1)^T$ to be?
- Plot the relative position of the four cameras (from above).



3.4 For each 3D point x that was matched to $left_1$ we have four associated pixel locations (on $left_0$, $right_0$, $left_1$, $right_1$) if these are correct and the transformation T we calculated is also correct, we can expect the projection of x to fall close to these pixel locations on all four images.



We consider a point x that projects close to the matched pixel locations in all four images a supporter of transformation T .

Use a distance threshold of 2 pixels to recognize the supporters.

- Plot on images $left_0$ and $left_1$ the matches, with supporters in different color.

3.5 Use a RANSAC framework, with PnP as the inner model, to find the 4 points that maximize the number of supporters. We call this maximal group the ‘inliers’.

Note: Implement RANSAC yourself, do not use ‘cv2.solvePnP’

Refine the resulting transformation by calculating transformation T for all the inliers.

Use the resulting T to transform the first point cloud (that belongs to pair 0)

- Plot the two 3D point clouds (pair 1 and pair 0 after T) from above. Use different colors for the two clouds.
Take care to crop the plot in a way that presents the point clouds in a meaningful manner (for example, no need to present points ‘at infinity’).
- Plot on images $left_0$ and $left_1$ the inliers and outliers in different colors.

Useful code: `numpy.random.randint, cv2.solvePnP`

```
def rodriguez_to_mat(rvec, tvec):
    rot, _ = cv2.Rodrigues(rvec)
    return numpy.hstack((rot, tvec))
```

3.6 Repeat steps 2.1-2.5 for the whole movie in: `VAN_ex\dataset\sequences\00\` for all the images.

- How long did the tracking take?

Keep track of all the relative transformations T and produce an estimation for the extrinsic matrix of all the left cameras.

- Plot a trajectory of all the left camera locations in the **coordinates of camera $left_0$** , as viewed from above.
- Read the ground-truth extrinsic matrices and add to the plot the ground truth locations in a different color.
 - The ground truth matrices are in `VAN_ex\dataset\poses\00.txt`
 - Each line has 12 numbers that represent the extrinsic matrix of the corresponding left camera in row-major order.

3.7 Let $x \sim N(\mu_x, \Sigma_x)$ and $y = Ax + b$ for a nonsingular matrix A .

- Prove that the mean and covariance of y are $\mu_y = A\mu_x + b$, $\Sigma_y = A\Sigma_x A^T$.
- Prove that y is normally distributed: $y \sim N(\mu_y, \Sigma_y)$.

- Use the **random vector transformation formula**:

For $x \sim f_x(x)$ and $y = g(x)$ we have the resulting pdf of y :

$$f_y(y) = f_x(g^{-1}(y)) |J_{g^{-1}}|$$

With $|J_{g^{-1}}|$ the determinant of the Jacobian of g^{-1} .

In our case $x \sim N(\mu_x, \Sigma_x)$, so $f_x(x) = \frac{1}{\sqrt{(2\pi)^n |\Sigma_x|}} \exp\left(-\frac{1}{2} \|x - \mu_x\|_{\Sigma_x}^2\right)$

and $g(x) = Ax + b$; We want to show that $f_y(y)$ is a normal distribution.

- Hint: Proving the following Mahalanobis norm property may be useful:

$$\|Mu\|_{\Sigma}^2 = \|u\|_{M^{-1}\Sigma M^{-T}}^2$$