Amitdeb Prasad Bhattacharya A20402789

Problem 1:

Given array is $A[1 \ldots n]$, $i \leq n$

Taking linear time "Select" algorithm and "Black-box" median to find "Median".

```
1. Select (A, l, r, i)
   // Here l is first number and r is last number
   // of array A.
2.     if (l == r)
3.         return A[l]
   // if there is only one element return the value
   // of that element
4.     x = black_box_median (A, l, r)
   // Using "black box" subroutine to find the
   // median (x) of the n elements
5.     q = Partition (x)
   // Partitioning input array around the
   // median x
6.     k = q - l + 1
   // k is the number of elements on the
   // low side of the partition and n-k is
   // the number of element on higher side.
```

```
7.      if i == k
8.          return A[q]
9.      else if i < k
10.         return Select (A, 1, q-1, i)
11.     else
12.         return Select (A, q+1, r, i-k)
```

x is the median of $A[1 \ldots n]$, each of the subarrays $A[1 \ldots q-1]$ and $A[q+1 \ldots n]$ has most half the number of elements of $A[1 \ldots n]$.

The recurrence for the worst-case running time of Select algorithm is

$$T(n) \leq T(n/2) + O(n)$$

$$= O(n)$$

Problem 2:

a) x_k is the median of $x_i = x_1, \ldots\ldots, x_n$

Let l is number of element for $x_i < x_k$ .... (I)

It is given that $\sum_{i=1}^{n} w_i = 1/n$, i.e. each x element has weight $1/n$ ..... (II)

From (I) & (II) we can say that,

$$\sum_{x_i < x_k} w_i = \frac{l}{n} \quad , \quad \sum_{x_i > x_k} w_i = \frac{n-l-1}{n}$$

The sum of weight should be $< 1/2$ and $\leq 1/2$ respectively.

The value of l ~~must~~ should be $\left(\frac{n}{2} - 1\right)$ to satisfy these sum $< 1/2$ and $\leq 1/2$ respectively. On other cases sum will not be $< 1/2$ and $\leq 1/2$ respectively. $\left(\frac{n}{2} - 1\right)$ is median as it has equal number of x_i which are smaller and larger.

We can conclude that median of x_i is weighted median of x_i ~~too~~.

Problem 2:

b) First we need to sort all the elements to compute weighted median. To sort all elements merge sort can be used in $O(n \log n)$ time. Sum of the weights of sorted element can be done in $O(1)$ time. Then we have to compute sum until we get a point k where $S_k \geq 1/2$. x_k is weighted median. Thus it can be done in $O(n)$ time. So, total running time is $O(n \log n)$.

c) $x_i = x_1, \ldots, x_n$ are the n number of elements with weights $w_i = w_1 \ldots w_n$ such that $\sum_{i=1}^{n} w_i = 1$. Then the weighted median should satisfy below conditions,

$$\sum_{x_i < x_k} w_i < 1/2$$

and

$$\sum_{x_i > x_k} w_i \leq 1/2$$

Algorithm to find weighted median:

1. K = randomized_partition of $x[x_1, x_n]$

// x_1 is the first element of the array
x_n is last element of the array.
We are partitioning the array randomly into two parts.

2. $LW = \sum_{i=x_1}^{K-1} x[i]$

// LW is weight on the left side of K after partitioning the array.

3. $RW = \sum_{i=K+1}^{n} x[i]$

// RW is weight of all elements of right side of K after partitioning the array.

4. if $(LW < 1/2)$ && $(RW \leq 1/2)$

5. Return K

// if $LW < 1/2$ and $RW \leq 1/2$ then the partitioning point from where partition is done is the weighted median, which is K.

6. if $LW \geq 1/2$

7. $w(k-1) = w(k-1) + RW$

8. Return weighted_median $x[x_1, x_{k-1}]$

// if LW is $\geq 1/2$ then we have to add RW to it and we need to run whole algorithm again with x_1 as first element and x_{k-1} as last element.

9. if $RW > 1/2$

10. $w(k+1) = w(k+1) + LW$

11. Return weighted_median $x[x_{k+1}, x_n]$

// if $RW > 1/2$ then we have to add LW to $(k+1)^{th}$ position and have to apply the algorithm again with x_{k+1} as first and x_n as last element.

Analysis for algorithm $\Rightarrow$

Recurrency for worst case running time is

$$T(n) = T(n/2 + 1) + \Theta(n)$$

$$= \Theta(n)$$

Problem 2:

a) $\sum_{i=1}^{n} w_i\, d(p, p_i) = \sum_{i=1}^{n} w_i |p - p_i|$

$$= \sum_{p \le p_i} w_i (p_i - p) + \sum_{p > p_i} w_i (p - p_i)$$

$$= \sum_{p \le p_i} w_i (p_i - p) - \sum_{p > p_i} w_i (p_i - p)$$

$$= (p_i - p)\left[\sum_{p \le p_i} w_i - \sum_{p > p_i} w_i\right]$$

It should be minimum but $p_i - p$ is constant for selected value of p.

$\therefore \sum_{p < p_i} w_i - \sum_{p > p_i} w_i \sim 0$

But $\sum_{i=1}^{n} w_i = 1$

$\therefore \sum_{p < p_i} w_i \sim \sum_{p > p_i} w_i \sim 1/2$

i.e. $\sum_{p < p_i} w_i \le 1/2$ & $\sum_{p > p_i} w_i < 1/2$

Thus p is weighted median.

Problem 2:

e) $\sum_{i=1}^{n} w_i\, d(p, p_i) = \sum_{i=1}^{n} w_i \left(d(p_x, p_{ix}) + d(p_y, p_{iy})\right)$

$$= \sum_{i=1}^{n} w_i \left(|p_x - p_{ix}| + |p_y - p_{iy}|\right)$$

$$= \sum_{i=1}^{n} w_i \left(|p_x - p_{ix}|\right) + \sum_{i=1}^{n} w_i \left(|p_y - p_{iy}|\right)$$

$$= \sum_{p_x > p_{ix}} w_i (p_x - p_{ix}) + \sum_{p_x \le p_{ix}} w_i (p_{ix} - p_x)$$

$$+ \sum_{p_y > p_{iy}} (p_y - p_{iy}) + \sum_{p_y \le p_{iy}} (p_{iy} - p_y)$$

$$= (p_x - p_{ix}) \left[\sum_{p_x > p_{ix}} w_i - \sum_{p_x \le p_{ix}} w_i \right] +$$

$$(p_y - p_{iy}) \left[\sum_{p_y > p_{iy}} w_i - \sum_{p_y \le p_{iy}} w_i \right]$$

But $\sum_{i=1}^{n} w_i = 1$

$\therefore \sum_{p_x > p_{ix}} w_i \sim \sum_{p_x \le p_{ix}} w_i \sim \frac{1}{2}$ &

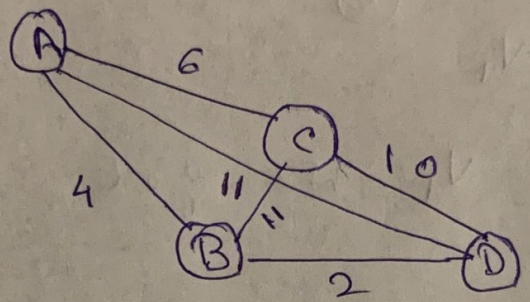
$\sum_{p_y > p_{iy}} w_i \sim \sum_{p_i \le p_{iy}} w_i \sim \frac{1}{2}$

With similar analogy as part (d), p_x must be weighted median of p_{ix} & p_y the weighted median of p_{iy}.

Problem 3:

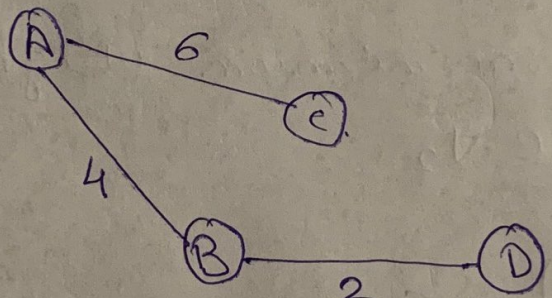
a) Here our aim is to prove that given algorithm should fail to produce MST.

Let us take an example and see ahead.



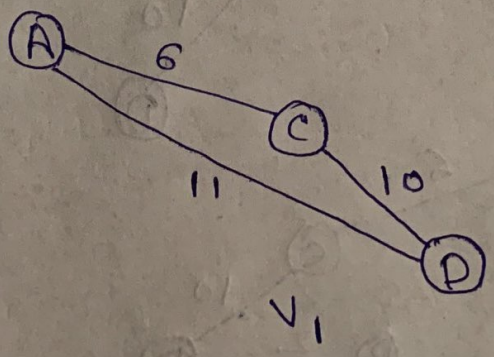
G_1

Finding MST by Krushkal's algorithm.

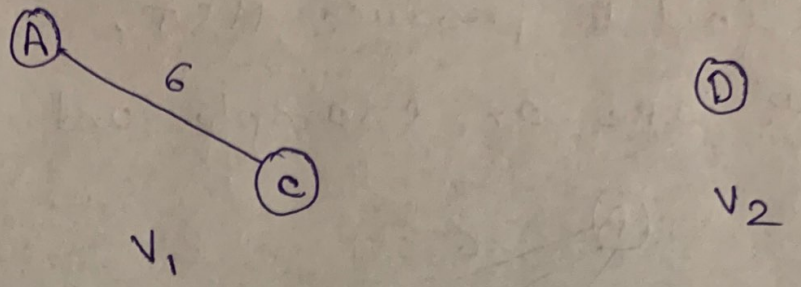


Cost of MST = 6 + 4 + 2 = 12

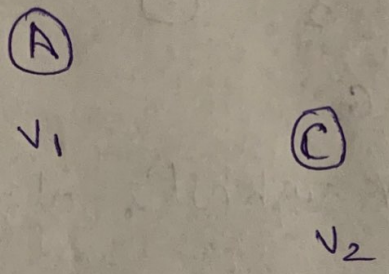
Now splitting the graph G_1 into V_1 & V_2



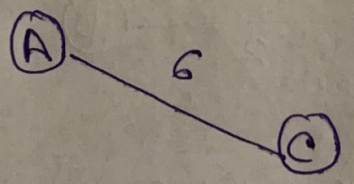
Now splitting V_1 further as algorithm is recursive.



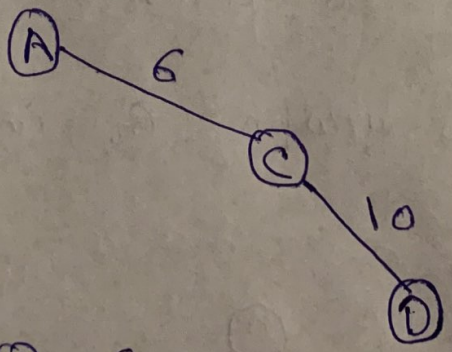
Again splitting V_1 we get



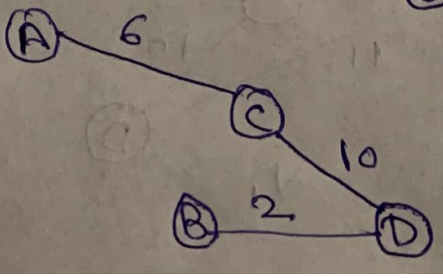
Now joining minimum weight edge that crosses V_1 & V_2



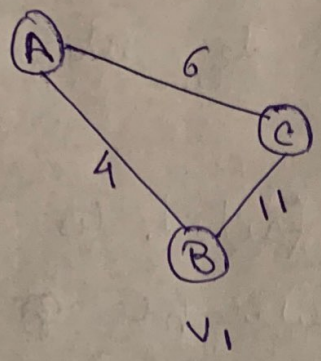
A → C



A → C → D



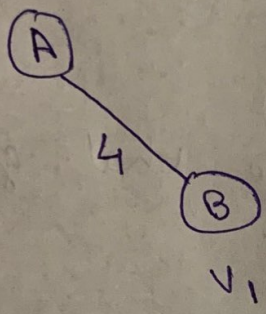
A → C → D → B



D

V_2

Splitting V_1 further,



C

V_2

Again by splitting V_1 we get

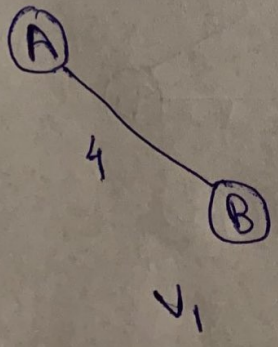
A

V_1

B

V_2

Now we are joining edge with minimum weight which cuts (V_1, V_2)



C

V_2

Weight of MST produced by algorithm is

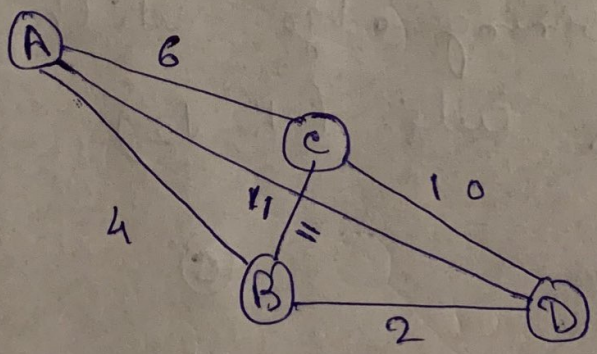
$6 + 10 + 2 = 18$

Thus, given algorithm has failed to provide MST as it gives total weight more than value given by Kruskal's algorithm $(18 > 12)$.

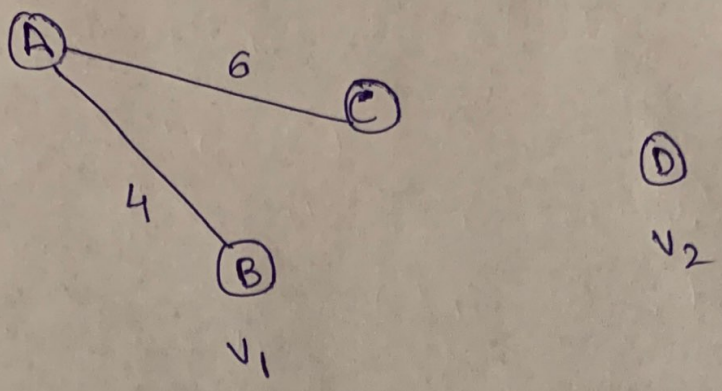
Problem 3:

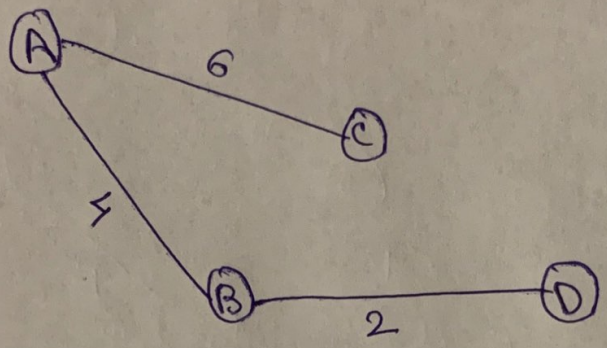
b) Here our aim is to prove that given algorithm should able to produce MST.

Let us take same graph G_2 but here we are going to consider different node as V_2 i.e partitioning different node to see whether the algorithm is ~~abortion~~ giving MST.



G_2





The cost of MST is $6+4+2$

$= 12$

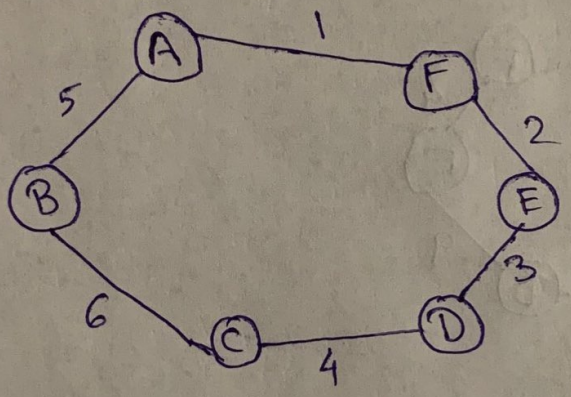
Here the algorithm gave MST. Here we have removed the edge which is having high value i.e. CD which is 10. So, to achieve MST by using given algorithm we need to remove the edge which is having highest weight. Here we removed the vertex D of CD is 10. This algorithm worked fine for above case.

Problem 4:

a) Let $G_2 = (V, E, C)$ be a weighted undirected graph where all the costs is c_e.

Let $G_2' = (V, E, C')$ be a weighted undirected graph where all costs are c_e' such that $c_e' = c_e^2$

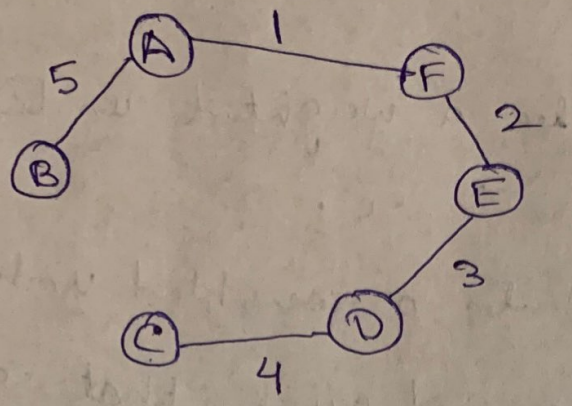
If we square the cost of each edge we are going to get same MST but shortest path may vary.



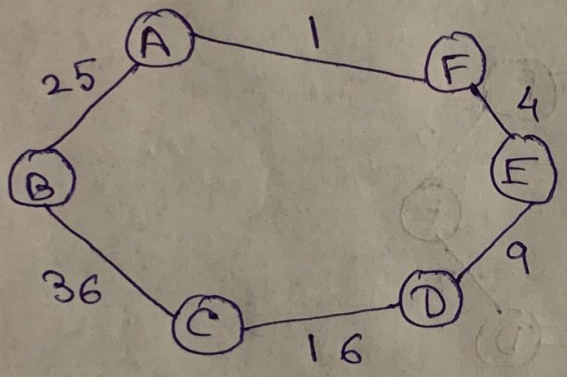
G_2

Here we are taking an example of graph as drawn above by name tag 'G_2'.

Now, we are going to apply Kruskal's algorithm to find minimum spanning tree for the graph (G_2).

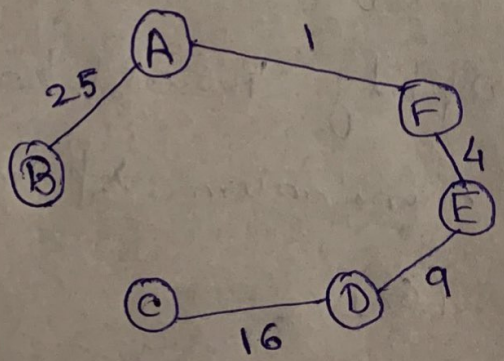


Now we are going to square cost of each edge of the graph G_2 and will give name of the graph as G_2'.



G_2'

Now we are going to apply Kruskal's algorithm to graph G_2' to find MST.



Here minimum spanning Tree is same even if cost of edges are squared and it stands for all cases.

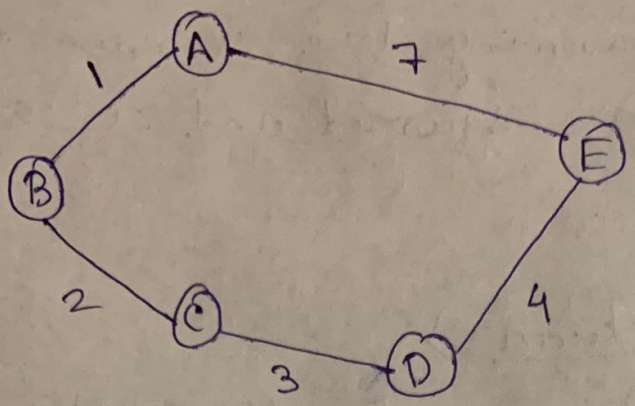
Proof by contradiction ⇒

If after squaring the weights, MST changes then there should be atleast one edge from old graph that should be replaced after the ~~weights~~ squaring ~~of squares~~ of weights, which means if for some ~~cost~~ cost say $c_1 < c_2$ becomes $c_1^2 > c_2^2$ after squaring which is not at all possible. Then in this case MST can change but practically it is not possible.

So, we always get MST even if the weights are squared.

Problem 4:

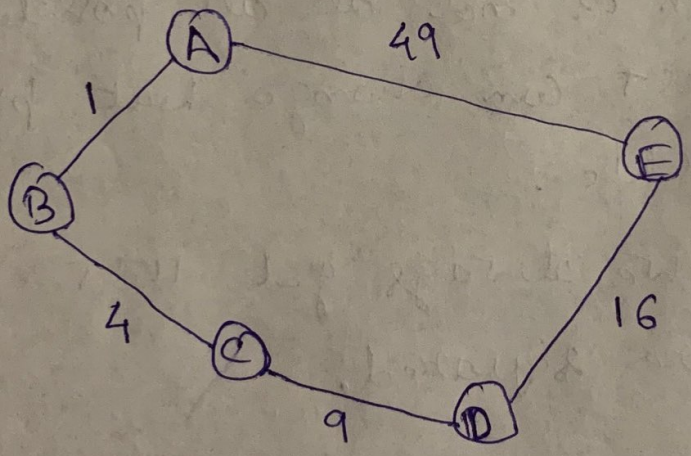
b) The shortest path may vary if we square the weights of the edges as we consider here the ratio of the edges. Let us check ~~with~~ by taking a random example of graph.



G

Here shortest path is A to E which is 7

Now we are going to square the weight of the edges to get graph G'.



Here shortest path is A → B → C → D → E

$= 1 + 4 + 9 + 16$

$= 30$

So, after squaring the weight of edges the route among the edges got change so shortest path is also changed.

College of Science Academic Integrity Pledge

Expectations

IIT requires instructors, teaching assistants, and students to maintain the highest standards of academic integrity and honesty. As a student, I will commit to academic integrity by adhering to the following requirements:

- I will ensure that all work I do is solely my own, and will collaborate with others only as is allowed in the course.
- I will make use of external resources only when they are permitted.
- I will properly give credit to any sources I make use of in my work, and will take credit only for work that I have personally produced.
- I will hold other students accountable to do the same.

During exams:

- I will not use any additional materials other than pen, pencil, or eraser, unless specifically allowed by the instructor.
- I will not look at another student's paper during the exam, or allow another student to look at mine.
- I will not communicate with anyone other than the TA or instructor during the exam.

Violations

Cheating, plagiarism, and dishonesty will not be tolerated. In the event that a violation of this policy comes to the attention of the instructor, the incident will be reported to academichonesty@iit.edu. According to the campus *Code of Academic Honesty*, a first incident may result in a zero for the assignment, and a second incident may result in failure for the course.

http://www.iit.edu/student_affairs/handbook/information_and_regulations/code_of_academic_honesty.shtml

Signature

I have read and understood this policy.

Name Amitdeb Prasad Bhattacharya Date 02/07/2019

http://www.iit.edu/student_affairs/handbook/information_and_regulations/code_of_academic_honesty.shtml

I. Code of Academic Honesty

With respect to matters covered by this Article I, students enrolled in or otherwise attending programs of IIT's Chicago-Kent College of Law, and only such students, are subject to the policies, procedures and provisions of the IIT Chicago-Kent College of Law Student Handbook (http://www.kentlaw.edu/depts/acadadm/handbook.html), and all such students should refer to the IIT Chicago-Kent College of Law Student Handbook for policies, procedures and provisions applicable to them.

A. Code

IIT expects students to maintain high standards of academic integrity. Students preparing for the practice of a profession are expected to conform to a code of integrity and ethical standards commensurate with the high expectations society places upon the practitioners of a learned profession. No student may seek to gain an unfair advantage over another.

It shall be a violation of this Code for a student to engage in conduct that violates the established standards of his or her major academic discipline, the established standards of the academic discipline in which she or he is engaged, the established standards of the profession of which she or he is training, or the standards of IIT set forth herein.
In addition, it is a violation of this Code for a student, whether or not currently enrolled at IIT, to knowingly engage or attempt to engage in the following:

- The misrepresentation of any work submitted for credit or otherwise as the product of a student's sole independent effort, such as using the ideas of others without attribution and other forms of plagiarism;
- The use of any unauthorized assistance in taking quizzes, tests or examinations;
- The acquisition, without permission, of tests, answers sheets, problem solutions or other academic material when such material has been withheld from distribution by the instructor;
- Deliberate and harmful obstruction of the studies, research or academic work of any member of the IIT community;
- Making material misrepresentation in any submission to or through any office of the University to a potential employer, professional society, meeting or organization; or
- The intentional assistance of others in the violation of the standards set forth in this Code.

B. Academic Discipline

Any member of the IIT community who is aware of a violation of the IIT Code of Academic Honesty, or who has evidence that a violation has occurred, is urged to inform the appropriate course instructor, academic unit head or dean or send an email to academichonesty@iit.edu. Faculty members or administrators who are aware of a violation, or who have good evidence of a violation, must inform the Chair of the Academic Honesty Committee for the appropriate academic unit and the Designated Dean for Academic Discipline ("DDAD"). The appropriate academic unit is the unit responsible for the course in which the alleged academic honesty violation occurred.

For the Main Campus, Rice Campus, Moffett and Institute of Design the DDAD is either the Vice Provost for Undergraduate Affairs or the Associate Dean for Graduate Academic Affairs, as appropriate given the status of the student. For Chicago-Kent College of Law, the DDAD is the Assistant Dean of Students, and for the Stuart School of Business it is the Dean.

The DDAD will provide a record of all incidents to the Office of Student Affairs. A student who believes that his or her record is factually inaccurate should inform the Dean of Students, who will investigate, as he or she deems appropriate, and, if he or she determines necessary, in his or her sole discretion, will correct any factual inaccuracies in the record. The decision of the Dean of Students as to such matters shall be final.

If a violation concerns conduct relating to coursework, the course instructor may meet with the student and impose one of the sanctions below if after the meeting the instructor has satisfied himself or herself that a violation has in fact occurred. The meeting may be held in consultation with the academic unit head, or with the Academic Honesty Committee of the academic unit. The course instructor will report the violation, a summary of the facts evidencing the violation and the sanction to the DDAD. Appropriate sanctions include:

- **Expulsion from a course**. The student is assigned a punitive failing grade of 'E' for the course and can no longer participate in the course or receive evaluation of coursework from the instructor.
- **Reduction in grade.** A reduction in grade for the assignment or exam involved or for the course may be applied.

Upon receipt of information about a violation of the Code, the DDAD will review the report and any record of prior violation of the Code by the student. If there are one or more prior violations, the DDAD will consult with the Chair of the Academic Grievance Committee of the University Faculty Council to determine whether the matter should be referred to a hearing before the Academic Grievance Committee for further sanctions. The DADD's decision to refer a matter to the Academic Grievance Committee shall be made in his or her sole discretion and shall be final. If the matter is referred for a hearing, the Academic Grievance Committee shall supplement its membership as necessary so that the Committee includes at least one faculty member from the Academic Honesty Committee of the academic unit where the violation occurred, one member of the student's major faculty, one other faculty member, two students and the Dean of Students or his or her designee. The Academic Grievance Committee so supplemented will form a disciplinary committee for the purpose of investigating the matter and making findings and a recommendation to the DDAD. The chair of the Academic Grievance Committee shall chair this disciplinary committee.

A student who is to appear before such a disciplinary committee shall be notified in writing by the Dean of Students of the charges, in summary fashion, placed against him or her and the time and place of the hearing. This notice shall be delivered to the student at least five days before the hearing. A student appearing before the disciplinary committee has the right to be accompanied by an advisor, witnesses on his or her behalf, and his or her parents. The advisor is limited to advising the student and may not participate in presenting the case, questioning witnesses, or making statements during the hearing. A student shall have the right to make an oral statement both at the start and the conclusion of the hearing and to submit written evidence to the committee. Further, as a general rule, a student shall have the right (i) to present and to question any witnesses of his or her choosing that have information relevant to the charges against him or her, provided that the chair may, in his or her sole discretion, disallow witnesses whose testimony would be redundant or not germane to the charges; and (ii) to cross-examine any witness that is called to testify in support of the charges, provided that the chair may, in his discretion, limit such cross-examination to the extent it is not germane to the charges, is redundant or becomes abusive or harassing. Members of the committee may, as each deems appropriate, ask questions of any witness called or regarding any evidence submitted. To the extent deemed necessary

to maintain decorum or to protect students from harassment or to ensure the integrity of the process, the chair may require a student to direct questions to the chair who will then ask them of a witness or witnesses.

An audio tape recording of the hearing will be made. The audio tape will remain the sole property of the University. No other record of the hearing may be made.

Once proper notification has been given, the University reserves the right to hold the hearing whether or not the student elects to participate. Email correspondence will be considered sufficient written notification in all instances where written notification is required.

Upon receipt of the findings of the disciplinary committee, which findings may include recommended sanctions, the DDAD shall determine the appropriate action to be taken and shall so notify the student. The disciplinary committee may recommend, and the DDAD may impose, any of the following sanctions:

- **Expulsion from a course.** The student is assigned a punitive failing grade of 'E' for the course and can no longer participate in the course or receive evaluation of coursework from the instructor.
- **Suspension.** Suspension is a status assigned for various periods of time in which a student's enrollment is interrupted. A suspended student may not attend day or evening classes, participate in student activities, or live in campus housing. A suspended student may apply for reinstatement at the end of the period of suspension. If reinstated, the student may be placed on disciplinary probation for a period of time designated by the DDAD.
- **Expulsion**. Expulsion is the complete severance of association with the University. Notation of the violation of the Code is made on the student's transcript.

In any matter where the DDAD's determination is to impose a sanction of suspension or expulsion, the DDAD's determination shall automatically be stayed and the matter referred to the Provost for review. With respect to such a proposed determination by the DDAD, the Provost may make any of the determinations set forth under the Appeals Procedures heading immediately following. Once the Provost's review has been completed, the DDAD's determination, as it may have been modified by the Provost, shall become effective. Notwithstanding any other provision of this Student Handbook, no matter so referred to the Provost shall be subject to any further appeals on the basis that the sanction imposed was inappropriate to the offense.

C. Appeal Procedures

An appeal of a decision on academic honesty must be submitted in writing to the Office of Student Affairs within five business days of notification of the decision. Supporting information must be submitted with the appeal. An appeal or submission made after this deadline will not be considered. Decisions rendered by the DDAD without referral to the University Faculty Council's Academic Grievance Committee and sanctions, excluding non-punitive grades, imposed by a course instructor may be appealed to the Academic Grievance Committee, which in the latter instance only may either affirm or overturn the course instructor's finding of academic dishonesty. Upon such an appeal, the Academic Grievance Committee will supplement its membership to form a disciplinary committee that is composed as described above and that follows hearing procedures as described above.

Decisions of the DDAD following a disciplinary committee hearing and decisions of a disciplinary committee hearing upholding a course instructor's finding of academic dishonesty may be appealed to the Provost, whose decision shall be final. An appeal to the Provost is limited to the following reasons:

- Appropriate procedures were not followed;
- The sanction imposed was inappropriate for the offense; and
- New evidence that was not available at the time of the original decision, not due to the fault of the student, has become available.

The Provost may make one of the following determinations:

- Uphold the decision and the sanction;
- Reverse the decision and the sanction; or
- Uphold the decision, but return the matter to the DDAD to reconsider the sanction.

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