ACM/IDS 104 - Problem Set 2 - MATLAB Problems

Before writing your MATLAB code, it is always good practice to get rid of any leftover variables and figures from previous scripts.

```
clc; clear; close all;
```

Problem 6 (10 points) Fundamental Matrix Subspaces

Your task for this problem is to write a function that takes a matrix `A` as its argument, and outputs four matrices: `K, I, cK` and `cI` where:

- Columns of `K` form a basis of the kernel of `A`. If $\ker A = \{0\}$, then `K` must be a zero vector of the appropriate dimension.
- Columns of `I` form a basis of the image of `A`. If $\mathrm{im} A = \{0\}$, then `I` must be a zero vector of the appropriate dimension.
- Columns of `cK` form a basis of the cokernel of `A`. If $\mathrm{coker} A = \{0\}$, then `cK` must be a zero vector of the appropriate dimension.
- Columns of `cI` form a basis of the coimage of `A`. If $\mathrm{coim} A = \{0\}$, then `cI` must be a zero vector of the appropriate dimension.

Move to the bottom of this livescript to write the function.

Now, let us test our function:

```
A = [2 0; 2 2; 20 24] % feel free to define A as you like
```

```
A = 3×2
     2     0
     2     2
    20    24
```

```
[K, I, cK, cI] = subspacer(A); % this is how you call a MATLAB function
disp(K);
```

```
     0
     0
```

```
disp(I);
```

```
   -0.0409   -0.9856
   -0.0897   -0.1597
   -0.9951    0.0549
```

```
disp(cK);
```

```
   -0.1638
    0.9831
   -0.0819
```

```
disp(cI);
```

```
   -0.6423   -0.7665
   -0.7665    0.6423
```

START HERE by writing the function:

```
function [K, I, cK, cI] = subspacer(A)
%{
This is the MATLAB function syntax.
-> [K, I, cK, cI] are the outputs of the function.
-> "subspacer" is the name of the function. (you can change that if
                            you wish but make sure you change
                            every function call as well!)
-> A is the argument of the function.
%}
[m, n] = size(A);
r = rank (A);
r2 = rank(A');
%{
We start by finding out the dimensions and rank of A.
Let us consider the matrix K. There exist 2 cases:
1) The kernel is trivial i.e. kerA = {0}
2) The kernel is not trivial -> Hint: use null()
Complete the following if/else statement.
%}
if r == n % this condition is done for you
    K = zeros(n,1);
else
    K = null(A);
end
%{
Now, let us consider the matrix cK.
As above, there exist 2 cases. Remember, you can use ' to
transpose a matrix.
Write a similar if/else statement to produce cK.
%}
if r2 == m
    cK = zeros(m,1);
else
    cK = null(A');
end

%{
For the image I and coimage cI, there exists only 1 condition
we must test, and that is if rankA = 0. With this in mind,
complete the following if/else statement.
-> Hint: orth() is useful here.
%}
if r == 0
    I = zeros(m,1);
```

```
    cI = zeros(n,1);

else
    I = orth(A);
    cI = orth(A');

end
end
```