ACM/IDS 104 - Problem Set 5 - MATLAB Problems

Before writing your MATLAB code, it is always good practice to get rid of any leftover variables and figures from previous scripts.

```
clc; clear; close all;
```

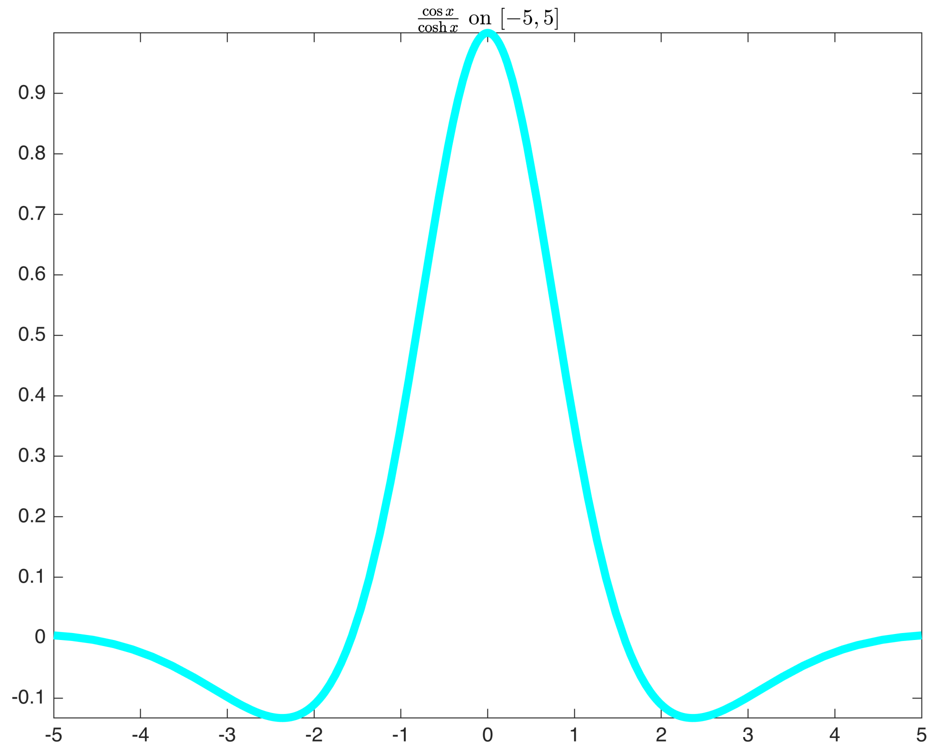
Problem 1 (10 points) Application of Projections to Approximation

In Problem 4 of PS4, we saw that even higher degree interpolating polynomials may not be accurate approximations to complex functions. We have the function:

$$f(x) = \frac{\cos x}{\cosh x}, \quad \text{on } [-a, a], \quad a = 5$$

Let us recall how this function looks like and how its interpolating polynomials of degree $(n-1)$ for $n = 3, 5, 10, 15$ behave:

```
%{
Setup
%}
f = @(x) cos(x)./cosh(x); % our function
a = 5; % setting the value of a
n = [3 5 10 15]; % setting the number of points
sub = 1; % subplot index

%{
How f(x) looks like on [-5, 5]
%}
figure;
fplot(f, [-a, a], "-c", "lineWidth", 4);
title("$\frac{\cos{x}}{\cosh{x}}$ on $[-5, 5]$","Interpreter","latex");
```

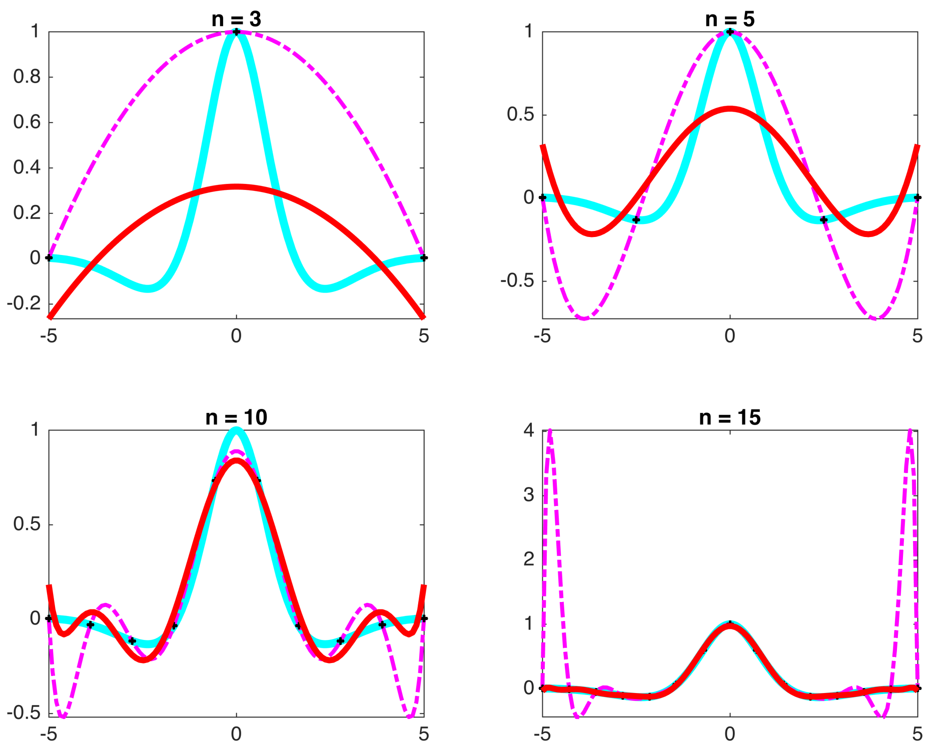



```
%{
Read the discussion below and complete the code
%}
figure;
for ival = a
    for degree = n-1
        %{
        INTERPOLATING POLYNOMIALS -- no changes needed
        -> Select degree+1 points in the interval
        -> Evaluate f(x) on these points
        -> Find the polynomial coefficients
        %}
        pts = ones(degree+1, 2); % initializing the points
        pts(:, 1) = linspace(-ival, ival, degree+1); % setting the x-values
        for i = 1 : degree+1
            pts(i, 2) = f(pts(i, 1)); % evaluating cos(x) /
cosh(x)
        end
        coeffs = polyfit(pts(:, 1), pts(:, 2), degree); % coefficients
        %{
        ORTHOGONAL PROJECTIONS -- TODO
        -> Get transformed Legendre polynomials
        -> Find alpha_k using L^2 inner product
        -> Evaluate alpha_k*Q_k
        %}
```

```
        x = linspace(-a, a);
        y = zeros(100 ,1);
        alpha_k = zeros(degree, 1);

        for d=0:degree
            numrgrand = @(x) (f(x) .* legendreP(d, x/a));
            denrgrand = @(x) (legendreP(d, x /a) .* legendreP(d,x/a));
            alpha_k(d+1) = integral(numrgrand, -a,a)/integral(denrgrand,-a,
a);
        end
        alpha_k

        for i = 1:100
         valati = 0;

         for d = 0:degree
            valati = valati + legendreP(d, x(i)/a) * alpha_k(d + 1);
         end

         y(i) = valati;
        end

        %{
        PLOTTING
        Plot f(x), the sampled points, interpolating and approximating
        polynomials
        Please use different colors and linestyles
        %}
        subplot(2, 2, sub);
        fplot(f, [-ival, ival], "-c", "lineWidth", 4);
        hold on
        interpoints = linspace(-ival, ival);
        p = polyval(coeffs, interpoints); % evaluating coeffs in interval
        plot(interpoints, p, "-.m", "lineWidth", 2);
        plot(pts(:, 1), pts(:, 2), "ok", "MarkerSize", 2, "lineWidth", 3);
        plot(x, y, "-r", "MarkerSize", 3, "lineWidth", 3);
        title(strcat("n = ", int2str(degree+1)));
        sub = sub + 1; % increase subplot index
    end
end
```

```
alpha_k = 3×1
    0.1235
   -0.0000
   -0.3888
alpha_k = 5×1
    0.1235
   -0.0000
   -0.3888
   -0.0000
    0.5881
alpha_k = 10×1
    0.1235
```

```
   -0.0000
   -0.3888
   -0.0000
    0.5881
   -0.0000
   -0.5815
    0.0000
    0.4415
    0.0000
alpha_k = 15×1
    0.1235
   -0.0000
   -0.3888
   -0.0000
    0.5881
   -0.0000
   -0.5815
    0.0000
    0.4415
    0.0000
      ⋮
```



Now, instead of interpolating polynomials, let us approximate $f(x)$ by its orthogonal projection onto the inner space $\mathscr{P}^{(n-1)}_{[-a,a]}$ of polynomials on $[-a,a]$, equipped with the L^2 inner product:

$$f(x) \approx p(x) = \mathrm{pr}_{\mathscr{P}^{(n-1)}_{[-a,a]}} f(x)$$

Recall (Lecture 10) that $p(x)$ is the closest (in the L^2 sense) polynomial to $f(x)$ in $\mathscr{P}^{(n-1)}_{[-a,a]}$, i.e.

$$p(x) = \arg \min_{q \in \mathscr{P}^{(n-1)}_{[-a,a]}} \| f(x) - q(x) \|$$

We know that the transformed Legendre polynomials $\widetilde{Q}_0(x), \cdots, \widetilde{Q}_{n-1}(x)$ form an orthogonal basis of $\mathscr{P}^{(n-1)}_{[-a,a]}$, and, therefore:

$$p(x) = \sum_{k=0}^{n-1} \alpha_k \widetilde{Q}_k(x)$$

where α_k are the coordinates of $p(x)$ in that basis.

Modify the above code to find the approximating polynomials as well. Plot each approximating polynomial on its corresponding subplot. Useful functions for this problem:

```
legendreP(), integral()
```