Inhomogeneous Recurrences

Example-4:

Solve the recurrence:

$t_n - 2t_{n-1} = 3^n$ ---------------- (1)

This recurrence is an inhomogeneous recurrence relation.

Compare recurrence with general form of inhomogeneous recurrence:

$$a0t_n + a1t_{n-1} + ...+ akt_{n-k} = b^n\ p(n)$$

Find value of k=1, a0=1,a1=-2,b=3, p(n)=1, d=0

Represent recurrence in characteristic equation:

$$(a0x^k + a1x^{k-1} + ...+ ak)\ (x-b)^{d+1}= 0$$

⇨(x-2) (x-3) =0

⇨Roots are: r1=2 and r2=3

General Solution is $tn = c1r1^n + c2r2^n$

$$tn = c1(2)^n+ c2(3)^n \quad ------------ (2)$$

Use initial condition to find c1 and c2.

Don't have initial value then put equation (2) into equation (1)

$t_n - 2t_{n-1} = 3^n$

$\Rightarrow c_1 2^n + c_2 3^n - 2(c_1 2^{n-1} + c_2 3^{n-1}) = 3^n$

$\Rightarrow c_1 2^n + c_2 3^n - c_1 2^n - 2c_2 3^{n-1} = 3^n$

$\Rightarrow c_2 3^n - (2/3)c_2 3^n = 3^n$

$\Rightarrow c_2 3^n[1 - (2/3)] = 3^n$

$\Rightarrow c2[1/3] = 1$

$\Rightarrow c2=3$

So, $\mathbf{t_n = O(3^n)}$

Example-5:

Solve the recurrence:

tn = 2tn-1 + n

tn -2tn-1 = n ----------- (1)

a0 = 1, a1 = -2, k =1

b=1, p(n) = n, d =1

$(x - 2)(x - 1)^2 = 0$

R1 = 2, m1 = 1, r2 = 1, m2 = 2

tn = c1r1^n + c2r2^n + c3nr2^n

tn = c12^n + c21^n + c3n1^n ------------ (2) (1^n always = 1)

Put eq 2 in eu 1

n = tn -2tn-1

= c12^n + c2 + c3n – 2 (c12^{n-1} + c2 + c3(n-1))

= c12^n + c2 + c3n - c12^n – 2c2 - 2c3n +2c3

= -c2 – c3n + 2c3

n = (2c3-c2) – c3n

If n = 0, **2c3 – c2 = 0**

If n =1, 2c3-c2-c3 = 1 → **c3 – c2 = 1**

If you solve these equations, you get c3 = -1 and c2 = -2

Therefore,

tn = c12^n - 2 – n

c1 must be positive because tn>=0

Thus answer is tn $t_n \in \theta(2^n)$, theta bcz we do not have exact value for c1