# Implementing logistic regression from scratch

The goal of this notebook is to implement your own logistic regression classifier. You will:

- Extract features from Amazon product reviews.
- · Convert an SFrame into a NumPy array.
- Implement the link function for logistic regression.
- Write a function to compute the derivative of the log likelihood function with respect to a single coefficient.
- · Implement gradient ascent.
- · Given a set of coefficients, predict sentiments.
- Compute classification accuracy for the logistic regression model.

Let's get started!

## Fire up GraphLab Create

Make sure you have the latest version of GraphLab Create. Upgrade by

```
pip install graphlab-create --upgrade
```

See this page (https://dato.com/download/) for detailed instructions on upgrading.

In [1]:

import graphlab

### Load review dataset

For this assignment, we will use a subset of the Amazon product review dataset. The subset was chosen to contain similar numbers of positive and negative reviews, as the original dataset consisted primarily of positive reviews.

In [2]:

```
products = graphlab.SFrame('amazon_baby_subset.gl/')
```

This non-commercial license of GraphLab Create for academic use is assigned to amitha353@gmail.com and will expire on May 07, 2019.

[INFO] graphlab.cython.cy\_server: GraphLab Create v2.1 started. Logging: C:\Users\Amitha\AppData\Local\Temp\graphlab\_server\_1532451135.log.0

One column of this dataset is 'sentiment', corresponding to the class label with +1 indicating a review with positive sentiment and -1 indicating one with negative sentiment.

```
In [3]:
```

Let us quickly explore more of this dataset. The 'name' column indicates the name of the product. Here we list the first 10 products in the dataset. We then count the number of positive and negative reviews.

```
In [4]:
```

```
products.head(10)['name']
Out[4]:
dtype: str
Rows: 10
["Stop Pacifier Sucking without tears with Thumbuddy To Love's Binky Fairy P
uppet and Adorable Book", "Nature's Lullabies Second Year Sticker Calendar",
"Nature's Lullabies Second Year Sticker Calendar", 'Lamaze Peekaboo, I Love
You', "SoftPlay Peek-A-Boo Where's Elmo A Children's Book", 'Our Baby Girl M
emory Book', 'Hunnt® Falling Flowers and Birds Kids Nursery Home Decor V
inyl Mural Art Wall Paper Stickers', 'Blessed By Pope Benedict XVI Divine Me
rcy Full Color Medal', 'Cloth Diaper Pins Stainless Steel Traditional Safety
Pin (Black)', 'Cloth Diaper Pins Stainless Steel Traditional Safety Pin (Bla
ck)']
In [5]:
print '# of positive reviews =', len(products[products['sentiment']==1])
print '# of negative reviews =', len(products[products['sentiment']==-1])
# of positive reviews = 26579
```

**Note:** For this assignment, we eliminated class imbalance by choosing a subset of the data with a similar number of positive and negative reviews.

## Apply text cleaning on the review data

In this section, we will perform some simple feature cleaning using **SFrames**. The last assignment used all words in building bag-of-words features, but here we limit ourselves to 193 words (for simplicity). We compiled a list of 193 most frequent words into a JSON file.

Now, we will load these words from this JSON file:

# of negative reviews = 26493

#### In [6]:

```
import json
with open('important_words.json', 'r') as f: # Reads the list of most frequent words
  important_words = json.load(f)
important_words = [str(s) for s in important_words]
```

#### In [7]:

```
print important_words
```

['baby', 'one', 'great', 'love', 'use', 'would', 'like', 'easy', 'little',
'seat', 'old', 'well', 'get', 'also', 'really', 'son', 'time', 'bought', ' oduct', 'good', 'daughter', 'much', 'loves', 'stroller', 'put', 'months', 'c ar', 'still', 'back', 'used', 'recommend', 'first', 'even', 'perfect', 'nic e', 'bag', 'two', 'using', 'got', 'fit', 'around', 'diaper', 'enough', 'mont h', 'price', 'go', 'could', 'soft', 'since', 'buy', 'room', 'works', 'made', 'child', 'keep', 'size', 'small', 'need', 'year', 'big', 'make', 'take', 'ea sily', 'think', 'crib', 'clean', 'way', 'quality', 'thing', 'better', 'witho ut', 'set', 'new', 'every', 'cute', 'best', 'bottles', 'work', 'purchased', 'right', 'lot', 'side', 'happy', 'comfortable', 'toy', 'able', 'kids', 'bi t', 'night', 'long', 'fits', 'see', 'us', 'another', 'play', 'day', 'money', 'monitor', 'tried', 'thought', 'never', 'item', 'hard', 'plastic', 'howeve r', 'disappointed', 'reviews', 'something', 'going', 'pump', 'bottle', 'cu p', 'waste', 'return', 'amazon', 'different', 'top', 'want', 'problem', 'kno w', 'water', 'try', 'received', 'sure', 'times', 'chair', 'find', 'hold', 'g ate', 'open', 'bottom', 'away', 'actually', 'cheap', 'worked', 'getting', 'o rdered', 'came', 'milk', 'bad', 'part', 'worth', 'found', 'cover', 'many', 'design', 'looking', 'weeks', 'say', 'wanted', 'look', 'place', 'purchase', 'looks', 'second', 'piece', 'box', 'pretty', 'trying', 'difficult', 'togethe r', 'though', 'give', 'started', 'anything', 'last', 'company', 'come', 'ret urned', 'maybe', 'took', 'broke', 'makes', 'stay', 'instead', 'idea', 'hea d', 'said', 'less', 'went', 'working', 'high', 'unit', 'seems', 'picture', 'completely', 'wish', 'buying', 'babies', 'won', 'tub', 'almost', 'either']

Now, we will perform 2 simple data transformations:

- 1. Remove punctuation using <u>Python's built-in (https://docs.python.org/2/library/string.html)</u> string functionality.
- 2. Compute word counts (only for important\_words)

We start with Step 1 which can be done as follows:

#### In [8]:

```
def remove_punctuation(text):
    import string
    return text.translate(None, string.punctuation)

products['review_clean'] = products['review'].apply(remove_punctuation)
```

Now we proceed with *Step 2*. For each word in **important\_words**, we compute a count for the number of times the word occurs in the review. We will store this count in a separate column (one for each word). The result of this feature processing is a single column for each word in **important\_words** which keeps a count of the number of times the respective word occurs in the review text.

**Note:** There are several ways of doing this. In this assignment, we use the built-in *count* function for Python lists. Each review string is first split into individual words and the number of occurances of a given word is counted.

```
In [9]:
```

```
for word in important_words:
    products[word] = products['review_clean'].apply(lambda s : s.split().count(word))
```

The SFrame **products** now contains one column for each of the 193 **important\_words**. As an example, the column **perfect** contains a count of the number of times the word **perfect** occurs in each of the reviews.

```
In [13]:
```

Now, write some code to compute the number of product reviews that contain the word **perfect**.

#### Hint:

- First create a column called contains\_perfect which is set to 1 if the count of the word **perfect** (stored in column **perfect**) is >= 1.
- Sum the number of 1s in the column contains\_perfect.

```
In [16]:
```

```
sum(products['perfect'] > 0)
Out[16]:
2955L
```

Quiz Question. How many reviews contain the word perfect?

## **Convert SFrame to NumPy array**

As you have seen previously, NumPy is a powerful library for doing matrix manipulation. Let us convert our data to matrices and then implement our algorithms with matrices.

First, make sure you can perform the following import.

```
In [17]:
```

```
import numpy as np
```

We now provide you with a function that extracts columns from an SFrame and converts them into a NumPy array. Two arrays are returned: one representing features and another representing class labels. Note that the feature matrix includes an additional column 'intercept' to take account of the intercept term.

```
In [18]:
```

```
def get_numpy_data(data_sframe, features, label):
    data_sframe['intercept'] = 1
    features = ['intercept'] + features
    features_sframe = data_sframe[features]
    feature_matrix = features_sframe.to_numpy()
    label_sarray = data_sframe[label]
    label_array = label_sarray.to_numpy()
    return(feature_matrix, label_array)
```

Let us convert the data into NumPy arrays.

```
In [19]:
```

```
# Warning: This may take a few minutes...
feature_matrix, sentiment = get_numpy_data(products, important_words, 'sentiment')
```

```
In [51]:
```

```
feature_matrix.shape
Out[51]:
```

```
(53072L, 194L)
```

Are you running this notebook on an Amazon EC2 t2.micro instance? (If you are using your own machine, please skip this section)

It has been reported that t2.micro instances do not provide sufficient power to complete the conversion in acceptable amount of time. For interest of time, please refrain from running get\_numpy\_data function. Instead, download the <a href="mailto:binary file">binary file</a> (<a href="https://s3.amazonaws.com/static.dato.com/files/coursera/course-3/numpy-arrays/module-3-assignment-numpy-arrays.npz">binary file</a> (<a href="https://s3.amazonaws.com/static.dato.com/files/coursera/course-3/numpy-arrays/module-3-assignment-numpy-arrays.npz">https://sa.amazonaws.com/static.dato.com/files/coursera/course-3/numpy-arrays</a> (<a href="https://sa.amazonaws.com/static.dato.com/files/coursera/course-3/numpy-arrays">https://sa.amazonaws.com/static.dato.com/st

```
arrays = np.load('module-3-assignment-numpy-arrays.npz')
feature_matrix, sentiment = arrays['feature_matrix'], arrays['sentiment']
```

```
In [20]:
```

```
feature_matrix.shape
```

Out[20]:

(53072L, 194L)

Quiz Question: How many features are there in the feature\_matrix?

```
In [24]:
```

```
len(important_words)
Out[24]:
```

193

**Quiz Question:** Assuming that the intercept is present, how does the number of features in **feature\_matrix** relate to the number of features in the logistic regression model?

Now, let us see what the **sentiment** column looks like:

```
In [25]:
```

```
sentiment
Out[25]:
array([ 1,  1,  1, ..., -1, -1], dtype=int64)
```

## Estimating conditional probability with link function

Recall from lecture that the link function is given by:

$$P(y_i = +1|\mathbf{x}_i, \mathbf{w}) = \frac{1}{1 + \exp(-\mathbf{w}^T h(\mathbf{x}_i))},$$

where the feature vector  $h(\mathbf{x}_i)$  represents the word counts of **important\_words** in the review  $\mathbf{x}_i$ . Complete the following function that implements the link function:

#### In [27]:

```
produces probablistic estimate for P(y_i = +1 | x_i, w).
estimate ranges between 0 and 1.

def predict_probability(feature_matrix, coefficients):
    # Take dot product of feature_matrix and coefficients
    # YOUR CODE HERE
    scores = feature_matrix.dot(coefficients)

# Compute P(y_i = +1 | x_i, w) using the link function
    # YOUR CODE HERE
    predictions = 1.0/(1.0+np.exp(-scores))

# return predictions
return predictions
```

Aside. How the link function works with matrix algebra

Since the word counts are stored as columns in **feature\_matrix**, each i-th row of the matrix corresponds to the feature vector  $h(\mathbf{x}_i)$ :

$$[\text{feature\_matrix}] = \begin{bmatrix} h(\mathbf{x}_1)^T \\ h(\mathbf{x}_2)^T \\ \vdots \\ h(\mathbf{x}_N)^T \end{bmatrix} = \begin{bmatrix} h_0(\mathbf{x}_1) & h_1(\mathbf{x}_1) & \cdots & h_D(\mathbf{x}_1) \\ h_0(\mathbf{x}_2) & h_1(\mathbf{x}_2) & \cdots & h_D(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ h_0(\mathbf{x}_N) & h_1(\mathbf{x}_N) & \cdots & h_D(\mathbf{x}_N) \end{bmatrix}$$

By the rules of matrix multiplication, the score vector containing elements  $\mathbf{w}^T h(\mathbf{x}_i)$  is obtained by multiplying **feature\_matrix** and the coefficient vector  $\mathbf{w}$ .

$$[score] = [feature\_matrix] \mathbf{w} = \begin{bmatrix} h(\mathbf{x}_1)^T \\ h(\mathbf{x}_2)^T \\ \vdots \\ h(\mathbf{x}_N)^T \end{bmatrix} \mathbf{w} = \begin{bmatrix} h(\mathbf{x}_1)^T \mathbf{w} \\ h(\mathbf{x}_2)^T \mathbf{w} \\ \vdots \\ h(\mathbf{x}_N)^T \mathbf{w} \end{bmatrix} = \begin{bmatrix} \mathbf{w}^T h(\mathbf{x}_1) \\ \mathbf{w}^T h(\mathbf{x}_2) \\ \vdots \\ \mathbf{w}^T h(\mathbf{x}_N) \end{bmatrix}$$

#### Checkpoint

Just to make sure you are on the right track, we have provided a few examples. If your predict\_probability function is implemented correctly, then the outputs will match:

#### In [28]:

# Compute derivative of log likelihood with respect to a single coefficient

Recall from lecture:

$$\frac{\partial \ell}{\partial w_j} = \sum_{i=1}^N h_j(\mathbf{x}_i) \left( \mathbf{1}[y_i = +1] - P(y_i = +1 | \mathbf{x}_i, \mathbf{w}) \right)$$

We will now write a function that computes the derivative of log likelihood with respect to a single coefficient  $w_j$ . The function accepts two arguments:

- errors vector containing  $\mathbf{1}[y_i = +1] P(y_i = +1 | \mathbf{x}_i, \mathbf{w})$  for all i.
- feature vector containing  $h_i(\mathbf{x}_i)$  for all i.

Complete the following code block:

#### In [29]:

```
def feature_derivative(errors, feature):
    # Compute the dot product of errors and feature
    derivative = errors.dot(feature)

# Return the derivative
    return derivative
```

In the main lecture, our focus was on the likelihood. In the advanced optional video, however, we introduced a transformation of this likelihood---called the log likelihood---that simplifies the derivation of the gradient and is more numerically stable. Due to its numerical stability, we will use the log likelihood instead of the likelihood to assess the algorithm.

The log likelihood is computed using the following formula (see the advanced optional video if you are curious about the derivation of this equation):

$$\mathscr{E}(\mathbf{w}) = \sum_{i=1}^{N} \left( (\mathbf{1}[y_i = +1] - 1)\mathbf{w}^T h(\mathbf{x}_i) - \ln\left(1 + \exp(-\mathbf{w}^T h(\mathbf{x}_i))\right) \right)$$

We provide a function to compute the log likelihood for the entire dataset.

#### In [30]:

```
def compute_log_likelihood(feature_matrix, sentiment, coefficients):
    indicator = (sentiment==+1)
    scores = np.dot(feature_matrix, coefficients)
    logexp = np.log(1. + np.exp(-scores))

# Simple check to prevent overflow
    mask = np.isinf(logexp)
    logexp[mask] = -scores[mask]

lp = np.sum((indicator-1)*scores - logexp)
    return lp
```

#### Checkpoint

Just to make sure we are on the same page, run the following code block and check that the outputs match.

#### In [31]:

```
dummy_feature_matrix = np.array([[1.,2.,3.], [1.,-1.,-1]])
dummy_coefficients = np.array([1., 3., -1.])
dummy_sentiment = np.array([-1, 1])
correct_indicators = np.array( [ -1==+1,
                                                                           1==+1 ] )
correct_scores = np.array([1.*1. + 2.*3. + 3.*(-1.),
                                                                           1.*1. + (-1)
correct_first_term = np.array( [ (correct_indicators[0]-1)*correct_scores[0],  (correct_ir
correct_second_term = np.array( [ np.log(1. + np.exp(-correct_scores[0])),
                                                                          np.log(1. +
                         sum( [ correct_first_term[0]-correct_second_term[0], correct_fir
correct 11
print 'The following outputs must match '
print 'correct_log_likelihood =', correct_ll
print 'output of compute log likelihood =', compute log likelihood(dummy feature matrix, du
The following outputs must match
_____
correct_log_likelihood
                              = -5.33141161544
output of compute log likelihood = -5.33141161544
```

## **Taking gradient steps**

Now we are ready to implement our own logistic regression. All we have to do is to write a gradient ascent function that takes gradient steps towards the optimum.

Complete the following function to solve the logistic regression model using gradient ascent:

#### In [35]:

```
from math import sqrt
def logistic_regression(feature_matrix, sentiment, initial_coefficients, step_size, max_ite
    coefficients = np.array(initial coefficients) # make sure it's a numpy array
    for itr in xrange(max iter):
        # Predict P(y_i = +1|x_i,w) using your predict_probability() function
        # YOUR CODE HERE
        predictions = predict_probability(feature_matrix, coefficients)
        # Compute indicator value for (y_i = +1)
        indicator = (sentiment==+1)
        # Compute the errors as indicator - predictions
        errors = indicator - predictions
        for j in xrange(len(coefficients)): # Loop over each coefficient
            # Recall that feature_matrix[:,j] is the feature column associated with coeffic
            # Compute the derivative for coefficients[j]. Save it in a variable called deri
            # YOUR CODE HERE
            derivative = feature_derivative(errors, feature_matrix[:, j])
            # add the step size times the derivative to the current coefficient
            ## YOUR CODE HERE
            coefficients[j] += step_size * derivative
        # Checking whether log likelihood is increasing
        if itr <= 15 or (itr <= 100 and itr % 10 == 0) or (itr <= 1000 and itr % 100 == 0)
        or (itr <= 10000 and itr % 1000 == 0) or itr % 10000 == 0:
            lp = compute_log_likelihood(feature_matrix, sentiment, coefficients)
            print 'iteration %*d: log likelihood of observed labels = %.8f' % \
                (int(np.ceil(np.log10(max_iter))), itr, lp)
    return coefficients
```

Now, let us run the logistic regression solver.

#### In [36]:

```
0: log likelihood of observed labels = -36780.91768478
iteration
           1: log likelihood of observed labels = -36775.13434712
iteration
iteration
           2: log likelihood of observed labels = -36769.35713564
iteration 3: log likelihood of observed labels = -36763.58603240
iteration 4: log likelihood of observed labels = -36757.82101962
iteration 5: log likelihood of observed labels = -36752.06207964
iteration 6: log likelihood of observed labels = -36746.30919497
iteration 7: log likelihood of observed labels = -36740.56234821
iteration 8: log likelihood of observed labels = -36734.82152213
          9: log likelihood of observed labels = -36729.08669961
iteration
iteration 10: log likelihood of observed labels = -36723.35786366
iteration 11: log likelihood of observed labels = -36717.63499744
iteration 12: log likelihood of observed labels = -36711.91808422
iteration 13: log likelihood of observed labels = -36706.20710739
iteration 14: log likelihood of observed labels = -36700.50205049
iteration 15: log likelihood of observed labels = -36694.80289716
iteration 20: log likelihood of observed labels = -36666.39512033
iteration 30: log likelihood of observed labels = -36610.01327118
iteration 40: log likelihood of observed labels = -36554.19728365
iteration 50: log likelihood of observed labels = -36498.93316099
iteration 60: log likelihood of observed labels = -36444.20783914
iteration 70: log likelihood of observed labels = -36390.00909449
iteration 80: log likelihood of observed labels = -36336.32546144
iteration 90: log likelihood of observed labels = -36283.14615871
iteration 100: log likelihood of observed labels = -36230.46102347
iteration 200: log likelihood of observed labels = -35728.89418769
iteration 300: log likelihood of observed labels = -35268.51212683
```

Quiz Question: As each iteration of gradient ascent passes, does the log likelihood increase or decrease?

## **Predicting sentiments**

Recall from lecture that class predictions for a data point **x** can be computed from the coefficients **w** using the following formula:

$$\hat{y}_i = \begin{cases} +1 & \mathbf{x}_i^T \mathbf{w} > 0 \\ -1 & \mathbf{x}_i^T \mathbf{w} \le 0 \end{cases}$$

Now, we will write some code to compute class predictions. We will do this in two steps:

- Step 1: First compute the scores using feature\_matrix and coefficients using a dot product.
- Step 2: Using the formula above, compute the class predictions from the scores.

Step 1 can be implemented as follows:

```
In [37]:
```

```
# Compute the scores as a dot product between feature_matrix and coefficients.
scores = np.dot(feature_matrix, coefficients)
```

Now, complete the following code block for **Step 2** to compute the class predictions using the **scores** obtained above:

```
In [38]:
```

```
class_predictions = np.array(graphlab.SArray(scores).apply(lambda x: 1 if x> 0 else -1))
print class_predictions
[ 1 -1 1 ..., -1 1 -1]
```

Quiz Question: How many reviews were predicted to have positive sentiment?

```
In [40]:
sum(class_predictions > 0)
Out[40]:
25126
In [41]:
unique, counts = np.unique(class_predictions, return_counts=True)
print unique, counts
```

## Measuring accuracy

[-1 1] [27946 25126]

We will now measure the classification accuracy of the model. Recall from the lecture that the classification accuracy can be computed as follows:

```
accuracy = \frac{\text{# correctly classified data points}}{\text{# total data points}}
```

Complete the following code block to compute the accuracy of the model.

```
In [45]:
```

Accuracy = 0.75

Quiz Question: What is the accuracy of the model on predictions made above? (round to 2 digits of accuracy)

# Which words contribute most to positive & negative sentiments?

Recall that in Module 2 assignment, we were able to compute the "most positive words". These are words that correspond most strongly with positive reviews. In order to do this, we will first do the following:

- Treat each coefficient as a tuple, i.e. (word, coefficient\_value).
- Sort all the (word, coefficient\_value) tuples by coefficient\_value in descending order.

#### In [46]:

```
coefficients = list(coefficients[1:]) # exclude intercept
word_coefficient_tuples = [(word, coefficient) for word, coefficient in zip(important_words
word_coefficient_tuples = sorted(word_coefficient_tuples, key=lambda x:x[1], reverse=True)
```

Now, **word\_coefficient\_tuples** contains a sorted list of (**word**, **coefficient\_value**) tuples. The first 10 elements in this list correspond to the words that are most positive.

## Ten "most positive" words

Now, we compute the 10 words that have the most positive coefficient values. These words are associated with positive sentiment.

```
In [47]:
```

```
word_coefficient_tuples[0:10]
```

#### Out[47]:

```
[('great', 0.066546084170457695),
  ('love', 0.065890762922123258),
  ('easy', 0.06479458680257838),
  ('little', 0.045435626308421372),
  ('loves', 0.044976401394906038),
  ('well', 0.030135001092107074),
  ('perfect', 0.029739937104968462),
  ('old', 0.020077541034775381),
  ('nice', 0.018408707995268992),
  ('daughter', 0.01770319990570169)]
```

Quiz Question: Which word is not present in the top 10 "most positive" words?

## Ten "most negative" words

Next, we repeat this exercise on the 10 most negative words. That is, we compute the 10 words that have the most negative coefficient values. These words are associated with negative sentiment.

```
In [49]:
```

```
word_coefficient_tuples[-10:]
Out[49]:

[('monitor', -0.024482100545891717),
    ('return', -0.026592778462247283),
    ('back', -0.027742697230661331),
    ('get', -0.028711552980192581),
    ('disappointed', -0.028978976142317068),
    ('even', -0.030051249236035808),
    ('work', -0.033069515294752737),
    ('money', -0.038982037286487109),
    ('product', -0.041511033392108897),
    ('would', -0.053860148445203121)]
```

Quiz Question: Which word is not present in the top 10 "most negative" words?

### Quiz

1.  How many reviews in amazon_baby_subset.gl contain the word perfect?				
2955				
1 point				
2.				
Consider the <b>feature_matrix</b> that was obtained by	y converting our data to NumPy format			
How many features are there in the <b>feature_matr</b>	ix?			
194				

Week 2 - Module 3 Linear Classifier Learning assignment
3. Assuming that the intercept is present, how does the number of features in
y = x - 1
y = x
y = x + 1
None of the above
1 point 4.
Run your logistic regression solver with provided parameters.
As each iteration of gradient ascent passes, does the log-likelihood increase or decrease?
It increases.
It decreases.
None of the above
5. We make predictions using the weights just learned.
How many reviews were predicted to have positive sentiment?
25126
1 point

What is the accuracy of the model on predictions made above? (round to 2 digits of accuracy)

0.75

7.	We look at "most positive" words, the words that correspond most strongly with positive reviews.		
	Which of the following words is <b>not</b> present in the top 10 "most positive" words?		
	$\circ$	love	
	$\circ$	easy	
	$\circ$	great	
	$\bigcirc$	perfect	
		cheap	
8.	Similarly, we look at "most negative" words, the words that correspond most strongly with negative reviews.		
	Which of the following words is <b>not</b> present in the top 10 "most negative" words?		
		need	
	$\circ$	work	
	$\circ$	disappointed	
	$\circ$	even	
		return	