Decision Trees in Practice

In this assignment we will explore various techniques for preventing overfitting in decision trees. We will extend the implementation of the binary decision trees that we implemented in the previous assignment. You will have to use your solutions from this previous assignment and extend them.

In this assignment you will:

- Implement binary decision trees with different early stopping methods.
- Compare models with different stopping parameters.
- Visualize the concept of overfitting in decision trees.

Let's get started!

Fire up GraphLab Create

Make sure you have the latest version of GraphLab Create.

In [1]:

import graphlab

Load LendingClub Dataset

This assignment will use the <u>LendingClub (https://www.lendingclub.com/)</u> dataset used in the previous two assignments.

In [2]:

```
loans = graphlab.SFrame('lending-club-data.gl/')
```

This non-commercial license of GraphLab Create for academic use is assigned to amitha353@gmail.com and will expire on May 07, 2019.

[INFO] graphlab.cython.cy_server: GraphLab Create v2.1 started. Logging: C:\Users\Amitha\AppData\Local\Temp\graphlab_server_1532846266.log.0

As before, we reassign the labels to have +1 for a safe loan, and -1 for a risky (bad) loan.

In [3]:

```
print loans.column_names()

(122607, 68)
['id', 'member_id', 'loan_amnt', 'funded_amnt', 'funded_amnt_inv', 'term',
    'int_rate', 'installment', 'grade', 'sub_grade', 'emp_title', 'emp_length',
    'home_ownership', 'annual_inc', 'is_inc_v', 'issue_d', 'loan_status', 'pymnt
    _plan', 'url', 'desc', 'purpose', 'title', 'zip_code', 'addr_state', 'dti',
    'delinq_2yrs', 'earliest_cr_line', 'inq_last_6mths', 'mths_since_last_delin
    q', 'mths_since_last_record', 'open_acc', 'pub_rec', 'revol_bal', 'revol_uti
    l', 'total_acc', 'initial_list_status', 'out_prncp', 'out_prncp_inv', 'total
    pymnt', 'total_pymnt_inv', 'total_rec_prncp', 'total_rec_int', 'total_rec_l
    ate_fee', 'recoveries', 'collection_recovery_fee', 'last_pymnt_d', 'last_pym
    nt_amnt', 'next_pymnt_d', 'last_credit_pull_d', 'collections_12_mths_ex_me
    d', 'mths_since_last_major_derog', 'policy_code', 'not_compliant', 'status',
    'inactive_loans', 'bad_loans', 'emp_length_num', 'grade_num', 'sub_grade_nu
    m', 'delinq_2yrs_zero', 'pub_rec_zero', 'collections_12_mths_zero', 'short_e
    mp', 'payment_inc_ratio', 'final_d', 'last_delinq_none', 'last_record_none',
    'last_major_derog_none']
```

In [4]:

```
loans['safe_loans'] = loans['bad_loans'].apply(lambda x : +1 if x==0 else -1)
loans = loans.remove_column('bad_loans')
```

We will be using the same 4 categorical features as in the previous assignment:

- 1. grade of the loan
- 2. the length of the loan term
- 3. the home ownership status: own, mortgage, rent
- 4. number of years of employment.

In the dataset, each of these features is a categorical feature. Since we are building a binary decision tree, we will have to convert this to binary data in a subsequent section using 1-hot encoding.

In [5]:

In [6]:

grade	term	home_ownership	emp_length	safe_loans
+ B	36 months	RENT	10+ years	+ 1
c	60 months	RENT	< 1 year	-1
c	36 months	RENT	10+ years	1
C	36 months	RENT	10+ years	1
Α	36 months	RENT	3 years	1

Subsample dataset to make sure classes are balanced

Just as we did in the previous assignment, we will undersample the larger class (safe loans) in order to balance out our dataset. This means we are throwing away many data points. We used seed = 1 so everyone gets the same results.

In [7]:

Percentage of safe loans : 0.502236174422 Percentage of risky loans : 0.497763825578

Total number of loans in our new dataset : 46508

Note: There are many approaches for dealing with imbalanced data, including some where we modify the learning approaches are beyond the scope of this course, but some of them are reviewed in this <u>paper (http://ieeexplore.ieetp=&arnumber=5128907&url=http%3A%2F%2Fieeexplore.ieee.org%2Fiel5%2F69%2F5173046%2F05128907.pdf For this assignment, we use the simplest possible approach, where we subsample the overly represented class to dataset. In general, and especially when the data is highly imbalanced, we recommend using more advanced method.</u>

Transform categorical data into binary features

Since we are implementing **binary decision trees**, we transform our categorical data into binary data using 1-hot encoding, just as in the previous assignment. Here is the summary of that discussion:

For instance, the **home_ownership** feature represents the home ownership status of the loanee, which is either own, mortgage or rent. For example, if a data point has the feature

```
{'home_ownership': 'RENT'}

we want to turn this into three features:

{
    'home_ownership = OWN' : 0,
    'home_ownership = MORTGAGE' : 0,
    'home_ownership = RENT' : 1
}
```

Since this code requires a few Python and GraphLab tricks, feel free to use this block of code as is. Refer to the API documentation for a deeper understanding.

```
In [8]:
```

```
print "original loan data shape = ", loans_data.shape

original loan data shape = (46508, 5)

In [9]:

loans_data = risky_loans.append(safe_loans)
for feature in features:
    loans_data_one_hot_encoded = loans_data[feature].apply(lambda x: {x: 1})
    loans_data_unpacked = loans_data_one_hot_encoded.unpack(column_name_prefix=feature)

# Change None's to 0's
for column in loans_data_unpacked.column_names():
    loans_data_unpacked[column] = loans_data_unpacked[column].fillna(0)

loans_data.remove_column(feature)
loans_data.add_columns(loans_data_unpacked)
```

The feature columns now look like this:

```
In [10]:
```

```
features = loans data.column names()
features.remove('safe_loans') # Remove the response variable
features
Out[10]:
['grade.A',
 'grade.B',
 'grade.C',
 'grade.D',
 'grade.E',
 'grade.F'
 'grade.G',
 'term. 36 months',
 'term. 60 months',
 'home_ownership.MORTGAGE',
 'home_ownership.OTHER',
 'home ownership.OWN',
 'home_ownership.RENT',
 'emp_length.1 year',
 'emp_length.10+ years',
 'emp_length.2 years',
 'emp length.3 years'
 'emp_length.4 years',
 'emp_length.5 years',
 'emp_length.6 years',
 'emp_length.7 years'
 'emp_length.8 years',
 'emp length.9 years',
 'emp_length.< 1 year',</pre>
 'emp_length.n/a']
```

Train-Validation split

We split the data into a train-validation split with 80% of the data in the training set and 20% of the data in the validation set. We use seed=1 so that everyone gets the same result.

```
In [11]:
train_data, validation_set = loans_data.random_split(.8, seed=1)
```

Early stopping methods for decision trees

In this section, we will extend the **binary tree implementation** from the previous assignment in order to handle some early stopping conditions. Recall the 3 early stopping methods that were discussed in lecture:

- 1. Reached a maximum depth. (set by parameter max_depth).
- 2. Reached a **minimum node size**. (set by parameter min_node_size).
- 3. Don't split if the gain in error reduction is too small. (set by parameter min error reduction).

For the rest of this assignment, we will refer to these three as early stopping conditions 1, 2, and 3.

Early stopping condition 1: Maximum depth

Recall that we already implemented the maximum depth stopping condition in the previous assignment. In this assignment, we will experiment with this condition a bit more and also write code to implement the 2nd and 3rd early stopping conditions.

We will be reusing code from the previous assignment and then building upon this. We will **alert you** when you reach a function that was part of the previous assignment so that you can simply copy and past your previous code.

Early stopping condition 2: Minimum node size

The function **reached_minimum_node_size** takes 2 arguments:

- 1. The data (from a node)
- 2. The minimum number of data points that a node is allowed to split on, min_node_size.

This function simply calculates whether the number of data points at a given node is less than or equal to the specified minimum node size. This function will be used to detect this early stopping condition in the **decision_tree_create** function.

Fill in the parts of the function below where you find ## YOUR CODE HERE. There is **one** instance in the function below.

```
In [12]:
```

```
def reached_minimum_node_size(data, min_node_size):
    # Return True if the number of data points is less than or equal to the minimum node si
## YOUR CODE HERE
    if len(data)<= min_node_size:
        return True</pre>
```

Quiz Question: Given an intermediate node with 6 safe loans and 3 risky loans, if the min_node_size parameter is 10, what should the tree learning algorithm do next?

- 6 + 3 = 9;
- 9 < 10
- stop when node size < min node size.
- · Create a leaf and return it.

Early stopping condition 3: Minimum gain in error reduction

The function error_reduction takes 2 arguments:

- 1. The error **before** a split, error before split.
- 2. The error after a split, error_after_split.

This function computes the gain in error reduction, i.e., the difference between the error before the split and that after the split. This function will be used to detect this early stopping condition in the **decision_tree_create** function.

Fill in the parts of the function below where you find ## YOUR CODE HERE. There is **one** instance in the function below.

```
In [13]:
```

```
def error_reduction(error_before_split, error_after_split):
    # Return the error before the split minus the error after the split.
## YOUR CODE HERE
error = error_before_split - error_after_split
return error
```

Quiz Question: Assume an intermediate node has 6 safe loans and 3 risky loans. For each of 4 possible features to split on, the error reduction is 0.0, 0.05, 0.1, and 0.14, respectively. If the **minimum gain in error reduction** parameter is set to 0.2, what should the tree learning algorithm do next?

- The minimum gain in error reduction (0.2) > all possible error reductions. [0.00, 0.05, 0.1, 0.14]
- · Tree learning algorithm should create leaf and stop.

Grabbing binary decision tree helper functions from past assignment

Recall from the previous assignment that we wrote a function intermediate_node_num_mistakes that calculates the number of **misclassified examples** when predicting the **majority class**. This is used to help determine which feature is best to split on at a given node of the tree.

Please copy and paste your code for intermediate node num mistakes here.

In [14]:

```
def intermediate_node_num_mistakes(labels_in_node):
    # Corner case: If labels_in_node is empty, return 0
    if len(labels_in_node) == 0:
        return 0

# Count the number of 1's (safe loans)
## YOUR CODE HERE
num_of_positive = (labels_in_node == +1).sum()

# Count the number of -1's (risky loans)
## YOUR CODE HERE
num_of_negative = (labels_in_node == -1).sum()

# Return the number of mistakes that the majority classifier makes.
## YOUR CODE HERE
return num_of_negative if num_of_positive > num_of_negative else num_of_positive
```

We then wrote a function best_splitting_feature that finds the best feature to split on given the data and a list of features to consider.

Please copy and paste your best splitting feature code here.

In [15]:

```
def best splitting feature(data, features, target):
    target_values = data[target]
    best_feature = None # Keep track of the best feature
    best_error = 10
                       # Keep track of the best error so far
    # Note: Since error is always <= 1, we should intialize it with something larger than 1
    # Convert to float to make sure error gets computed correctly.
    num_data_points = float(len(data))
    # Loop through each feature to consider splitting on that feature
    for feature in features:
        # The left split will have all data points where the feature value is 0
        left_split = data[data[feature] == 0]
        # The right split will have all data points where the feature value is 1
        ## YOUR CODE HERE
        right_split = data[data[feature] == 1]
        # Calculate the number of misclassified examples in the left split.
        # Remember that we implemented a function for this! (It was called intermediate_nod
        # YOUR CODE HERE
        left_mistakes = intermediate_node_num_mistakes(left_split[target])
        # Calculate the number of misclassified examples in the right split.
        ## YOUR CODE HERE
        right_mistakes = intermediate_node_num_mistakes(right_split[target])
        # Compute the classification error of this split.
        # Error = (# of mistakes (left) + # of mistakes (right)) / (# of data points)
        ## YOUR CODE HERE
        error = (left_mistakes + right_mistakes) / num_data_points
        # If this is the best error we have found so far, store the feature as best_feature
        ## YOUR CODE HERE
        if error < best error:</pre>
            best_feature = feature
            best_error = error
    return best feature # Return the best feature we found
```

Finally, recall the function create_leaf from the previous assignment, which creates a leaf node given a set of target values.

Please copy and paste your create leaf code here.

In [16]:

```
def create leaf(target values):
    # Create a leaf node
    leaf = {'splitting_feature' : None,
            'left' : None,
            'right' : None,
            'is_leaf': True
                                   ## YOUR CODE HERE
                               }
    # Count the number of data points that are +1 and -1 in this node.
    num ones = len(target values[target values == +1])
    num_minus_ones = len(target_values[target_values == -1])
    # For the leaf node, set the prediction to be the majority class.
    # Store the predicted class (1 or -1) in leaf['prediction']
    if num_ones > num_minus_ones:
        leaf['prediction'] = +1 ## YOUR CODE HERE
    else:
        leaf['prediction'] = -1 ## YOUR CODE HERE
    # Return the Leaf node
    return leaf
```

Incorporating new early stopping conditions in binary decision tree implementation

Now, you will implement a function that builds a decision tree handling the three early stopping conditions described in this assignment. In particular, you will write code to detect early stopping conditions 2 and 3. You implemented above the functions needed to detect these conditions. The 1st early stopping condition, **max_depth**, was implemented in the previous assignment and you will not need to reimplement this. In addition to these early stopping conditions, the typical stopping conditions of having no mistakes or no more features to split on (which we denote by "stopping conditions" 1 and 2) are also included as in the previous assignment.

Implementing early stopping condition 2: minimum node size:

- Step 1: Use the function reached_minimum_node_size that you implemented earlier to write an if condition to detect whether we have hit the base case, i.e., the node does not have enough data points and should be turned into a leaf. Don't forget to use the min node size argument.
- **Step 2:** Return a leaf. This line of code should be the same as the other (pre-implemented) stopping conditions.

Implementing early stopping condition 3: minimum error reduction:

Note: This has to come after finding the best splitting feature so we can calculate the error after splitting in order to calculate the error reduction.

• Step 1: Calculate the classification error before splitting. Recall that classification error is defined as:

classification error =
$$\frac{\text{\# mistakes}}{\text{\# total examples}}$$

• **Step 2:** Calculate the **classification error after splitting**. This requires calculating the number of mistakes in the left and right splits, and then dividing by the total number of examples.

- **Step 3:** Use the function **error_reduction** to that you implemented earlier to write an if condition to detect whether the reduction in error is less than the constant provided (min_error_reduction). Don't forget to use that argument.
- **Step 4:** Return a leaf. This line of code should be the same as the other (pre-implemented) stopping conditions.

Fill in the places where you find ## YOUR CODE HERE. There are **seven** places in this function for you to fill in.

In [17]:

```
def decision_tree_create(data, features, target, current_depth = 0,
                        max_depth = 10, min_node_size=1,
                        min_error_reduction=0.0):
    remaining_features = features[:] # Make a copy of the features.
   target_values = data[target]
    print "-----"
    print "Subtree, depth = %s (%s data points)." % (current_depth, len(target_values))
   # Stopping condition 1: All nodes are of the same type.
    if intermediate_node_num_mistakes(target_values) == 0:
       print "Stopping condition 1 reached. All data points have the same target value."
       return create_leaf(target_values)
   # Stopping condition 2: No more features to split on.
    if remaining_features == []:
       print "Stopping condition 2 reached. No remaining features."
       return create_leaf(target_values)
   # Early stopping condition 1: Reached max depth limit.
    if current depth >= max depth:
       print "Early stopping condition 1 reached. Reached maximum depth."
       return create_leaf(target_values)
   # Early stopping condition 2: Reached the minimum node size.
   # If the number of data points is less than or equal to the minimum size, return a leaf
    if reached_minimum_node_size(data, min_node_size):
                                                              ## YOUR CODE HERE
       print "Early stopping condition 2 reached. Reached minimum node size."
       return create_leaf(target_values) ## YOUR CODE HERE
   # Find the best splitting feature
   splitting_feature = best_splitting_feature(data, features, target)
   # Split on the best feature that we found.
    left split = data[data[splitting feature] == 0]
   right_split = data[data[splitting_feature] == 1]
   # Early stopping condition 3: Minimum error reduction
   # Calculate the error before splitting (number of misclassified examples
   # divided by the total number of examples)
    error_before_split = intermediate_node_num_mistakes(target_values) / float(len(data))
   # Calculate the error after splitting (number of misclassified examples
   # in both groups divided by the total number of examples)
                     intermediate node num mistakes(left split[target]) ## YOUR CODE HERE
    right_mistakes = intermediate_node_num_mistakes(right_split[target]) ## YOUR CODE HERE
    error_after_split = (left_mistakes + right_mistakes) / float(len(data))
   # If the error reduction is LESS THAN OR EQUAL TO min_error_reduction, return a leaf.
    if error reduction(error before split, error after split) <= min error reduction:</pre>
       print "Early stopping condition 3 reached. Minimum error reduction."
       return create leaf(target values)
    remaining_features.remove(splitting_feature)
    print "Split on feature %s. (%s, %s)" % (\
                     splitting_feature, len(left_split), len(right_split))
```

```
# Repeat (recurse) on Left and right subtrees
left_tree = decision_tree_create(left_split, remaining_features, target,
                                 current_depth + 1, max_depth, min_node_size,
                                 min_error_reduction)
## YOUR CODE HERE
right_tree = decision_tree_create(right_split, remaining_features, target,
                                 current_depth + 1, max_depth, min_node_size,
                                  min_error_reduction)
return {'is_leaf'
                          : False,
        'prediction'
                          : None,
        'splitting_feature': splitting_feature,
                          : left_tree,
        'right'
                          : right_tree}
```

Here is a function to count the nodes in your tree:

In [18]:

```
def count_nodes(tree):
    if tree['is_leaf']:
        return 1
    return 1 + count_nodes(tree['left']) + count_nodes(tree['right'])
```

Run the following test code to check your implementation. Make sure you get 'Test passed' before proceeding.

```
In [19]:
```

```
Subtree, depth = 0 (37224 data points).
Split on feature term. 36 months. (9223, 28001)
______
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
Subtree, depth = 2 (9122 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 2 (101 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 1 (28001 data points).
Split on feature grade.D. (23300, 4701)
______
Subtree, depth = 2 (23300 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 2 (4701 data points).
Early stopping condition 1 reached. Reached maximum depth.
Test passed!
```

Build a tree!

Now that your code is working, we will train a tree model on the train data with

```
max_depth = 6min_node_size = 100,min error reduction = 0.0
```

Warning: This code block may take a minute to learn.

```
In [20]:
```

```
my_decision_tree_new = decision_tree_create(train_data, features, 'safe_loans', max_depth =
                     min_node_size = 100, min_error_reduction=0.0)
______
Subtree, depth = 0 (37224 data points).
Split on feature term. 36 months. (9223, 28001)
______
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
Subtree, depth = 2 (9122 data points).
Early stopping condition 3 reached. Minimum error reduction.
______
Subtree, depth = 2 (101 data points).
Split on feature emp length.n/a. (96, 5)
______
Subtree, depth = 3 (96 data points).
Early stopping condition 2 reached. Reached minimum node size.
______
Subtree, depth = 3 (5 data points).
Early stopping condition 2 reached. Reached minimum node size.
______
Subtree, depth = 1 (28001 data points).
Split on feature grade.D. (23300, 4701)
______
Subtree, depth = 2 (23300 data points).
Split on feature grade.E. (22024, 1276)
______
Subtree, depth = 3 (22024 data points).
Split on feature grade.F. (21666, 358)
Subtree, depth = 4 (21666 data points).
Split on feature emp_length.n/a. (20734, 932)
______
Subtree, depth = 5 (20734 data points).
Split on feature grade.G. (20638, 96)
______
Subtree, depth = 6 (20638 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (96 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 5 (932 data points).
Split on feature grade.A. (702, 230)
______
Subtree, depth = 6 (702 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (230 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 4 (358 data points).
Split on feature emp_length.8 years. (347, 11)
______
Subtree, depth = 5 (347 data points).
Early stopping condition 3 reached. Minimum error reduction.
______
```

Subtree, depth = 5 (11 data points).

```
Early stopping condition 2 reached. Reached minimum node size.

Subtree, depth = 3 (1276 data points).

Early stopping condition 3 reached. Minimum error reduction.

Subtree, depth = 2 (4701 data points).

Early stopping condition 3 reached. Minimum error reduction.
```

Let's now train a tree model **ignoring early stopping conditions 2 and 3** so that we get the same tree as in the previous assignment. To ignore these conditions, we set min_node_size=0 and min_error_reduction=-1 (a negative value).

In [21]:

```
my decision tree old = decision_tree_create(train_data, features, 'safe_loans',
                                 max_depth = 6,
                        min_node_size = 0, min_error_reduction=-1)
Subtree, depth = 0 (37224 data points).
Split on feature term. 36 months. (9223, 28001)
______
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
Subtree, depth = 2 (9122 data points).
Split on feature grade.B. (8074, 1048)
______
Subtree, depth = 3 (8074 data points).
Split on feature grade.C. (5884, 2190)
Subtree, depth = 4 (5884 data points).
Split on feature grade.D. (3826, 2058)
Subtree, depth = 5 (3826 data points).
Split on feature grade.E. (1693, 2133)
______
```

Making predictions

Recall that in the previous assignment you implemented a function classify to classify a new point x using a given tree.

Please copy and paste your classify code here.

```
In [22]:
```

```
def classify(tree, x, annotate = False):
    # if the node is a Leaf node.
    if tree['is_leaf']:
        if annotate:
            print "At leaf, predicting %s" % tree['prediction']
        return tree['prediction']
    else:
        # split on feature.
        split_feature_value = x[tree['splitting_feature']]
        if annotate:
            print "Split on %s = %s" % (tree['splitting_feature'], split_feature_value)
        if split_feature_value == 0:
            return classify(tree['left'], x, annotate)
        else:
            return classify(tree['right'], x, annotate) ### YOUR CODE HERE
```

Now, let's consider the first example of the validation set and see what the my_decision_tree_new model predicts for this data point.

```
In [23]:
validation set[0]
Out[23]:
{'emp_length.1 year': 0L,
 'emp_length.10+ years': 0L,
 'emp_length.2 years': 1L,
 'emp length.3 years': 0L,
 'emp_length.4 years': 0L,
 'emp_length.5 years': 0L,
 'emp_length.6 years': 0L,
 'emp_length.7 years': 0L,
 'emp_length.8 years': 0L,
 'emp_length.9 years': 0L,
 'emp_length.< 1 year': 0L,</pre>
 'emp_length.n/a': 0L,
 'grade.A': 0L,
 'grade.B': 0L,
 'grade.C': 0L,
 'grade.D': 1L,
 'grade.E': 0L,
 'grade.F': 0L,
 'grade.G': 0L,
 'home_ownership.MORTGAGE': 0L,
 'home_ownership.OTHER': 0L,
 'home ownership.OWN': 0L,
 'home ownership.RENT': 1L,
 'safe loans': -1L,
 'term. 36 months': 0L,
 'term. 60 months': 1L}
In [24]:
print 'Predicted class: %s ' % classify(my_decision_tree_new, validation_set[0])
Predicted class: -1
```

Let's add some annotations to our prediction to see what the prediction path was that lead to this predicted class:

```
In [25]:
```

```
classify(my_decision_tree_new, validation_set[0], annotate = True)

Split on term. 36 months = 0

Split on grade.A = 0

At leaf, predicting -1

Out[25]:
-1
```

Let's now recall the prediction path for the decision tree learned in the previous assignment, which we recreated here as my_decision_tree_old.

In [26]:

```
classify(my_decision_tree_old, validation_set[0], annotate = True)
Split on term. 36 months = 0
Split on grade.A = 0
Split on grade.B = 0
Split on grade.C = 0
Split on grade.D = 1
Split on grade.E = 0
At leaf, predicting -1
Out[26]:
-1
In [41]:
print "Number of nodes (my_decision_tree_new):", count_leaves(my_decision_tree_new)
print "----"
print "Number of nodes (my_decision_tree_old):", count_leaves(my_decision_tree_old)
Number of nodes (my_decision_tree_new): 11
Number of nodes (my decision tree old): 41
```

Quiz Question: For my_decision_tree_new trained with max_depth = 6, min_node_size = 100, min_error_reduction=0.0, is the prediction path for validation_set[0] shorter, longer, or the same as for my decision tree old that ignored the early stopping conditions 2 and 3?

my_decision_tree_new has fewer leafs than my_decision_tree_old when tested by vali dation_set[0].

Quiz Question: For my_decision_tree_new trained with max_depth = 6, min_node_size = 100, min_error_reduction=0.0, is the prediction path for **any point** always shorter, always longer, always the same, shorter or the same, or longer or the same as for my_decision_tree_old that ignored the early stopping conditions 2 and 3?

```
always shorter,
always longer,
always the same,
shorter or the same, [correct]
longer or the same
```

Quiz Question: For a tree trained on **any** dataset using max_depth = 6, min_node_size = 100, min_error_reduction=0.0, what is the maximum number of splits encountered while making a single prediction?

```
maximum number of splits = max_depth = 6
```

Evaluating the model

Now let us evaluate the model that we have trained. You implemented this evaluation in the function evaluate classification error from the previous assignment.

Please copy and paste your evaluate_classification_error code here.

```
In [27]:
```

```
def evaluate_classification_error(tree, data, target):
    # Apply the classify(tree, x) to each row in your data
    prediction = data.apply(lambda x: classify(tree, x))

# Once you've made the predictions, calculate the classification error and return it
    ## YOUR CODE HERE
    num_of_mistakes = (prediction != data[target]).sum()/float(len(data))
    return num_of_mistakes
```

Now, let's use this function to evaluate the classification error of my_decision_tree_new on the **validation_set**.

```
In [28]:
```

```
evaluate_classification_error(my_decision_tree_new, validation_set, target)
Out[28]:
```

0.38367083153813014

Now, evaluate the validation error using my_decision_tree_old.

```
In [29]:
```

```
evaluate_classification_error(my_decision_tree_old, validation_set, target)
Out[29]:
```

0.3837785437311504

Quiz Question: Is the validation error of the new decision tree (using early stopping conditions 2 and 3) lower

than, higher than, or the same as that of the old decision tree from the previous assignment?

```
In [42]:
```

```
my_decision_tree_new_error = evaluate_classification_error(my_decision_tree_new, validatior
my_decision_tree_old_error = evaluate_classification_error(my_decision_tree_old, validatior
print my_decision_tree_new_error - my_decision_tree_old_error
```

-0.00010771219302

In [43]:

```
my_decision_tree_new < my_decision_tree_old</pre>
```

Out[43]:

False

Exploring the effect of max_depth

We will compare three models trained with different values of the stopping criterion. We intentionally picked models at the extreme ends (too small, just right, and too large).

Train three models with these parameters:

- 1. model_1: max_depth = 2 (too small)
- 2. model_2: max_depth = 6 (just right)
- 3. model_3: max_depth = 14 (may be too large)

For each of these three, we set min_node_size = 0 and min_error_reduction = -1.

Note: Each tree can take up to a few minutes to train. In particular, mode1_3 will probably take the longest to train.

```
In [30]:
```

```
model_1 = decision_tree_create(train_data, features, 'safe_loans', max_depth = 2,
                           min_node_size = 0, min_error_reduction=-1)
model 2 = decision_tree_create(train_data, features, 'safe_loans', max_depth = 6,
                           min node size = 0, min error reduction=-1)
model_3 = decision_tree_create(train_data, features, 'safe_loans', max_depth = 14,
                           min_node_size = 0, min_error_reduction=-1)
Subtree, depth = 0 (37224 data points).
Split on feature term. 36 months. (9223, 28001)
______
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
______
Subtree, depth = 2 (9122 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 2 (101 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 1 (28001 data points).
Split on feature grade.D. (23300, 4701)
Subtree, depth = 2 (23300 data points).
Early stopping condition 1 reached. Reached maximum depth.
```

Evaluating the models

Let us evaluate the models on the **train** and **validation** data. Let us start by evaluating the classification error on the training data:

```
In [31]:
```

```
print "Training data, classification error (model 1):", evaluate_classification_error(model
print "Training data, classification error (model 2):", evaluate_classification_error(model
print "Training data, classification error (model 3):", evaluate_classification_error(model
Training data, classification error (model 1): 0.400037610144
Training data, classification error (model 2): 0.381850419084
Training data, classification error (model 3): 0.374462712229
```

Now evaluate the classification error on the validation data.

```
In [32]:
```

```
print "validation_set, classification error (model 1):", evaluate_classification_error(mode
print "validation_set, classification error (model 2):", evaluate_classification_error(mode
print "validation_set, classification error (model 3):", evaluate_classification_error(mode
validation_set, classification error (model 1): 0.398104265403
validation_set, classification error (model 2): 0.383778543731
validation_set, classification error (model 3): 0.380008616975
```

Quiz Question: Which tree has the smallest error on the validation data?

- Model 3

Quiz Question: Does the tree with the smallest error in the training data also have the smallest error in the validation data?

- Yes

Quiz Question: Is it always true that the tree with the lowest classification error on the **training** set will result in the lowest classification error in the **validation** set?

- No

Measuring the complexity of the tree

Recall in the lecture that we talked about deeper trees being more complex. We will measure the complexity of the tree as

```
complexity(T) = number of leaves in the tree T
```

Here, we provide a function count_leaves that counts the number of leaves in a tree. Using this implementation, compute the number of nodes in model_1, model_2, and model_3.

In [33]:

```
def count_leaves(tree):
    if tree['is_leaf']:
        return 1
    return count_leaves(tree['left']) + count_leaves(tree['right'])
```

Compute the number of nodes in model_1, model_2, and model_3.

```
In [34]:
```

```
print "Number of nodes (model 1):", count_leaves(model_1)
print "Number of nodes (model 2):", count_leaves(model_2)
print "Number of nodes (model 3):", count_leaves(model_3)

Number of nodes (model 1): 4
Number of nodes (model 2): 41
Number of nodes (model 3): 341
```

Quiz Question: Which tree has the largest complexity?

```
- Model 3 (More Leaves)
```

Quiz Question: Is it always true that the most complex tree will result in the lowest classification error in the **validation_set?**

- Not always

Exploring the effect of min error

We will compare three models trained with different values of the stopping criterion. We intentionally picked models at the extreme ends (**negative**, **just right**, and **too positive**).

Train three models with these parameters:

- model_4: min_error_reduction = -1 (ignoring this early stopping condition)
- 2. model_5: min error reduction = 0 (just right)
- 3. model_6: min_error_reduction = 5 (too positive)

For each of these three, we set max_depth = 6, and min_node_size = 0.

Note: Each tree can take up to 30 seconds to train.

C..htman danth C /1002 data maintal

In [35]:

```
model_4 = decision_tree_create(train_data, features, 'safe_loans', max_depth = 6,
                          min_node_size = 0, min_error_reduction=-1)
model_5 = decision_tree_create(train_data, features, 'safe_loans', max_depth = 6,
                          min_node_size = 0, min_error_reduction=0)
model 6 = decision tree create(train data, features, 'safe loans', max depth = 6,
                          min_node_size = 0, min_error_reduction=5)
-----
Subtree, depth = 0 (37224 data points).
Split on feature term. 36 months. (9223, 28001)
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
-----
Subtree, depth = 2 (9122 data points).
Split on feature grade.B. (8074, 1048)
Subtree, depth = 3 (8074 data points).
Split on feature grade.C. (5884, 2190)
______
Subtree, depth = 4 (5884 data points).
Split on feature grade.D. (3826, 2058)
_____
Subtree, depth = 5 (3826 data points).
Split on feature grade.E. (1693, 2133)
```

Calculate the accuracy of each model (model_4, model_5, or model_6) on the validation set.

In [36]:

```
print "Validation data, classification error (model 4):", evaluate_classification_error(model print "Validation data, classification error (model 5):", evaluate_classification_error(model print "Validation data, classification error (model 6):", evaluate_classification_error(model validation data, classification error (model 4): 0.383778543731
Validation data, classification error (model 5): 0.383778543731
Validation data, classification error (model 6): 0.503446790177
```

Using the count_leaves function, compute the number of leaves in each of each models in (**model_4**, **model_5**, and **model_6**).

In [37]:

```
print "Number of nodes (model 4):", count_leaves(model_4)
print "Number of nodes (model 5):", count_leaves(model_5)
print "Number of nodes (model 6):", count_leaves(model_6)
Number of nodes (model 4): 41
```

```
Number of nodes (model 4): 41
Number of nodes (model 5): 13
Number of nodes (model 6): 1
```

Quiz Question: Using the complexity definition above, which model (**model_4**, **model_5**, or **model_6**) has the largest complexity? -model 4 has greatest complexity.

Did this match your expectation?

- yes. When the impact of reducing the parameter "min_error_reduction" is consider ed, the impact of smaller min_error_reduction producing more mode and more complex ity is expected.

Quiz Question: model_4 and model_5 have similar classification error on the validation set but model_5 has lower complexity. Should you pick model_5 over model_4?

```
- yes, pick model_5 over model_4 because lower complexity is good. Occums Razor etc.
```

Exploring the effect of min_node_size

We will compare three models trained with different values of the stopping criterion. Again, intentionally picked models at the extreme ends (too small, just right, and just right).

Train three models with these parameters:

```
    model_7: min_node_size = 0 (too small)
    model_8: min_node_size = 2000 (just right)
    model_9: min_node_size = 50000 (too large)
```

For each of these three, we set max depth = 6, and min error reduction = -1.

Note: Each tree can take up to 30 seconds to train.

```
In [38]:
```

```
______
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
______
Subtree, depth = 2 (9122 data points).
Split on feature grade.B. (8074, 1048)
Subtree, depth = 3 (8074 data points).
Split on feature grade.C. (5884, 2190)
______
Subtree, depth = 4 (5884 data points).
Split on feature grade.D. (3826, 2058)
______
Subtree, depth = 5 (3826 data points).
Split on feature grade.E. (1693, 2133)
______
Subtree, depth = 6 (1693 data points).
Early stopping condition 1 reached. Reached maximum depth.
  Subtree, depth = 6 (2133 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 5 (2058 data points).
Split on feature grade.E. (2058, 0)
Subtree, depth = 6 (2058 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 4 (2190 data points).
Split on feature grade.D. (2190, 0)
______
Subtree, depth = 5 (2190 data points).
Split on feature grade.E. (2190, 0)
-----
Subtree, depth = 6 (2190 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 3 (1048 data points).
Split on feature emp_length.5 years. (969, 79)
```

```
Subtree, depth = 4 (969 data points).
Split on feature grade.C. (969, 0)
Subtree, depth = 5 (969 data points).
Split on feature grade.D. (969, 0)
______
Subtree, depth = 6 (969 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
-----
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 4 (79 data points).
Split on feature home_ownership.MORTGAGE. (34, 45)
______
Subtree, depth = 5 (34 data points).
Split on feature grade.C. (34, 0)
-----
Subtree, depth = 6 (34 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 5 (45 data points).
Split on feature grade.C. (45, 0)
-----
                       Subtree, depth = 6 (45 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
_____
Subtree, depth = 2 (101 data points).
Split on feature emp_length.n/a. (96, 5)
______
Subtree, depth = 3 (96 data points).
Split on feature emp_length.< 1 year. (85, 11)
-----
Subtree, depth = 4 (85 data points).
Split on feature grade.B. (85, 0)
Subtree, depth = 5 (85 data points).
Split on feature grade.C. (85, 0)
______
Subtree, depth = 6 (85 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
-----
Subtree, depth = 4 (11 data points).
Split on feature grade.B. (11, 0)
______
Subtree, depth = 5 (11 data points).
```

```
Split on feature grade.C. (11, 0)
Subtree, depth = 6 (11 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 3 (5 data points).
Split on feature grade.B. (5, 0)
-----
Subtree, depth = 4 (5 data points).
Split on feature grade.C. (5, 0)
Subtree, depth = 5 (5 data points).
Split on feature grade.D. (5, 0)
______
Subtree, depth = 6 (5 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 4 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 1 (28001 data points).
Split on feature grade.D. (23300, 4701)
______
Subtree, depth = 2 (23300 data points).
Split on feature grade.E. (22024, 1276)
______
Subtree, depth = 3 (22024 data points).
Split on feature grade.F. (21666, 358)
_____
Subtree, depth = 4 (21666 data points).
Split on feature emp_length.n/a. (20734, 932)
-----
Subtree, depth = 5 (20734 data points).
Split on feature grade.G. (20638, 96)
Subtree, depth = 6 (20638 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (96 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 5 (932 data points).
Split on feature grade.A. (702, 230)
-----
Subtree, depth = 6 (702 data points).
Early stopping condition 1 reached. Reached maximum depth.
  ______
Subtree, depth = 6 (230 data points).
Early stopping condition 1 reached. Reached maximum depth.
```

```
Subtree, depth = 4 (358 data points).
Split on feature emp length.8 years. (347, 11)
______
Subtree, depth = 5 (347 data points).
Split on feature grade.A. (347, 0)
-----
Subtree, depth = 6 (347 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 5 (11 data points).
Split on feature home_ownership.OWN. (9, 2)
______
Subtree, depth = 6 (9 data points).
Early stopping condition 1 reached. Reached maximum depth.
_____
Subtree, depth = 6 (2 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 3 (1276 data points).
Split on feature grade.A. (1276, 0)
______
Subtree, depth = 4 (1276 data points).
Split on feature grade.B. (1276, 0)
______
Subtree, depth = 5 (1276 data points).
Split on feature grade.C. (1276, 0)
Subtree, depth = 6 (1276 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 4 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 2 (4701 data points).
Split on feature grade.A. (4701, 0)
______
Subtree, depth = 3 (4701 data points).
Split on feature grade.B. (4701, 0)
-----
                         -----
Subtree, depth = 4 (4701 data points).
Split on feature grade.C. (4701, 0)
Subtree, depth = 5 (4701 data points).
Split on feature grade.E. (4701, 0)
______
Subtree, depth = 6 (4701 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
```

```
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
Subtree, depth = 4 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 3 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 0 (37224 data points).
Split on feature term. 36 months. (9223, 28001)
-----
Subtree, depth = 1 (9223 data points).
Split on feature grade.A. (9122, 101)
______
Subtree, depth = 2 (9122 data points).
Split on feature grade.B. (8074, 1048)
______
Subtree, depth = 3 (8074 data points).
Split on feature grade.C. (5884, 2190)
______
Subtree, depth = 4 (5884 data points).
Split on feature grade.D. (3826, 2058)
Subtree, depth = 5 (3826 data points).
Split on feature grade.E. (1693, 2133)
______
Subtree, depth = 6 (1693 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (2133 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 5 (2058 data points).
Split on feature grade.E. (2058, 0)
Subtree, depth = 6 (2058 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 4 (2190 data points).
Split on feature grade.D. (2190, 0)
Subtree, depth = 5 (2190 data points).
Split on feature grade.E. (2190, 0)
______
Subtree, depth = 6 (2190 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
-----
Subtree, depth = 3 (1048 data points).
Early stopping condition 2 reached. Reached minimum node size.
______
Subtree, depth = 2 (101 data points).
```

```
Early stopping condition 2 reached. Reached minimum node size.
Subtree, depth = 1 (28001 data points).
Split on feature grade.D. (23300, 4701)
______
Subtree, depth = 2 (23300 data points).
Split on feature grade.E. (22024, 1276)
______
Subtree, depth = 3 (22024 data points).
Split on feature grade.F. (21666, 358)
______
Subtree, depth = 4 (21666 data points).
Split on feature emp_length.n/a. (20734, 932)
______
Subtree, depth = 5 (20734 data points).
Split on feature grade.G. (20638, 96)
Subtree, depth = 6 (20638 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 6 (96 data points).
Early stopping condition 1 reached. Reached maximum depth.
______
Subtree, depth = 5 (932 data points).
Early stopping condition 2 reached. Reached minimum node size.
______
Subtree, depth = 4 (358 data points).
Early stopping condition 2 reached. Reached minimum node size.
------
Subtree, depth = 3 (1276 data points).
Early stopping condition 2 reached. Reached minimum node size.
______
Subtree, depth = 2 (4701 data points).
Split on feature grade.A. (4701, 0)
-----
Subtree, depth = 3 (4701 data points).
Split on feature grade.B. (4701, 0)
-----
Subtree, depth = 4 (4701 data points).
Split on feature grade.C. (4701, 0)
_____
Subtree, depth = 5 (4701 data points).
Split on feature grade.E. (4701, 0)
_____
Subtree, depth = 6 (4701 data points).
Early stopping condition 1 reached. Reached maximum depth.
Subtree, depth = 6 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 5 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
______
Subtree, depth = 4 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
-----
Subtree, depth = 3 (0 data points).
Stopping condition 1 reached. All data points have the same target value.
Subtree, depth = 0 (37224 data points).
Early stopping condition 2 reached. Reached minimum node size.
```

Now, let us evaluate the models (model_7, model_8, or model_9) on the validation_set.

```
In [39]:
```

```
print "Validation data, classification error (model 7):", evaluate_classification_error(model 7):", evaluate_classification_error(model 8):", evaluate_classification_error(model 7):", evaluate_classification_error(model 7):", evaluate_classification_error(model 7): 0.383778543731
Validation data, classification error (model 7): 0.384532529082
Validation data, classification error (model 9): 0.503446790177
```

Using the count_leaves function, compute the number of leaves in each of each models (model_7, model_8, and model 9).

In [40]:

```
print "Number of nodes (model 7):", count_leaves(model_7)
print "Number of nodes (model 8):", count_leaves(model_8)
print "Number of nodes (model 9):", count_leaves(model_9)

Number of nodes (model 7): 41
Number of nodes (model 8): 19
Number of nodes (model 9): 1
```

Quiz Question: Using the results obtained in this section, which model (**model_7**, **model_8**, or **model_9**) would you choose to use?

- Model 8

Quiz

1.	Given an intermediate node with 6 safe loans and 3 risky loans, if the min_node_size parameter is 10, what should the tree learning algorithm do next?					
	Create a leaf and return it					
	Continue building the tree by finding the best splitting feature					
2.	Assume an intermediate node has 6 safe loans and 3 risky loans. For each of 4 possible features to split on, the error reduction is 0.0, 0.05, 0.1, and 0.14, respectively. If the minimum gain in error reduction parameter is set to 0.2, what should the tree learning algorithm do next?					
	Create a leaf and return it					
	Continue building the tree by using the splitting feature that gives 0.14 error reduction					
3.	Consider the prediction path validation_set[0] with my_decision_tree_old and my_decision_tree_new. For my_decision_tree_new trained with					
	1 max_depth = 6, min_node_size = 180, min_error_reduction=0.0					
	is the prediction path shorter, longer, or the same as the prediction path using my_decision_tree_old that ignored the early stopping conditions 2 and 3?					
	Shorter					
	Longer					
	○ The same					
4.	Consider the prediction path for ANY new data point. For my_decision_tree_new trained with					
	1 max_depth = 6, min_node_size = 180, min_error_reduction=8.8					
	is the prediction path for a data point always shorter, always longer, always the same, shorter or the same, or longer or the same as for my_decision_tree_old that ignored the early stopping conditions 2 and 3?					
	Always shorter					
	Always longer					
	Always the same					
	Shorter or the same					
	Longer or the same					
5.	For a tree trained on any dataset using parameters					
	1 max_depth = 6, min_node_size = 180, min_error_reduction=0.0					
	what is the maximum possible number of splits encountered while making a single prediction?					
	6					

6.	Is the validation error of the new decision tree (using early stopping conditions 2 and 3) lower than, higher than, or the same as that of the old decision tree from the previous assigment?			
	Higher than			
	Lower than			
	The same			
7.	Which tree has the smallest error on the validation data?			
	modeL1			
	modeL2			
	model3			
8.	Does the tree with the smallest error in the training data also have the smallest error in the validation data?			
	Yes			
	○ No			
9.	Is it always true that the tree with the lowest classification error on the training set will result in the lowest classification error in the validation set?			
	Yes, this is ALWAYS true.			
	No, this is NOT ALWAYS true.			
10.	Which tree has the largest complexity?			
	model_1			
	model_2			
	model_3			
11.	Is it always true that the most complex tree will result in the lowest classification error in the validation_set?			
	Yes, this is always true.			
	No, this is not always true.			
12.	Using the complexity definition, which model (model_4, model_5, or model_6) has the largest complexity?			
	model_4			
	model_5			
	model_6			

13.	model_4 and model_5 have similar classification error on the validation set but model_5 has lower complexity. Should you pick model_5 over model_4?					
		Pick model_5 over model_4				
	\circ	Pick model_4 over model_5				
14. Using the results obtained in this section, which model (model_7, model_8, or model_9) would you choose to use?						
	\circ	model_7				
		model_8				
	\bigcirc	model_9				