Count data and Poisson distribution

GENERALIZED LINEAR MODELS IN PYTHON

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Count data

- Count the **number of occurrences** in a specified **unit of** time, distance, area or volume

Examples:

- Goals in a soccer match
- Number of earthquakes
- Number of crab satellites
- Number of awards won by a person
- Number of bike crossings over the bridge

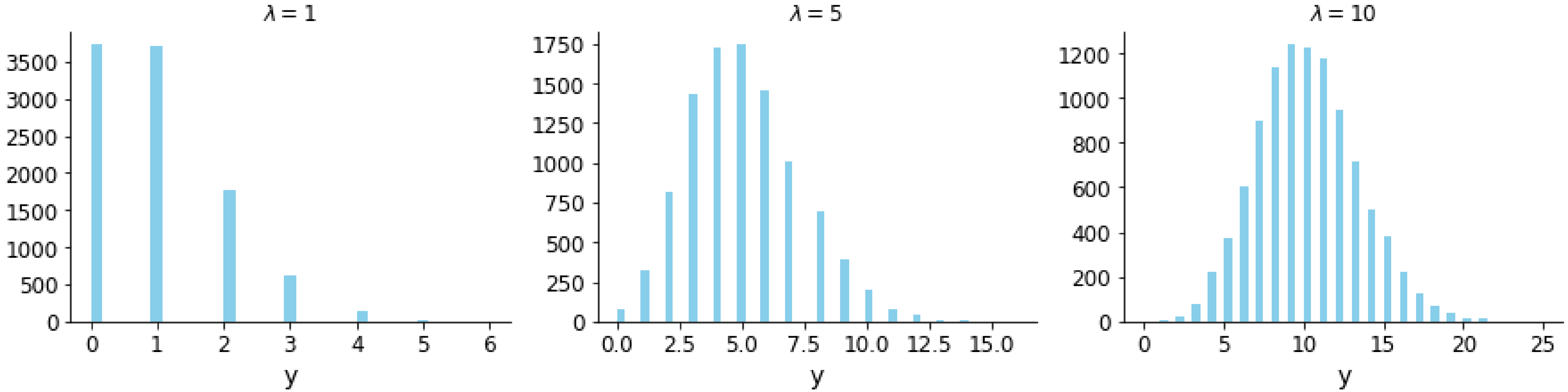
Poisson random variable

- Events occur independently and randomly
- Poisson distribution

$$P(y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

  - λ : mean and variance
  - $y = 0, 1, 2, 3, ...$
    - Always positive
    - Discrete (*not continuous*)
    - $\text{Lower bound at zero}$, but no upper bound

Understanding the parameter of the Poisson distribution



Visualizing the response

```
import seaborn as sns
```

```
sns.distplot('y')
```

Poisson regression

- Response variable

$$y \sim Poisson(\lambda)$$

- Mean of the response

$$E(y) = \lambda$$

- Poisson regression model

$$log(\lambda) = \beta_0 + \beta_1 x_1$$

Explanatory variables

- Continuous and/or categorical $\longrightarrow$ **Poisson regression model**
- Categorical $\longrightarrow$ **log-linear model**

GLM with Poisson in Python

```
import statsmodels.api as sm
from statsmodels.formula.api import glm
```

```
glm('y ~ x',
    data = my_data,
    family = sm.families.Poisson())
```

Let's practice!

Interpreting model fit

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Parameter estimation

- Maximum likelihood estimation (MLE)
- Iteratively reweighted least squares (IRLS)

The response function

- Poisson regression model

$$log(\lambda) = \beta_0 + \beta_1 x_1$$

- The response function:

$$\lambda = exp(\beta_0 + \beta_1 x_1)$$

or

$$\lambda = exp(\beta_0) \times exp(\beta_1 x_1)$$

The response function

- Poisson regression model

$$log(\lambda) = \beta_0 + \beta_1 x_1$$

- The response function:

$$\lambda = exp(\beta_0 + \beta_1 x_1)$$

or

$$\lambda = exp(\beta_0) \times exp(\beta_1 x_1)$$

Interpretation of parameters

- $exp(\beta_0)$
  - The effect on the mean λ when $x = 0$
- $exp(\beta_1)$
  - The **multiplicative effect** on the mean λ for a **1-unit increase** in x

Interpreting coefficient effect

- If $\beta_1 > 0$
  - $exp(\beta_1) > 1$
  - λ is $exp(\beta_1)$ times larger than when $x = 0$
- If $\beta_1 = 0$
  - $exp(\beta_1) = 1$
  - $\lambda = exp(\beta_0)$
  - Multiplicative factor is 1
  - y and x are not related
- If $\beta < 0$
  - $exp(\beta_1) < 1$
  - λ is $exp(\beta_1)$ times smaller than when $x = 0$

Example

```
model = glm('sat ~ weight', data = crab,
            family = sm.families.Poisson()).fit()
```

```
                 Generalized Linear Model Regression Results (print cut)
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -0.4284      0.179     -2.394      0.017      -0.779      -0.078
weight         0.5893      0.065      9.064      0.000       0.462       0.717
==============================================================================
```

Example - interpretation of beta

- Extract model coefficients

```
model.params
```

```
Intercept    -0.428405
weight        0.589304
```

- Compute the effect

```
np.exp(0.589304)
```

```
1.803
```

Confidence interval for ...

- β_1

```
print(model.conf_int())
```

```
                  0         1
Intercept -0.779112 -0.077699
weight     0.461873  0.716735
```

- The multiplicative effect on mean

```
print(np.exp(crab_fit.conf_int()))
```

```
                  0         1
Intercept  0.458813  0.925243
weight     1.587044  2.047737
```

Let's practice!

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The Problem of Overdispersion

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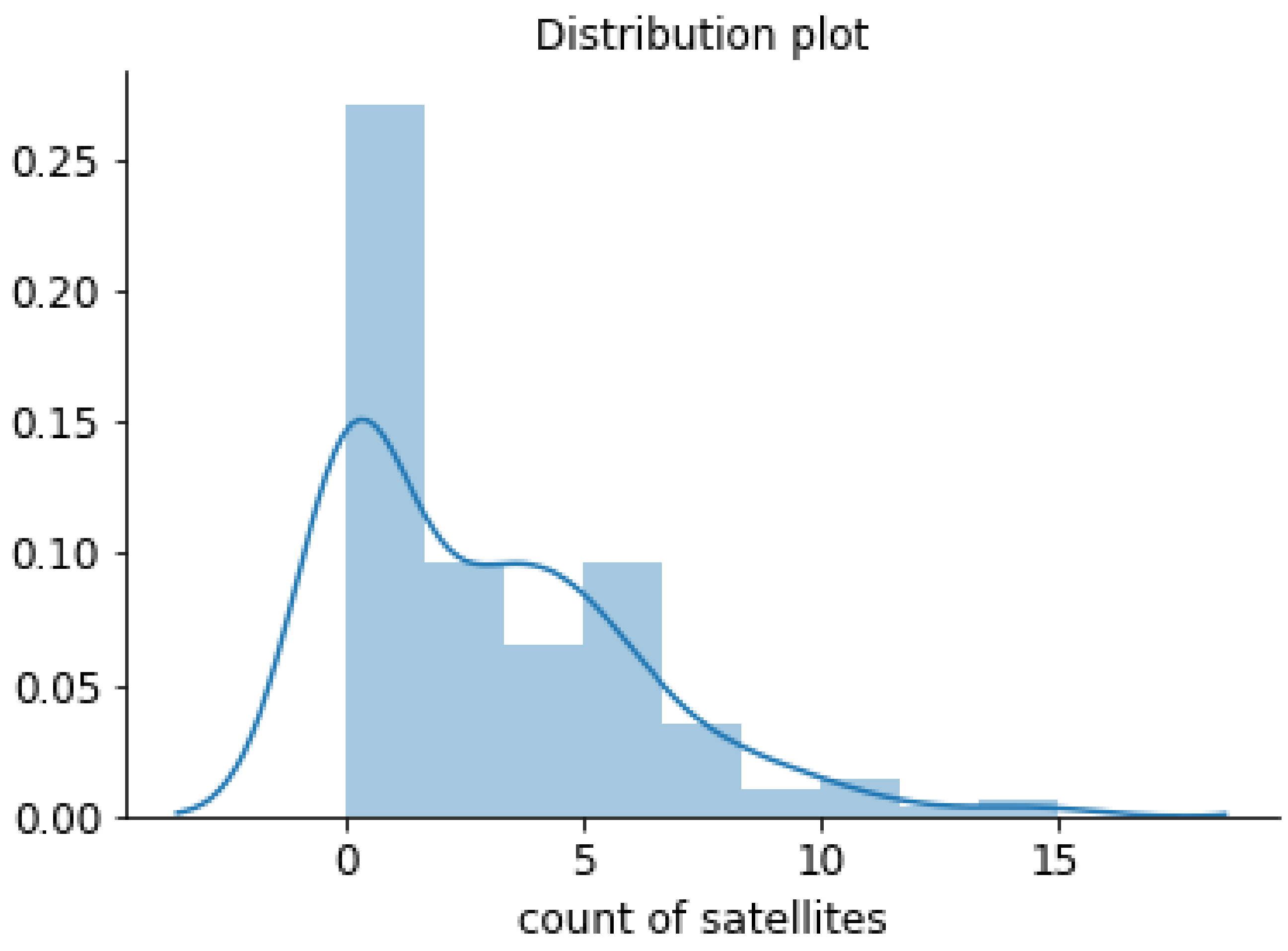
Understanding the data




```
# mean of y
y_mean = crab['sat'].mean()
```

```
2.919
```

```
# variance of y
y_variance = crab['sat'].var()
```

```
9.912
```

Mean not equal to variance

- $variance > mean \rightarrow$ **overdispersion**
- $variance < mean \rightarrow$ **underdispersion**

Consequences:

- Small standard errors
- Small p-value

How to check for overdispersion?

```
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                    sat   No. Observations:                  173
Model:                            GLM   Df Residuals:                      171
Model Family:                 Poisson   Df Model:                            1
Link Function:                    log   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -458.08
Date:                Tue, 05 Mar 2019   Deviance:                       560.87
Time:                        18:21:13   Pearson chi2:                     536.
No. Iterations:                     5   Covariance Type:             nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -0.4284      0.179     -2.394      0.017      -0.779      -0.078
weight         0.5893      0.065      9.064      0.000       0.462       0.717
==============================================================================
```

Compute estimated overdispersion

```
ratio = crab_fit.pearson_chi2 / crab_fit.df_resid
print(ratio)
```

```
3.134
```

- Ratio $= 1 \rightarrow$ approximately Poisson
- Ratio $< 1 \rightarrow$ underdispersion
- Ratio $> 1 \rightarrow$ overdispersion

Negative Binomial Regression

- $E(y) = \lambda$
- $Var(y) = \lambda + \alpha\lambda^2$
- α - dispersion parameter

GLM negative Binomial in Python

```
import statsmodels.api as sm
from statsmodels.formula.api import glm
```

```
model = glm('y ~ x', data = my_data,
            family = sm.families.NegativeBinomial(alpha = 1)).fit()
```

Let's practice!

Plotting a regression model

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Import libraries

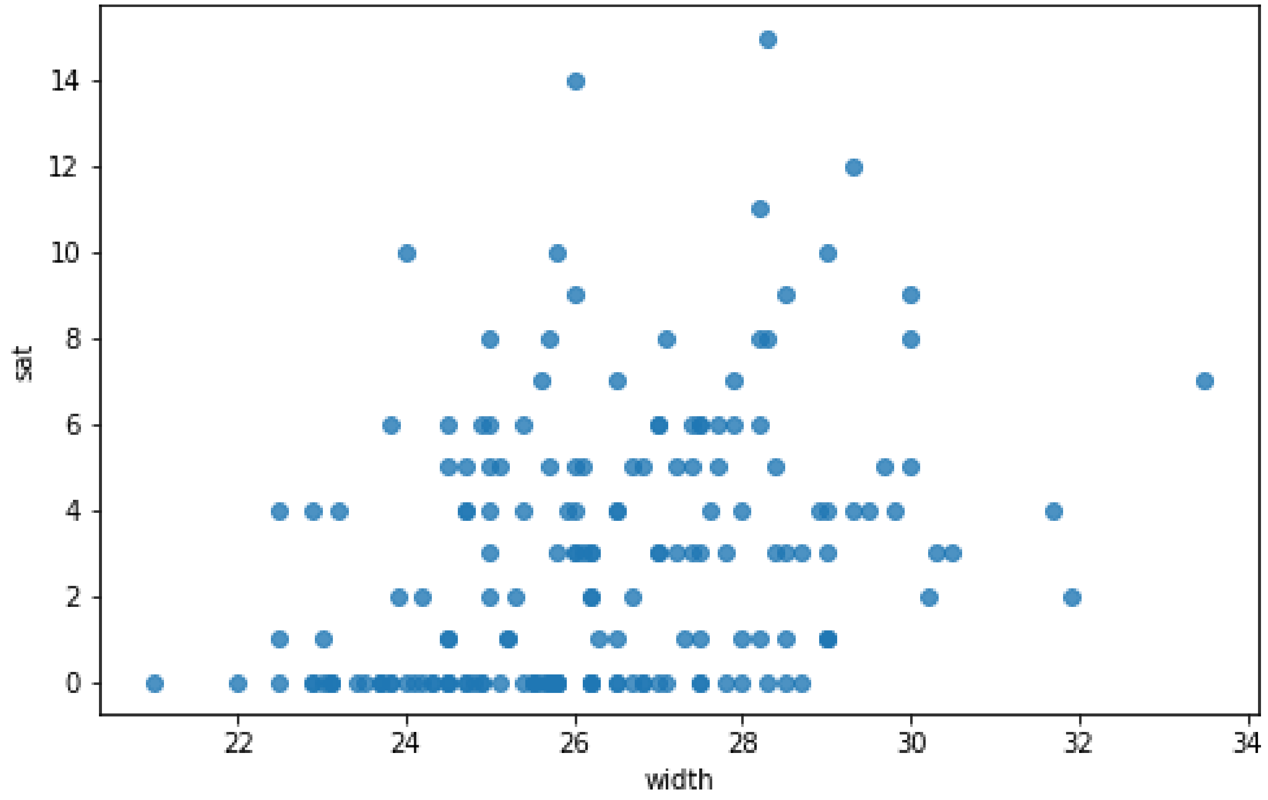
```
import seaborn as sns
import matplotlib.pyplot as plt
```

- Crab model `'sat ~ width'` is saved as `model`

Plot data points

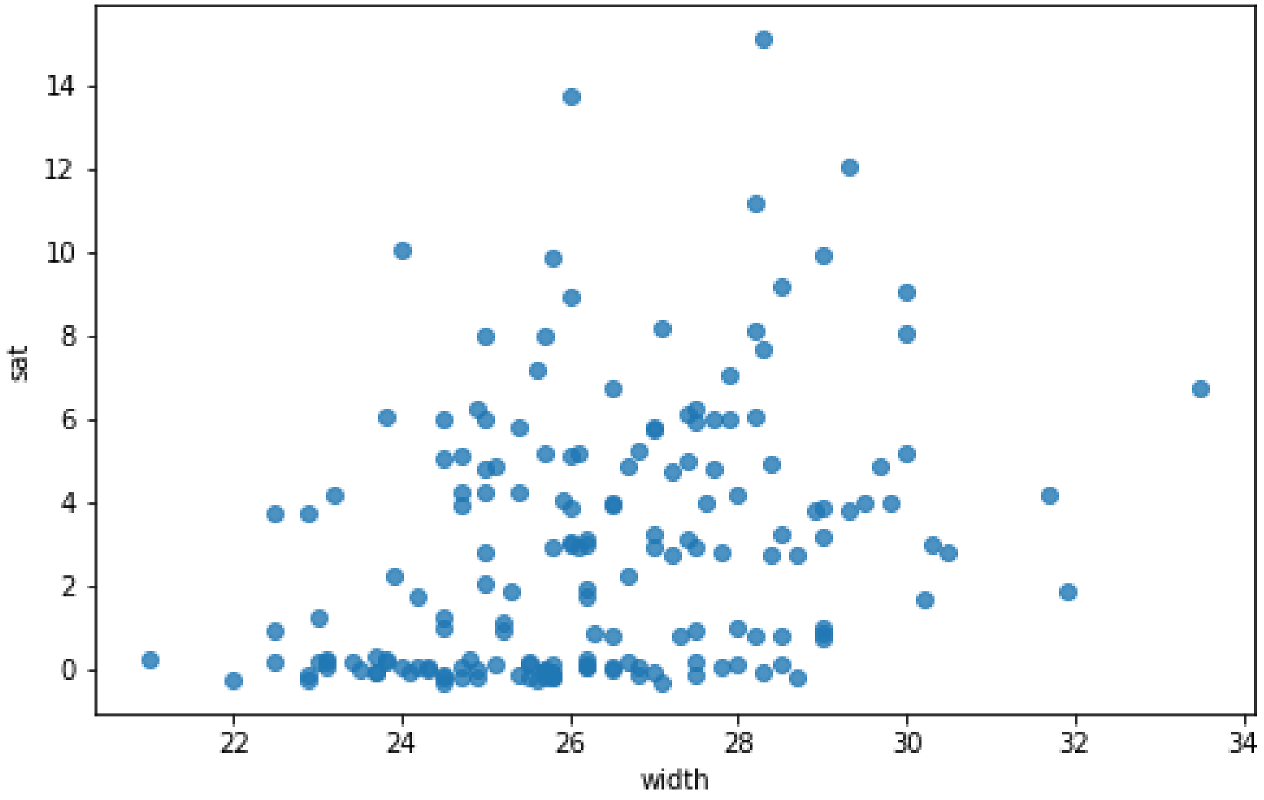
```
# Adjust figure size
plt.subplots(figsize = (8, 5))
```

```
# Plot data points
sns.regplot('width', 'sat',
            data = crab,
            fit_reg = False)
```



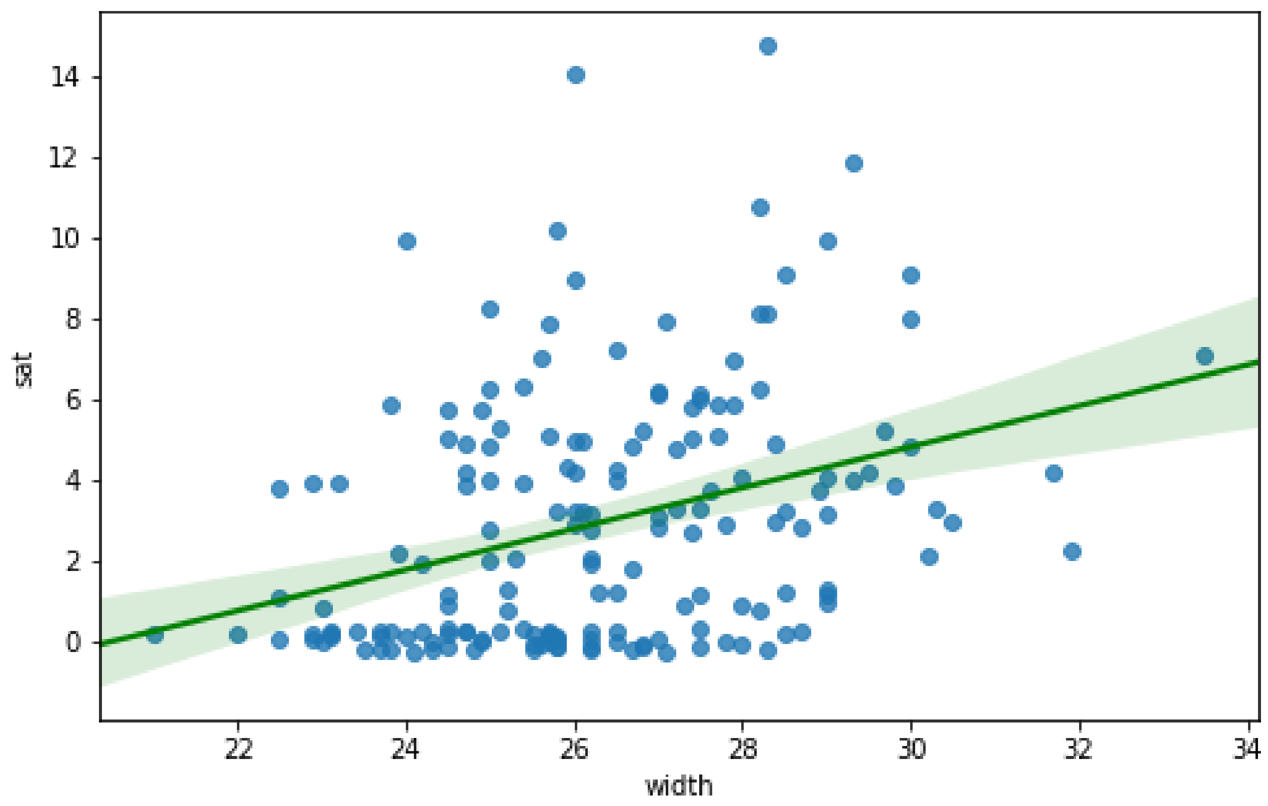
Add jitter

```
sns.regplot('width', 'sat',
            data = crab,
            fit_reg = False,
            y_jitter = 0.3)
```



Add linear fit

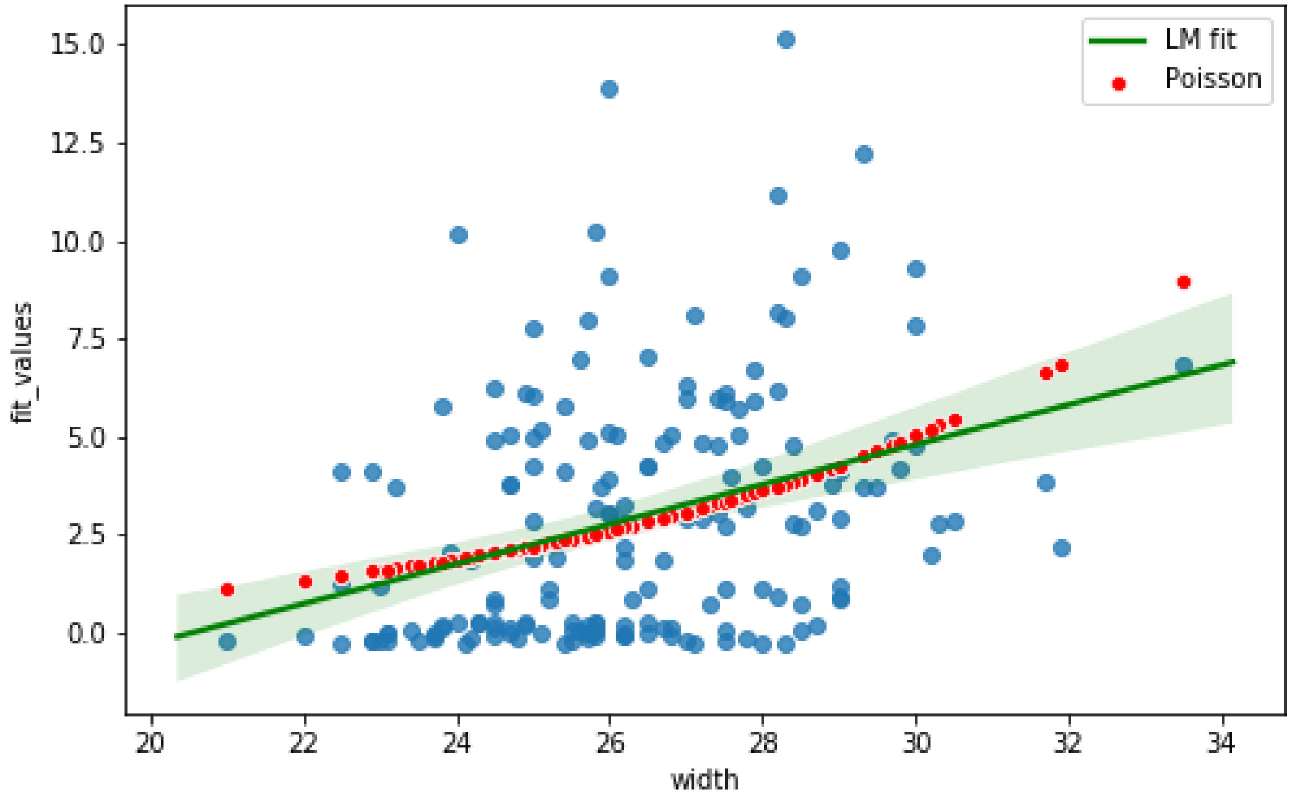
```
sns.regplot('width', 'sat',
            data = crab,
            y_jitter = 0.3,
            fit_reg = True,
            line_kws = {'color':'green',
                        'label':'LM fit'})
```



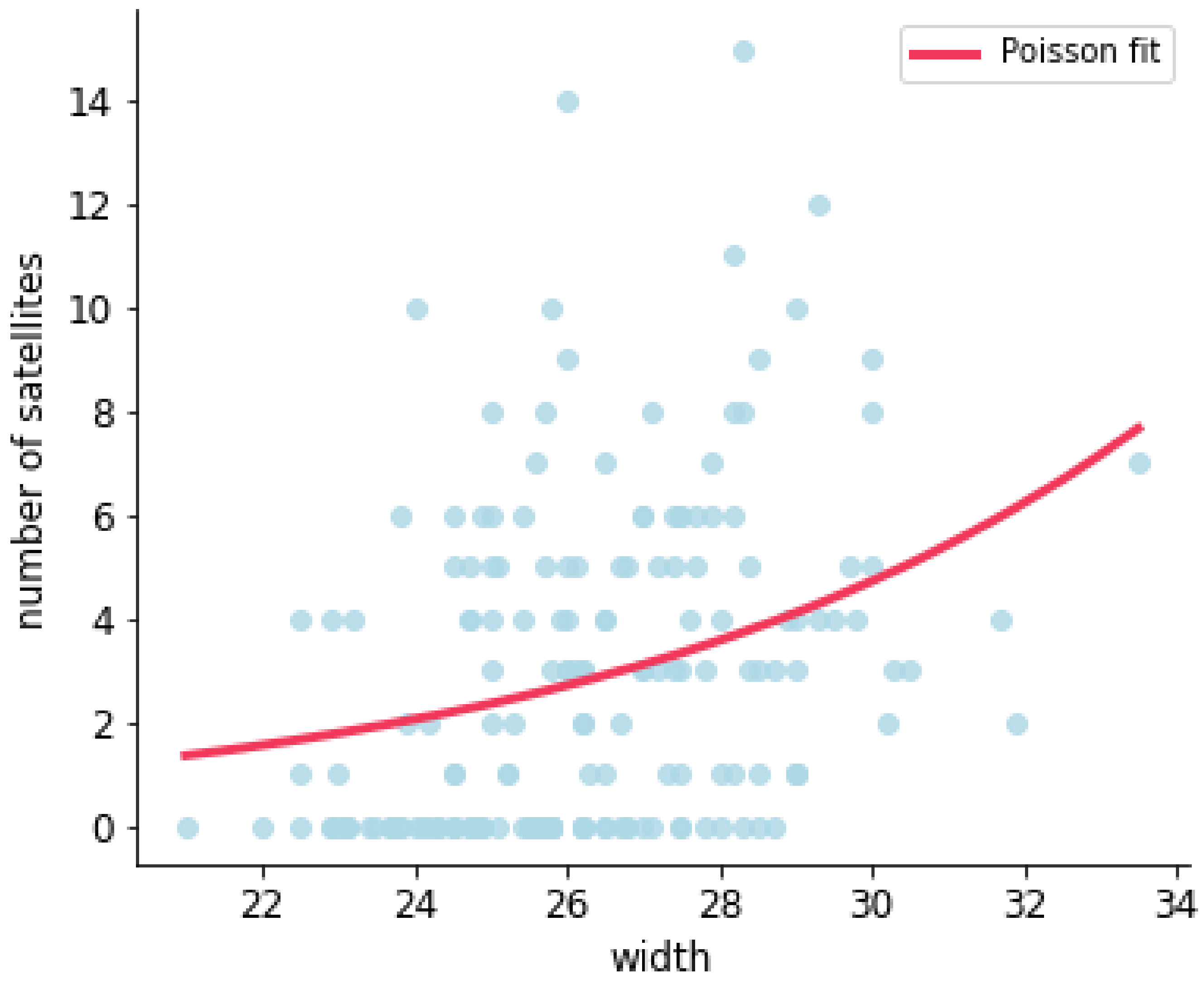
Add Poisson GLM estimated values

```
crab['fit_values'] = model.fittedvalues
```

```
sns.scatterplot('width','fit_values',
                data = crab,
                color = 'red',
                label = 'Poisson')
```



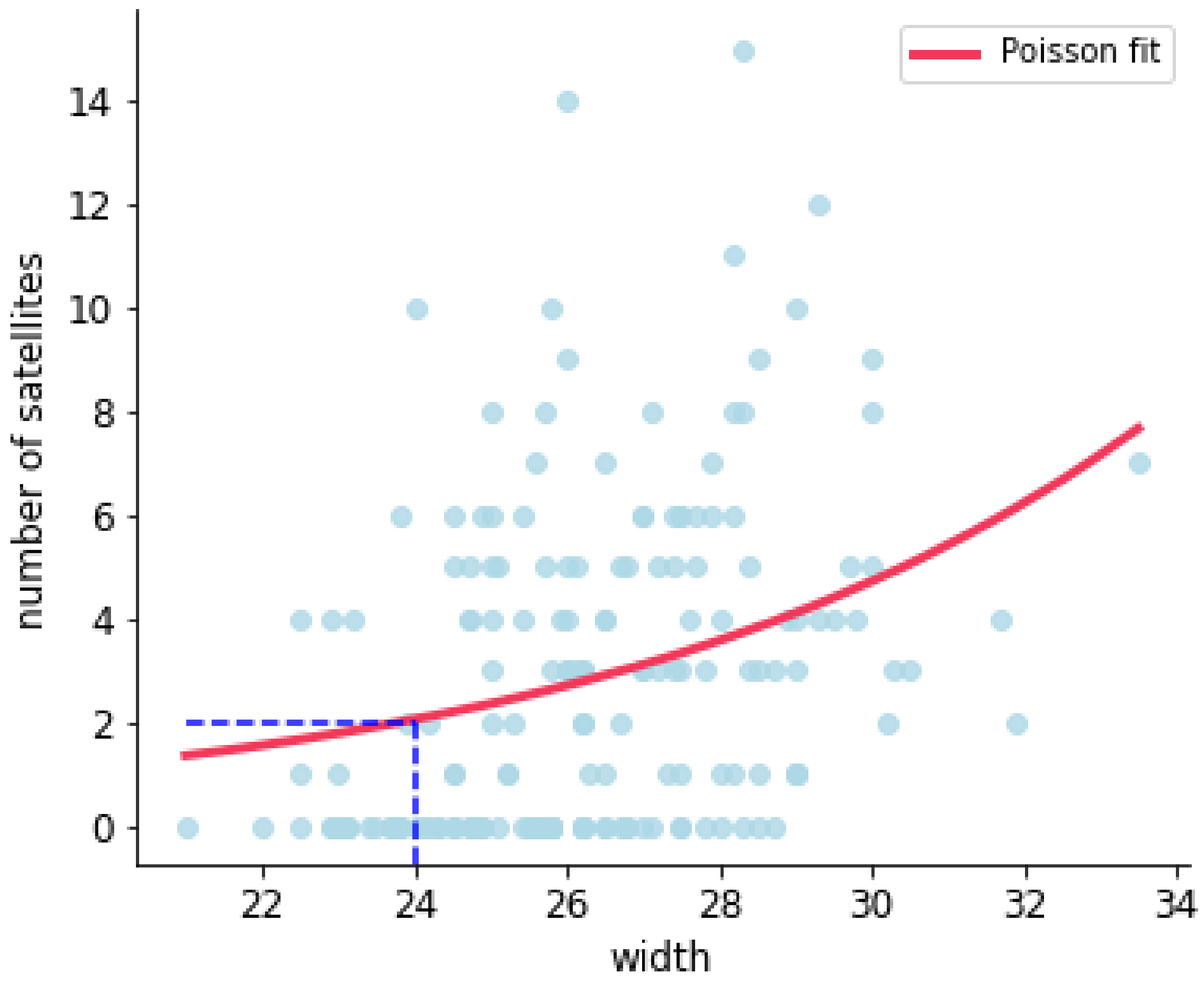
Predictions



Predictions

```
new_data = pd.DataFrame({'width':[24, 28, 32]})
model.predict(new_data)
```

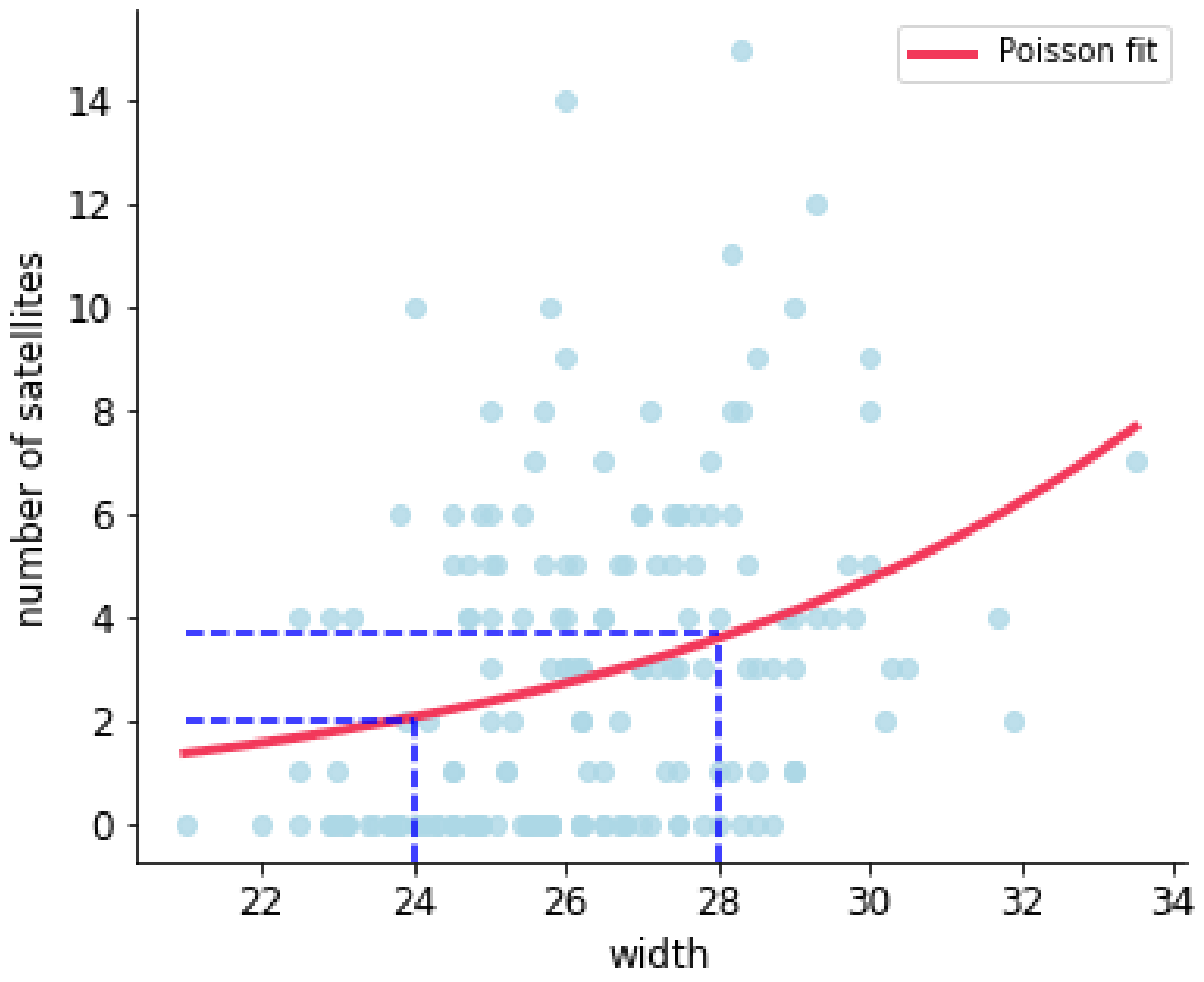
```
0    1.881981
```



Predictions

```
new_data = pd.DataFrame({'width':[24, 28, 32]})
model.predict(new_data)
```

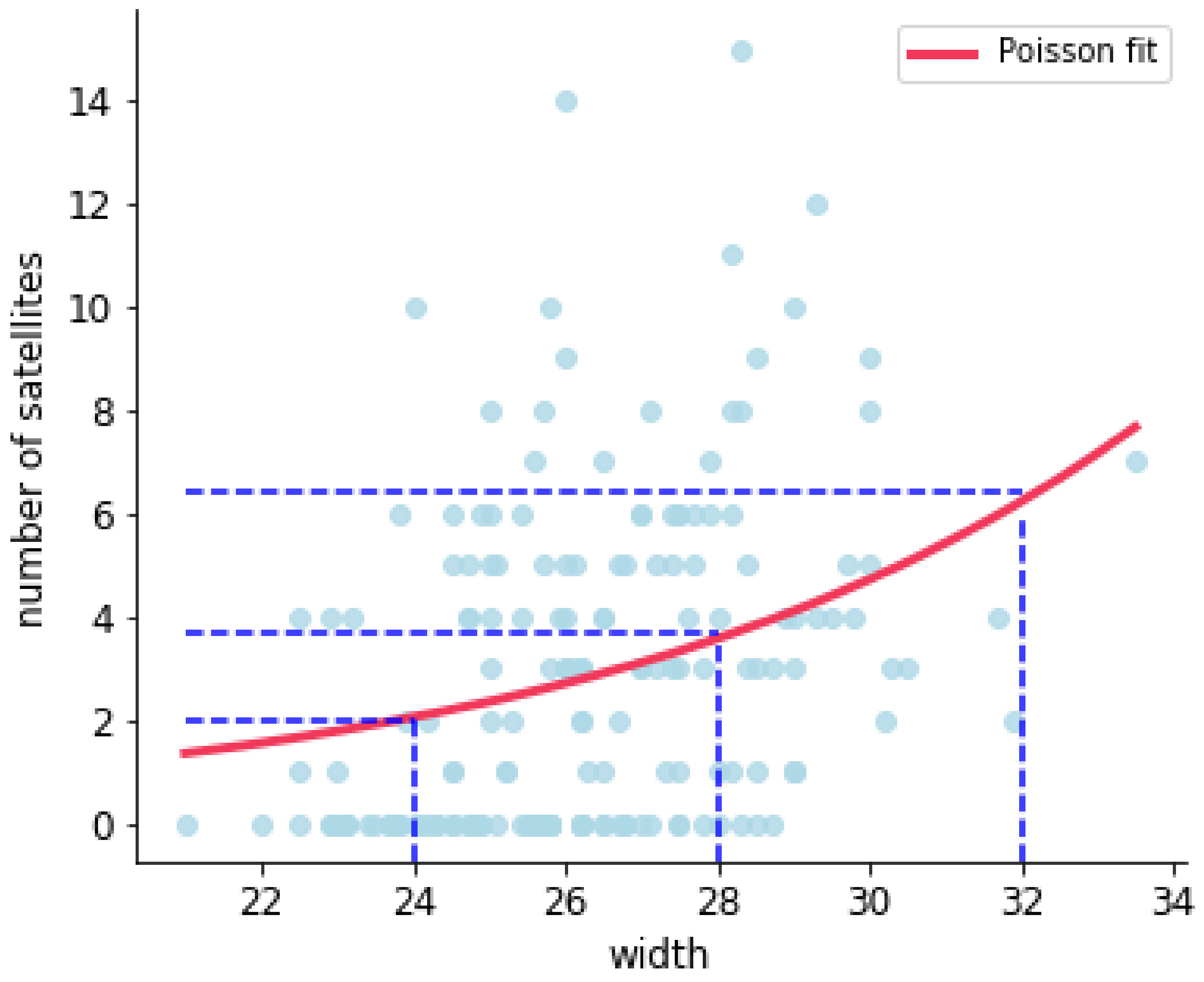
```
0    1.881981
1    3.627360
```



Predictions

```python
new_data = pd.DataFrame({'width':[24, 28, 32]})
model.predict(new_data)
```

```
0    1.881981
1    3.627360
2    6.991433
```



Let's practice!

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