Short Assignment 3

This is an individual assignment.

Due: Friday, November 4 @ 11:59PM

Problem 1

Consider the two classes represented by Gaussians distributions P1 and P2 in Figures 1 and 2. Calculate Fisher's univariate separation indices to answer the following questions.

In [2]:
```
from IPython.display import Image
Image('figures/two-gaussian-distributions.png',width=800)
```

Out[2]:

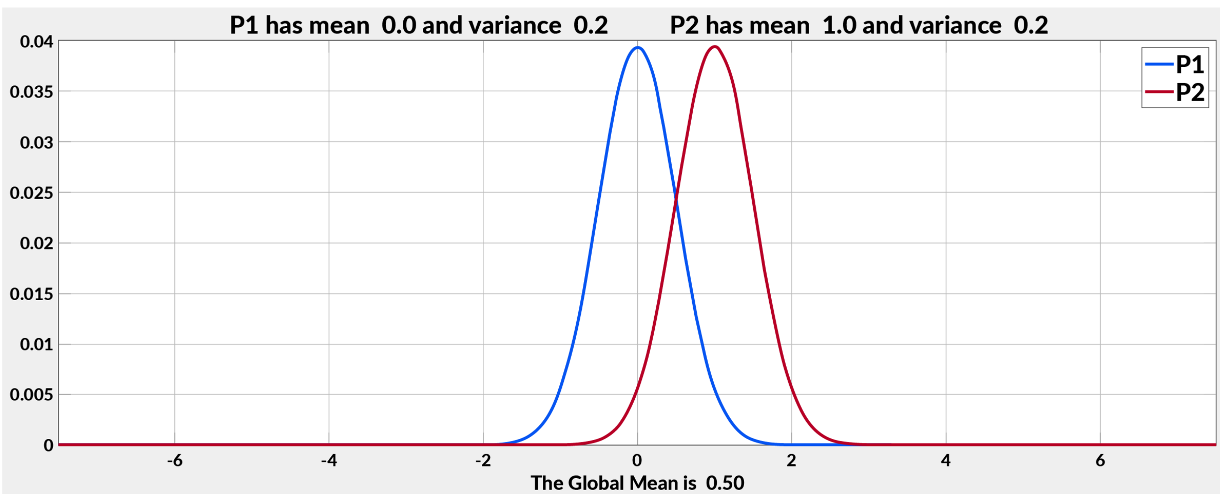


Figure 1: Two Gaussian Classes

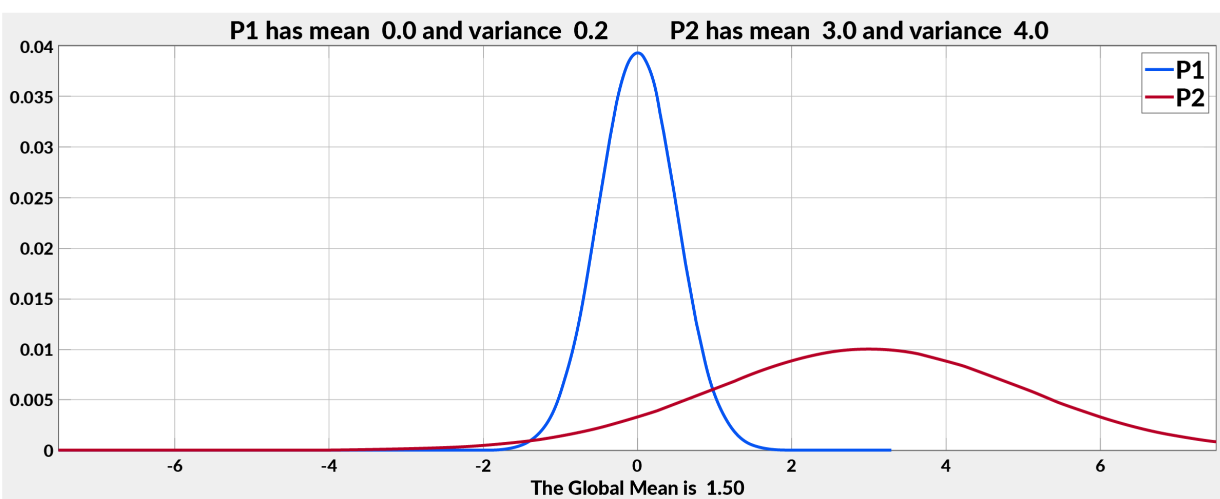


Figure 2: Two Gaussian Classes

1. What is the Within-Class Separation in Figure 1?

2. What is the Within-Class Separation in Figure 2?
3. What is the Between-Class Separation in Figure 1?
4. What is the Between-Class Separation in Figure 2?
5. Which Distributions are more separated: (A) P1,P2 in Figure 1 or (B) P1,P2 in Figure 2? Justify your answer.

The with in class seperation in Figure 1 is

Ans-01: Objective function to maximize the distance b/w mean of the distributions & minimize the sparcity in the distribution is

$$J(w) = \frac{M_2 - M_1}{S_1^2 + S_2^2} \longrightarrow M_2, M_1 = \text{Mean}$$

$$s_1, s_2 = \sqrt{\text{variance}}$$

An (a) Within class seperation for figure 1

$= M_2 - M_1$ ($M_2 \to$ Mean P_2, $M_1 \to$ Mean P_1)

$= 1 - 0$

$= 1$

(b) Within class seperation for figure - 2

$= M_2 - M_1$

$= 3 - 0$

$= 3$

(c) Between class seperation figure 1 $= S_1^2 + S_2^2$

$\Rightarrow 0.2 + 0.2$

$\Rightarrow 0.4$

(d) Between class seperation figure - 2 $\Rightarrow S_1^2 + S_2^2$

$\Rightarrow 0.2 + 4$

$= 4.2$

(15) $J_1(w) = \frac{1-0}{0.2+0.2} \Rightarrow \cancel{\frac{1}{0.4}} \Rightarrow \cancel{\frac{20 \times 10}{4}}$

(figure 1)

$= \frac{1}{0.4} \Rightarrow \frac{10}{4} \Rightarrow 2.5$ Ans

$J_2(w) = \frac{3-0}{0.2+4} = \frac{3}{4.2} = \frac{30}{42} \Rightarrow 0.714$ Ans

figure (2)

Here the objective function in figure 1 is 2.5 while in figure 2 is 0.714

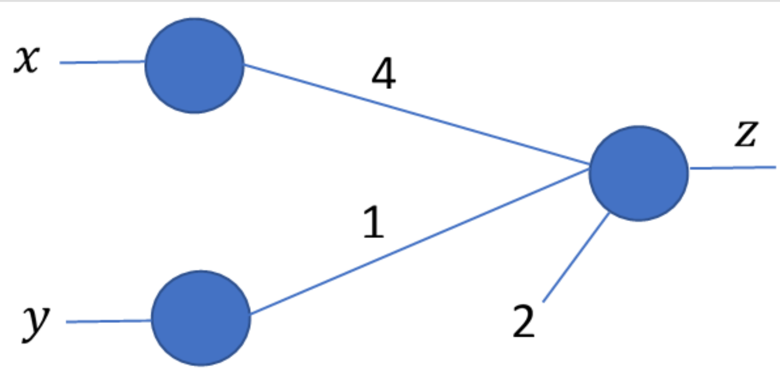
Then $J_1(w)$ is more separate compared to $J_2(w)$

Problem 2

Consider the following perceptron:

In [3]:
```
Image('figures/Perceptron.png', width=400)
```

Out[3]:



Recall that the perceptron uses the activation function:

$$\phi(x) = \begin{cases} -1 & x \leq 0 \\ 1 & x > 0 \end{cases}$$

And the cost function is:

$$E_p(\mathbf{w}, b) = -\sum_{m \in \mathcal{M}} (\mathbf{w}^T \mathbf{x}_n + b)^T t_n$$

where is the set of all misclassified points. The update equations for the weights and bias term are:

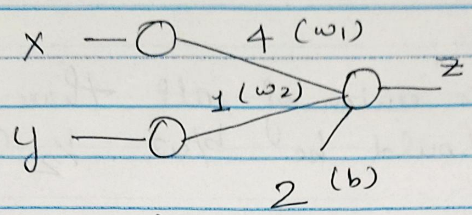
$$\begin{aligned} \mathbf{w}^{(t+1)} \leftarrow & \quad \mathbf{w}^{(t)} - \eta \frac{\partial E_p(\mathbf{w}, b)}{\partial \mathbf{w}} = \mathbf{w}^{(t)} + \eta \mathbf{x}_n t_n \\ b^{(t+1)} \leftarrow & \quad b^{(t)} - \eta \frac{\partial E_p(\mathbf{w}, b)}{\partial b} = b^{(t)} + \eta t_n \end{aligned}$$

Suppose you have the following 5 data samples (x, y) and their corresponding labels t:

$$\begin{gathered} (x_1, y_1) = (1, 0) \text{ with } t_1 = 1 \\ (x_2, y_2) = (4, 2) \text{ with } t_2 = 1 \\ (x_3, y_3) = (0, -1) \text{ with } t_3 = -1 \\ (x_4, y_4) = (-1, -1) \text{ with } t_4 = -1 \\ (x_5, y_5) = (-2, 1) \text{ with } t_5 = -1 \end{gathered}$$

What is the smallest value for the learning rate η such that the updated network will result in zero misclassified points using only one iteration?

②



activation function = $\begin{cases} -1 \le 0 \\ 1 > 0 \end{cases}$

~~Cost function~~

$\therefore \quad w^{(t+1)} \longrightarrow w^{(t)} + \eta\, x_n t_n \quad \longrightarrow ①$

$b^{(t+1)} \longrightarrow b^{(t)} + \eta\, t_n . \quad — ⑪$

from the Equation

$\Rightarrow$

$f(w_1 x_n + w_2 y_n + b_n) = t_n$

$\Rightarrow \quad x = \begin{bmatrix} 1 \\ 4 \\ 0 \\ -1 \\ -2 \end{bmatrix} \quad y = \begin{bmatrix} 0 \\ 2 \\ -1 \\ -1 \\ 1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix} \quad t = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$

$\Rightarrow$ from ① & ⑪

$f_n(w_1^{t+1} x_n + w_2^{t+1} y_n + b^{t+1}) = t_n$

$\Rightarrow \begin{bmatrix} 2n+7 > 0 \\ 21n + 20 > 0 \\ 1 - 2n \le 0 \\ -3 - 3n \le 0 \\ 2n - 5 \le 0 \end{bmatrix} \quad \therefore$

$n > -7/2 \quad — ①$

$n > \frac{-20}{21} \quad — ⑪$

$n \geq \frac{1}{2} \quad — ⑪⑪$

$n \geq -1 \quad — ⑪V$

$n \le \frac{5}{2} \quad — V$

taking the union of all three the value of x should be b/w $\frac{1}{2}$ & $\frac{5}{2}$

$\therefore$ the smallest value of $x = \frac{1}{2}$ for this equation to be true.

In [ ]:

Submit Your Solution

Confirm that you've successfully completed the assignment.

Along with the Notebook, include a PDF of the notebook with your solutions.

`add` and `commit` the final version of your work, and `push` your code to your GitHub repository.

Submit the URL of your GitHub Repository as your assignment submission on Canvas.
