Location, Gesture and Activity Sensing

Part I: The Fundamentals of Motion Tracking

Department of Computer Science and Engineering

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We are living in the edge of wearables ...

Wearables -- Head to Toe

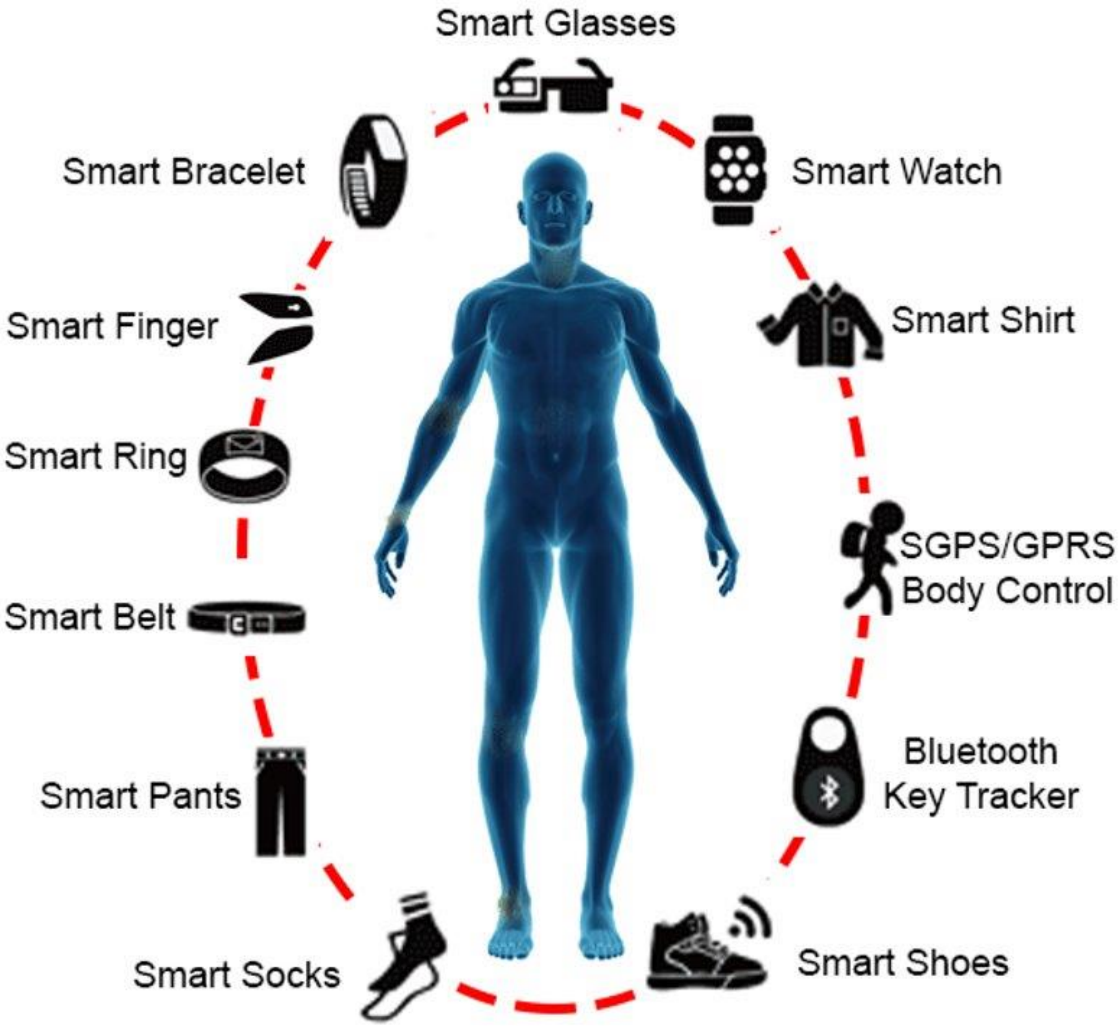


Image source: https://doi.org/10.1109/ACCESS.2017.2789329

Range of Devices

Image source: https://doi.org/10.1249/jsr.0000000000000680

Sensors on Wearables

IMU
Altimeter
Bio-acoustic
Blood Pressure
Brightness
Camera
ECG
EDA
EMG
Fiber Optic Sensors
Compass

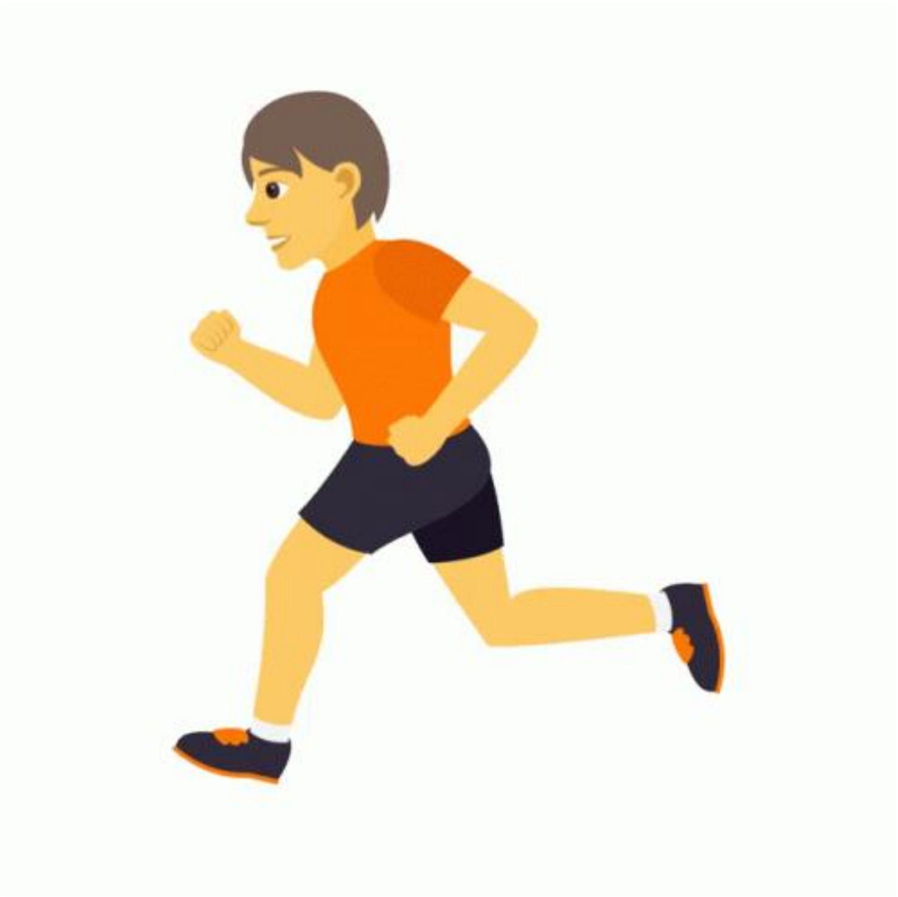
GPS
GSR
Humidity
Magnetometer
IR Proximity
IR Temperature
LED
Glucometer
Pressure
Microphone
Piezoelectric

RFID
Spectrometer
Thermometer
Textile sensors
Ultrasound
Weight
Optical sensor
mmWave sensor
Electrodes
Inductive wire
IR Photo Reflective

Our First Problem ...

vs

with a



IMU / Inertial Sensors

- Accelerometer, Gyroscope, and sometime Magnetometer
- Measures
  - Specific force of a body
  - Angular Rate
  - Orientation
- Detects the linear acceleration and the rotation rate (angular rate)
- Also used for measuring the orientation of a device (one of the primary use-cases for wearables)
- Largely used for maneuver in a 3D space and navigation purpose

Motion and the Frame of Reference

- **Rest Frame**: The coordinate system in which the particle is at rest
- **Proper acceleration**: Rate of change of velocity of a body in its instantaneous rest frame
  - Acceleration relative to an observer who is in ***free fall*** (relative to an inertial frame of reference)
  - Different from coordinate acceleration
  - Accelerometer measures the proper acceleration

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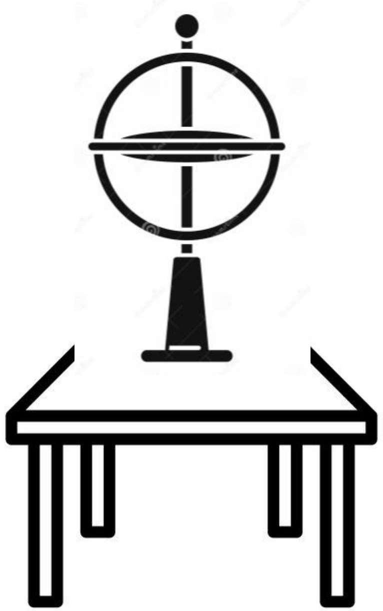
- **What acceleration value would be reported by an accelerometer placed on a table at our classroom?**



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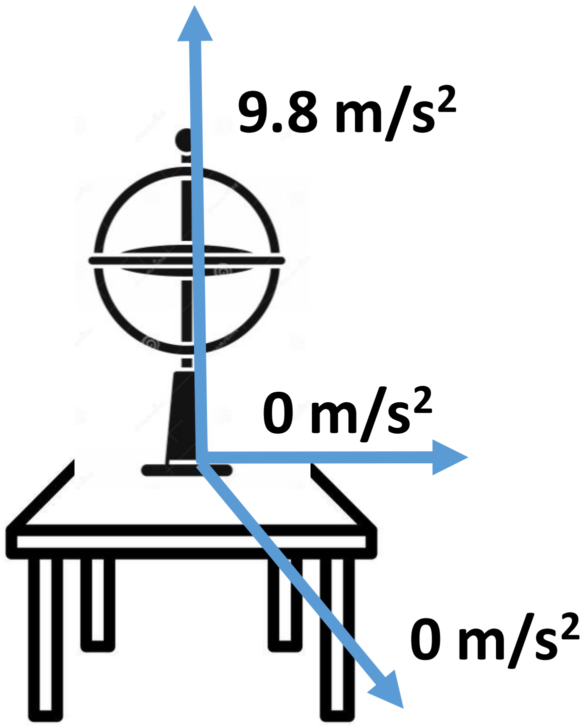

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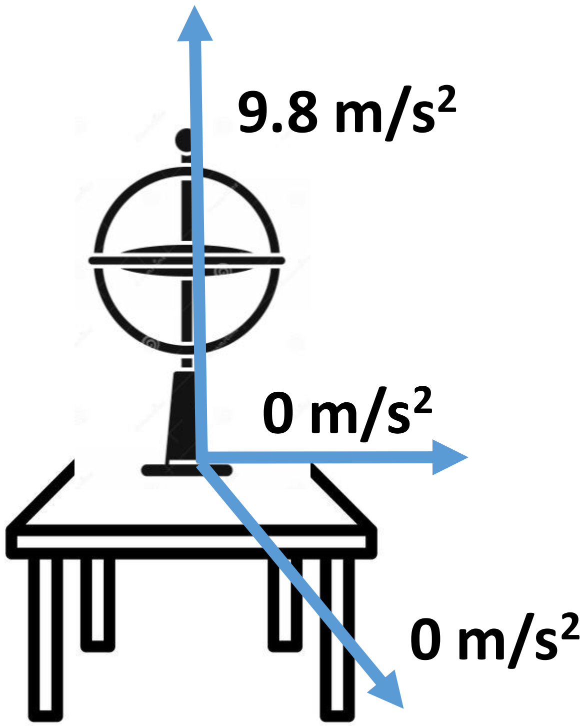
- A free-fall accelerometer will read zero acceleration!



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Assume that the ball is in rest

Motion and the Frame of Reference

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Can we represent the motion of every object with this principle?

Consider the following scenario ...

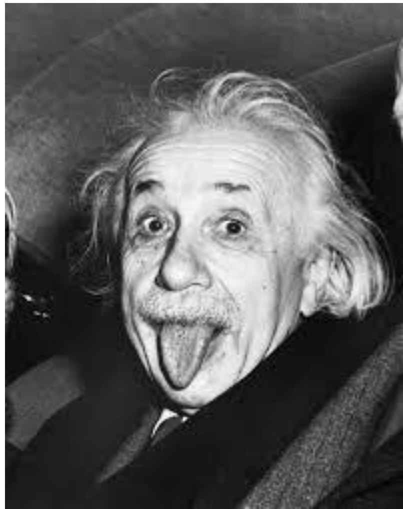
With an accelerometer mounted on Mr. Bean's headphone, can you detect that he is enjoying the music?

Equivalence Principle and Its Impact

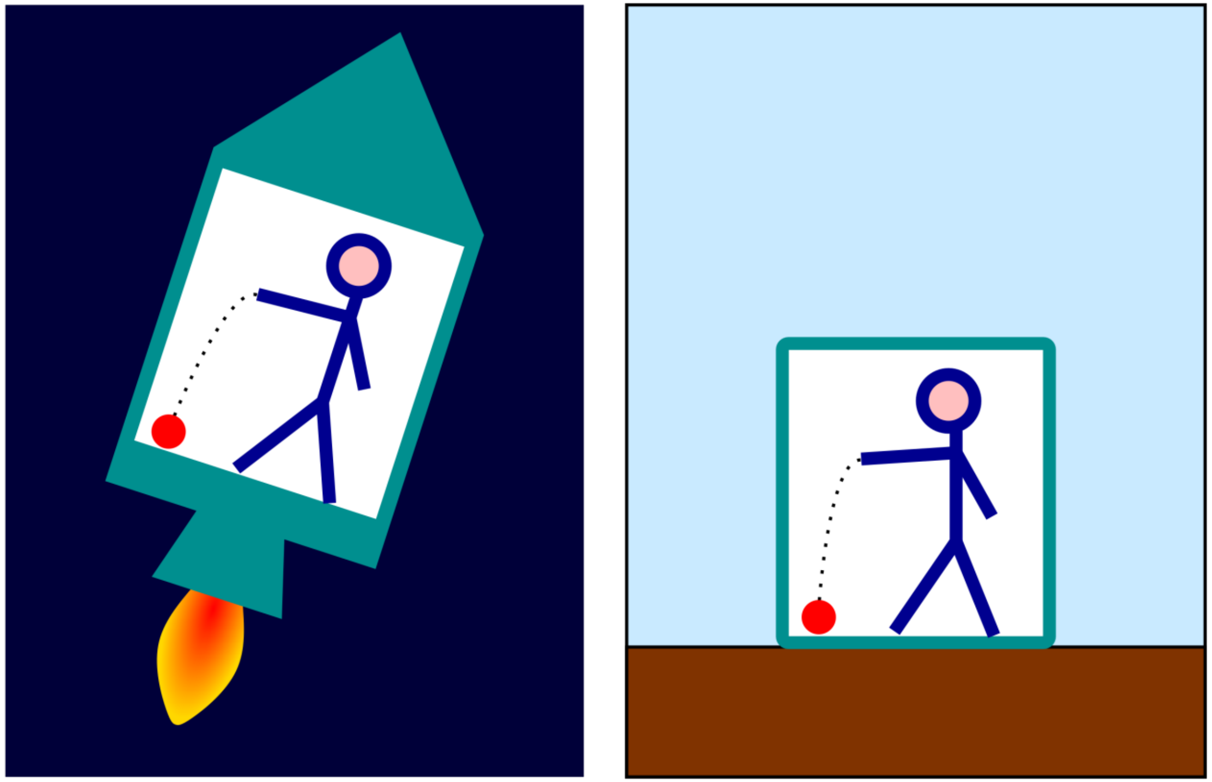
"A little reflection will show that the **law of the equality of the inertial and the gravitational mass** is **equivalent to the assertion** that the **acceleration imparted to a body by a gravitational field** is **independent of** the **nature of the body**. For Newton's equation in motion in a gravitational field, written out in full, it is:

(inertial mass).(acceleration) = (intensity of the gravitational field).(Gravitational mass)

It is only when there is **numerical equality between the inertial and gravitational mass** that the **acceleration is independent of the nature of the body**."



Equivalence Principle and Its Impact

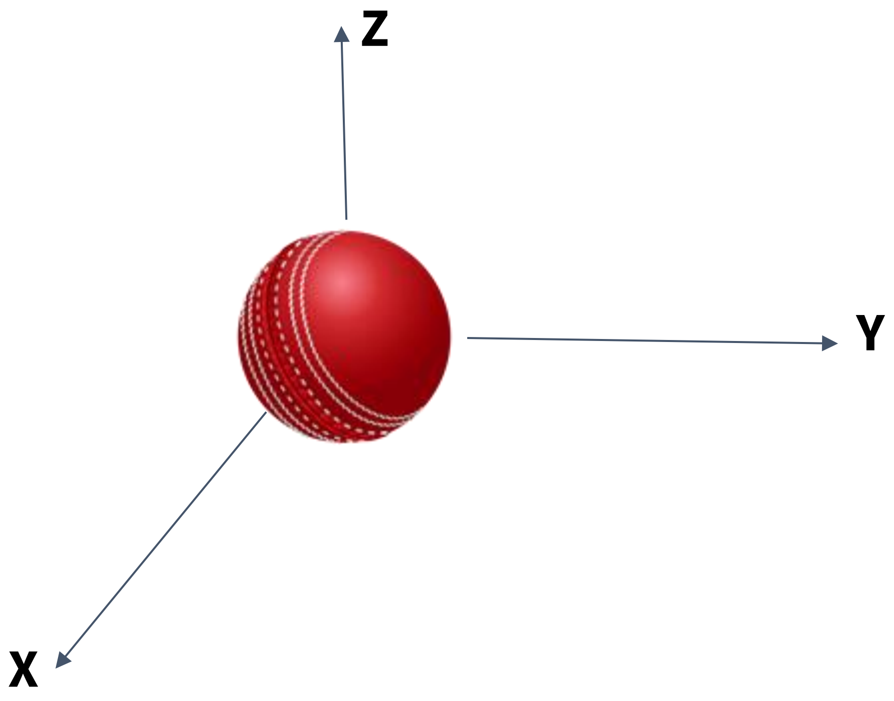


A falling object behaves exactly the same on a planet or in an equivalent accelerating frame of reference.

Image Source: Wikipedia

Equivalence Principle and Its Impact

- **In its simplified form**: The effects of gravity on an object are indistinguishable from its acceleration



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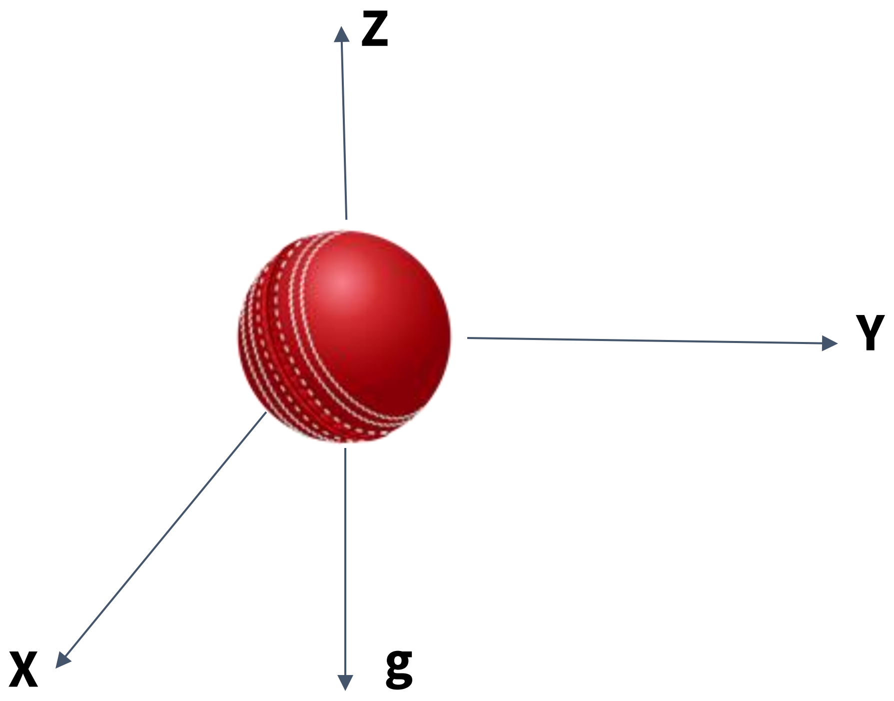
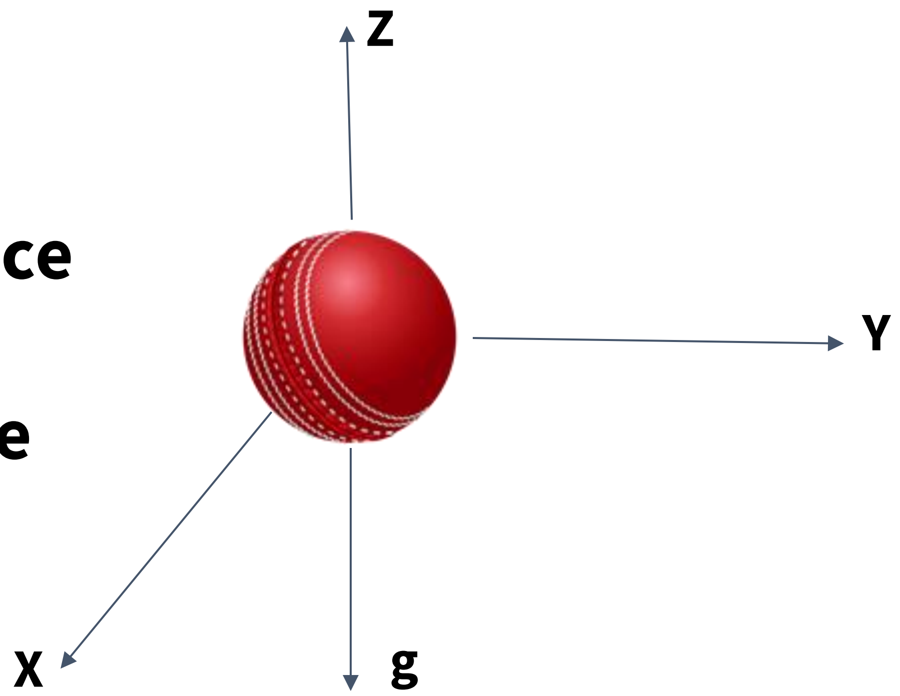


Equivalence Principle and Its Impact

- **In its simplified form**: The effects of gravity on an object are indistinguishable from its acceleration

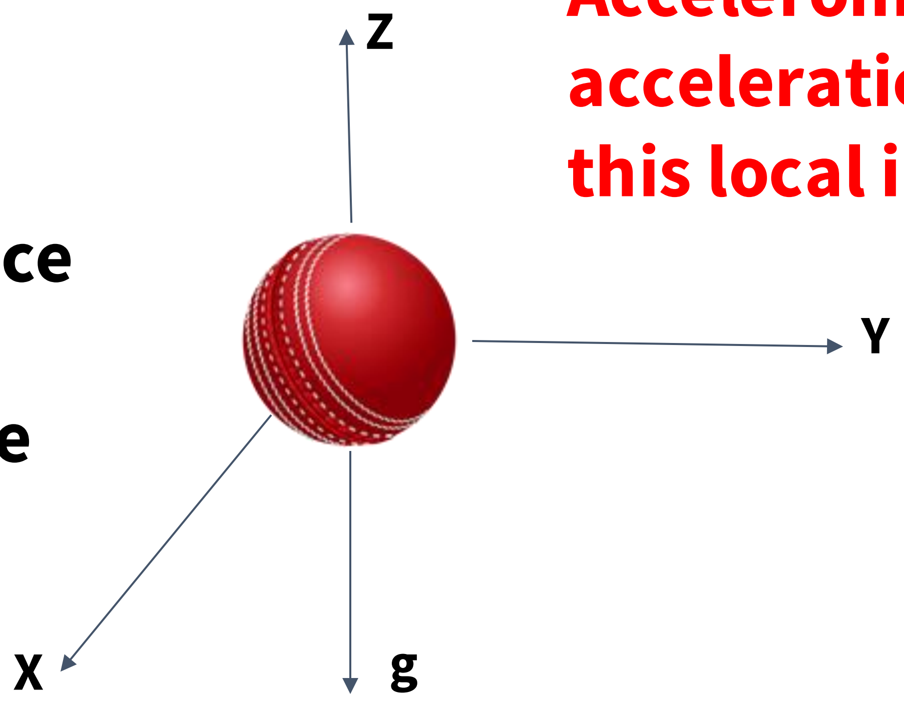
We call this reference frame as the local inertial frame of the object



Equivalence Principle and Its Impact

- **In its simplified form**: The effects of gravity on an object are indistinguishable from its acceleration

Accelerometer measures the acceleration with respect to this local inertial frame



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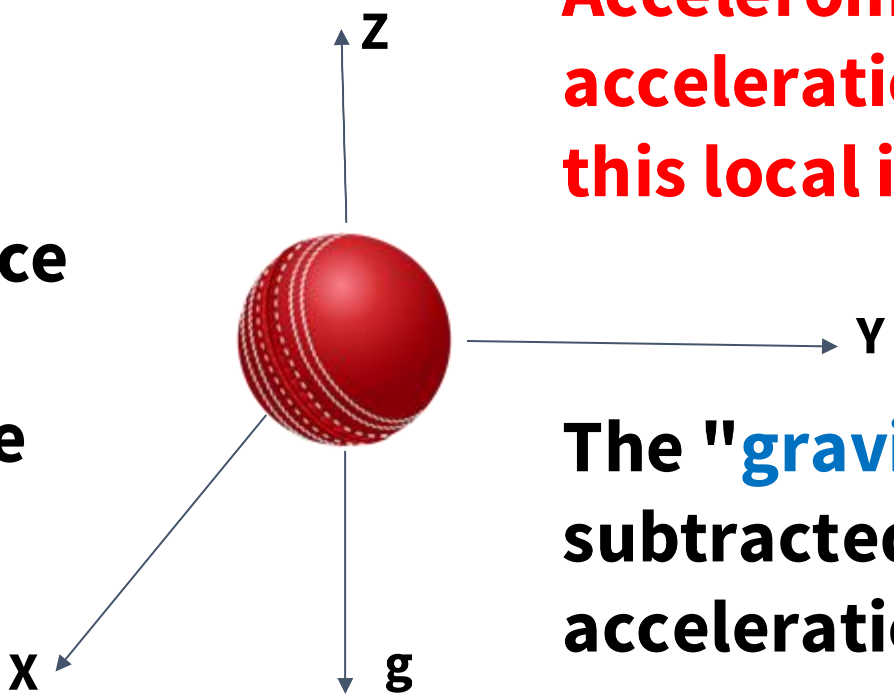


The "gravity offset" must be subtracted to obtain the actual acceleration, when the object moves on the earth's surface

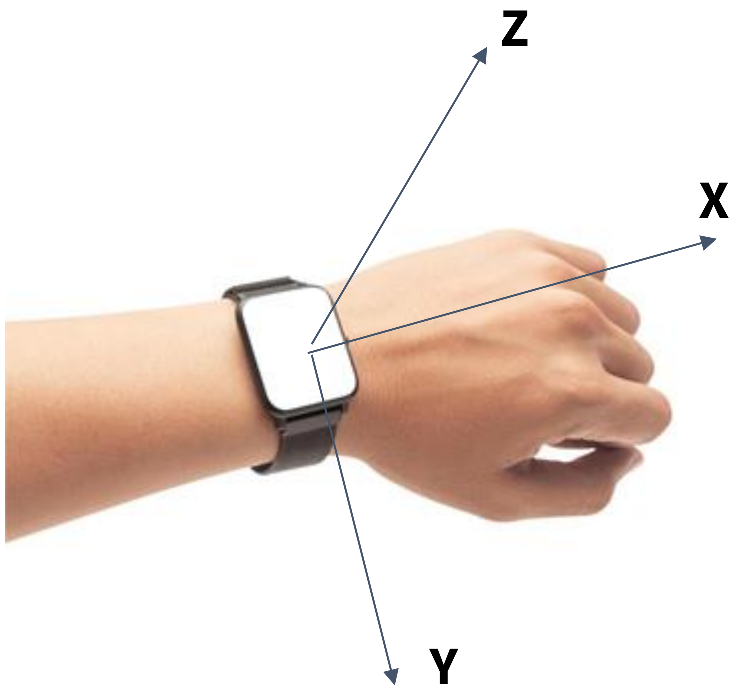
Impact of Rotation

Impact of Rotation

The gravity offset is towards the y-axis

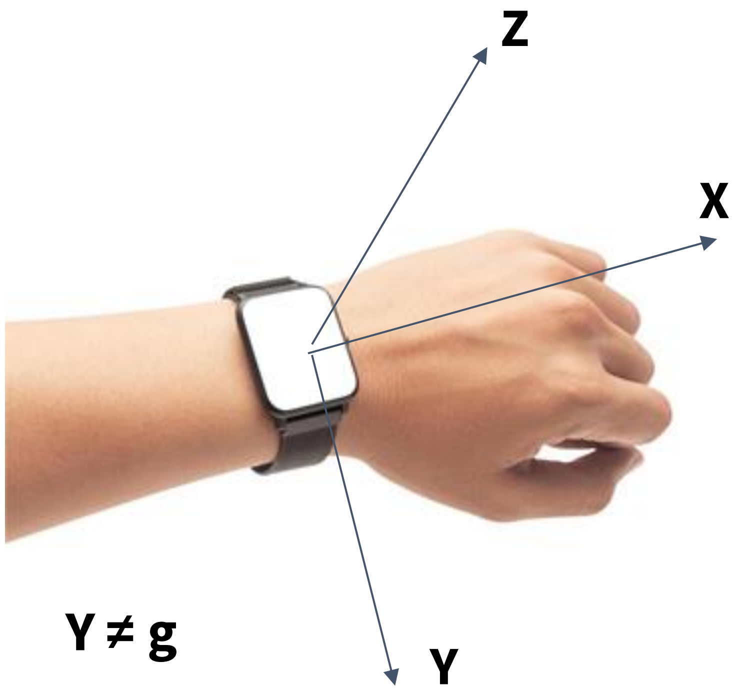
Impact of Rotation

The local inertial frame changes due to the rotation

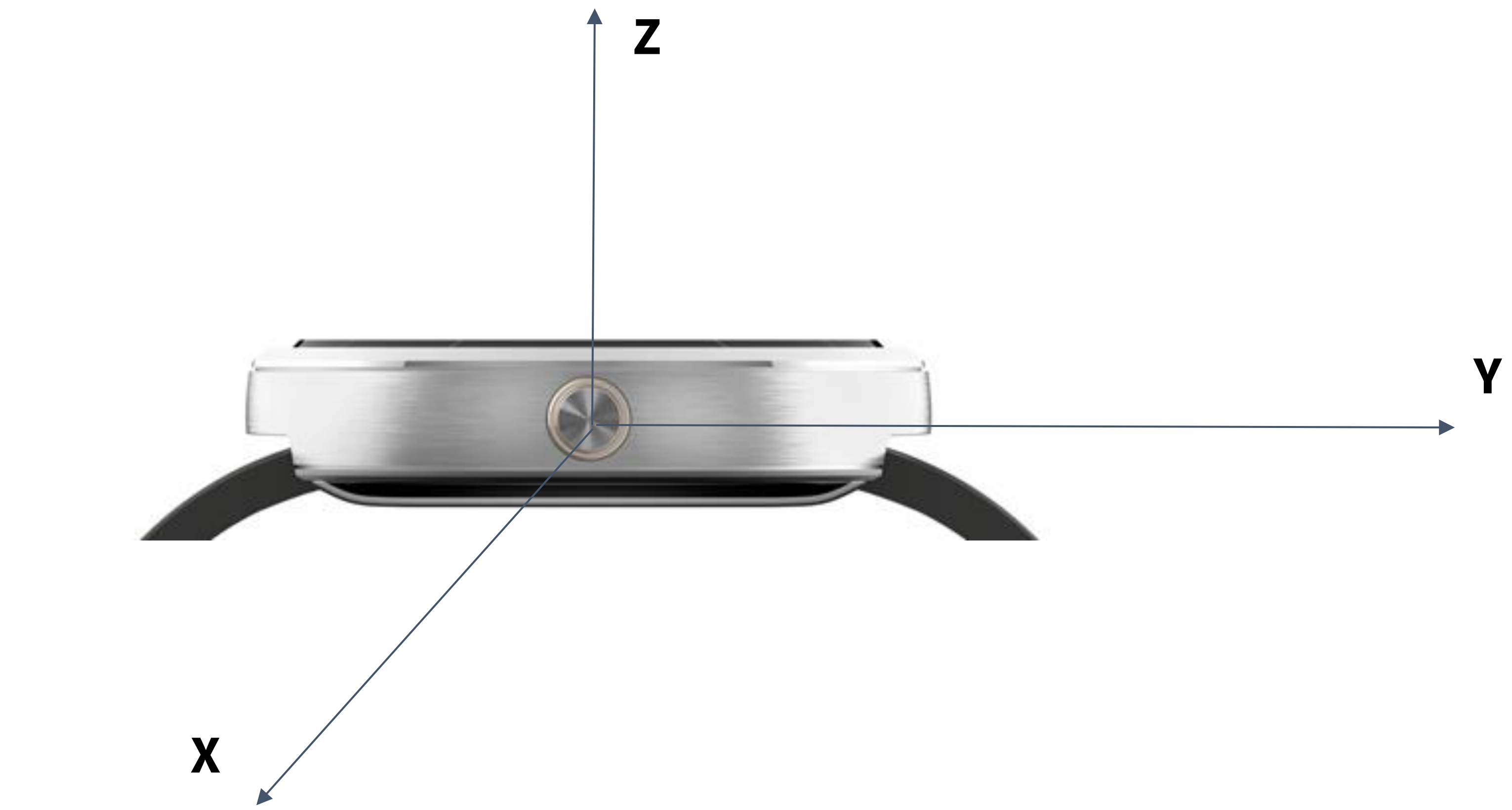


The gravity offset gets different impact on different axes -- how to subtract the gravity offset?

Impact of Rotation

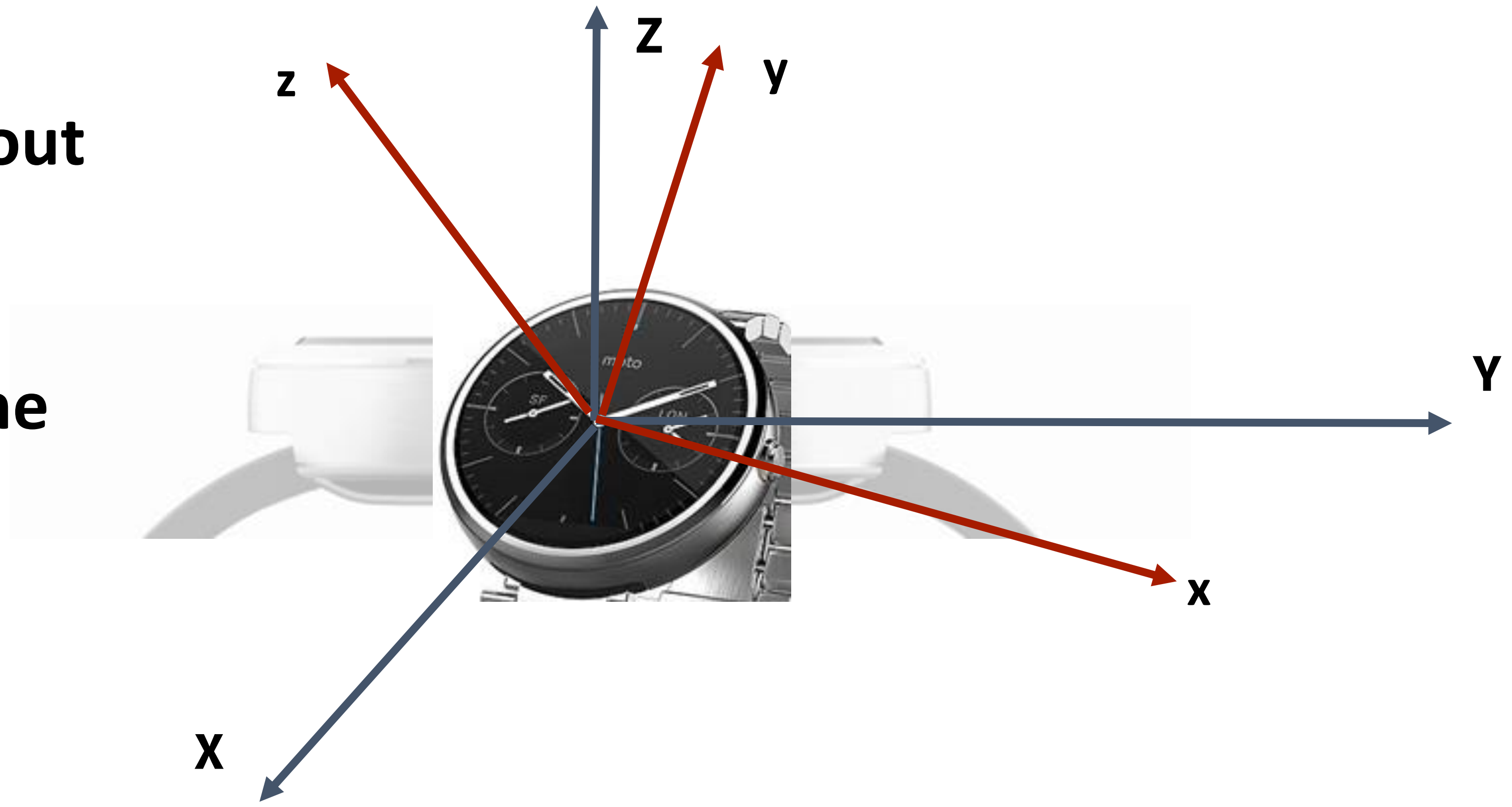


Angle Transformation



Angle Transformation

We need to figure out the components of the gravity offset on each of the components of the local inertial frame



Orientation Update

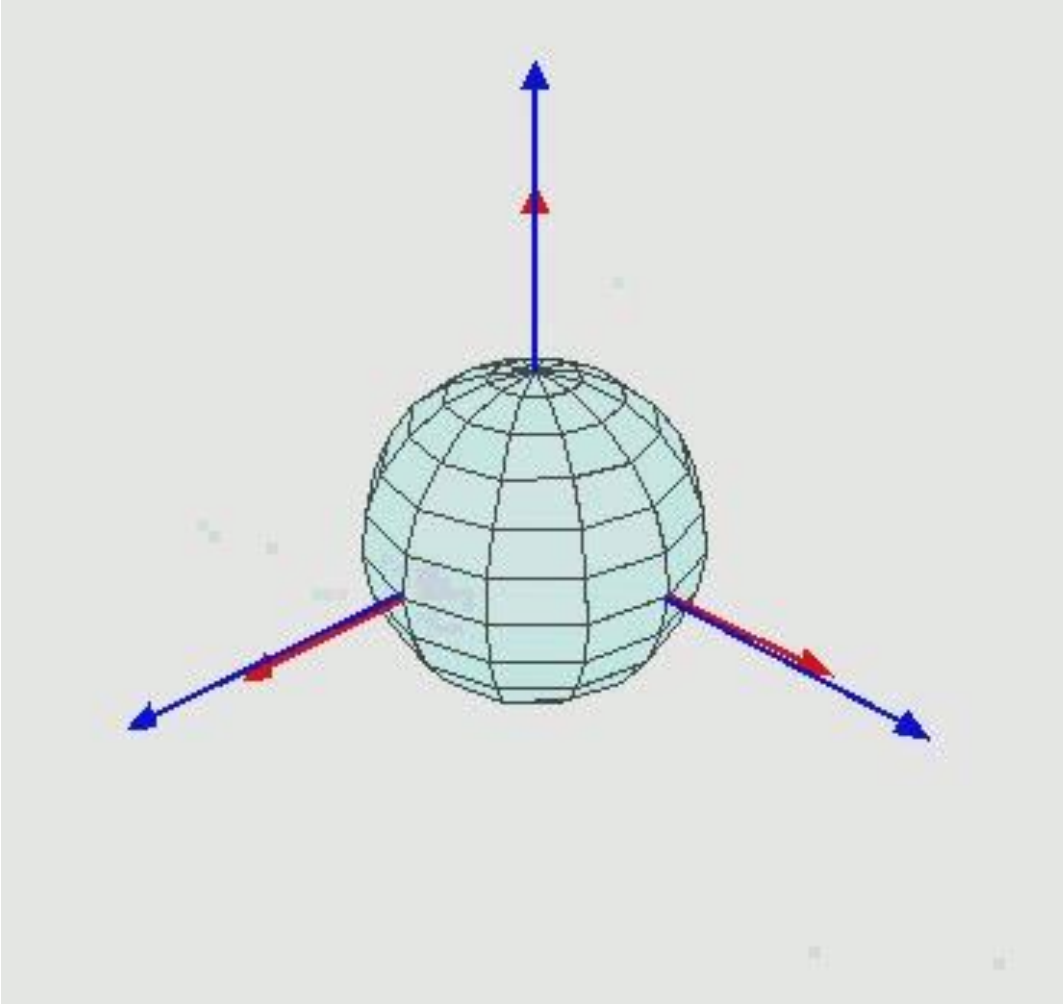
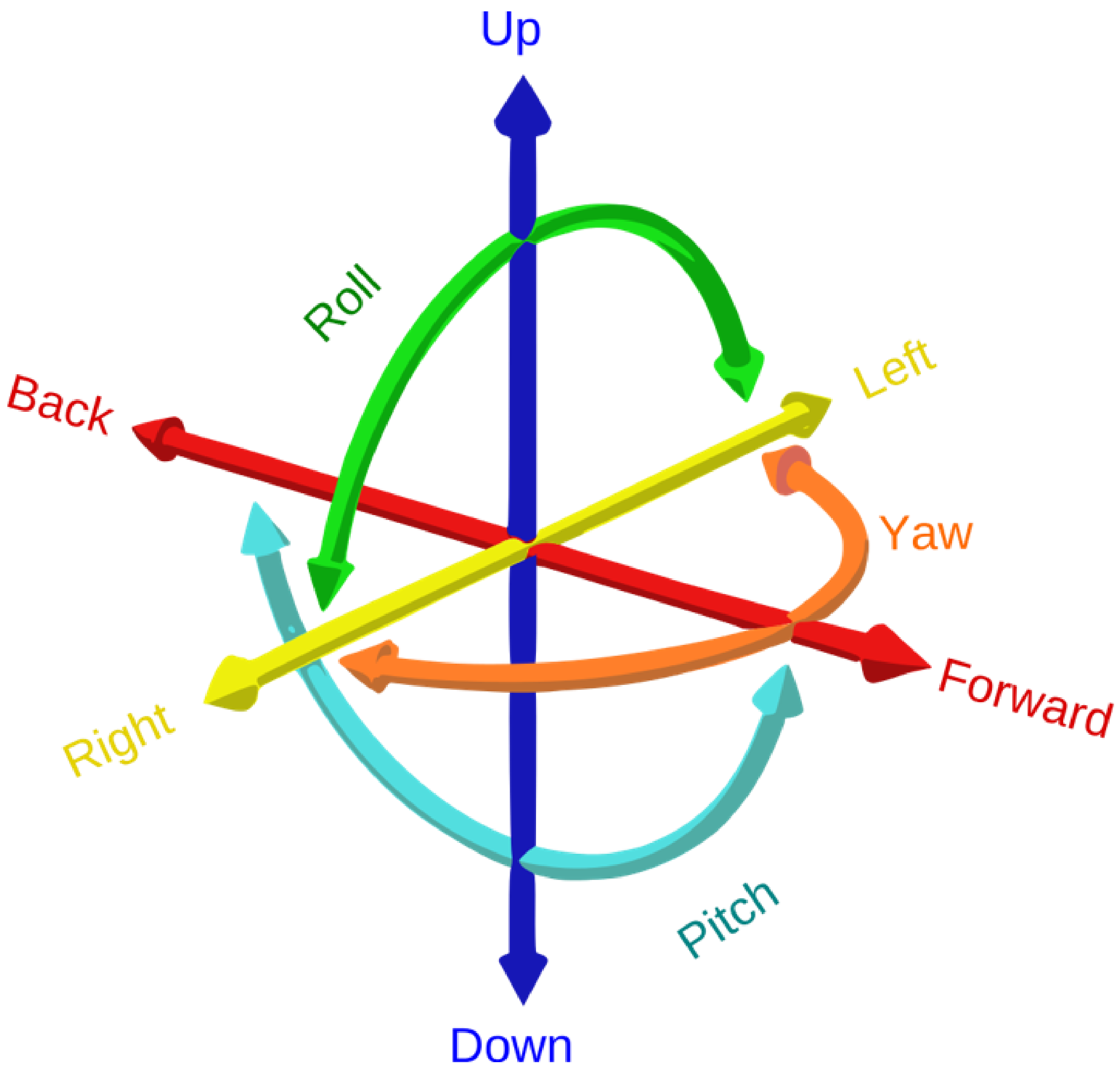


Image source: Wikipedia

Six Degrees of Freedom

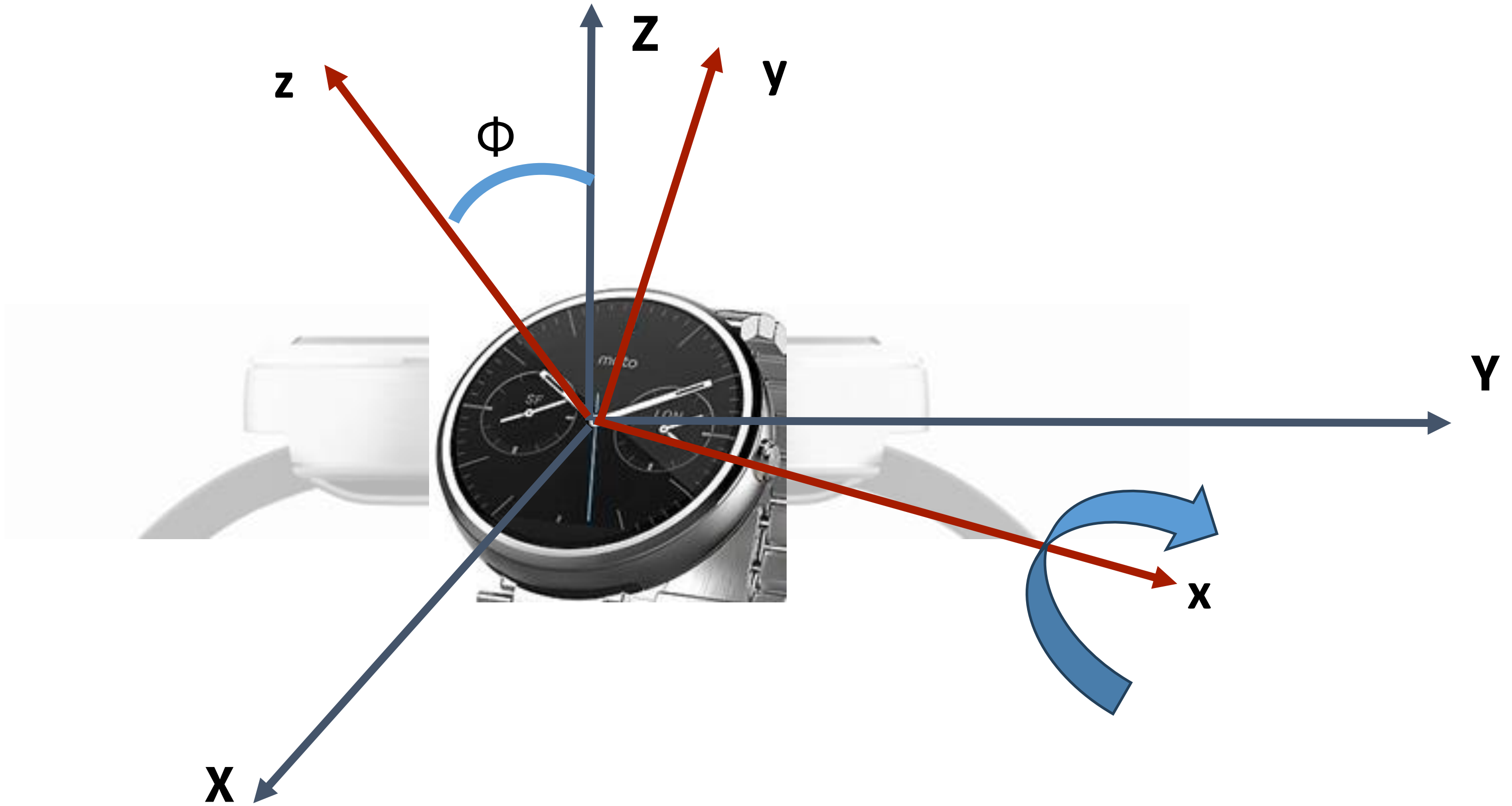


The three angles, roll, pitch and yaw are called the Euler angles to denote the orientation of an object, with respect to a frame of reference

Image Source: Wikipedia

Euler Angle and Angle Transformation

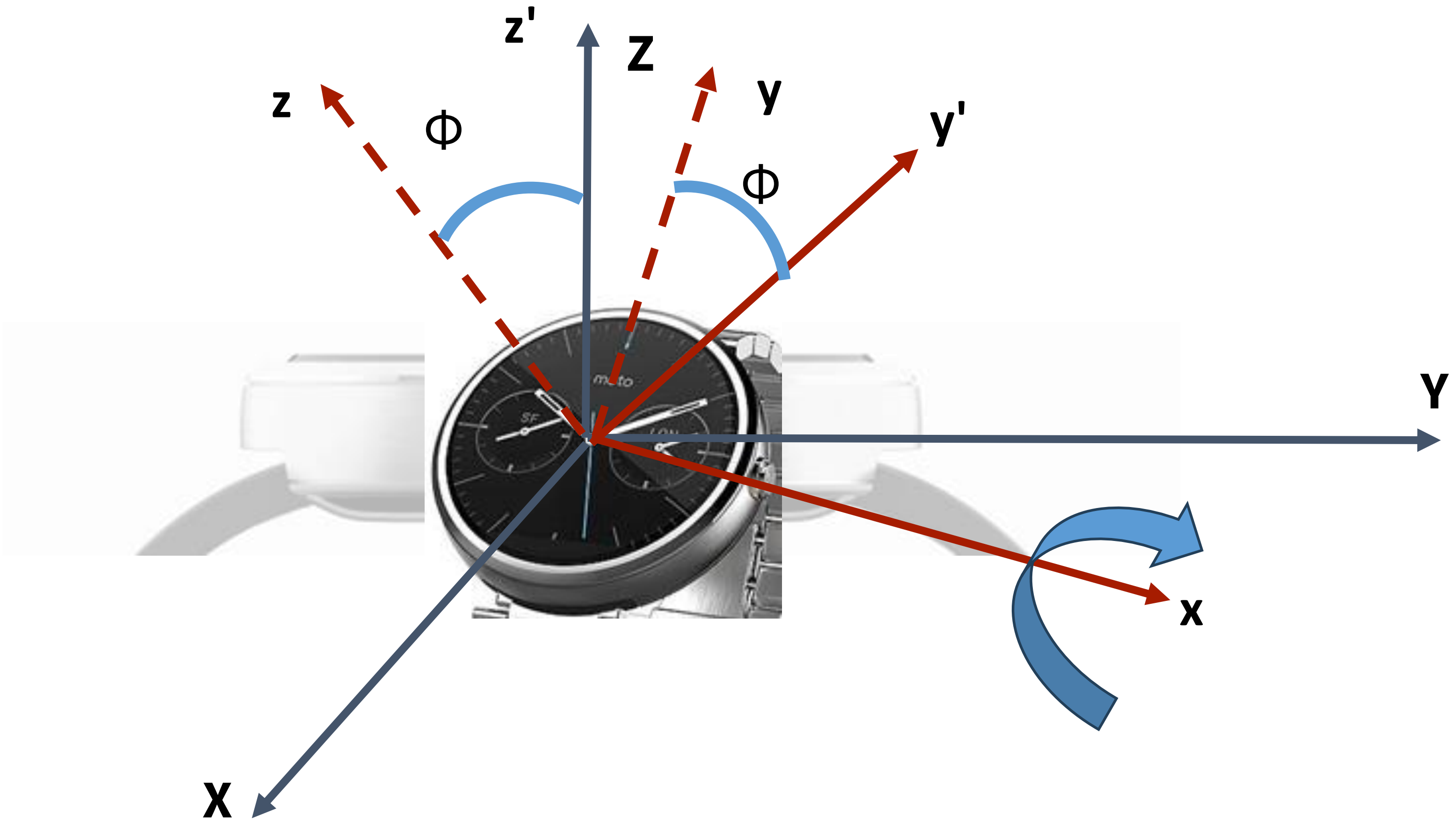
Φ = Roll
Θ = Pitch
Ψ = Yaw



Step 1: Rotate the Z-Y plane on the roll angle to adjust the Z axis (rotation across X-axis)

Euler Angle and Angle Transformation

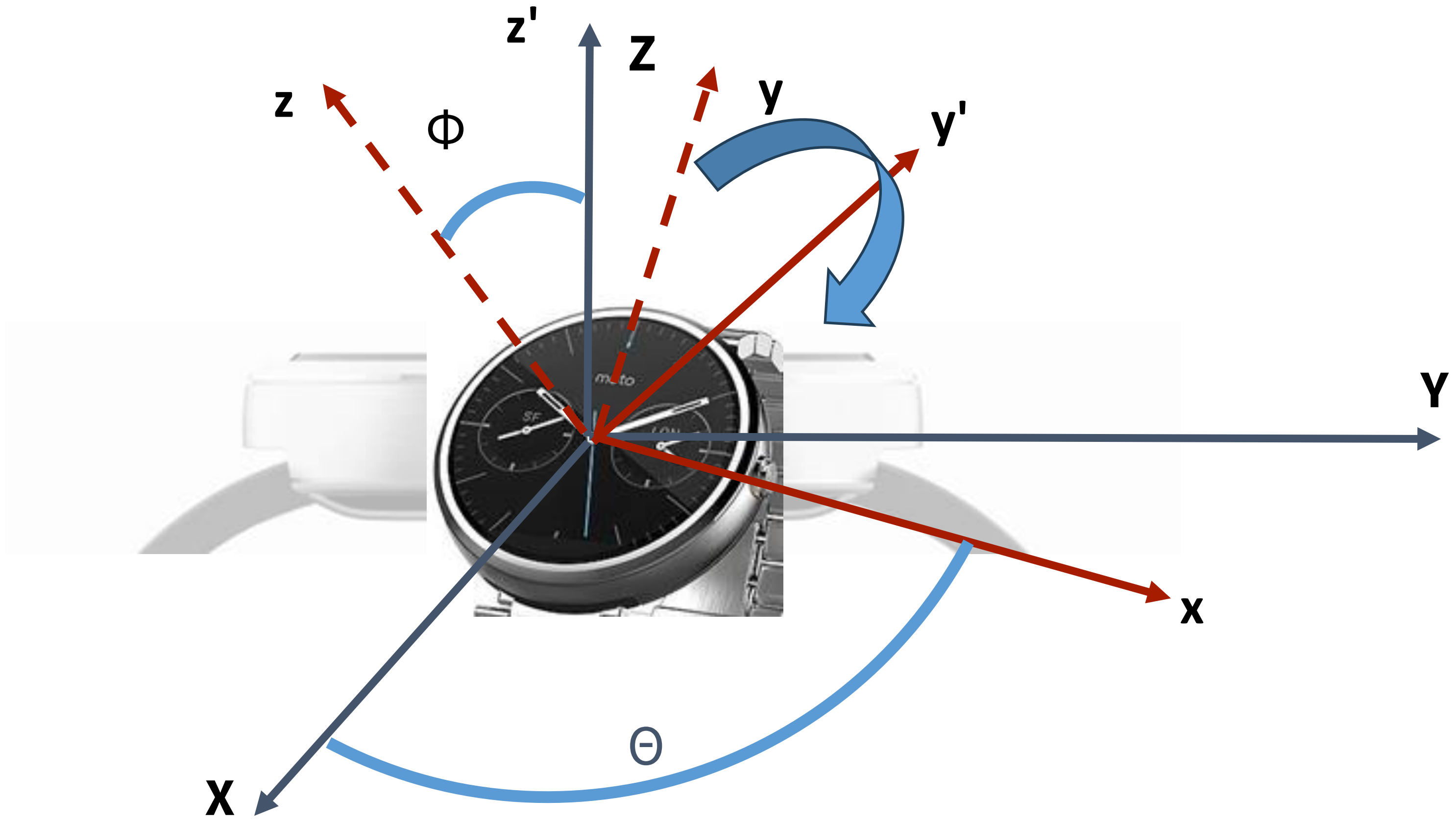
Φ = Roll
Θ = Pitch
Ψ = Yaw



When you try to adjust the Z-axis, the Y-axis also observes an additional rotation on the roll angle

Euler Angle and Angle Transformation

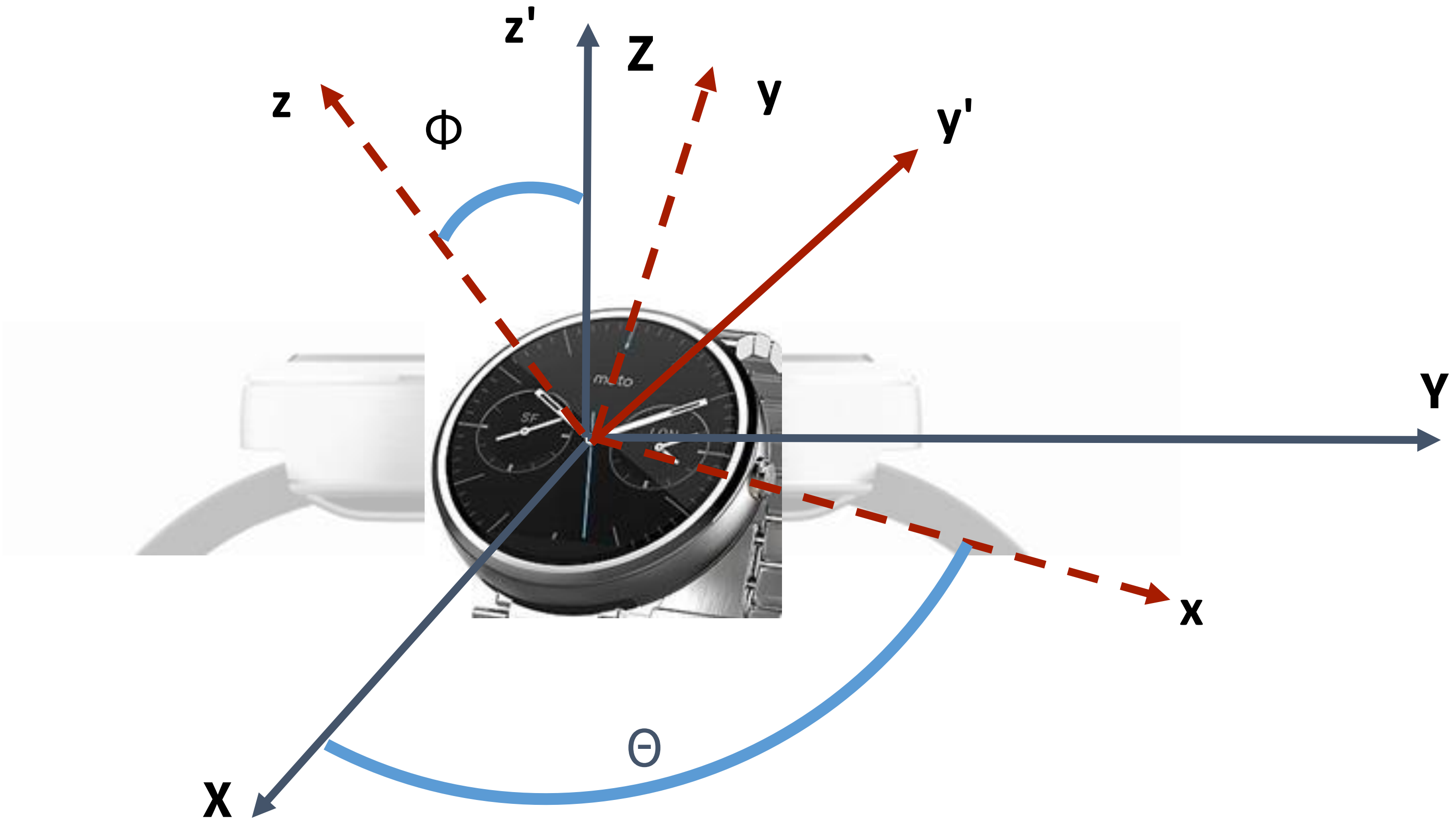
Φ = Roll
Θ = Pitch
Ψ = Yaw



Step 2: Rotate the X-Z plane on the pitch angle to adjust the X-axis (Rotation across Y-axis)

Euler Angle and Angle Transformation

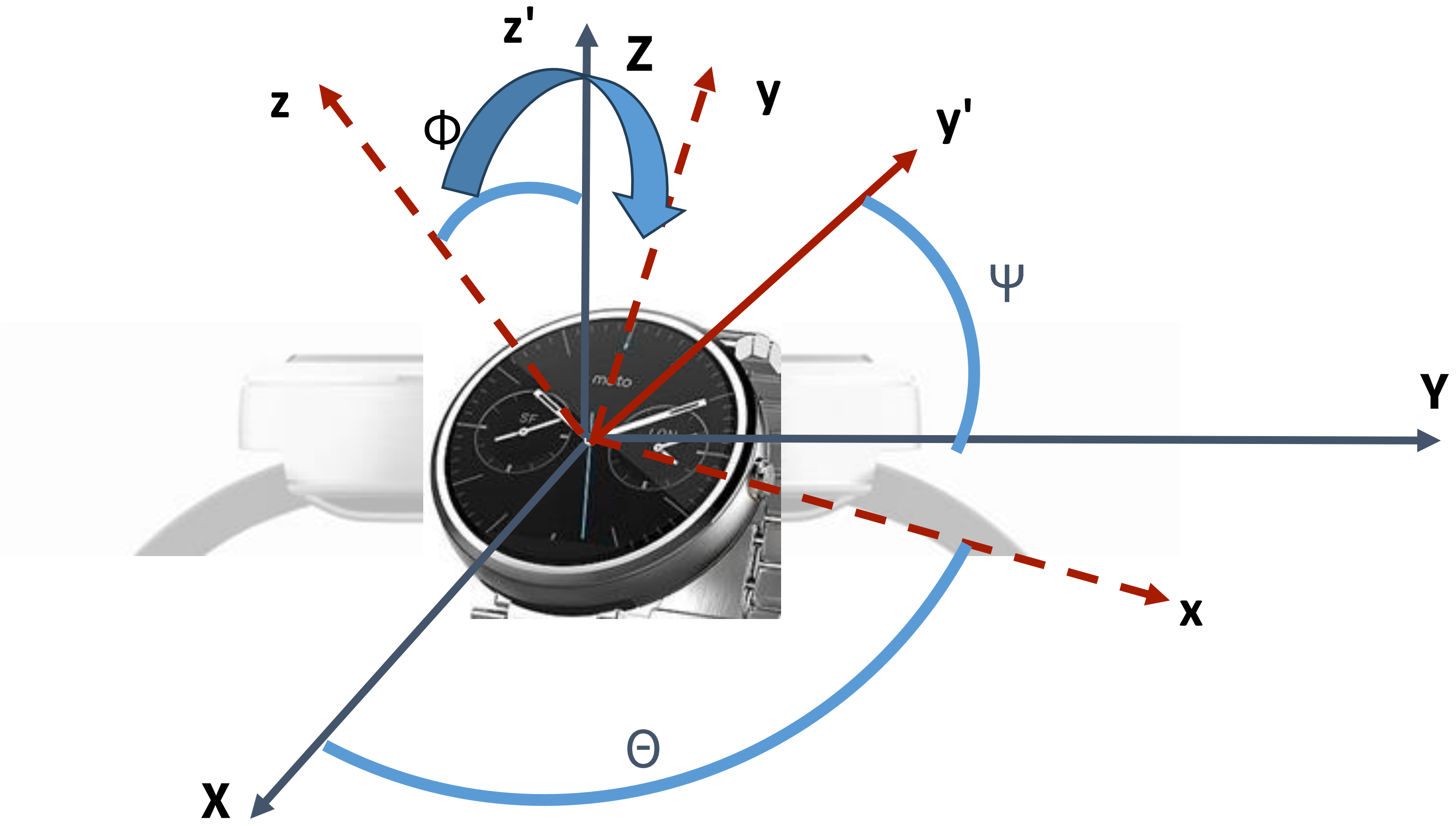
Φ = Roll
Θ = Pitch
Ψ = Yaw



The Y-axis needs to be adjusted now!

Euler Angle and Angle Transformation

Φ = Roll
Θ = Pitch
Ψ = Yaw



Step 3: Rotate the Y-X plane for the yaw angle with respect to the Z-axis

Computing the Euler Angles

- Consider two reference frames R_1 and R_2. Let R_1 be the fixed reference frame of the observer and R_2 be the local reference frame of the object. To figure out the orientation of the object, we need to rotate the object to transform R_2 to R_1
  - The rotations follow right hand thumb rules, and always in the right (clockwise) direction
  - The first rotation with respect to any one of the three axes x, y or z
  - The second rotation along with a different axis
  - The third rotation either w.r.t. the first axis or the leftover axis
  - We have 12 possible combinations in this way: 1-2-1, 1-3-1, 2-1-2, 2-3-2, 3-1-3, 3-2-3, 1-2-3, 1-3-2, 2-3-1, 2-1-3, 3-1-2, 3-2-1
  - The 3-2-1 (yaw, pitch, roll) is one of the most popular ones, and used widely in aircraft navigation

Computing the Euler Angles

- The rotational matrices across the three axes (corresponds to the three rotations is given by):

$$M_1(\alpha) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix}$$

$$M_2(\alpha) = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

$$M_3(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We can replace α with the corresponding roll, pitch or the yaw angles depending on the rotations. For example, for 3-2-1 rotations, the angles corresponding to M1, M2, and M3 would be the roll, the pitch, and the yaw angles

Computing the Euler Angles

- Subsequently, the Direction Cosine Matrix (DCM) is computed as the multiplication of the three rotational matrices across the three axes reading from right two left.
- Ex: The DCM for the 2-3-1 Euler angles will be evaluated as follows:

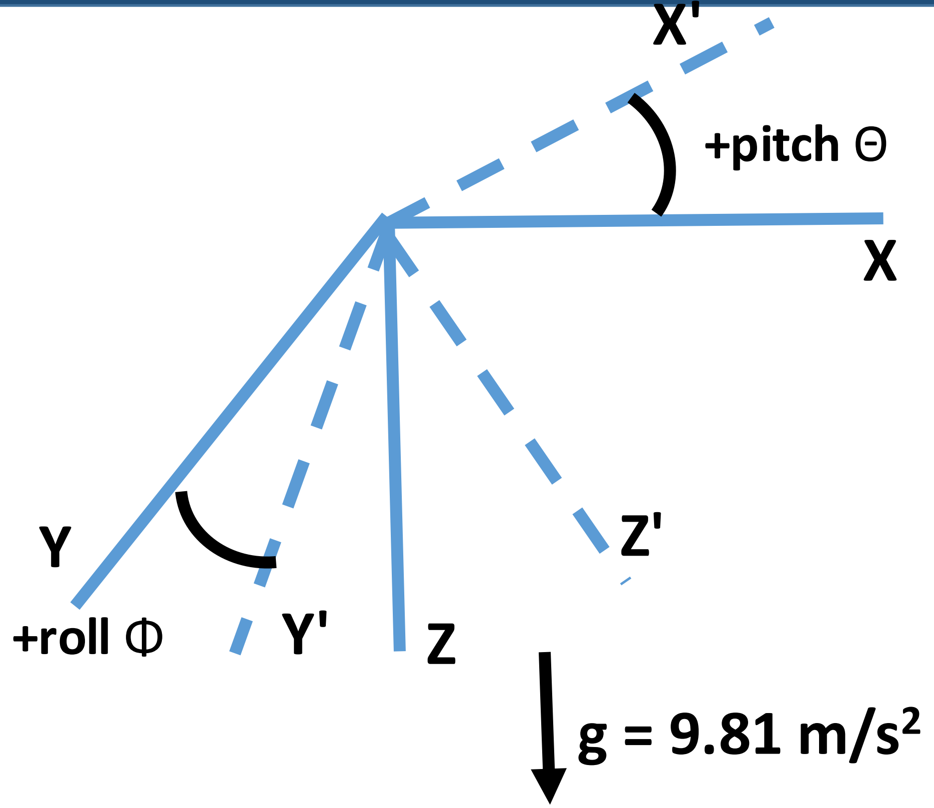
$$B = M_1(\alpha_3)M_3(\alpha_2)M_2(\alpha_1)$$

- Similarly, the DCM for a 3-2-1 Euler angle will look as follows.

$$B = \begin{pmatrix} \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ \sin(\phi)\sin(\theta)\cos(\psi) - \cos(\phi)\sin(\psi) & \sin(\phi)\sin(\theta)\sin(\psi) + \cos(\phi)\cos(\psi) & \sin(\phi)\cos(\theta) \\ \cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi) & \cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi) & \cos(\phi)\cos(\theta) \end{pmatrix}$$

Computing the Euler Angles

- Let [a_x, a_y, a_z] be the acceleration reading on the tilted frame T(X'Y'Z')
- Acceleration on the level frame L(XYZ) is [0 0 g]
- We consider 3-2-1 Euler angle (Yaw Pitch Roll)
- **Can we observe gravity's impact on the three axes of the accelerometer, and based on the acceleration values, measure the Euler angles?**



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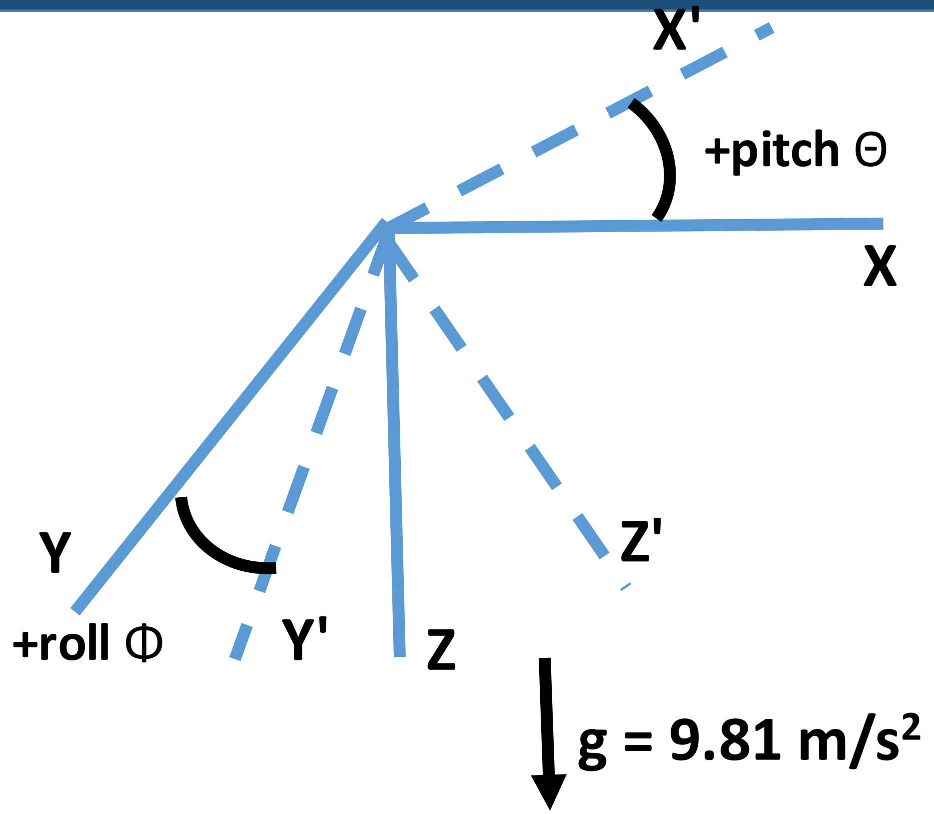
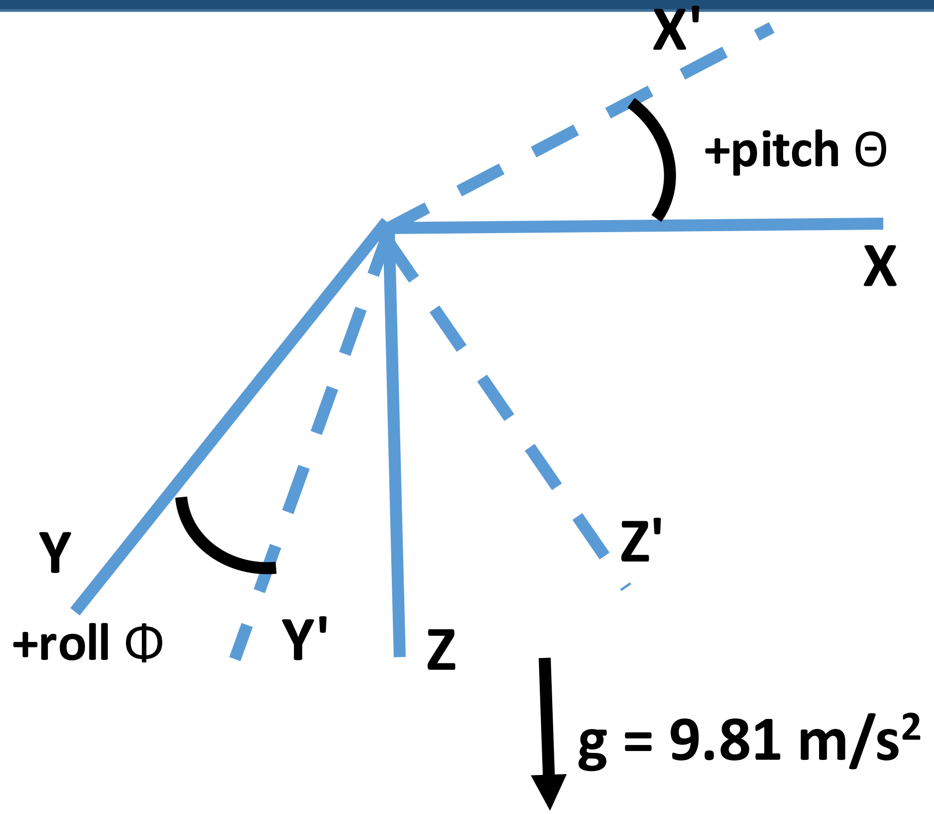
  - This will help us to compute the orientation of the object when in static



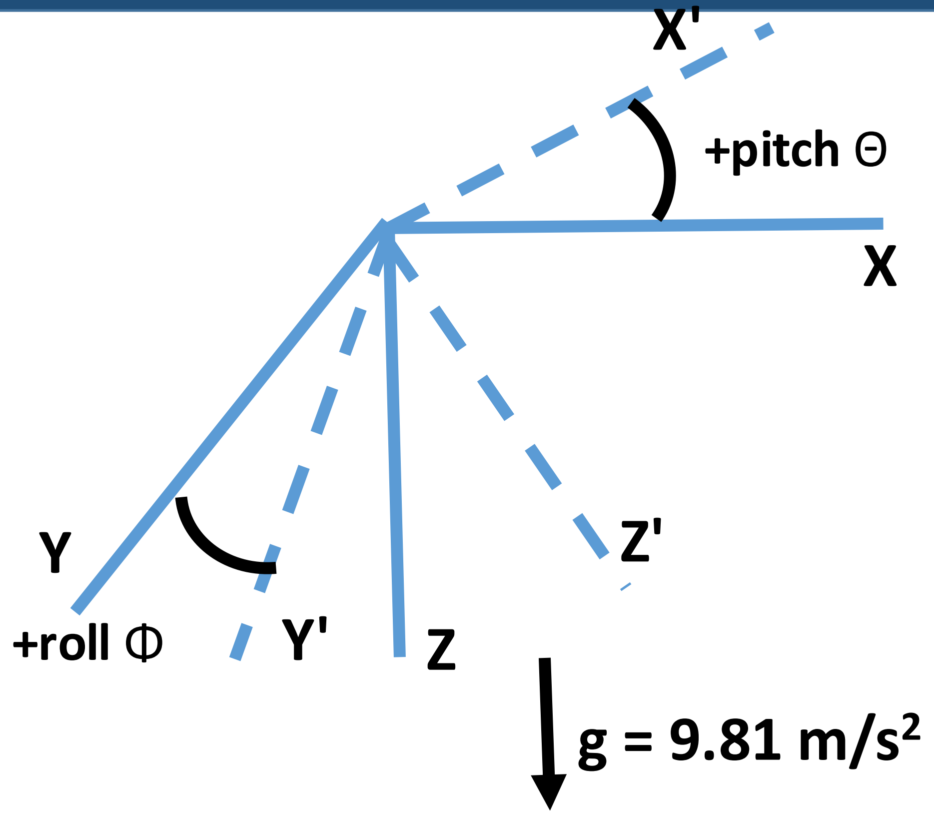
Computing the Euler Angles

- However, using acceleration, we can only compute the roll and the pitch angles, but now the yaw angle
  - As gravity is always downwards (z-axis), a rotation across the z-axis will not have any impact on the accelerometer readings



Computing the Euler Angles

- However, using acceleration, we can only compute the roll and the pitch angles, but now the yaw angle
  - As gravity is always downwards (z-axis), a rotation across the z-axis will not have any impact on the accelerometer readings
  - So, we ignore the yaw angle



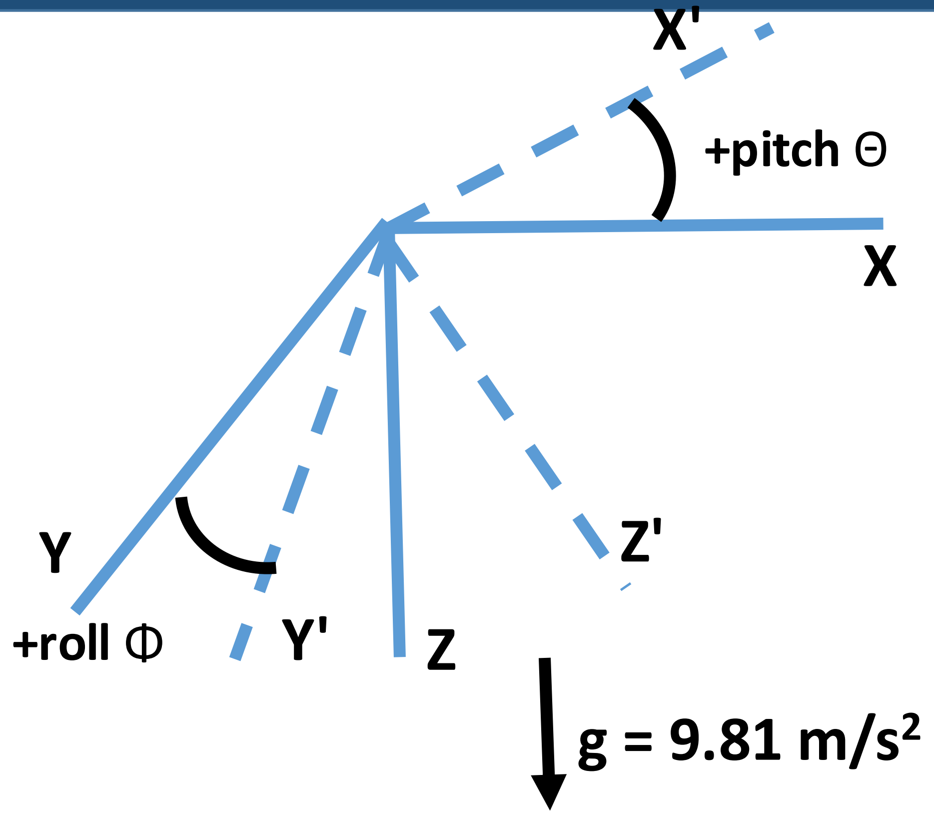
Computing the Euler Angles

- Let us assume the yaw angle to be zero
- We obtain the following DCM:

$$\begin{bmatrix} \cos\phi & 0 & -\sin\theta \\ \sin\theta\sin\phi & \cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & -\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

- Now, we can have the kinematic equations as follows:

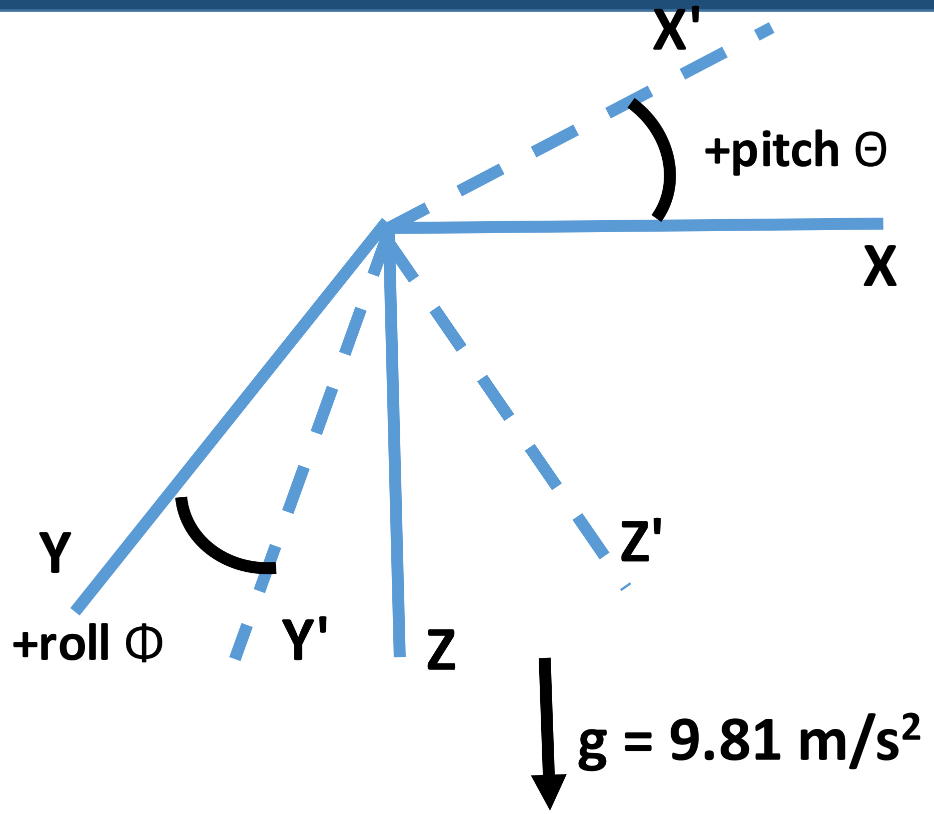
$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 & -\sin\theta \\ \sin\theta\sin\phi & \cos\phi & \cos\theta\sin\phi \\ \sin\theta\cos\phi & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}$$



Computing the Euler Angles

- We obtain the following equations by multiplying the matrices:

$$a_x = g \sin\theta$$
$$a_y = -g \sin\phi \cos\theta$$
$$a_z = -g \cos\phi \cos\theta$$



Computing the Euler Angles

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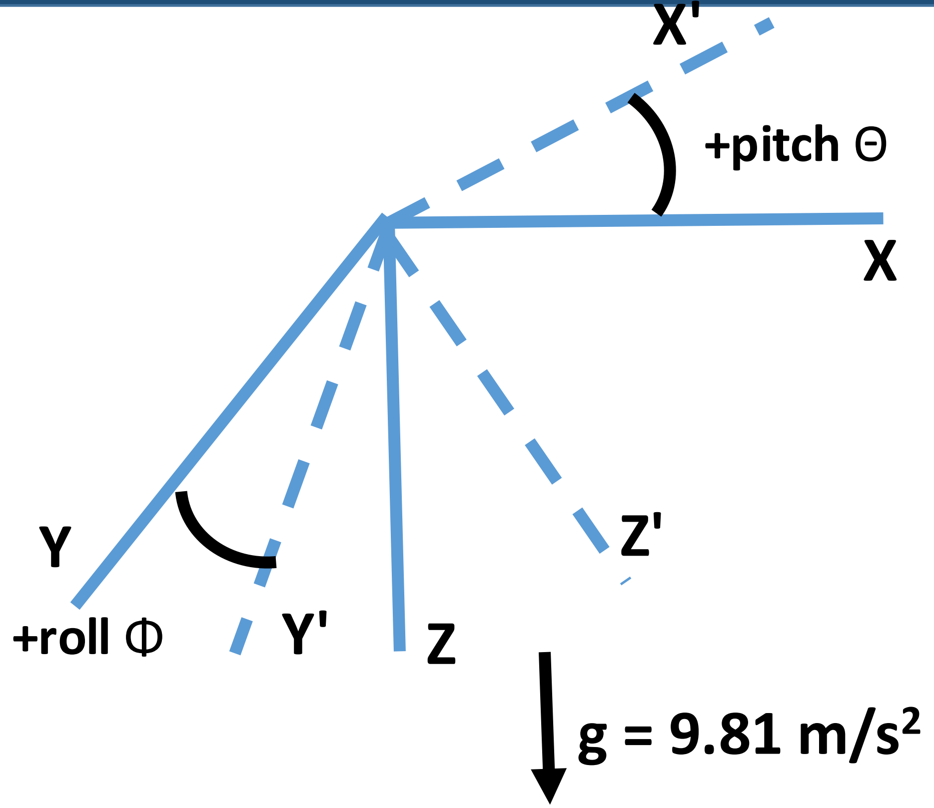
$$a_x = g \sin\theta$$
$$a_y = -g \sin\phi \cos\theta$$
$$a_z = -g \cos\phi \cos\theta$$

- Finally, we obtain the roll and the pitch angles as follows:

$$\theta = \sin^{-1}\frac{a_x}{g}$$

$$\phi = \tan^{-1}\frac{a_y}{a_z}$$

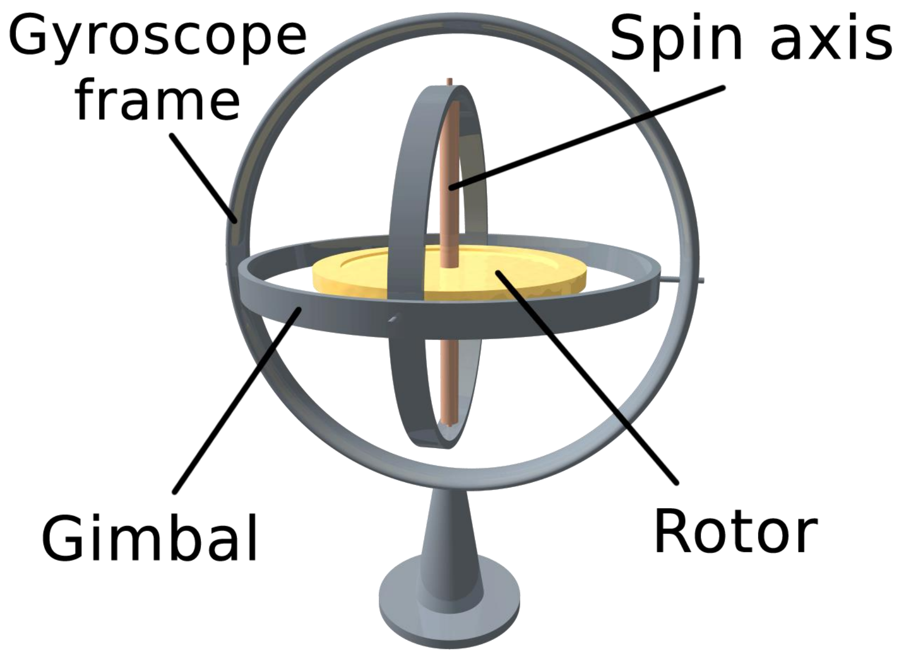


Computing the Euler Angles

- **But how do we compute the yaw angle?**

Computing the Euler Angles

- **But how do we compute the yaw angle?**
- We need a second modality: **Gyroscope**



Gyroscopes

- Measures the **angular velocity** and the **orientation** of the object
  - Angular velocity is measured based on the rotation rate with respect to an inertial frame
  - We already know the roll and the pitch angle from the accelerometer
  - The measured angular velocities across the three axes are then used to compute the orientation (the yaw angle)

Image Source: Wikipedia

Putting them all together ...

- We know,

$$g = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

- Accordingly, we can compute the roll and the pitch angles as;

$$\theta = \tan^{-1} \frac{-a_x}{\sqrt{a_y^2 + a_z^2}}$$

$$\phi = \tan^{-1} \frac{a_y}{a_z}$$

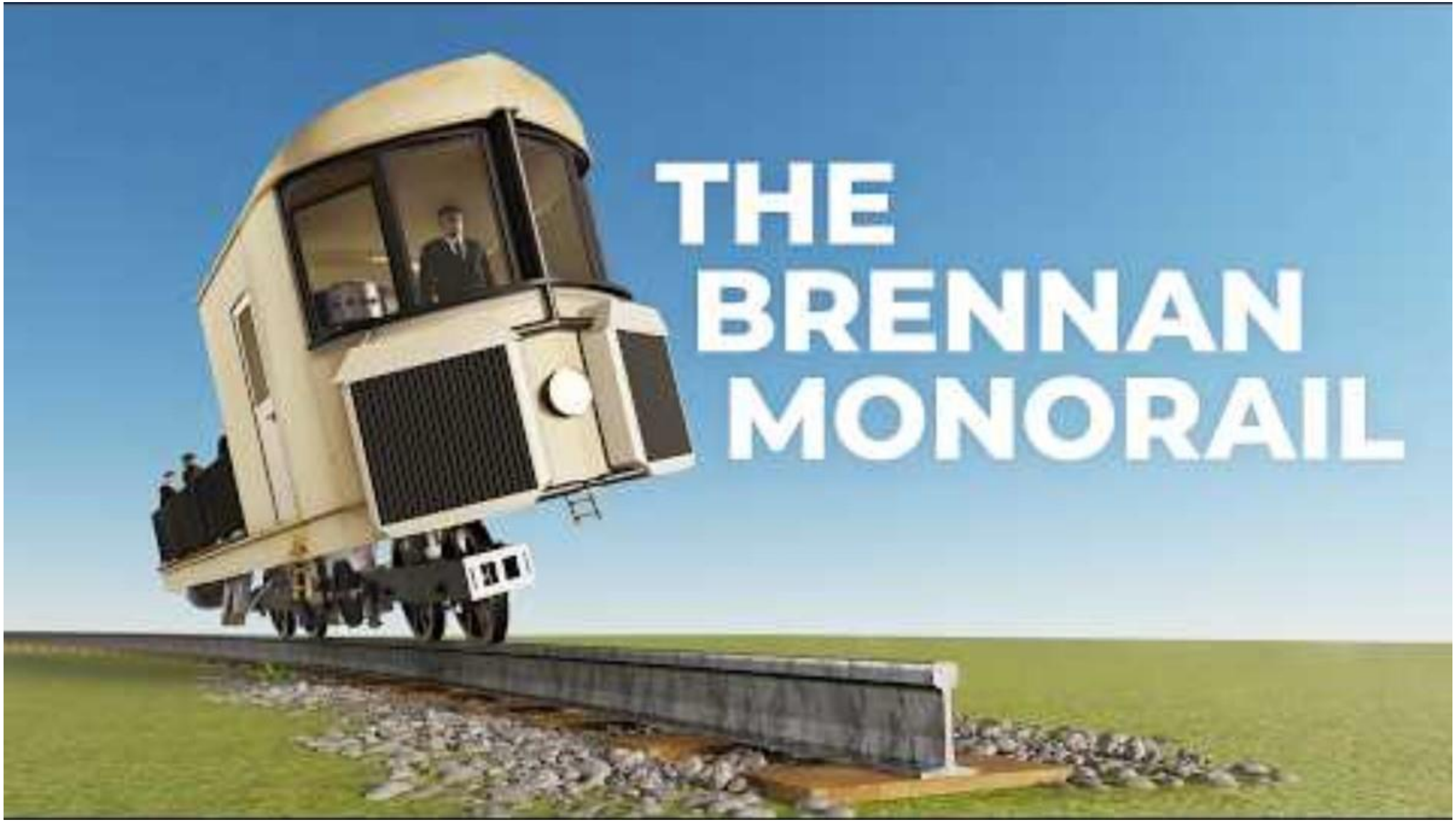
Putting them all together ...

- Gyroscope measures the angular velocity [ω_x, ω_y, ω_z]
  - Multiplying these values with the time (δ_t) will give the angular movement within these time unit, which is indeed the three angles -- roll, pitch and yaw
- However, the orientation angles calculated with this method return noisy values (due to minute calculation of the time)
  - So, IMU (Inertial Measurement Units – combine acclerometer, gyroscope, and magnetometer) uses a Kalman Filter or Complimentary filter based approach to reduce the measured noises with the help of roll and pitch values computed from the accelerometer
    - Indeed, it also takes help of the yaw computed from magnetometer as

$$\tan^{-1}\frac{m_y}{m_x}$$

An Interesting Use-case of Gyroscope

How Can We Use Gyroscope for Self-Balancing?



From Motion to Gesture

Can you think of a method to generate a digital representation of the boardwork from the IMU sensor readings obtained from the smartwatch?

Image generated by ChatGPT 4o

Happy Learning!

Some resources related to this topic