**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans:**

A normal distribution with mu = 45 and sigma = 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour it must have X ≤ 50 so requirement is to find Pr(X > 50).

i.e Pr(X > 50) = 1 - Pr(X ≤ 50).

> 1-pnorm(50,45,8)

[1] 0.2659855

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans:**

1. **FALSE** (For std dev of 6 and mean of 38, most of the employee age will be in 38+/-6 i.e. 32-44.)
2. **TRUE** (Probability of no of employees below 30 is P(<30)=0.0912. so no of employees having age below 30=0.0912\*400=~36.

pnorm(30,38,6)

[1] 0.09121122

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans:**

As the data is normally distributed, the 2 X1 and X1+X2, will be normal.

2X1 is scaled version of X1

In the case the Xi ’s are already normal, then the associated sums and random samples are exactly (and not just approximately) normal, with the appropriate parameters. So X1+X2 is also Normal distributed.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:

The probability value is 0.99 i.e 99%.

The standard deviation rule is **68-95-99.7** for 1 std. dev, 2 std. dev and 3 std. dev.

So the case here need to consider is mu+3sigma=99.

So Range of distribution will be mu +/- 3 sigma = 100 +/- 3\*20 = 160-40

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**Ans:**

1. 95 % probability means 2 std dev from mean.

Mean of profit in rupees in million of two division = 45 \*(5+7) =540

St dev. of two division is = sqrt(var1+var2)= sqrt(9+16)=5

And in million rupees is st. dev. of devision= 45\*5=225

So 95 % probability range= mu+/- 2 Std. Dev= 540+/-225= (315-765) Million

1. For 5th percentile the z score is -1.645.

So X=mean + Z sigma

=540+(-1.645)\*225

=169.875 million

1. To check chance of loss we test the x=0 in z test

So for division 1: > pnorm(0,5,3)

[1] 0.04779035

division 2: > pnorm(0,7,4)

[1] 0.04005916

Hence division two will incur more loss