

Binomial Dist.

E.g. In the recent survey it was found that 85% of household in USA have a high speed internet. If you take the sample of 18 household, what is the probability that exactly 15 will have high internet.

Solⁿ

① If the exp. being repeated - yes

② Are the trials independent - yes

$$n = 18$$

$$X = 15$$

$$Pr = 85 = 0.85$$

$$\begin{aligned}
 P(X=15) &= {}^nC_X p^X (1-p)^{n-X} \\
 &= {}^{18}C_{15} (0.85)^{15} (1-0.85)^{18-15} \\
 &= 0.234 \Rightarrow 23.4\%
 \end{aligned}$$

Z-test

In a population the Avg IQ. $\mu = 100$ with $\sigma = 15$ then the doctor tested a new medication to find out it increase the IQ or decrease the IQ

After medication

> 100

< 100

After one month sample of 30 participant were taken and 30 participant had mean is 140

Did this medication effect intelligence given is significant value α is 0.05.

Solⁿ

$$\mu > \bar{x}$$

$$\mu < \bar{x}$$

$$\mu = 100, \sigma = 15, n = 30$$

$$\bar{x} = 140$$

$$\alpha = 0.05$$

$$Z_{table} - 1 - 0.05 = 0.95$$

$$Z_{\alpha} = 1.6 + 0.05 = 1.65$$

= Z-test

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{140 - 100}{15/\sqrt{30}}$$

✓ $Z = 14.65$

$$1.96$$

$$\text{lower} \Rightarrow \bar{X} - z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

$$140 - 1.65 \times \frac{15}{\sqrt{30}}$$

$$\Rightarrow 135.48$$

$$\text{Hight} \Rightarrow 140 + 1.65 \times \frac{15}{\sqrt{30}}$$

$$\Rightarrow 144.51$$

$$n=100$$

$$135 \text{ — } 144$$

Conclusion - We reject null hypothesis and accept alternate hypothesis.

T-test

① on the verbal section of CAT
 Sample of 25 test taken has
 a mean of 520 with standard
 Deviation of sample is 80.
 CI = 95%.

Soln

$$\bar{x} = 520$$

$$n = 25$$

$$S = 80$$

$$C.I. = 95$$

$$\alpha = 100 - 95 = 5\%$$

$$= 0.05/2 = 0.025$$

$$T\text{-table} = 0.025$$

$$\text{Degree of freedom} = n - 1$$

$$= 25 - 1$$

$$= 24$$

value from T-table = 2.391

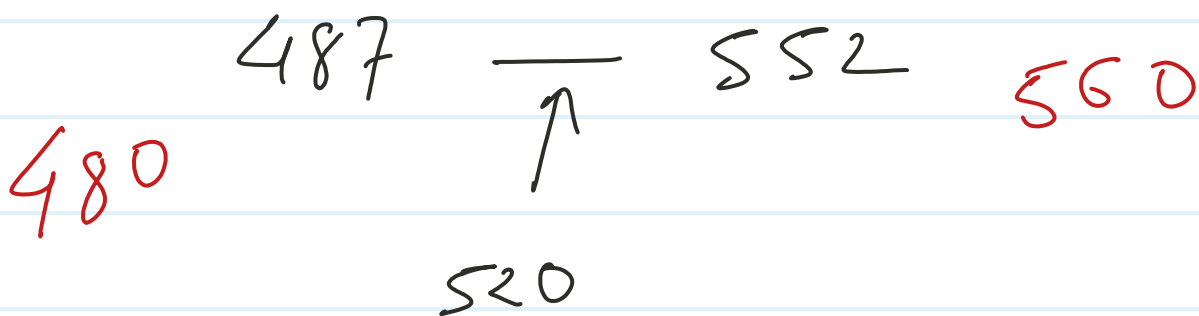
$$\underline{\text{T-test}}_{\text{lower}} = \bar{x} \pm t_{\alpha/2} (S/\sqrt{n})$$

$$= 520 - 2.064 \left(\frac{80}{\sqrt{25}} \right)$$

$$= 487.61$$

$$\text{upper} = 520 + 2.064 \left(\frac{80}{\sqrt{25}} \right)$$

$$= 552.38$$



We fail to reject null hypothesis.

we reject null hypothesis and accept alternate hypothesis.

★ Discrete probability Dist.

↓

③ Poisson probability Dist.

Ex: A small business receive an average 12 customer per day. What is the probability that the business will receive exactly 8 customers.

euler number

Soln

$$\mu = 12$$

$$x = 8$$

Formula $P_x(X=x) = \frac{\mu^x e^{-\mu}}{x!}$

$$\Rightarrow \frac{12^8 e^{-12}}{8!}$$

$$\Rightarrow \frac{429981696 \times 0.000000614}{40320}$$

$$\Rightarrow 0.0654$$

$$\boxed{8 \Rightarrow 6.54 \%}$$

e.g.

A small business receive on average 8 calls per hour.

- (a) Probability that business receive exactly 7 calls in an hour.
- (b) probability that the business will receive at most 5 call in an hour.
- (c) Probability that the business will receive more than 6 calls in an hour.

$$\textcircled{1} \quad \mu = 8, \quad x = 7$$

$$\Rightarrow \frac{8^7 e^{-8}}{7!} \Rightarrow 0.139 = 13.9\%$$

$$\textcircled{2} \quad \mu = 8 \quad p(x \leq 5)$$

$$p(x \leq 5) = p(x=0) + p(1) + p(2) + p(3) + p(4) + p(5)$$

$$p(x=x) = \frac{\mu^x e^{-\mu}}{x!} = e^{-\mu} \left[\frac{\mu^x}{x!} \right]$$

$$\Rightarrow e^{-8} \left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} \right]$$

$$p(x \leq 5) = 0.1917 = 19.17\%$$

$$\textcircled{3} \quad \mu = 8 \quad P(X > 6)$$

$$P(X > 6) = 1 - P(X \leq 6)$$

$$\Rightarrow 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5) + P(6)]$$

$$\Rightarrow 1 - e^{-8} \left[\frac{8^0}{0!} + \frac{8^1}{1!} + \frac{8^2}{2!} + \frac{8^3}{3!} + \frac{8^4}{4!} + \frac{8^5}{5!} + \frac{8^6}{6!} \right]$$

$$\Rightarrow 1 - 0.31$$

$$\boxed{\Rightarrow 68.6\%} \text{ more than 6 calls.}$$

