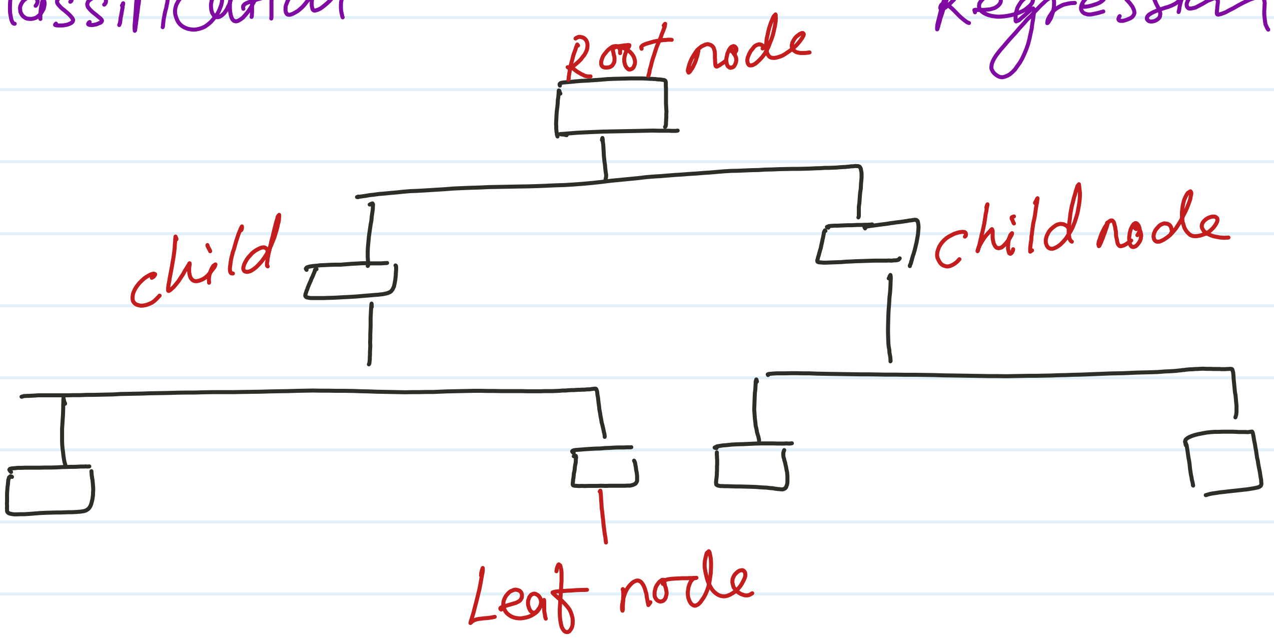
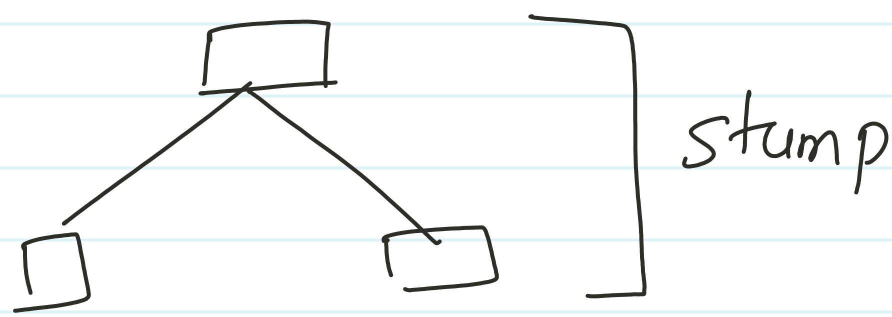
Decision tree

classification | Regression





method of solve decision Tree algorithm -

1. ID3 — Entropy, Information Gain
2. Cart — Gini Index

Entropy OR Gini Index

Information Gain

* In DT we do not required to label categorical data into numerical

* Entropy and Gini Index → purity split in dataset

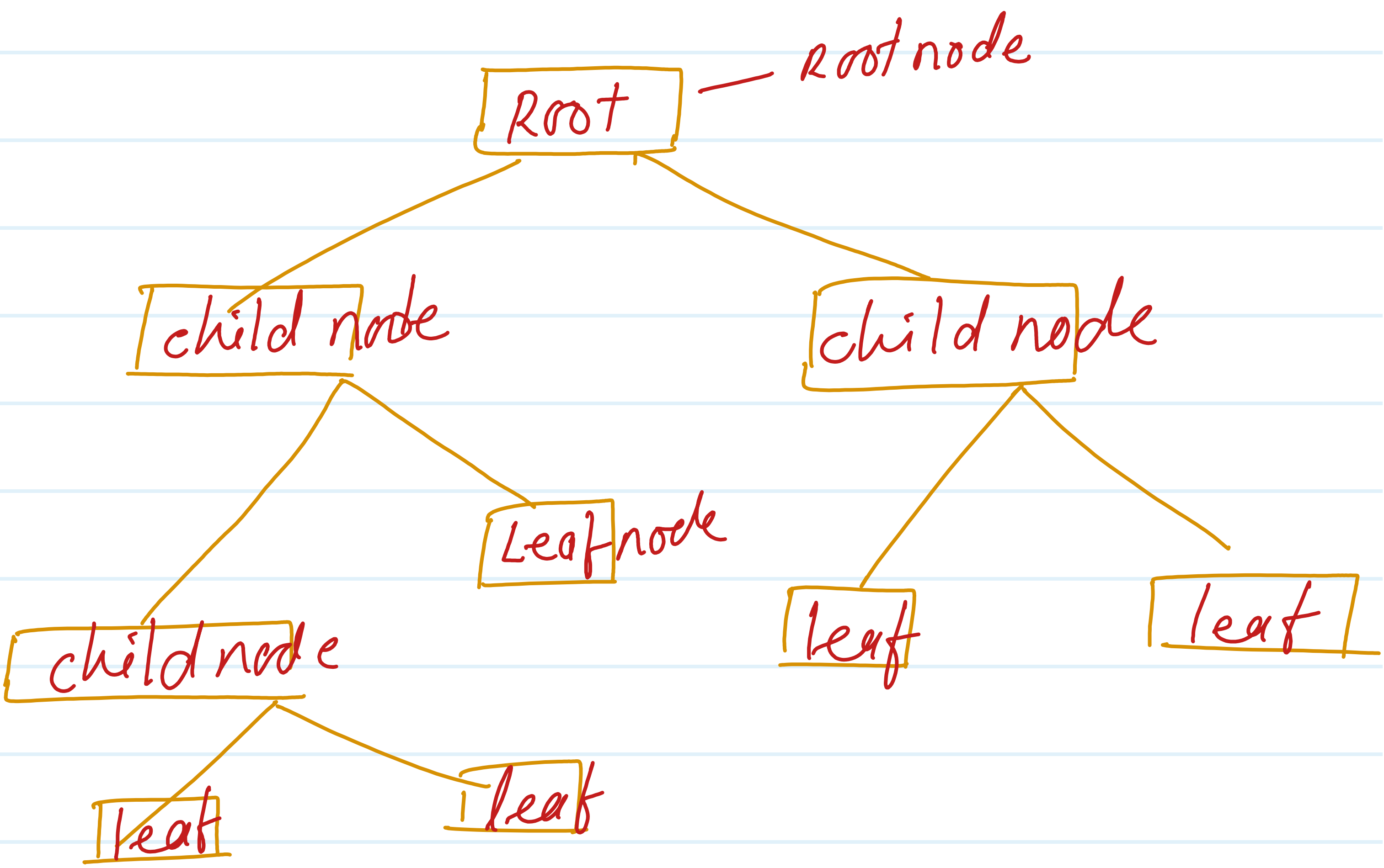
* Information Gain → DT feature split

weight	height	o/p obese/No obese.
60	160	ob
70	170	No
80	180	ob
90	190	No
100	200	No

(11) DT Regressor

Regression we use standard deviation / MSE / MAE

weight	height	BMI
60	160	21
70	170	22
80	180	20
85	190	23
90	195	24

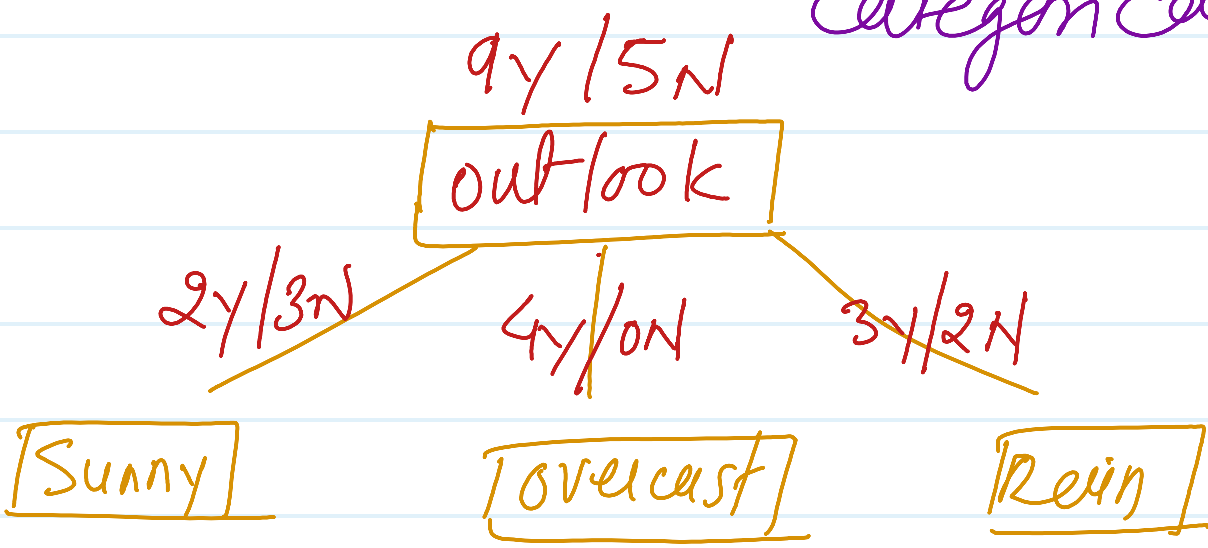


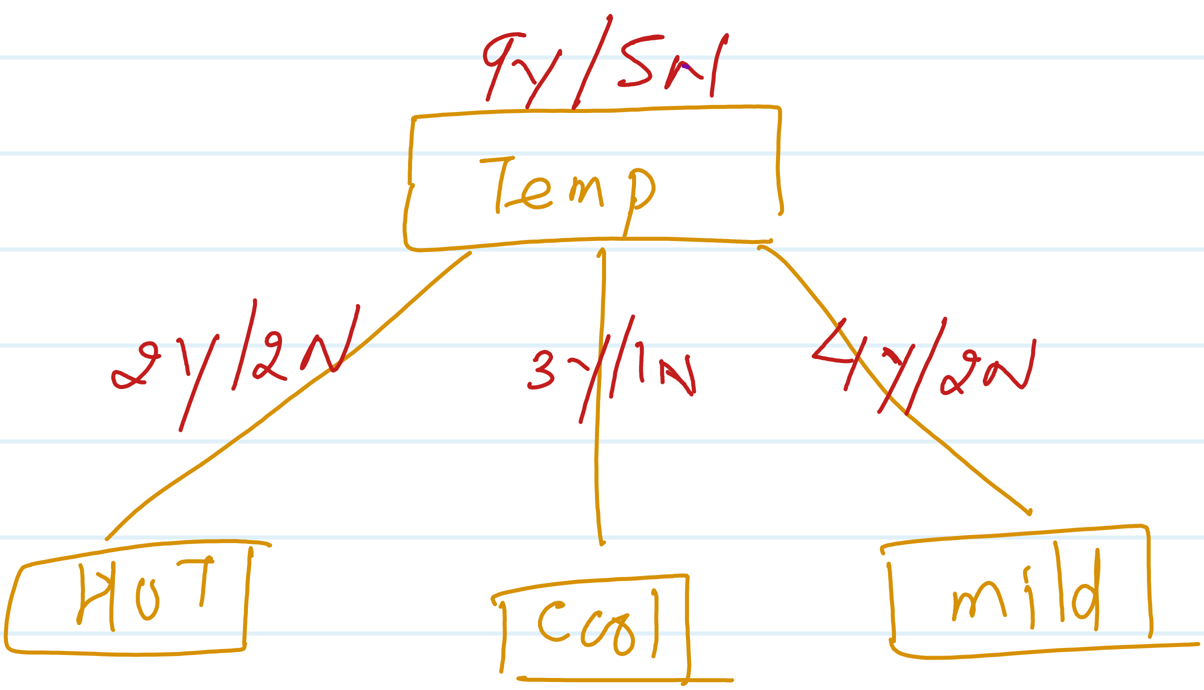
Decision Tree Classifies

outlook	Temp	humidity	wind	play
sunny	H	High	weak	N
sunny	H	H	strong	N
overcast	H	H	W	Y
rain	M	H	W	Y
rain	C	Normal	W	Y
rain	C	N	S	N
overcast	C	N	W	Y
sunny	M	H	W	N
sunny	C	N	W	Y
rain	M	N	W	Y
sunny	M	N	S	Y
overcast	M	high	S	Y
overcast	H	N	W	Y
rain	M	H	S	N

// Feature can be numeric and categorical

// Output can be numeric and categorical





(a) Entropy $H(s)$

⇒ **Formula** (Binary class)

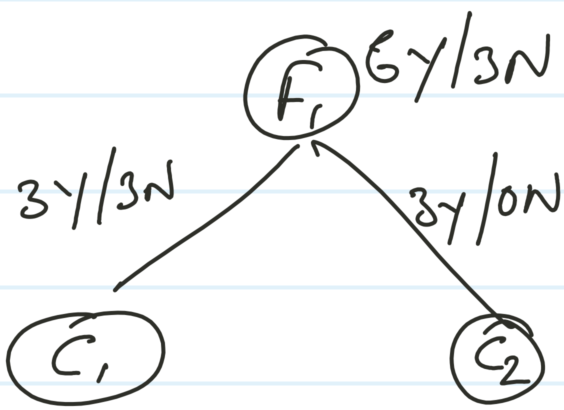
$$H(s) = -P_{yes}\log_2(P_{yes}) - P_{No}\log_e(P_{No})$$

multiclass

$$H(s) = -P_{c_1}\log_2(P_{c_1}) - P_{c_2}\log_2(P_{c_2}) - P_{c_3}\log_2(P_{c_3})$$

Example

F_1 F_2 F_3 o/p



$$C_1 \Rightarrow H(s) = -\frac{3}{6}\log\left(\frac{3}{6}\right) - \frac{3}{6}\log\frac{3}{6}$$

⇒ 1 impure split

$$C_2 \Rightarrow H(s) = -\frac{3}{3}\log\frac{3}{3} - \frac{0}{3}\log\frac{0}{3}$$

⇒ 0 pure split

For the pure split of feature

pure entropy should be zero (0)

for impure split = 1

③ Gini index (Impurities).

main formula -

$$G.I. = 1 - \sum_{i=1}^{n} (P)^2$$

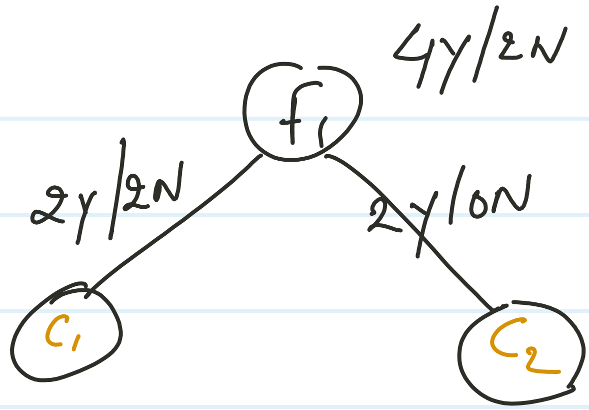
binary class.

$$G.I. = 1 - \sum_{i=1}^{n} \left[(P_{c_1})^2 + (P_{c_2})^2 \right]$$

multiclass

$$G.I. = 1 - \sum_{i=1}^{n} \left[(P_{c_1})^2 + (P_{c_2})^2 + (P_{c_3})^2 + \cdots \right]$$

Example



$$C_1 \Rightarrow G.I. = 1 - \left[\left(\frac{2}{4}\right)^2 + \left(\frac{2}{4}\right)^2 \right]$$

$$\boxed{= 0.5}$$

$$C_2 \Rightarrow G.I. = 1 - \left[\left(\frac{2}{2}\right)^2 + \left(\frac{0}{2}\right)^2 \right]$$

$$\boxed{= 0}$$

Range of entropy = 0 to 1

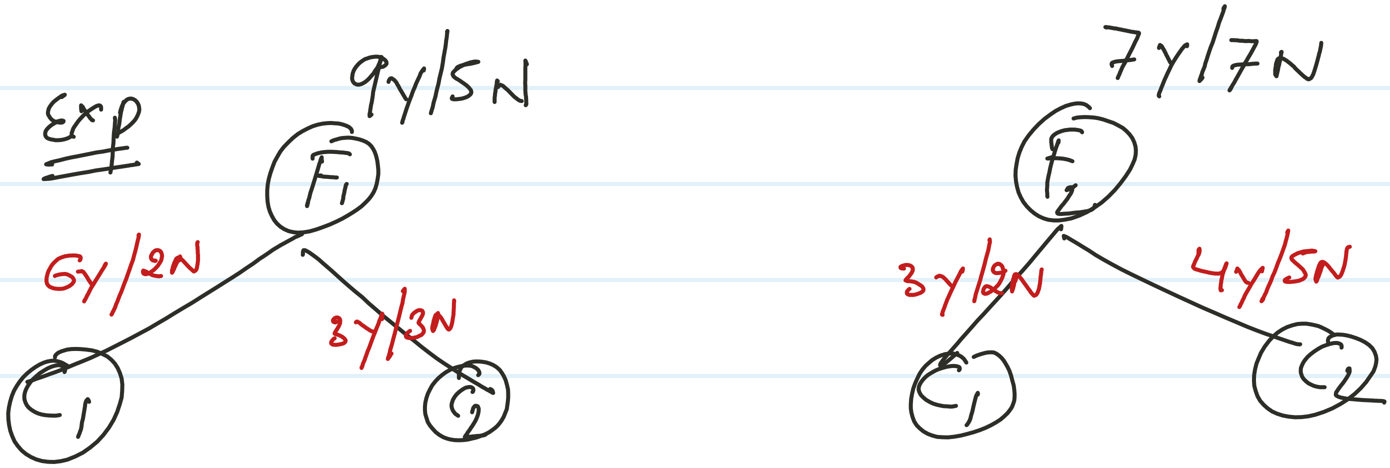
Gini impurity (index) = 0 to 0.5

(c) Information Gain

Formula –

$$gain(S, f_1) = H(S) - \sum \frac{|S_v|}{|S|} H(S_v)$$

Exp



⇒ For F_1 ⇒

$$H(S) = -\frac{9}{14}\log\frac{9}{14} - \frac{5}{14}\log\frac{5}{14}$$

$$\boxed{H(S) = 0.94}$$

C_1 ⇒ $H(S) = -\frac{6}{8}\log\frac{6}{8} - \frac{2}{8}\log\frac{2}{8}$

$$\boxed{H(S) = 0.81}$$

$C_2 \Rightarrow$

$$H(s) = -\frac{3}{3}\log\frac{3}{3} - \frac{0}{3}\log\frac{0}{3}$$

$$\boxed{H(s) = 1}$$

gain of $f_1 \Rightarrow$

$$gain(s, f_1) = 0.94 - \left[\frac{8}{14} \times 0.81 + \frac{6}{14} \times 1\right]$$

$$gain(s, f_1) = 0.049$$

$f_2 \rightarrow H(s) = -\frac{7}{7}\log\frac{7}{7} - \frac{7}{7}\log\frac{7}{7}$

$$= 0$$

$C_1 \rightarrow H(s) = -\frac{3}{5}\log\frac{3}{5} - \frac{2}{5}\log\frac{2}{5}$

$$= 0.133 + 0.159$$

$$\boxed{= 0.29}$$

$C_2 \Rightarrow H(S) = -\frac{4}{9}\log\frac{4}{9} - \frac{5}{9}\log\frac{5}{9}$

$$\boxed{= 0.014}$$

f_2 $\text{gain}(S, f_2) = 0 - \left[\frac{5}{14} \times 0.29 + \frac{9}{14} \times 0.014\right]$

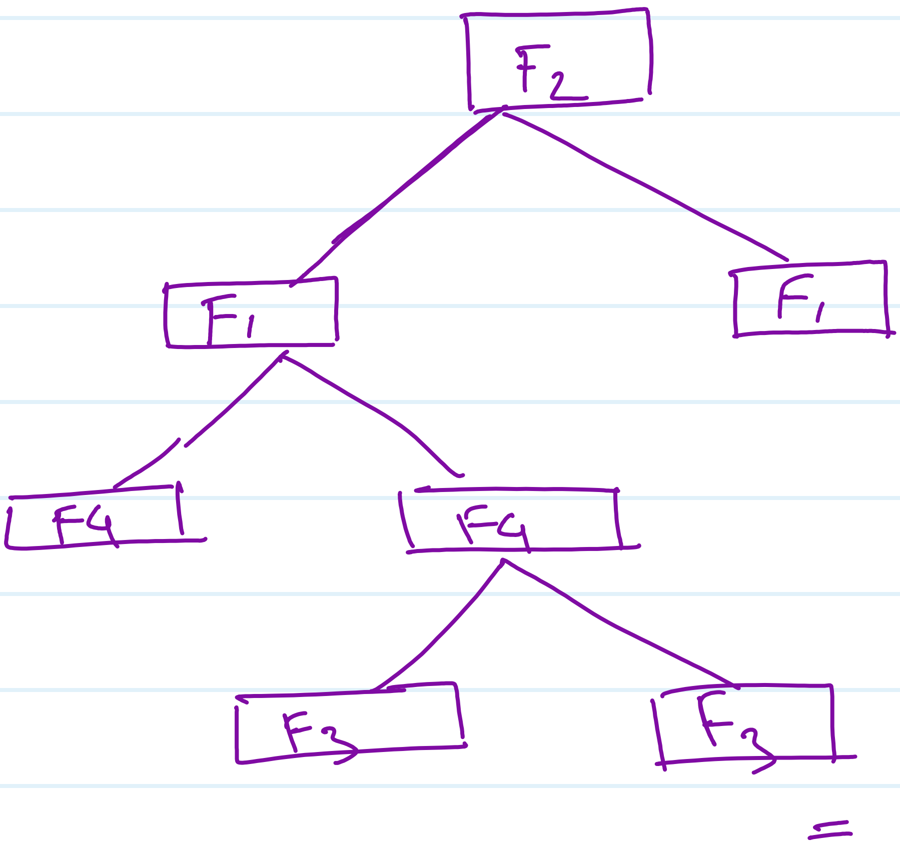
$= 0 - [0.10 + 0.009]$

$$\boxed{= -0.10}$$

F_1	F_2	F_3	F_4
0.49	0.56	0.025	0.10

Since F_2 has highes value of information gain's among the all feature so that It will be our root node.

F_1	F_2	F_3	F_4
0.49	0.56	0.025	0.10



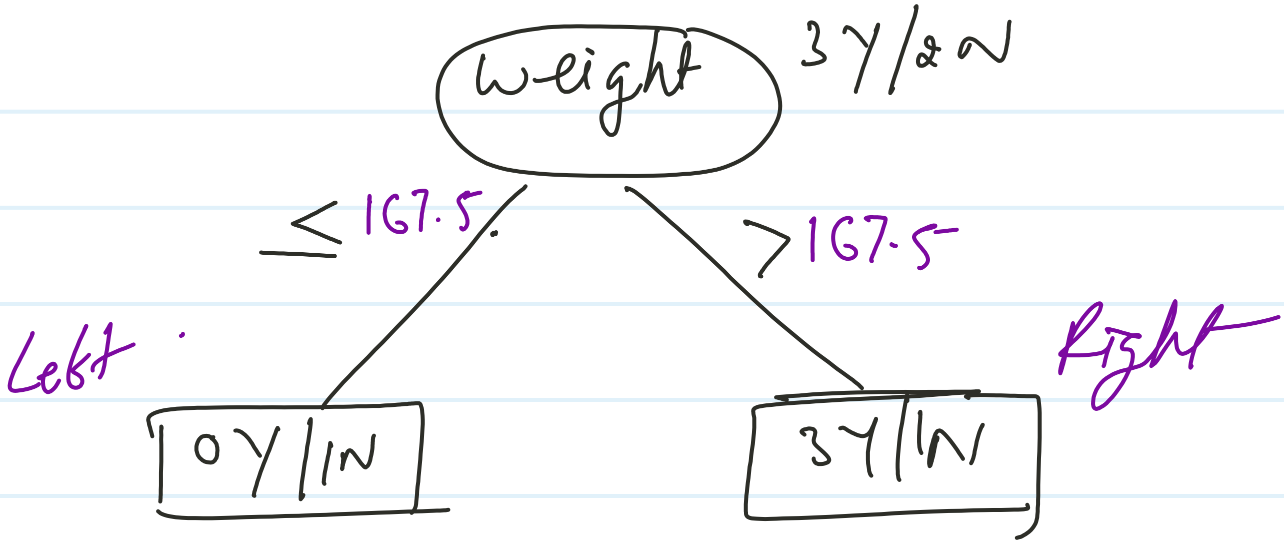
★ Independent analysis befor making DT

build DT with numerical feature

weight	heart De.
220	Y
180	Y
225	Y
190	N
155	N

weight		Heart
155		N
	167.5	
180		Y
	185	
190		N
	204	
220		Y
	222.4	
225		Y

with respect to every point avg. value need to find out gini index / Entropy



$$\text{Gini impurity} = 1 - \sum_{i=1}^{n} P_i^2$$

$$\text{gini (Left)} = 0$$

$$\text{gini (Right)} = 1 - \left[\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2\right]$$

$$= 0.375$$

$$\text{Information gain} = G.I.[\text{Root}] - \sum_{\text{Value}} \frac{|S_v|}{|S|} \, G.I.[\text{child}]$$

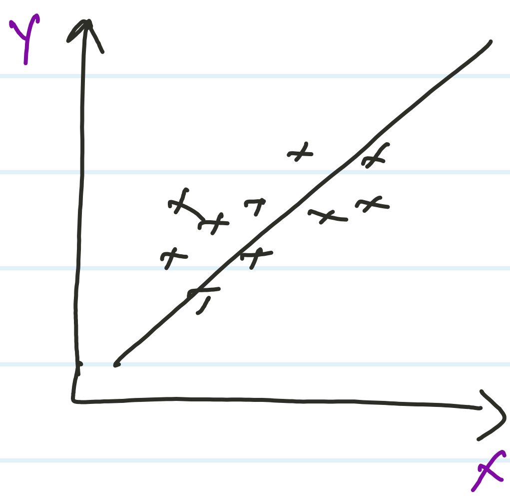
$$G.I.[\text{Root}] = 0.48$$

$$I.G.[167.5] = 0.48 \left[\frac{1}{5} \times 0 + \frac{4}{5} \times 0.375\right]$$

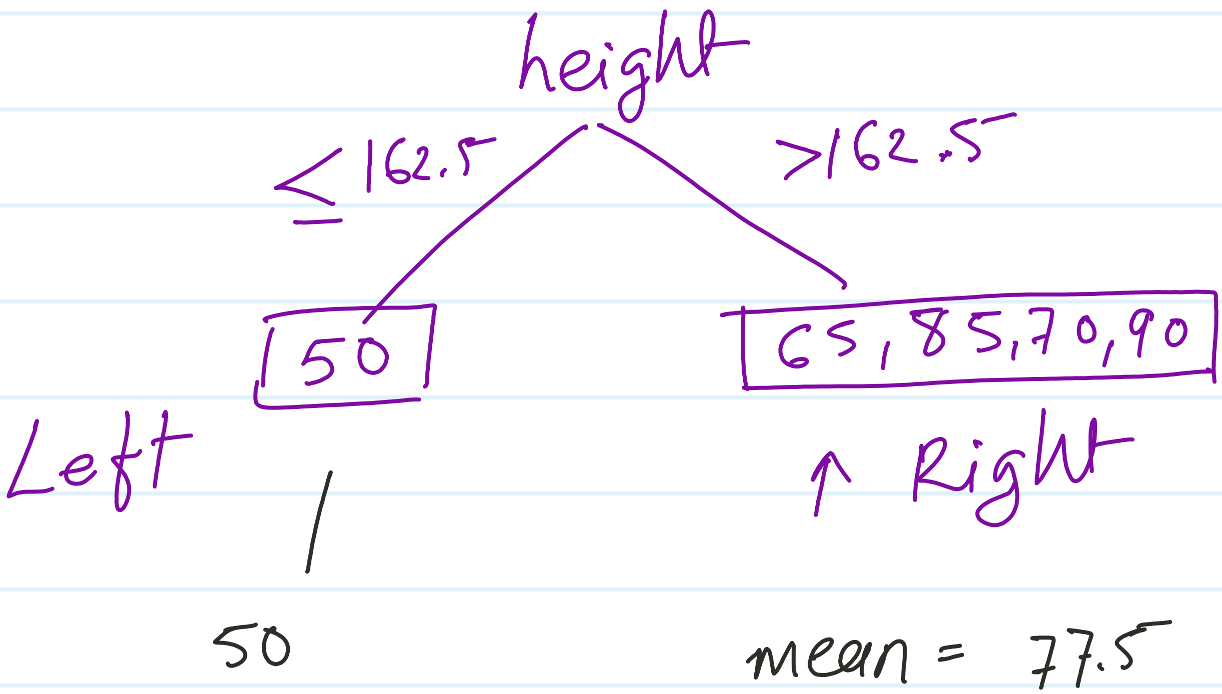
$$I.G.[167.5] = 0.18$$

Information Gain should be high and Gini Index should be low.

☆ DT Regression



height		weight
160		50
	162.5	
165		65
	167.5	
170		85
	172.5	
175		70
	177.5	
180		90



Ex

	height	weight
	165	65
162.5		
	160	50
167.5		
	180	90
172.5		
	170	85
177.5		
	175	70

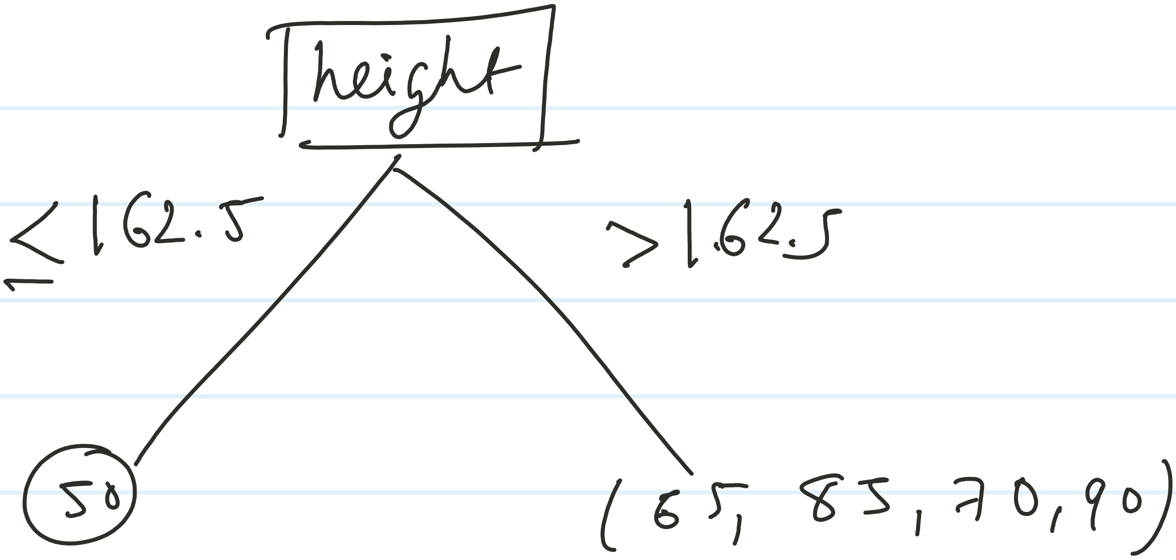
160
165
170
175
180

Regression problem weight calculated with respect to height

① Step - Short the value of height column (x Featur)

② step - Find Adjcent Avg. value b/w data point

③ step - Find Information gain with help of entropy and Gini Index.



mean = 77.5

Regression -

1. mean = 77.5
2. MSE, RMSE, MAE

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y - \hat{y})^2$$

overall mean $= \dfrac{50 + 65 + 85 + 70 + 90}{5} = 72.$

$$\text{height (variance)} = \frac{(72-50)^2 + (72-65)^2 + (72-85)^2 + (72-70)^2 + (72-90)^2}{5}$$

hight (variance) = 206

$$var(Right) = \frac{(77.5-65)^2 + (77.5-85)^2 + (77.5-70)^2 + (77.5-90)^2}{4}$$

var (Right) = 106.25

var (left) = 50

Reduction in variance

$$= var(root) - \sum_{i=1}^{n} w_i \times var[child]$$

$$= 206 - \left[\frac{1}{5} \times 0 + \frac{4}{5} \times 106.25\right]$$

Reduction variance = 121

We calculate MSE for all the datapoint whichever is less will be thresold.

height	gender	weight
160	M	65
165	F	70
170	M	80
175	M	90
180	F	100

from height / Gender, . choose root node

for height MSE = 55.5
for gender MSE = 53

so value of gender MSE is less
It will be our root node.

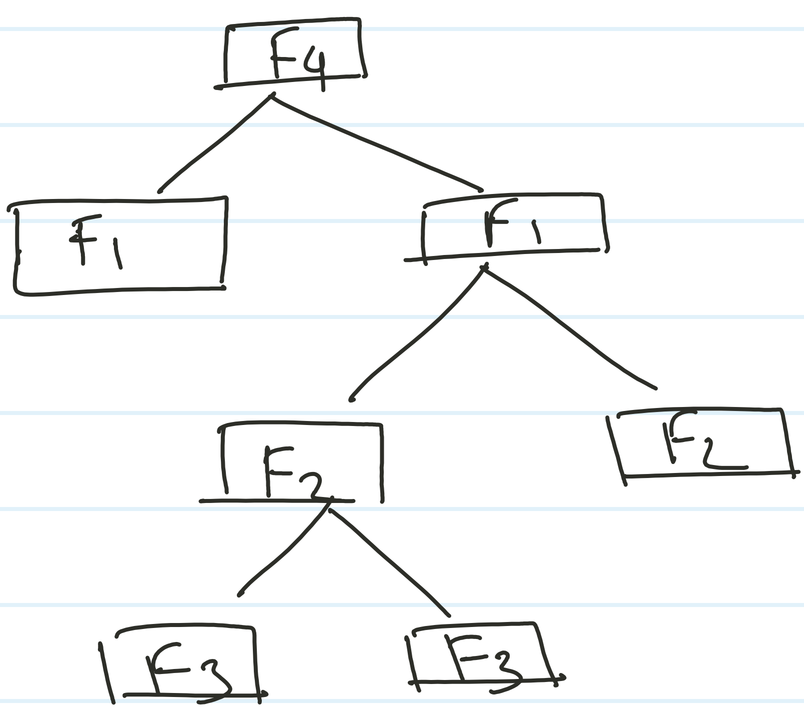
F_1	F_2	F_3	F_4	Target(y)
—	—	—	—	
—	—	—	—	
—	—	—	—	
—	—	—	—	

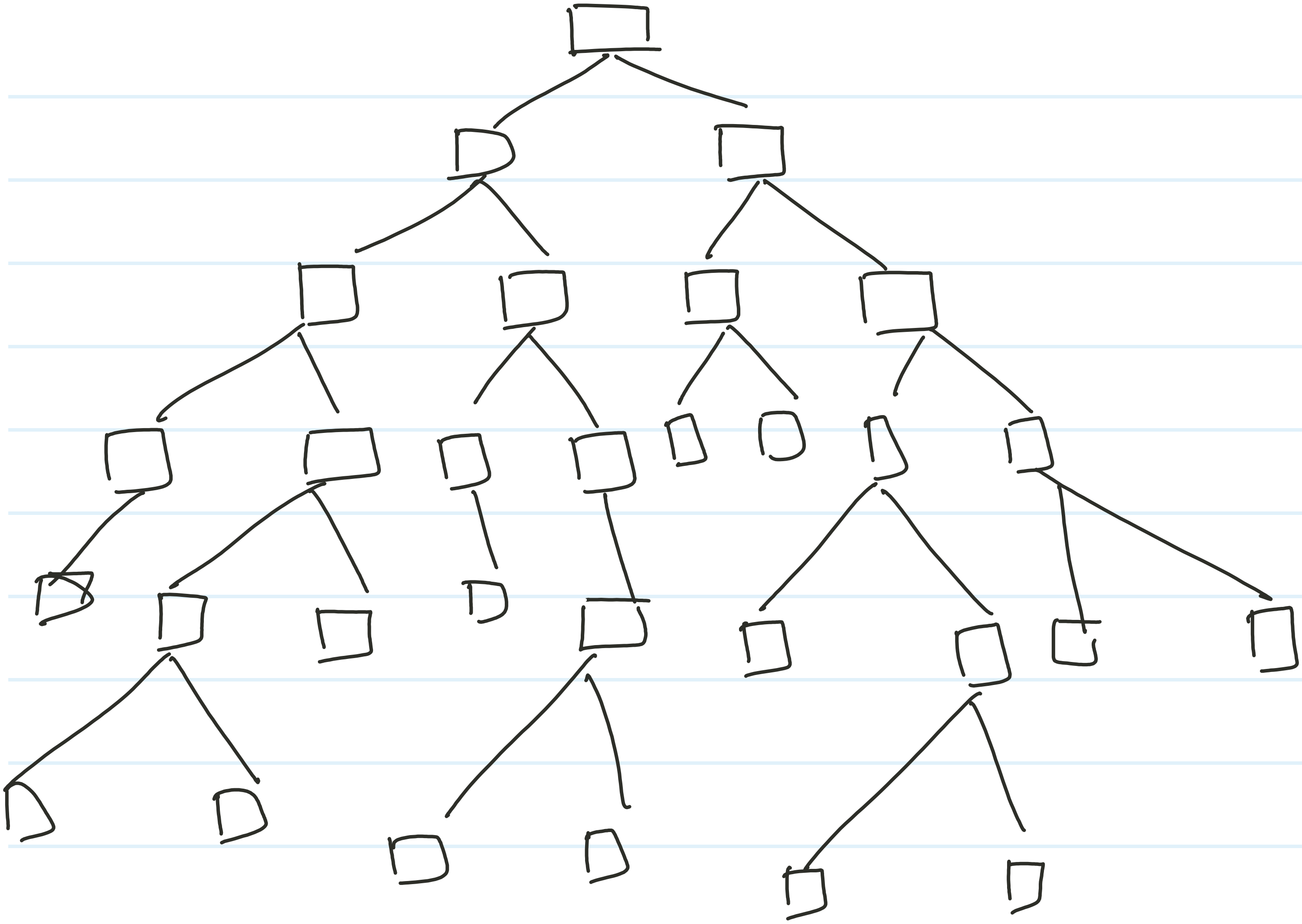
$F_1 \Rightarrow MSE = 49$

$F_2 \Rightarrow MSE = 55$

$F_3 \Rightarrow MSE = 60$

$F_4 \Rightarrow MSE = 43$





Pre-pruning and post-pruning

max-Depth = 5
mini-sample-leaf = 10
min-sample-split = 8
max-feature = 6

These 4 hyperparameters selected for pre-pruning before build DT algorithms.

Post pruning =>

① make DT till end
② cut DT using ccp value.
③ ccp value is nothing but thresold for gini / Entropy.

ccp value is responsible for depth of Tree. If ccp is less, the depth will be less.
High ccp value the depth will be more.

ccp = [0.4, 0.5, 0.6, 0.01]

For model training either we can use pre-prunning or post-prunning.

① When we have large dataset at this time we use pre-prunning.

② When we have small dataset at this time we use postprunning.

Why we use post or pre-pruning?

⇒ To avoid model from overfitting.

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